MECHANICS OF FLIGHT I
Project no. 2 – Aerodynamic Characteristics of the Airplane

3.1 Introduction

Estimation of the aerodynamic characteristics of an airplane usually requires computation of characteristics of main parts of the aircraft, excluding of course the wing. Following parts of the aircraft should be taken into account:

- fuselage,
- horizontal and vertical tailplane
- nacelles of engines mounted on wing
- crew canopy
- undercarriage
- external tanks and weapon

Experiments in wind tunnel shows that lift, drag and pitching moment of the complete aircraft can be with enough accuracy expressed as a sum of forces and moments of all parts of airplane:

$$D=\sum_{j=1}^{n} D_j \qquad L=\sum_{j=1}^{n} L_j \qquad M_C=\sum_{j=1}^{n} M_{C\ j} \quad ,$$

where "n" is the number of main parts of the airplane including the wing. According to the definition of the non dimensional aerodynamic coefficients of the airplane

$$C_D=\frac{D}{\frac{1}{2}\rho\cdot V^2\cdot S} \qquad C_L=\frac{L}{\frac{1}{2}\rho\cdot V^2\cdot S} \qquad C_{m\ C}=\frac{M_{m\ C}}{\frac{1}{2}\rho\cdot V^2\cdot S\cdot c_a} \quad ,$$

we obtain aerodynamic coefficients of the aircraft as follows:

$$C_D=\frac{\sum_{j=1}^{n} C_{D\ j}\cdot S_j}{S} \qquad C_L=\frac{\sum_{j=1}^{n} C_{L\ j}\cdot S_j}{S} \qquad C_{m\ C}=\frac{\sum_{j=1}^{n} C_{m\ C\ j}\cdot S_j\cdot l_j}{S\cdot c_a} \quad ,$$

where

$$C_{D\ j}\ , \qquad C_{L\ j}\ , \qquad C_{m\ C\ j} \quad ,$$

denotes aerodynamic coefficients of particular parts or the airplane,

$$S_j\ , \qquad l_j\ , \qquad c_a$$

are: reference area and reference length of each part, and mean aerodynamic chord of the wing respectively.

Remark: Drag forces of parts of the airplane other than the wing are usually called as ***parasite drag.***

3.2. Drag coefficients of parts of the airplane

A. Fuselage and nacelles

For all aircraft with the engines mounted on the wing or for well designed single engine jet aircrafts the drag coefficient of the fuselage at C_L=0 can be calculated using following *Ostoslavsky* formula:

$$C_{D\ f}=C_{friction}\cdot\eta_f\cdot\eta_{Ma}\cdot\frac{S_{wet}}{S_f}$$

where (see fig. 1):

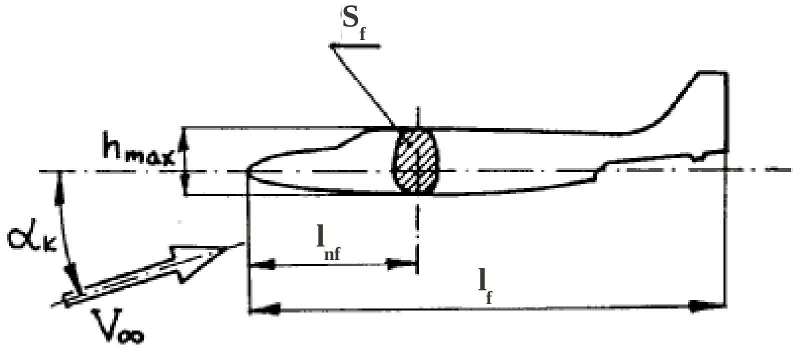


Figure 1

$C_{friction}$ - friction coefficient due to viscosity of the air flowing around the fuselage body, given as a function of the Reynolds number (see fig. 2),

η_f - correctional coefficient adding the influence of shape of the fuselage, given as a function of so called fuselage aspect ratio defined bellow, see fig. 3,

η_{Ma} - influence of the compressibility on body drag; for airplanes with maximum speed up to 500 km/h assume $\eta_{Ma}=1.0$, for faster (but subsonic) airplanes assume $\eta_{Ma}=1.05$,

S_{wet} - so called “wet area” - external area of the fuselage body in contact with the air flow; the area should be calculated based on the accurate geometrical data and drawings of the fuselage; for less accuracy calculations following formula can be used:

$$S_{wet}=2.8\cdot l_f\cdot\sqrt{S_f} \quad ,$$

l_f - total length of the fuselage,

S_f - largest cross section area of the fuselage measured at l_{nf} from nose of the fuselage,

Λ_f - aspect ratio of the fuselage defined as:

$$\Lambda_f=\frac{l_f}{\sqrt{\frac{4\cdot S_f}{\pi}}} \quad ,$$

The Reynolds number must be calculated using the length l_f of the fuselage as the reference dimension:

$$\mathrm{Re}_f=\frac{V_\infty\cdot l_f}{\nu} \quad .$$

<u>Remark:</u> the value of C_{Df} should not be less than 0.08. If it is, please assume C_{Df} = 0.08 .

