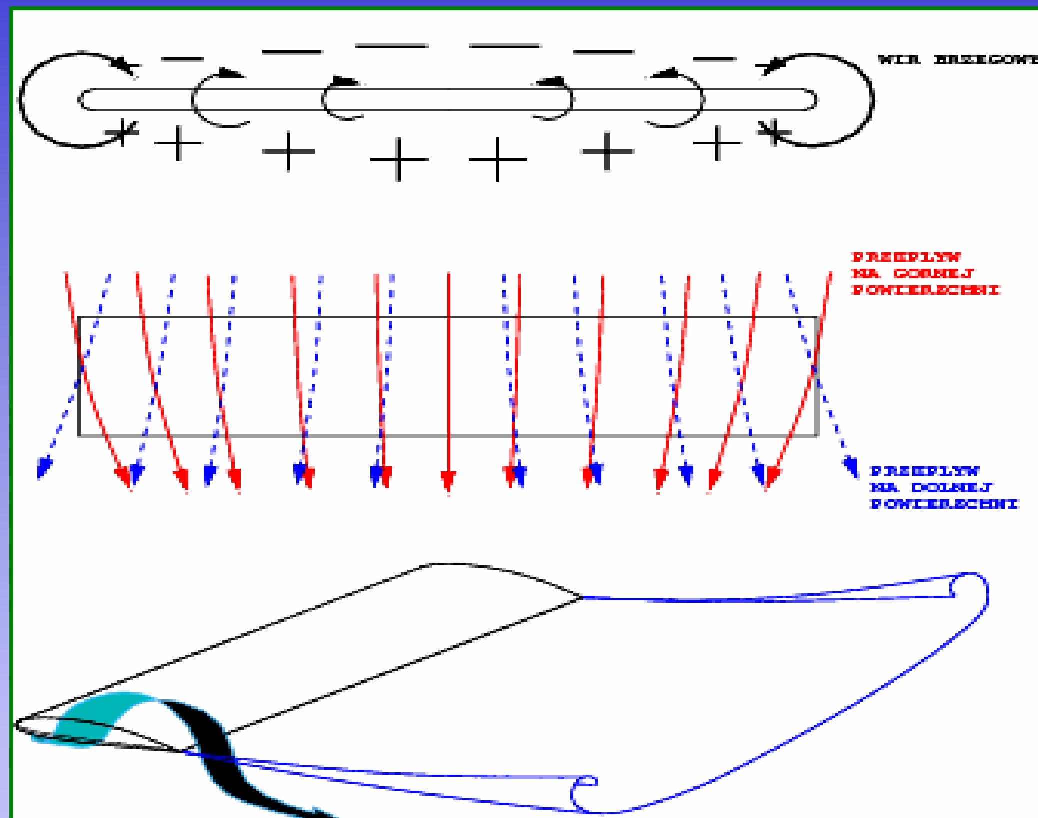
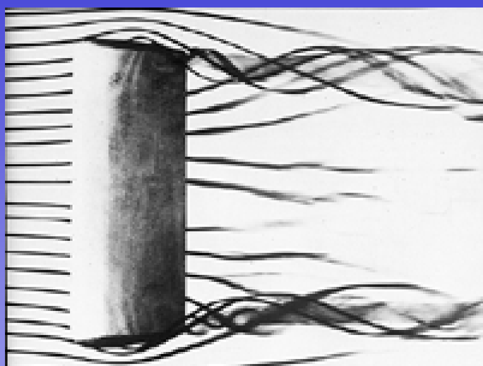
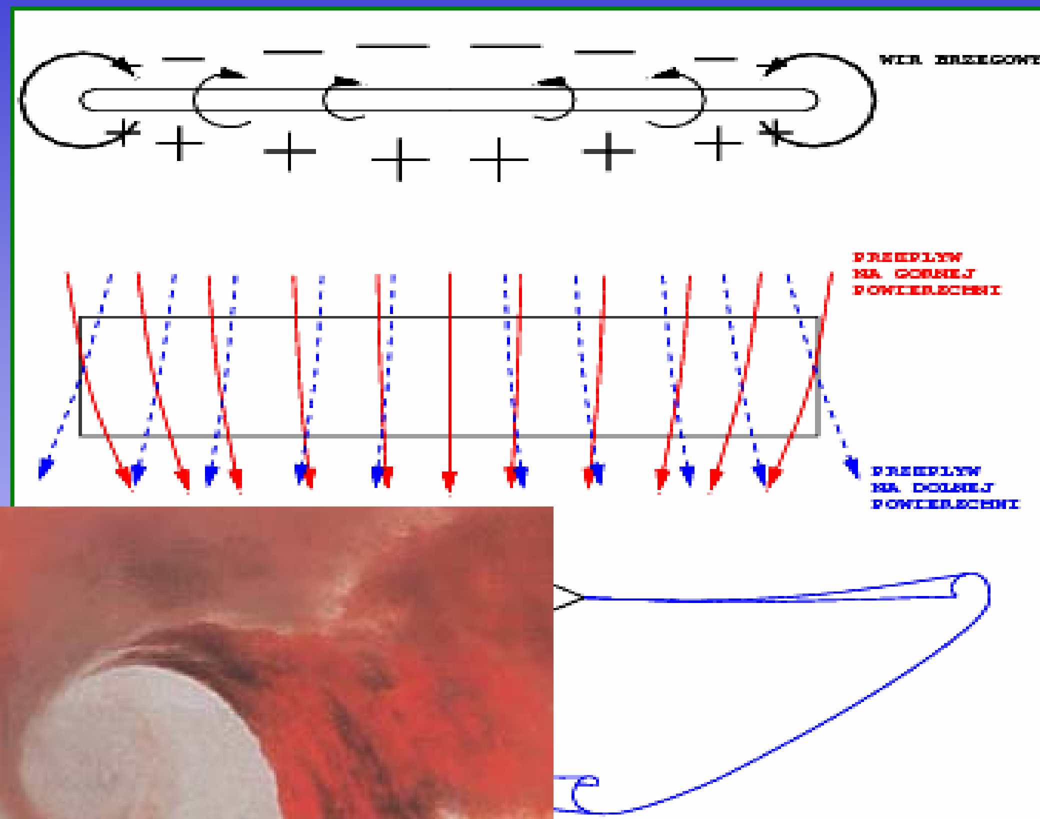
Lifting Line Theory (Prandtl)



Lifting Line Theory (Prandtl)





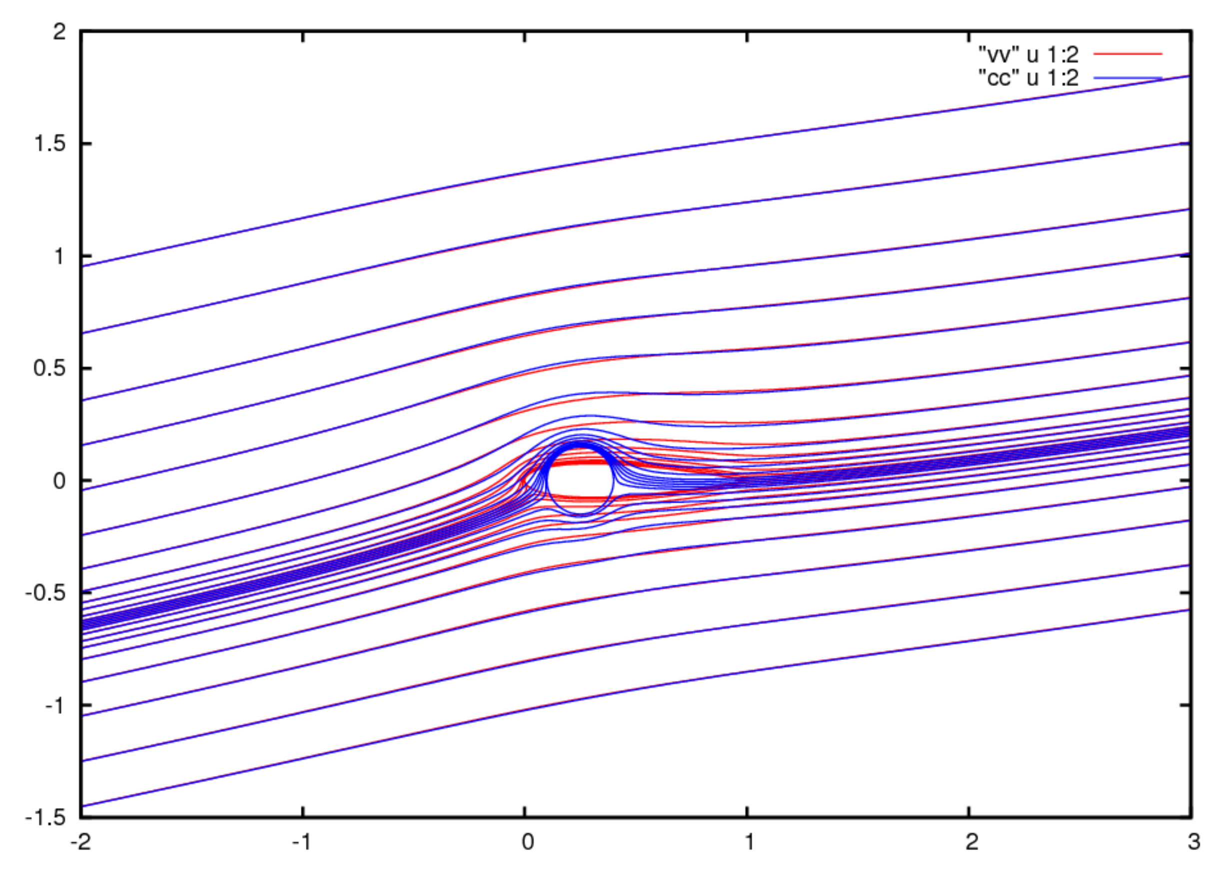
Lifting Line Theory (Prandtl)

$$L = C_L \cdot S \cdot \frac{\rho_\infty V_\infty^2}{2} = C_L \cdot c \cdot b \cdot \frac{\rho_\infty V_\infty^2}{2} = \rho_\infty V_\infty \cdot \Gamma \cdot b$$

$$\Gamma = \frac{c \cdot C_L}{2} V_\infty$$

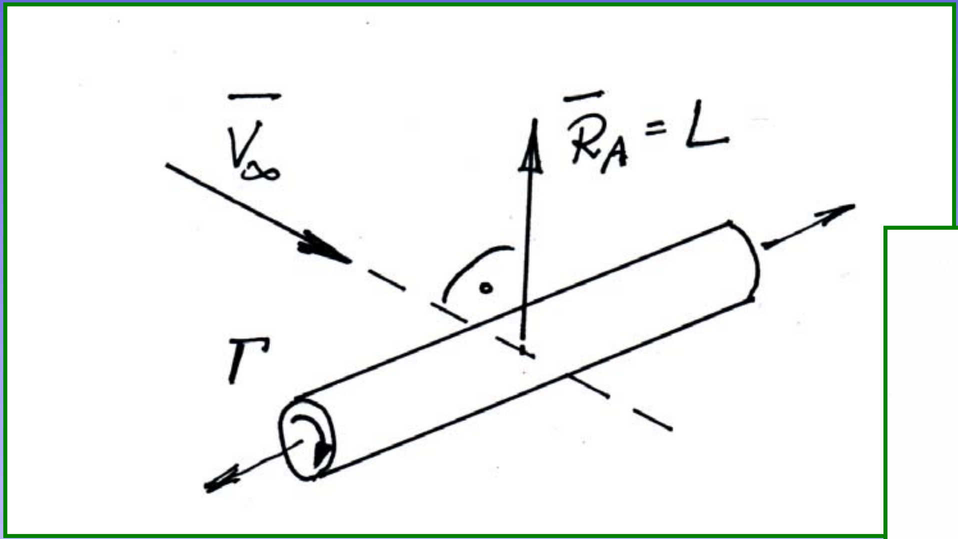
Lifting Line Theory (Prandtl)