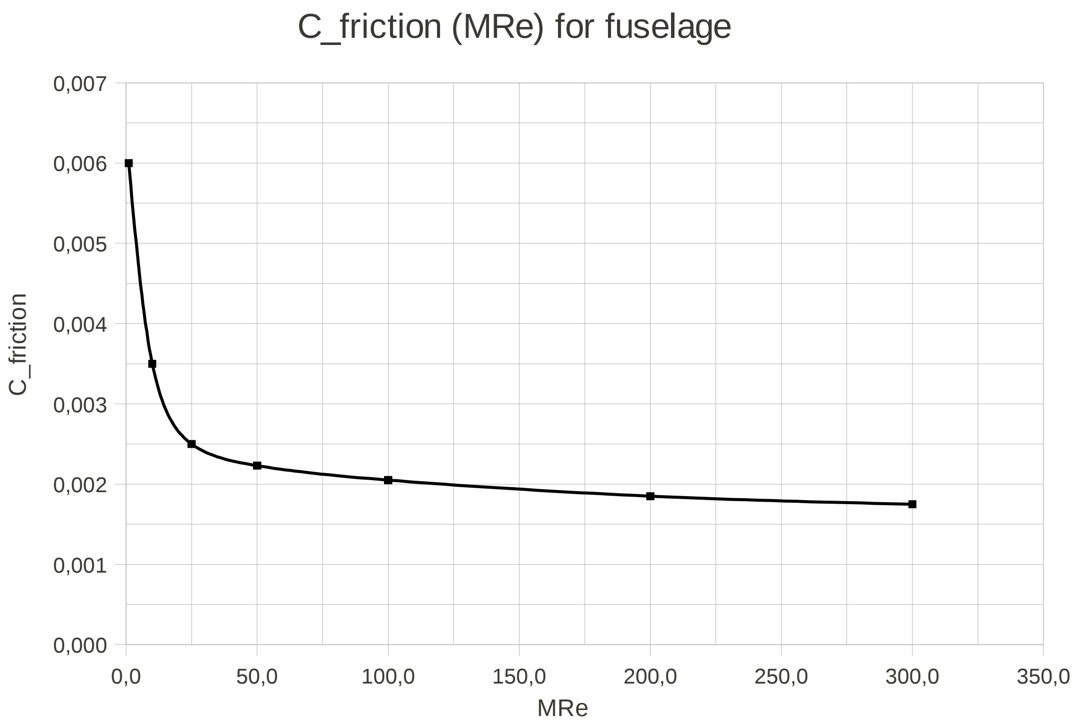


Figure 2

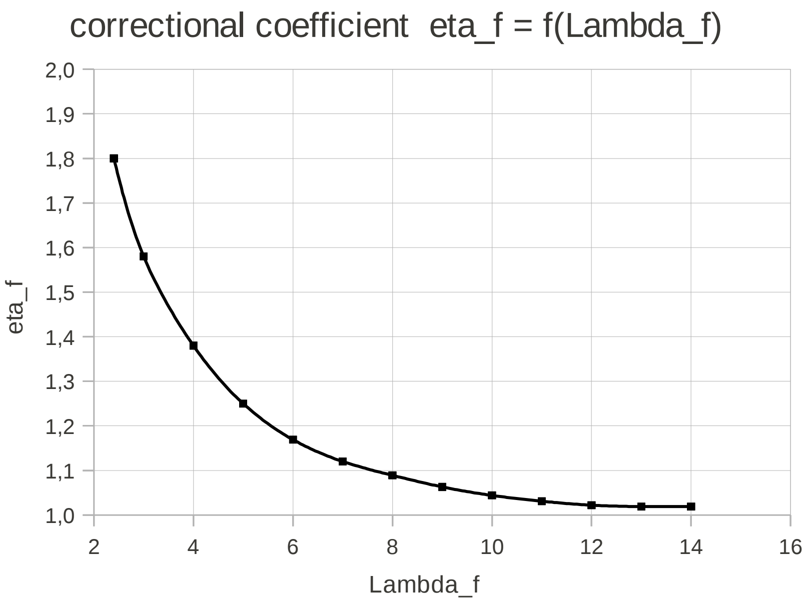
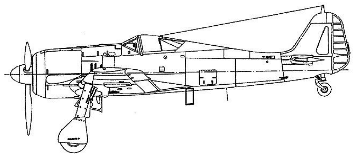
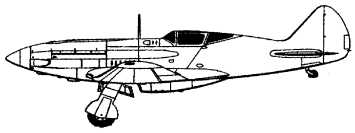
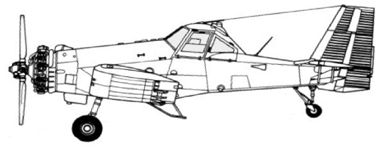
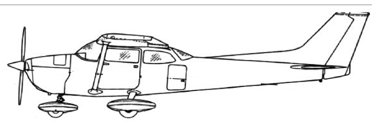
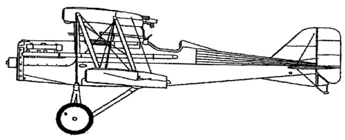