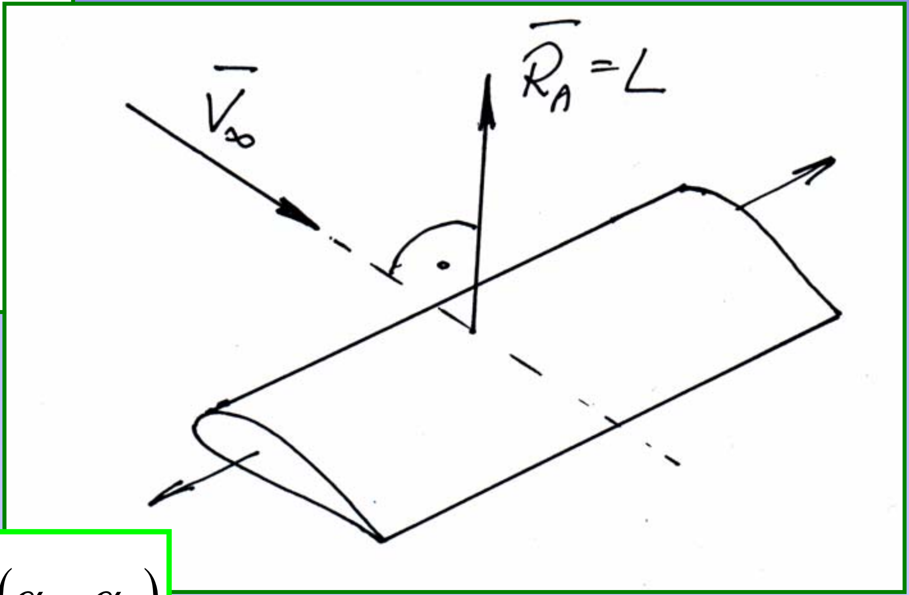
$$\Gamma = \frac{c \cdot c_L}{2} V$$



Lifting Line Theory (Prandtl)

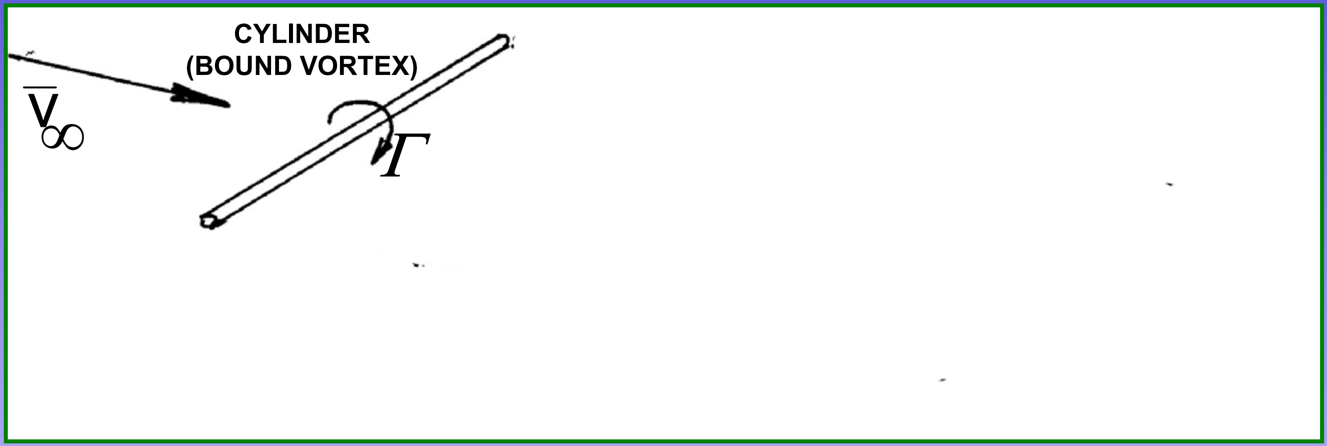
$$\frac{c \cdot c_L}{2} V = \Gamma$$



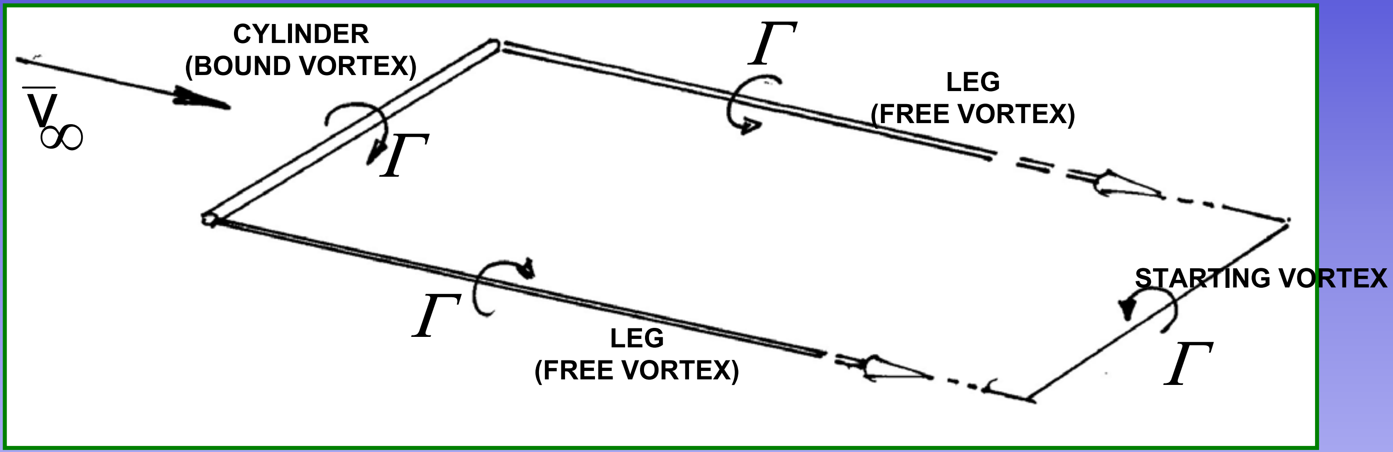


$$\Gamma = \frac{c \cdot C_L}{2} V = \frac{c \cdot V}{2} C_L = \frac{c \cdot V}{2} \cdot \frac{dC_L}{d\alpha} \cdot (\alpha - \alpha_0)$$

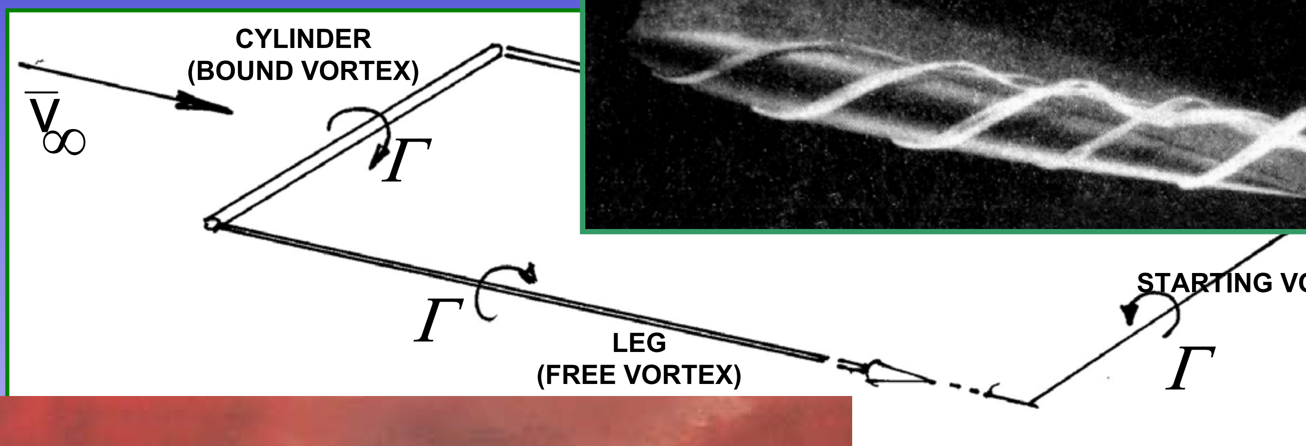
Lifting Line Theory (Prandtl)

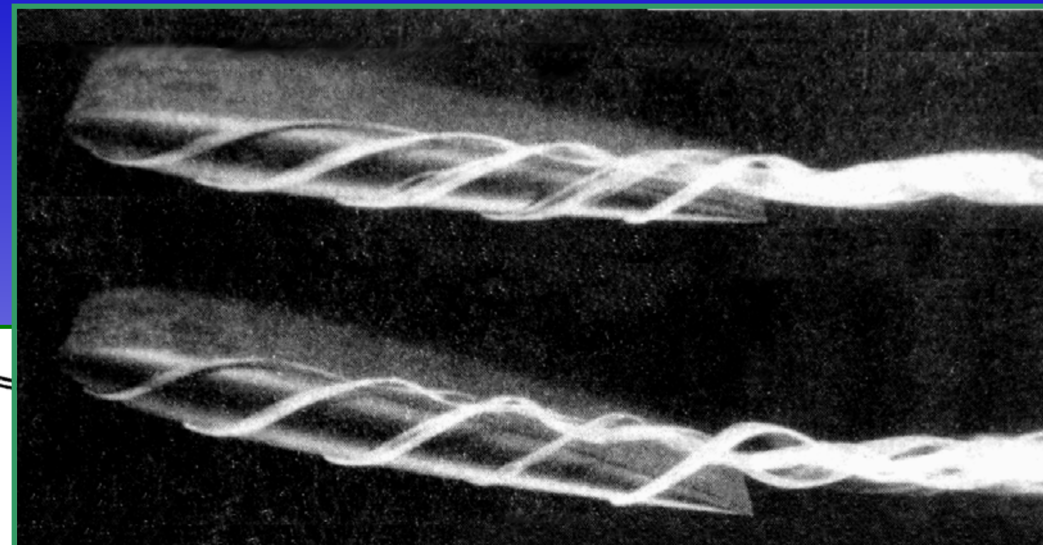


Lifting Line Theory (Prandtl)