Figure 3

For single reciprocate engine airplanes fuselage drag coefficient may be estimated using following table.

<u>Remark:</u> all drag coefficients are referenced to maximal fuselage cross section area S_f.

Aircraft family	Example aircraft	C_D
World War II fighter, radial engine	 Focke Wulf FW-190 (Germany)	0.25 up to 0.35
World War II fighter, in-line inverted engine	 Migoyan-Gurievich MiG-3 (USSR)	0.15 up to 0.25
Modern agriculture, radial engine	 PZL M-18 Dromader (Poland)	0.3 up to 0.45
Popular training and tourist, horizontal oppose engine	 Cessna 172 Skyhawk (USA)	0.2 up to 0.3
World War I fighter, in-line engine	 Royal Aircraft Factory S.E.5 (United Kingdom)	0.3 up to 0.5

B. Horizontal tailplane

Aerodynamic drag force of the horizontal tailplane depends on longitudinal equilibrium of the aircraft and consists of two components: a constant part and a part depends on angle of attack (lift force).

Equation of motion for horizontal steady flight with constant speed can be written as (see fig. 4):

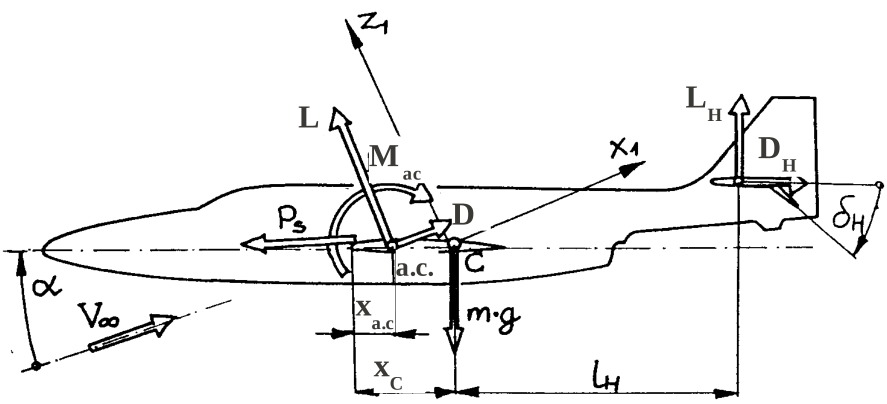


Figure 4

$$L - m \cdot g = 0 \ , \qquad D - P_s = 0 \ , \qquad M_{a.c.} + L \cdot (x_c - x_{a.c.}) - L_H \cdot l_H = 0 \ .$$

After transferring above equations into non-dimensional form, third equation we can obtain as:

$$C_{m\ a.c.} + C_L \cdot \left(\bar{x}_C - \bar{x}_{a.c.} \right) = \kappa'_H \cdot C_{L\ H}$$

where:

- $C_{m\ a.c.}$ - pitching moment coefficient of the wing measured in aerodynamic center of the wing
- $\bar{x}_{a.c.} = \dfrac{x_{a.c.}}{c_a}$ - relative coordinate of the aerodynamic center (same as for wing section)
- $\bar{x}_C = \dfrac{x_C}{c_a}$ - relative coordinate of the center of mass ot the airplane
- $\kappa'_H = \dfrac{S_H * l_H}{S * c_a} * \left(\dfrac{V_{H\infty}}{V\infty} \right)^2$ - horizontal tail volume ratio
- S_H, l_H - horizontal tail area and the distance of the tail from center of mass
- $\left(\dfrac{V_{H\infty}}{V_\infty} \right)^2$ - a square of relationship of mean flow speed close to the horizontal tail to the speed of undisturbed flow far before the aircraft; one of the following values should be assumed according to the vertical position of tailplane:
  - 0.98 for T-type of horizontal tail (mounted on the top of vertical tail),
  - 0.90 for horizontal tail mounted on lower part of the fuselage,
  - 0.85 for tail mounted in the centerline of fuselage.

The horizontal tail lift coefficient computed from this equation is a linear function of wing lift coefficient and depend on the position of center of mass:

$$C_{L\ H} = \frac{C_{m\ a.c.}}{\kappa'_H} + C_L \cdot \frac{\bar{x}_C - \bar{x}_{a.c.}}{\kappa'_H} \ .$$

The non dimensional position of center of mass (referred to wing mean aerodynamic chord c_a) can be usually find in the technical description of the aircraft. Typical value: 0.15 up to 0.30.

The drag coefficient of horizontal tail should be calculated using formula:

$$C_{D\ H} = \left(C_{D\ H_\infty} \right)_{min} + \Delta C_{D\ gap} + \frac{C_{L\ H}^2}{\pi \cdot \Lambda_{e\ H}} \ ,$$

where:

- $\left(C_{D\ H_\infty} \right)_{min}$ - minimal drag coefficient of the tailplane airfoil
- $\Delta C_{D\ gap}$ - drag coefficient increment due to gaps between horizontal stabilizer (fixed part of the tailplane) and horizontal control surface; assume:
  $$\Delta C_{D\ gap} = 0.5 \cdot \left(C_{D\ H_\infty} \right)_{min}$$
- $\Lambda_{e\ H} = \dfrac{b_H^2}{S_H} \cdot e_H$ - effective horizontal tail aspect ratio; e_H is the *Osvald's* correction factor for tailplane; assume: e_H=0.7.

C. Vertical tailplane

The vertical tail drag coefficient can be calculated similar to horizontal tail coefficient neglecting of course induced drag part:

$$C_{D\,V} = \left(C_{D\,V_\infty}\right)_{min} + \Delta C_{D\,gap} \;.$$

<u>Remark:</u> In case of lack of information on the type of airfoil of horizontal and/or vertical tailplane, following types can be used for calculation of tail drag coefficients:

- *NACA 0012 for airplanes with maximal speed not exceed 500 km/h,*
- *NACA 0009 for faster airplanes.*

D. Other parts of the airplane

The drag coefficients of other parts of airplane (undercarriage, crew canopy, antennas, struts) can be estimated using results of the measurement in wind tunnels or flight tests. Useful information on this matter can be found in many NACA and NASA reports (see *Collection* folder at American National Aeronautical and Space Administration repository web home page http://ntrs.nasa.gov).

E. Airplane's parasite drag coefficient

Results of calculations of drag coefficients as well as its reference areas (excluding data for horizontal tail!) should be collected in the table. The list of airplane's parts depends of course on particular type of airplane (single engine or multiple engines, fixed or retractable undercarriage). Please note that all drag coefficients have been estimated for small angles of attack (for C_L=0).

Part No.	Part Name	$C_{D\,j}$	$S_{ref\,j}$	$C_{D\,j}$*$S_{ref\,j}$	Data source
1	Fuselage				
2	Vertical tail				
3	Undercarriage (fixed type only!)				
4	Engine nacelles (wing mounted!)				
5	Crew canopy/windshield				
6	External fuel tanks				
.....	....				
n					
			$\Sigma C_{D\,j}$*S_j		-

<u>Remark:</u> In some sources of data (reports, textbooks) instead of drag coefficient of a part of airplane, it can be found dimensioned drag force measured in wind tunnel, conditions of measurements (wind speed, air temperature and static air pressure) and drawings and dimensions of part's model. For this case it can be easy computed the value of $C_{D\,j}$ $S_{ref\;j}$ as follows:*

$$C_{D\;j} \cdot S_{ref\;\;j} = \frac{D_j}{\frac{1}{2} \cdot \rho \cdot V^2_{wind}}$$

Test in wind tunnels show that parasite drag of parts of aircraft mentioned above depend on angle of attack. This effect can be taken into account using following simple formulas:

- calculate minimal parasite drag coefficient

$$\left(C_{D\;parasite}\right)_{min} = \frac{\sum_{j=1}^{n} C_{D'\,j} \cdot S_j}{S}$$

- changes of the parasite drag coefficient with changes of angle of attack can be estimated using linear function of wing lift coefficient (instead of angle of attack):

$$C'_{D\ parasite} = \left(C_{D\ parasite}\right)_{min} \cdot \left(1 + \frac{|C_L|}{\zeta}\right) \quad ,$$

where:

ζ - proportional factor depend on the aerodynamic properties of the airplane; assume a value between 6.0 for streamlined aircrafts (military jet aircrafts, modern transport and passengers aircrafts, modern training and sport propeller driven light aircrafts) up to 3.0 for agriculture, home-build as well as old biplane aircrafts.

3.4 Drag coefficient of complete aircraft

Flight tests as well a lot of investigations in wind tunnels show that drag coefficient of an airplane is usually greater than simple sum of drag coefficients of all parts of the aircraft. This additional component of the aerodynamic drag have its source in unfavorable influence between aircraft's parts and is called as *interference drag.* Interference effect can be estimated using formula:

$$C_{D\ airplane} = \left(C_{D\ wing} + C'_{D\ parasite} + \frac{S_H}{S} \cdot C_{D\ H}\right) \cdot \left(1 + K_{interfer}\right) \quad ,$$

where:

$K_{interfer}$ - empirical interference drag factor; assume:

- 0.02 for jet airplanes
- 0.04 for well-designed single and multi engines propeller driven light aircrafts
- 0.06 do 0.15 for other aircrafts.

Remark: *Calculations of the final aerodynamic characteristics of the airplane should be performed by continuing of the table (spreadsheet) from Project No. 2. Note that the independent variable for aerodynamic characteristics must be the lift coefficient* C_L.

3.4 Lift coefficient of the airplane

According to initial assumptions, the influence of horizontal tail lift force on total lift of the aircraft should be taken into account using simple formula:

$$C_{L\ airplane} = C_{L\ wing} + \frac{S_H}{S} \cdot C_{L\ H} \quad ,$$

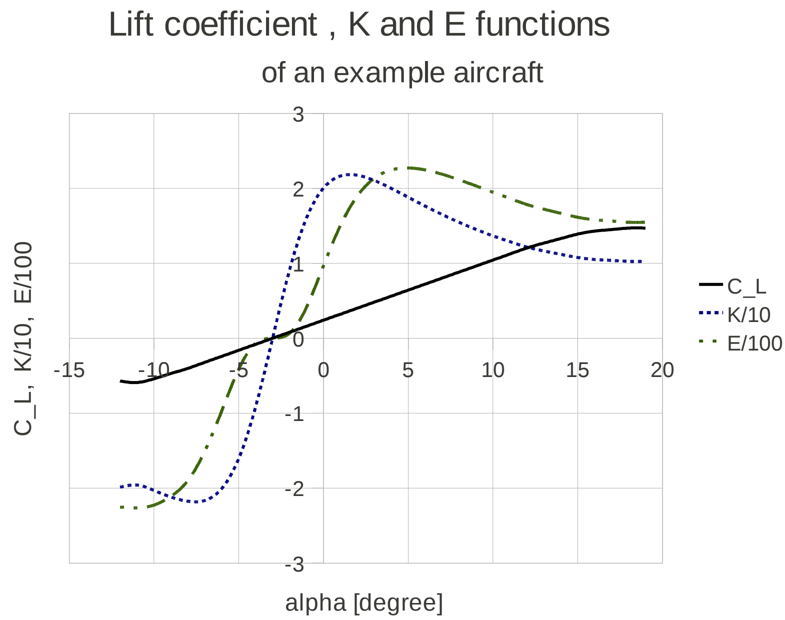
3.5 Additional aerodynamic functions

Besides basic aerodynamic characteristics of the airplane $C_L=f(\alpha)$ and $C_D=f(C_L)$ following two additional functions of wing angle of attack should be calculated:

$$K(\alpha) = \frac{C_{L\ airplane}}{C_{D\ airplane}}$$ - lift-to-drag ratio,

$$E(\alpha) = \frac{C^3_{L\ airplane}}{C^2_{D\ airplane}}$$ - aerodynamic energy function.

All characteristics should be presented on graphs (see fig. 5).