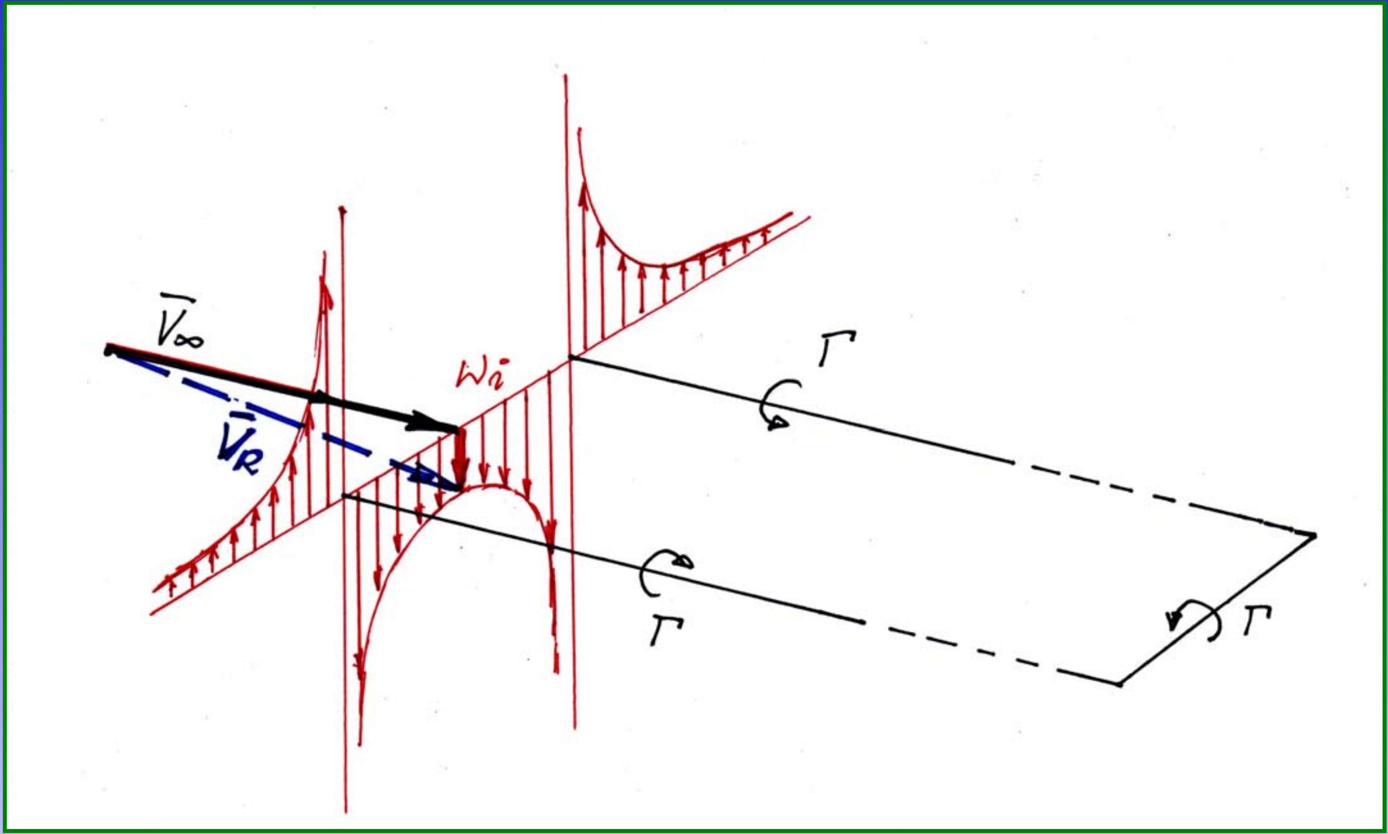


Lifting Line Theory (Prandtl)

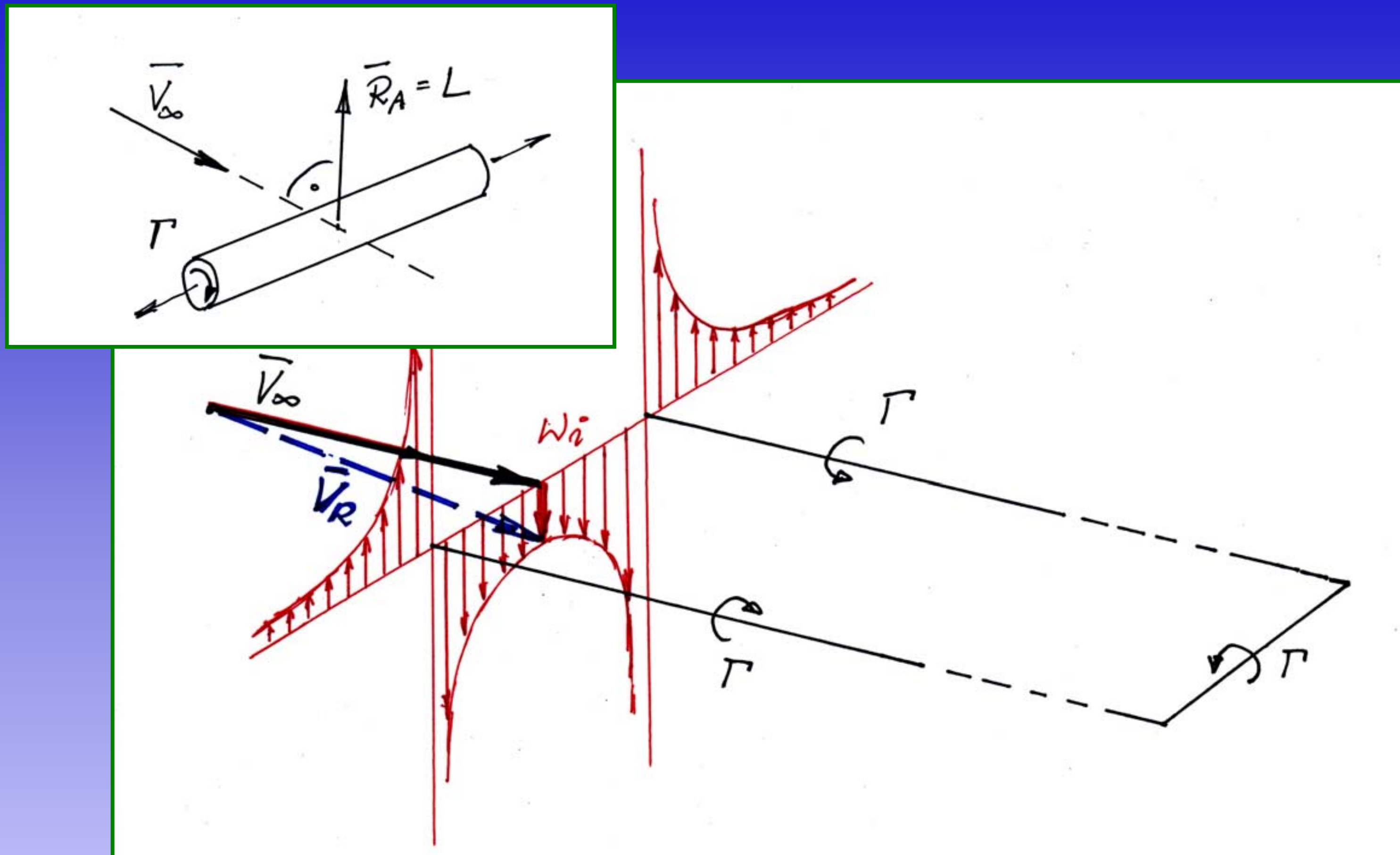




Lifting Line Theory (Prandtl)



Lifting Line Theory (Prandtl)



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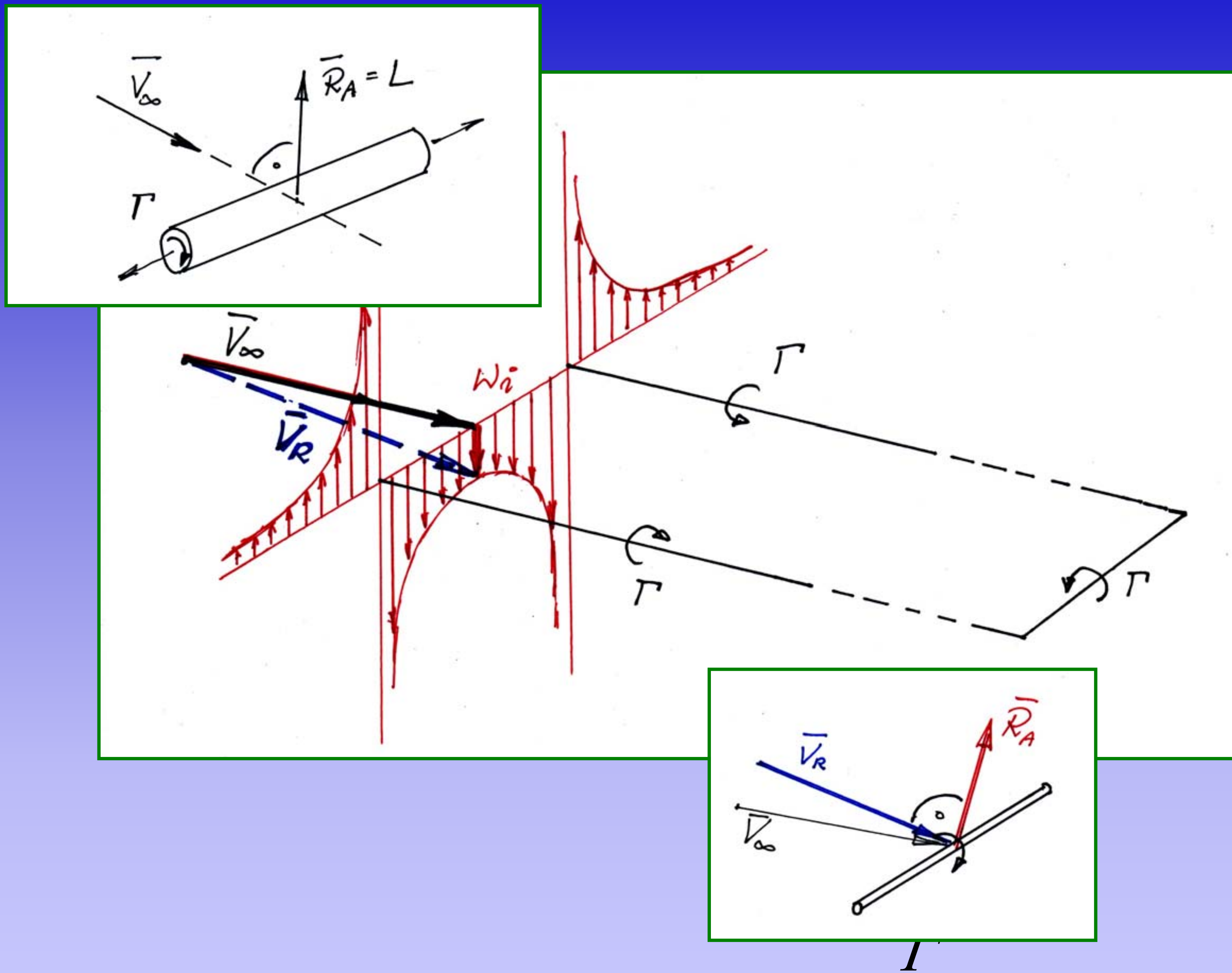
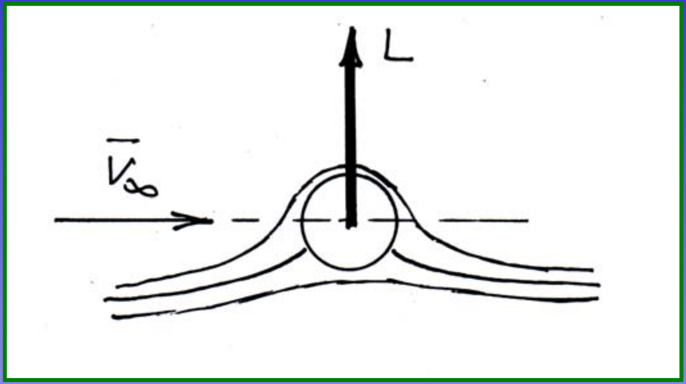
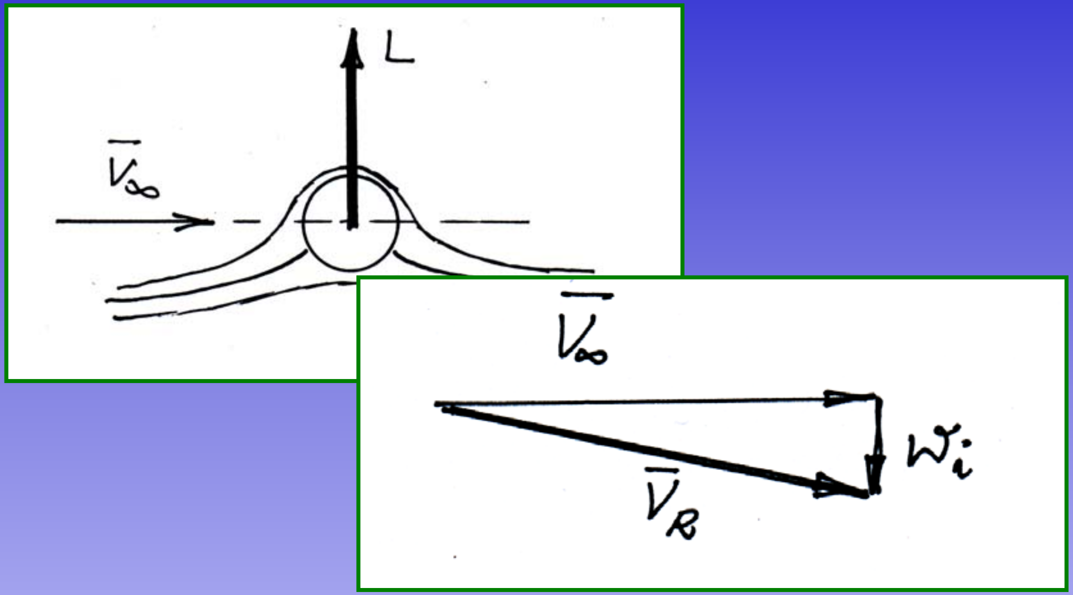
Lifting Line Theory (Prandtl)



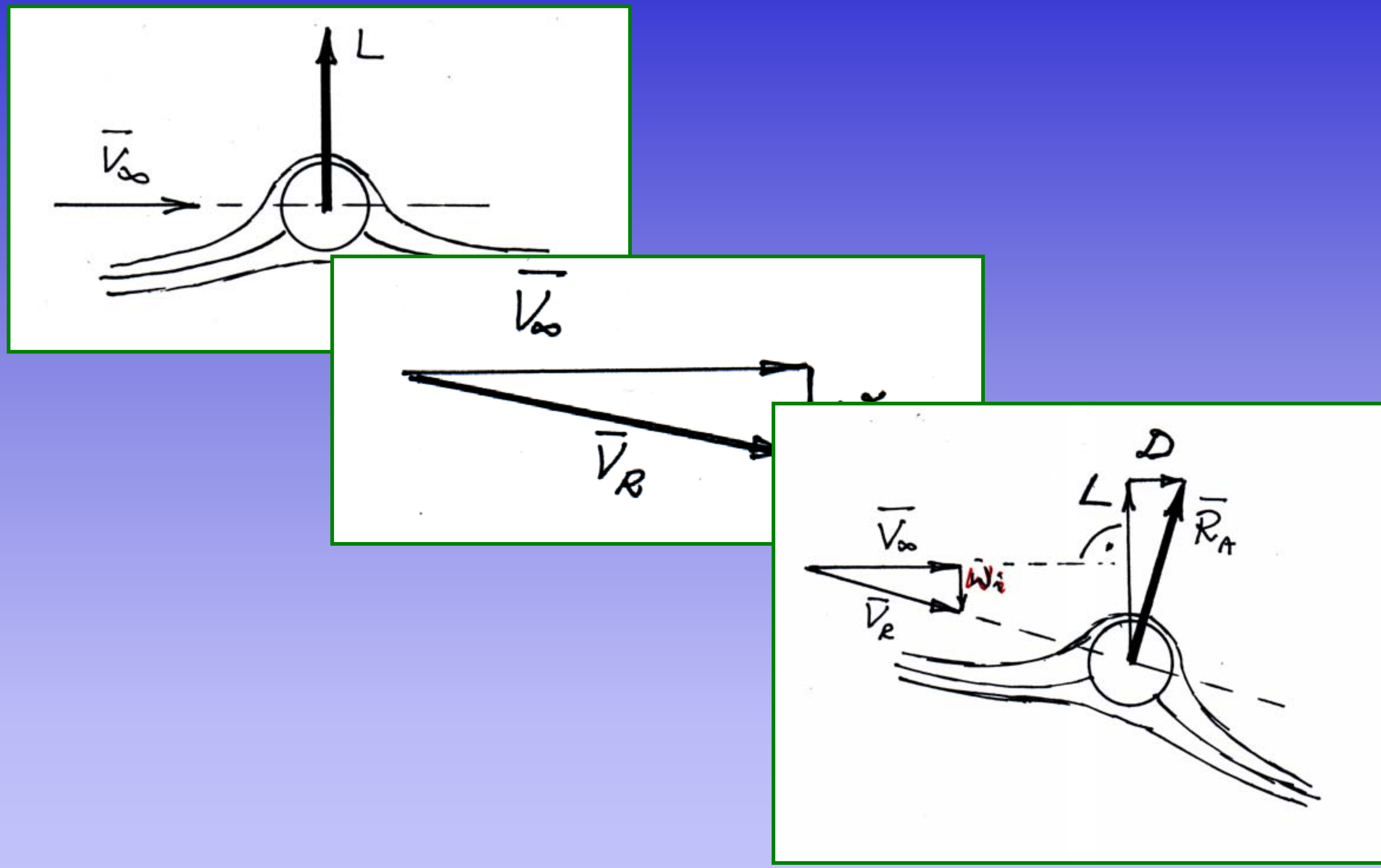
Lifting Line Theory (Prandtl)



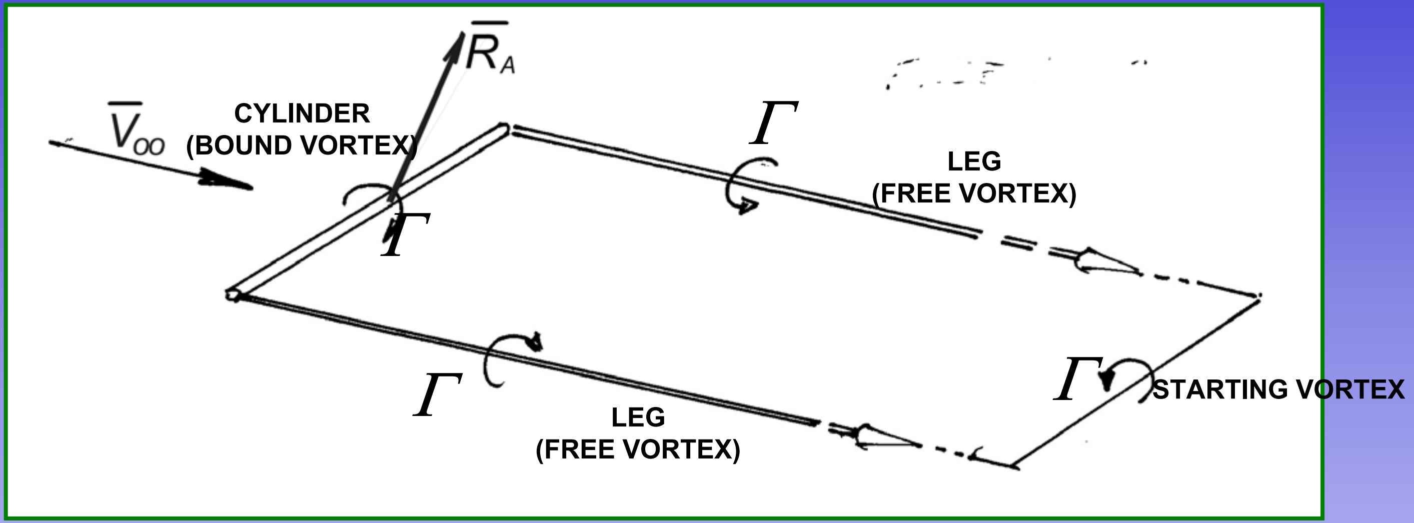
Lifting Line Theory (Prandtl)