Lift coefficient , K and E functions
of an example aircraft
C_L, K/10, E/100
alpha [degree]
3
2
1
0
-1
-2
-3
-15
-10
-5
0
5
10
15
20
C_L
K/10
E/100

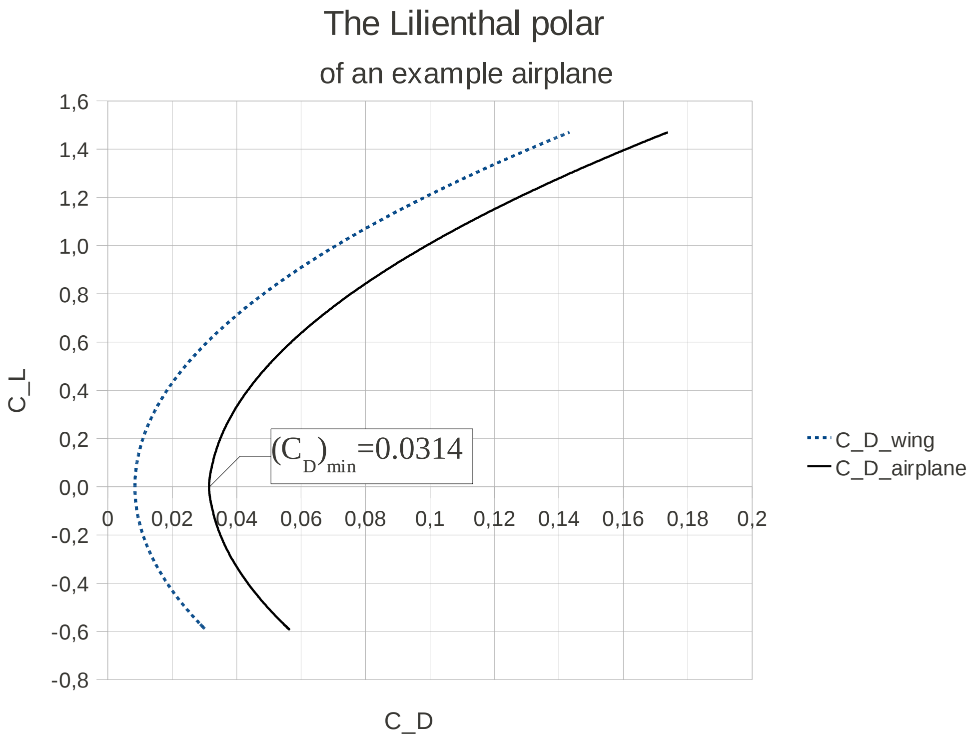

The Lilienthal polar
of an example airplane
C_L
C_D
1,6
1,4
1,2
1,0
0,8
0,6
0,4
0,2
0,0
-0,2
-0,4
-0,6
-0,8
0
0,02
0,04
0,06
0,08
0,1
0,12
0,14
0,16
0,18
0,2
$(C_D)_{min}$=0.0314
C_D_wing
C_D_airplane

Figure 5

3.6 Analytical form of aerodynamic characteristics of the airplane

For fast (but less accurate) estimation of some performance parameters of an aircraft (i.e. gliding optimal parameters, maximum horizontal speed), following algebraical approximation of the lift and drag coefficients would be very useful:

$$C_L = a \cdot (\alpha - \alpha_0),$$
$$C_D = C_{D\,0} + \frac{C_L^2}{\pi \cdot \Lambda_e},$$

Four approximation coefficients:

$$a\,, \quad \alpha_0\,, \quad C_{D\,0}\,, \quad \frac{1}{\pi \cdot \Lambda_e}\,,$$

should be obtained using one of numerical approximation methods (last square method, min-max method). For example, Open Office Calc package includes REGLINP(*y_vector, x_vector*) function for linear approximation of a discrete function *y_vector(x_vector).*

Remark

Approximation coefficients must be evaluated.

Lift slope $a = d\,C_L / d\,\alpha$ *should be practically equal to the value obtained using formula:*

$$a = \frac{a_\infty}{1 + \frac{a_\infty}{\pi \cdot \Lambda} \cdot (1 + \tau)}.$$

(a_∞ *denotes lift slope calculated for the wing section, see Project No. 2).*
Zero lift angle α_0 *must be equal this same value as for wing section (see source data).*

Parameters of the polar curve should be checked as follows:

- $C_{D\,0}$ *must be very close to the minimal value of drag coefficient of the airplane (see fig. 5),*
- *Osvald's factor*

$$e = \frac{\Lambda_e}{\Lambda}$$

  mus be a value 0.7 up to 0.95.

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