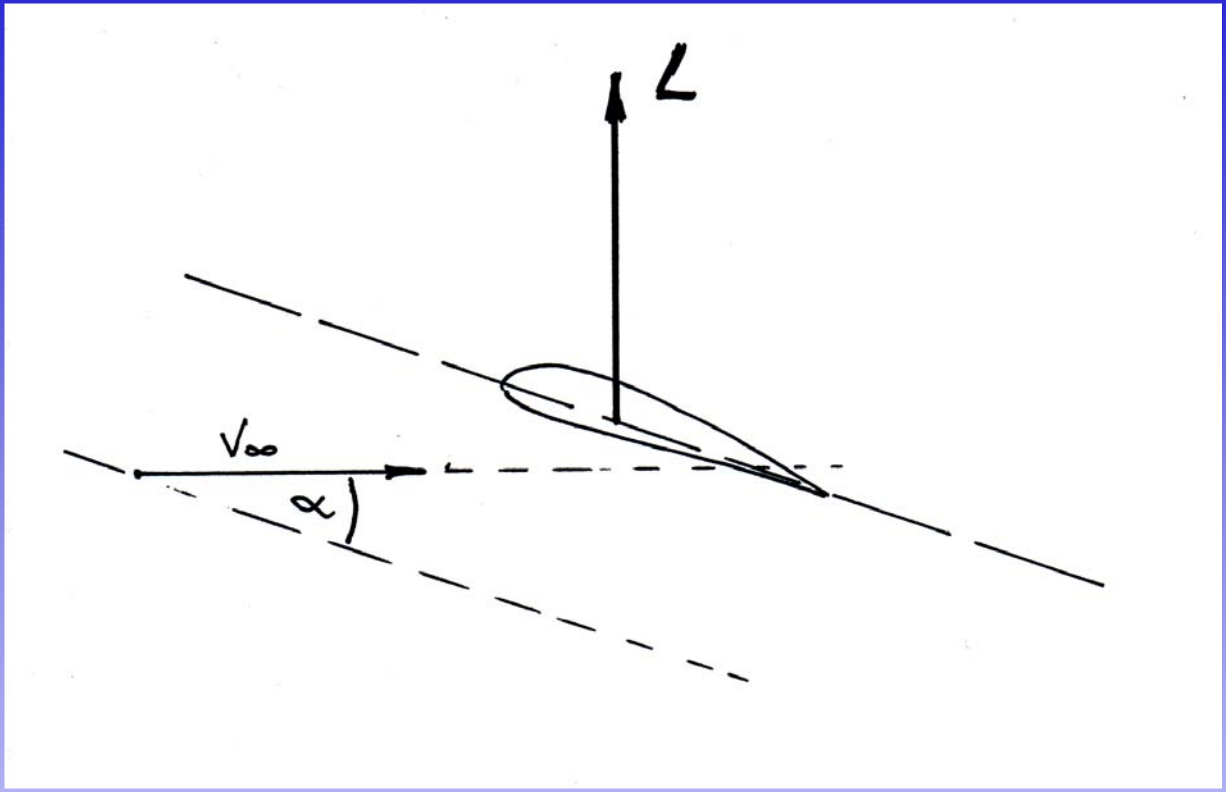
Lifting Line Theory (Prandtl)



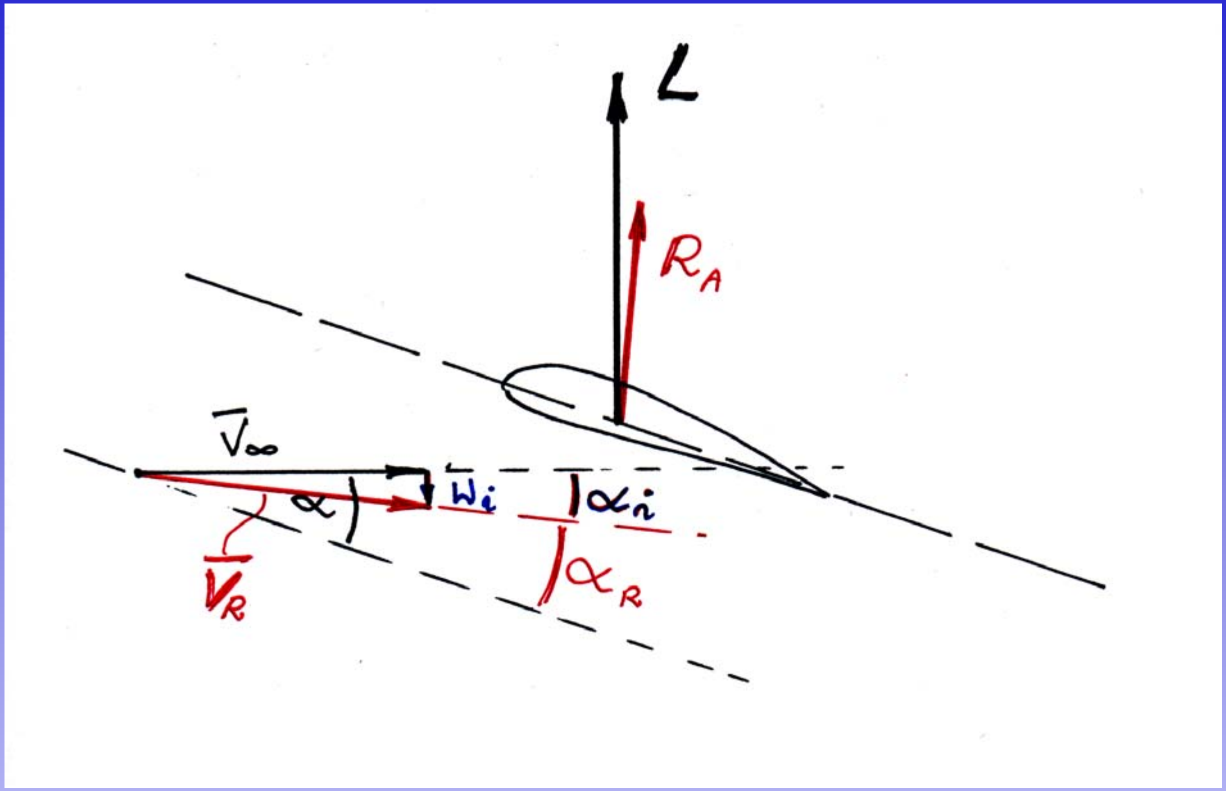
Lifting Line Theory (Prandtl)



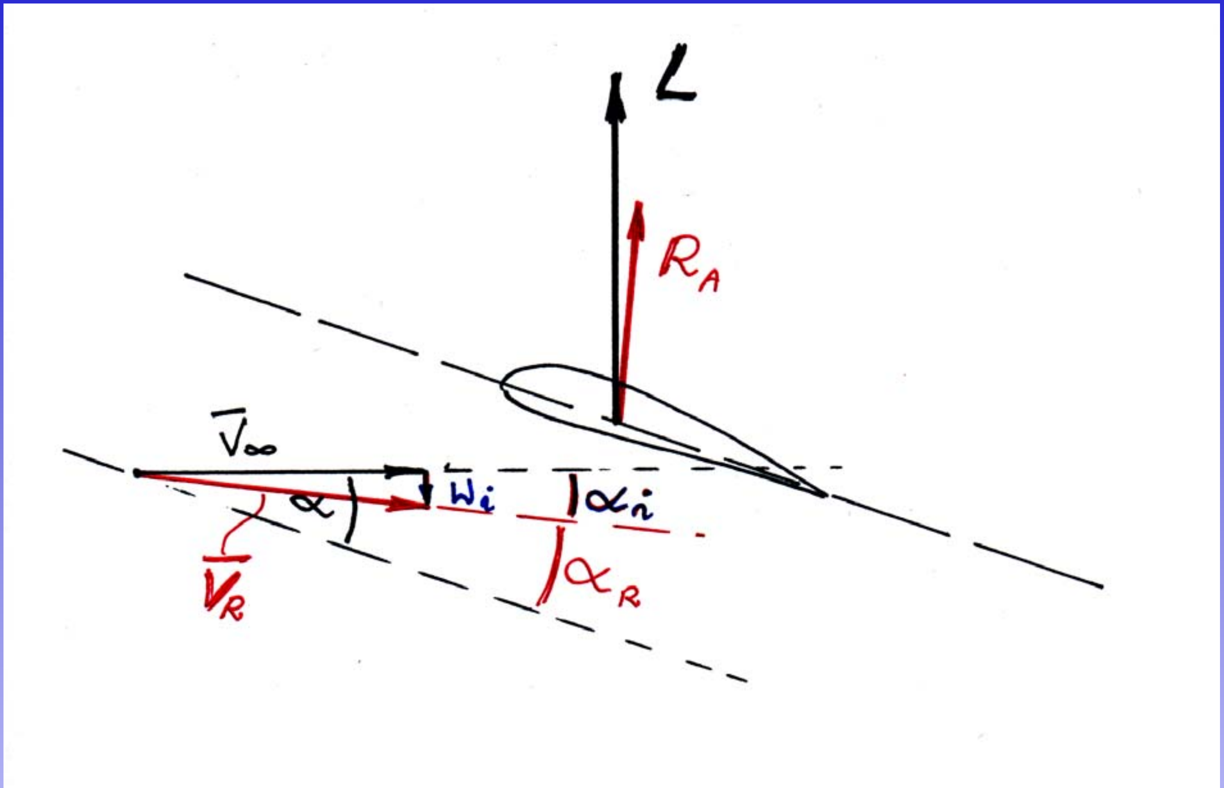
Lifting Line Theory (Prandtl)



Lifting Line Theory (Prandtl)

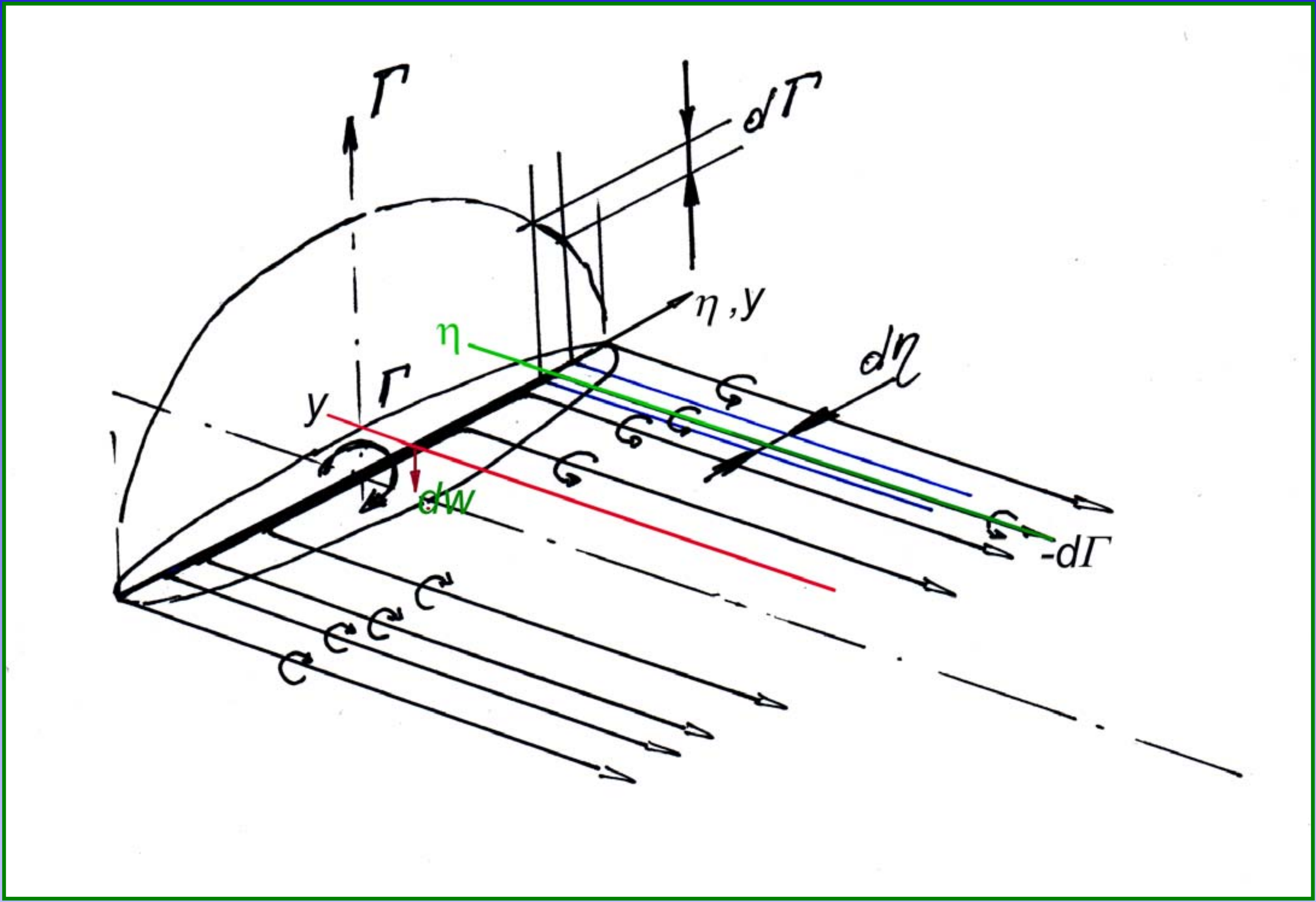


Lifting Line Theory (Prandtl)

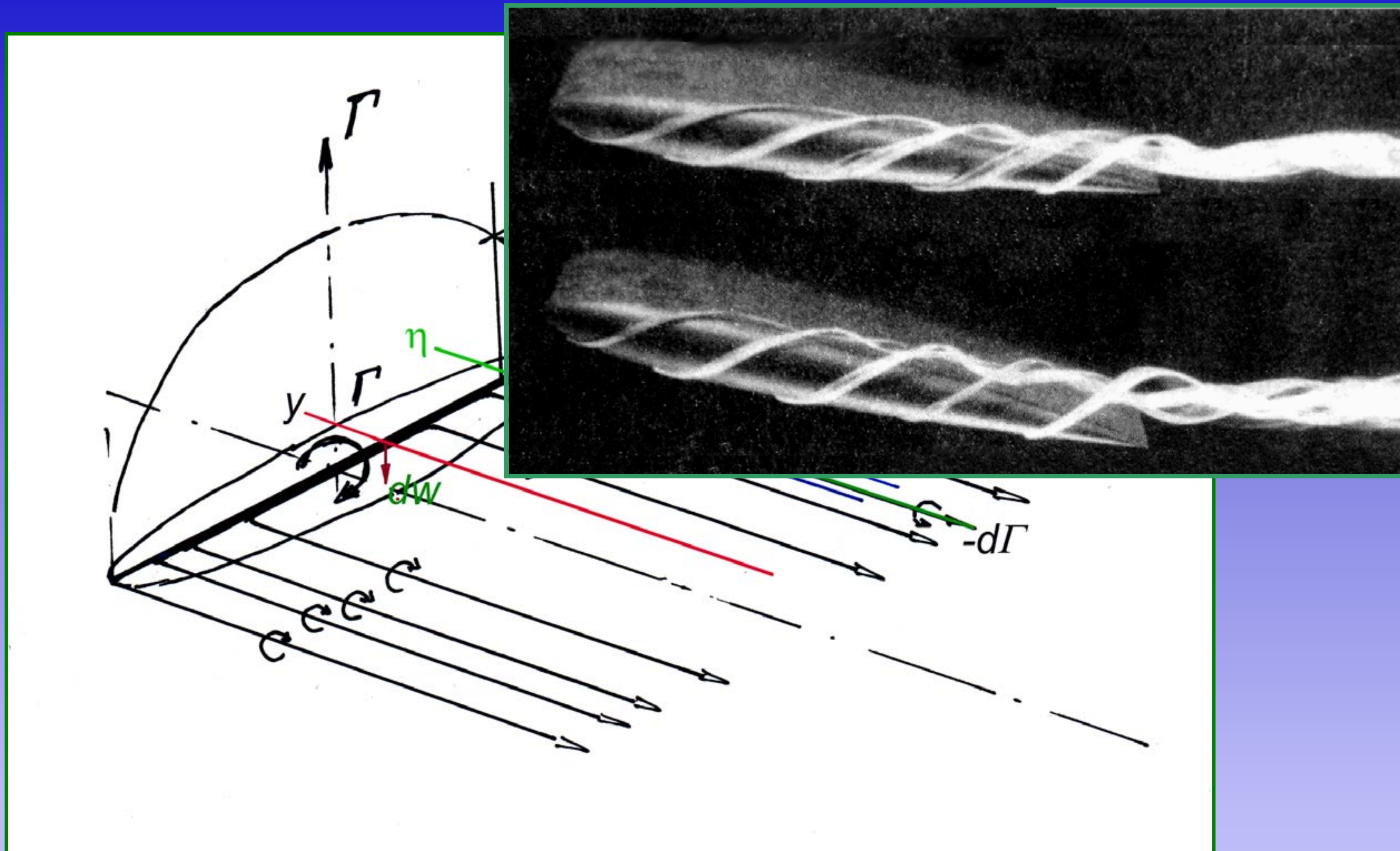


$$\Delta L = \frac{dC_L}{d\alpha} \cdot \alpha_i$$

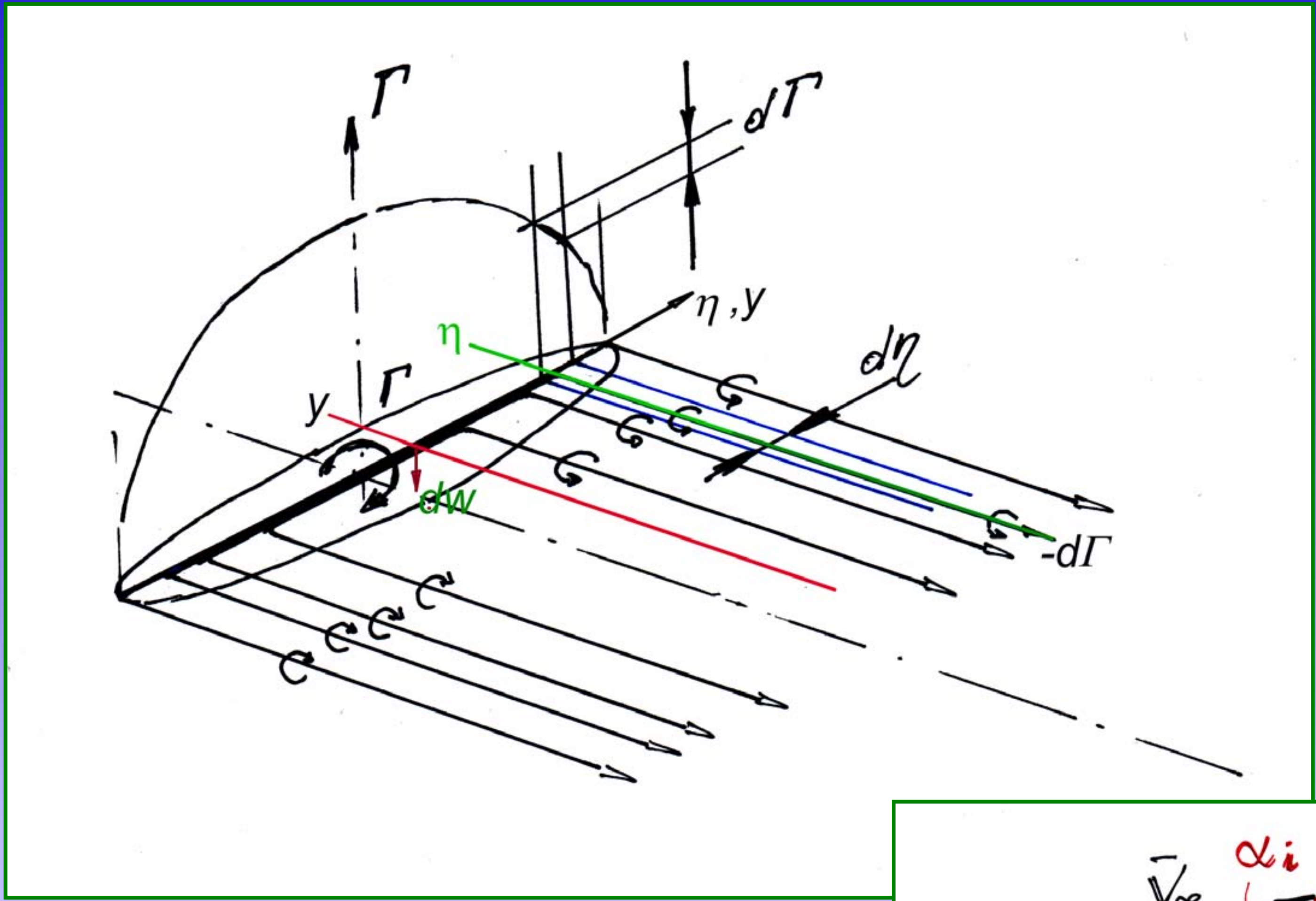
Lifting Line Theory (Prandtl)

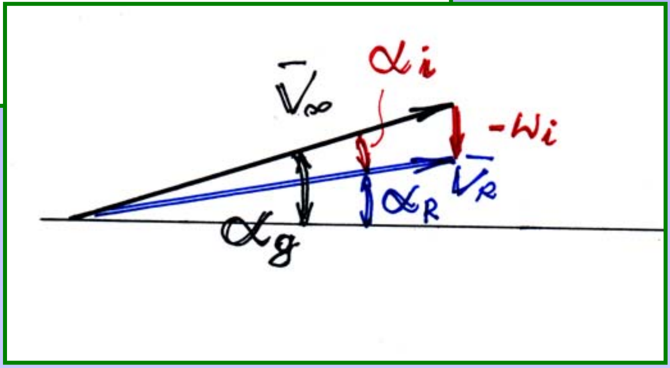


Lifting Line Theory (Prandtl)

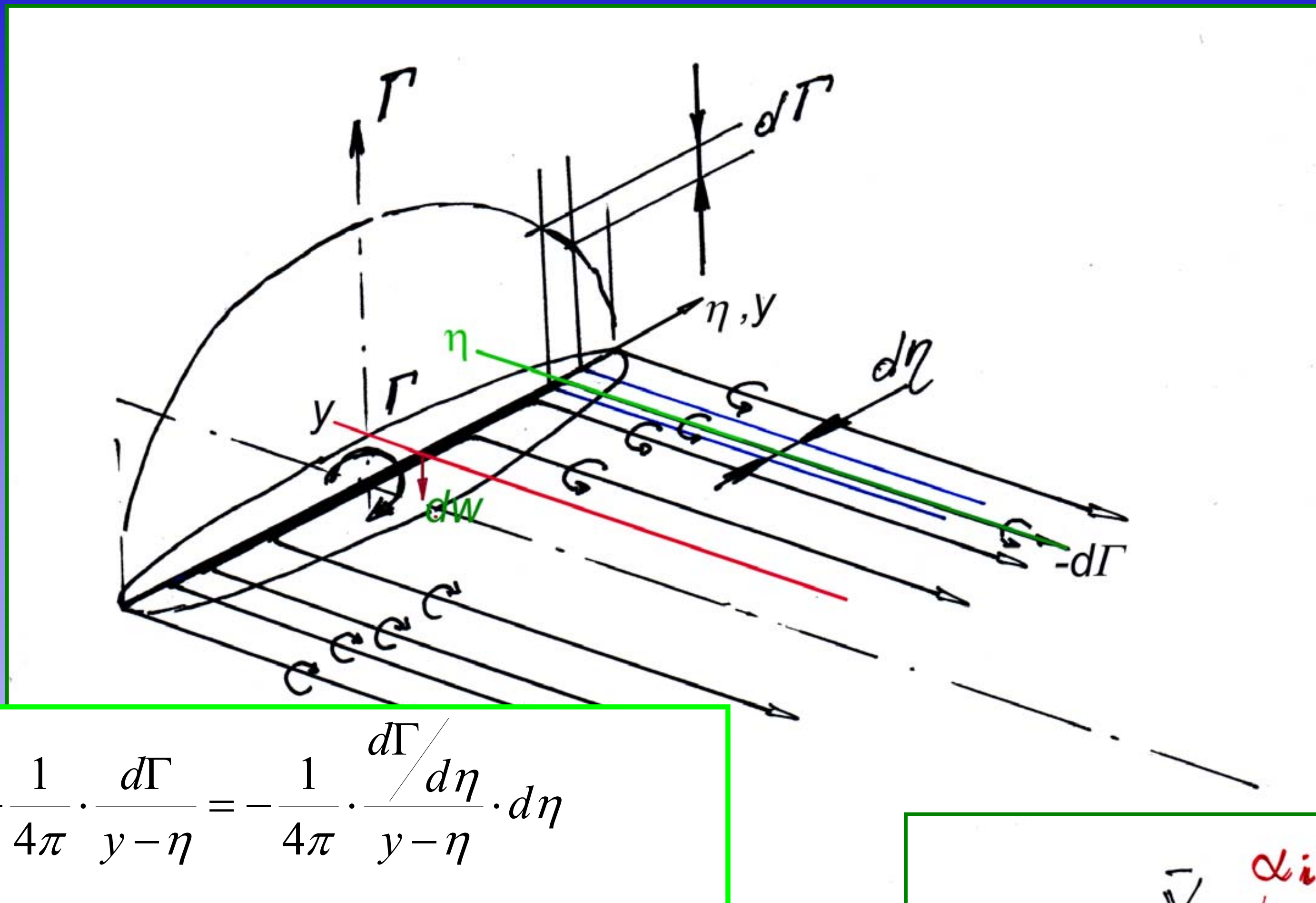


Lifting Line Theory (Prandtl)



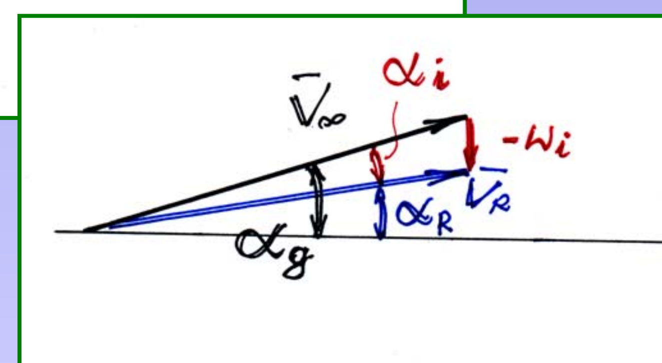


Lifting Line Theory (Prandtl)

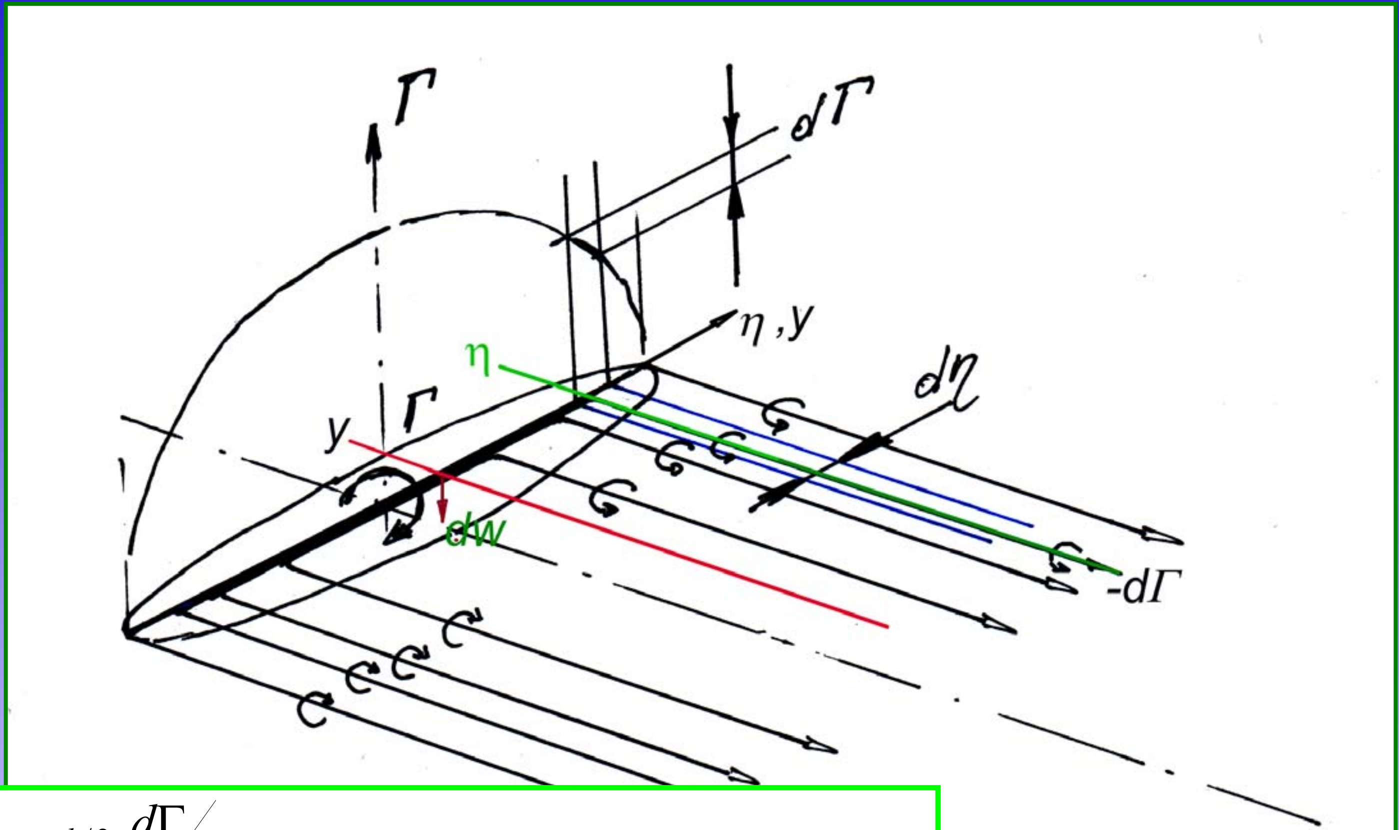


$$dw_i(y) = -\frac{1}{4\pi}\cdot\frac{d\Gamma}{y-\eta} = -\frac{1}{4\pi}\cdot\frac{d\Gamma/d\eta}{y-\eta}\cdot d\eta$$

$$w_i(y) = -\frac{1}{4\pi}\int_{-b/2}^{b/2}\frac{d\Gamma/d\eta}{y-\eta}\cdot d\eta \qquad \alpha_i = -w_i / V_\infty$$

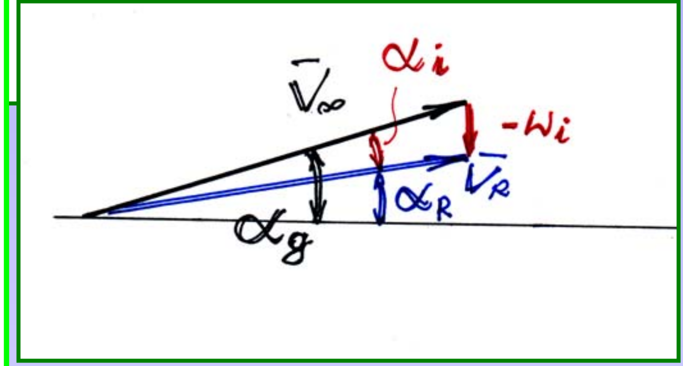


Lifting Line Theory (Prandtl)



$$w_i(y) = -\frac{1}{4\pi}\int_{-b/2}^{b/2}\frac{d\Gamma/d\eta}{y-\eta}\cdot d\eta \qquad \alpha_i = -w_i / V_\infty$$

$$\alpha_R = \alpha_G - \alpha_i = \alpha_G - \frac{1}{4\pi V_\infty}\int_{-b/2}^{b/2}\frac{d\Gamma/d\eta}{y-\eta}\cdot d\eta$$



Lifting Line Theory (Prandtl)

$$\Gamma = \frac{1}{2} V_\infty \cdot c \cdot C_L$$

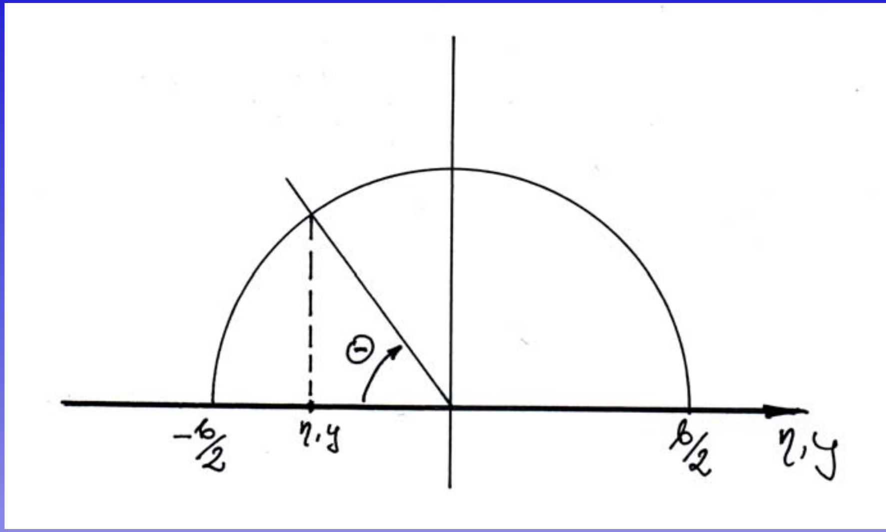
$$C_L = \left.\frac{dC_L}{d\alpha}\right|_\infty \cdot \left(\alpha_R - \alpha_0\right) = a_\infty \cdot \left(\alpha_R - \alpha_0\right)$$

$$\Gamma(y) \; = \; \frac{1}{2} V_\infty \cdot c(y) \cdot a_\infty \cdot \left(\left(\alpha_G(y) - \alpha_0(y)\right) - \frac{1}{4\pi} \int_{-b/2}^{b/2} \frac{d\Gamma / d\eta}{y - \eta} \cdot d\eta \right)$$

$$GIVEN: \quad c(y),\ \alpha_G(y),\ \alpha_0(y)$$

$$TO\ FIND: \quad \Gamma(y)\ \left(c_L(y)\right)$$

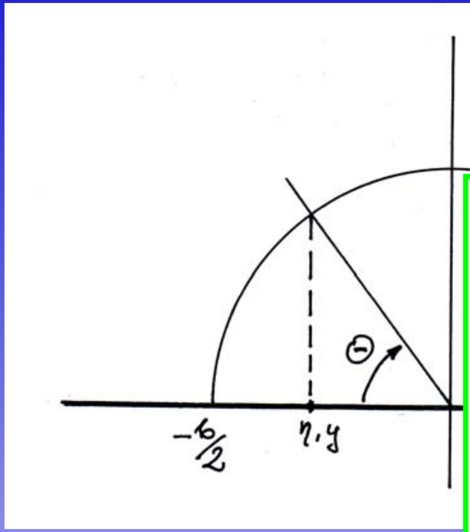
Lifting Line Theory (Prandtl)



$$\eta = -\frac{b}{2} \cdot \cos(\Theta) \qquad y = -\frac{b}{2} \cdot \cos(\Theta_0)$$

$$\Gamma(y) = 2V_\infty \cdot b \cdot \sum_n A_n \cdot \sin(n\Theta)$$

Lifting Line Theory (Prandtl)



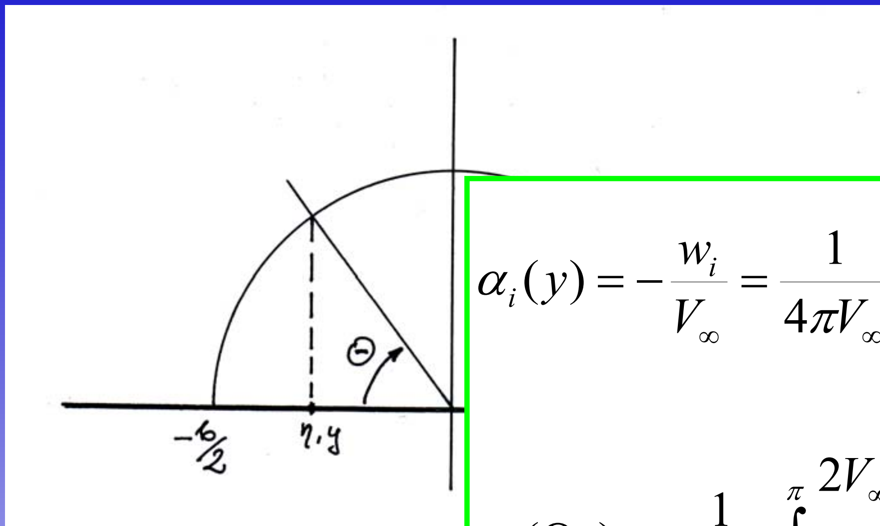
$$\alpha_i(y) = -\frac{w_i}{V_\infty} = \frac{1}{4\pi V_\infty}\int_{-b/2}^{b/2}\frac{d\Gamma/d\eta}{y-\eta}\cdot d\eta$$

$$\alpha_i(\Theta_0) = \frac{1}{4\pi V_\infty}\int_0^{\pi}\frac{2V_\infty\cdot b\cdot\sum_n n\cdot A_n\cdot\cos(n\Theta)}{-\frac{b}{2}\left(\cos(\Theta_0)-\cos(\Theta)\right)}\cdot d\Theta$$

$$\eta = -\frac{b}{2}\cdot\cos(\Theta) \qquad y = -\frac{b}{2}\cdot\cos(\Theta_0)$$

$$\Gamma(y) = 2V_\infty\cdot b\cdot\sum_n A_n\cdot\sin(n\Theta)$$

Lifting Line Theory (Prandtl)



$$\alpha_i(y) = -\frac{w_i}{V_\infty} = \frac{1}{4\pi V_\infty}\int_{-b/2}^{b/2}\frac{d\Gamma/d\eta}{y-\eta}\cdot d\eta$$

$$\alpha_i(\Theta_0) = \frac{1}{4\pi V_\infty}\int_0^\pi \frac{2V_\infty \cdot b \cdot \sum_n n\cdot A_n \cdot \cos(n\Theta)}{-\frac{b}{2}\left(\cos(\Theta_0)-\cos(\Theta)\right)}\cdot d\Theta =$$

$$-\frac{1}{\pi}\int_0^\pi \frac{\sum_n n\cdot A_n \cdot \cos(n\Theta)}{\cos(\Theta_0)-\cos(\Theta)}\cdot d\Theta = -\frac{1}{\pi}\sum_n n\cdot A_n \cdot \underbrace{\int_0^\pi \frac{\cos(n\Theta)}{\cos(\Theta_0)-\cos(\Theta)}\cdot d\Theta}_{-\pi\frac{\sin(n\Theta_0)}{\sin(\Theta_0)}}$$

$$\alpha_i(\Theta_0) = \sum_n n\cdot A_n \cdot \frac{\sin(n\Theta_0)}{\sin(\Theta_0)}$$

$$\eta = -\frac{b}{2}\cdot\cos(\Theta) \qquad y = -$$

$$\Gamma(y) = 2V_\infty \cdot b \cdot \sum_n A_n \cdot \mathrm{si}$$

Lifting Line Theory (Prandtl)

$$L_{WING} = \rho_\infty \cdot V_\infty \cdot \int_{-b/2}^{b/2} \Gamma(\eta) \cdot d\eta =$$

$$\rho_\infty \cdot V_\infty \cdot \int_0^\pi 2V_\infty \cdot b \cdot \left(\sum_n A_n \cdot \sin(n\Theta) \right) \cdot \frac{b}{2} \cdot \sin(\Theta) \cdot d\Theta =$$

$$\rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \sum_n A_n \cdot \int_0^\pi \sin(n\Theta) \cdot \sin(\Theta) \cdot d\Theta$$

$$\eta = -\frac{b}{2} \cdot \cos(\Theta) \qquad d\eta = \frac{b}{2} \cdot \sin(\Theta) \cdot d\Theta$$

$$\Gamma(y) = 2V_\infty \cdot b \cdot \sum_n A_n \cdot \sin(n\Theta)$$

Lifting Line Theory (Prandtl)

$$L_{WING} = \rho_\infty \cdot V_\infty \cdot \int_{-b/2}^{b/2} \Gamma(\eta) \cdot d\eta =$$

$$\rho_\infty \cdot V_\infty \cdot \int_0^\pi 2V_\infty \cdot b \cdot \left(\sum_n A_n \cdot \sin(n\Theta) \right) \cdot \frac{b}{2} \cdot \sin(\Theta) \cdot d\Theta =$$

$$\rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \sum_n A_n \cdot \int_0^\pi \sin(n\Theta) \cdot \sin(\Theta) \cdot d\Theta$$

$$\int_0^\pi \sin(i\Theta) \cdot \sin(n\Theta) \cdot d\Theta = \begin{cases} \frac{\pi}{2} & i = n \\ 0 & i \neq n \end{cases}$$

$$\eta = -\frac{b}{2} \cdot \cos(\Theta) \qquad d\eta = \frac{b}{2} \cdot \sin(\Theta) \cdot d\Theta$$

$$\Gamma(y) = 2V_\infty \cdot b \cdot \sum_n A_n \cdot \sin(n\Theta)$$

Lifting Line Theory (Prandtl)

$$L_{WING} = \rho_\infty \cdot V_\infty \cdot \int_{-b/2}^{b/2} \Gamma(\eta) \cdot d\eta =$$

$$\rho_\infty \cdot V_\infty \cdot \int_0^\pi 2V_\infty \cdot b \cdot \left(\sum_n A_n \cdot \sin(n\Theta) \right) \cdot \frac{b}{2} \cdot \sin(\Theta) \cdot d\Theta =$$

$$\rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot A_1 \cdot \frac{\pi}{2}$$

$$\int_0^\pi \sin(i\Theta) \cdot \sin(n\Theta) \cdot d\Theta = \begin{cases} \frac{\pi}{2} & i = n \\ 0 & i \neq n \end{cases}$$

$$\eta = -\frac{b}{2} \cdot \cos(\Theta) \qquad d\eta = \frac{b}{2} \cdot \sin(\Theta) \cdot d\Theta$$

$$\Gamma(y) = 2V_\infty \cdot b \cdot \sum_n A_n \cdot \sin(n\Theta)$$

Lifting Line Theory (Prandtl)

$$\int_0^{\pi} \sin(i\Theta)\cdot\sin(n\Theta)\cdot d\Theta = \begin{cases} \dfrac{\pi}{2} & i = n \\ 0 & i \neq n \end{cases}$$

$$L_{WING} = \rho_\infty \cdot V_\infty \cdot \int_{-b/2}^{b/2} \Gamma(\eta)\cdot d\eta =$$

$$\rho_\infty \cdot V_\infty \cdot \int_0^{\pi} 2V_\infty \cdot b \cdot \left(\sum_n A_n \cdot \sin(n\Theta) \right) \cdot \frac{b}{2} \cdot \sin(\Theta) \cdot d\Theta =$$

$$\rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot A_1 \cdot \frac{\pi}{2} = \frac{\rho_\infty \cdot V_\infty^2}{2} \cdot S \cdot C_L$$

$$C_L = \pi \cdot \frac{b^2}{S} \cdot A_1 = \pi \cdot \Lambda \cdot A_1$$

$$\eta = -\frac{b}{2}\cdot\cos(\Theta) \qquad d\eta = \frac{b}{2}\cdot\sin(\Theta)\cdot d\Theta$$

$$\Gamma(y) = 2V_\infty \cdot b \cdot \sum_n A_n \cdot \sin(n\Theta)$$

Lifting Line Theory (Prandtl)

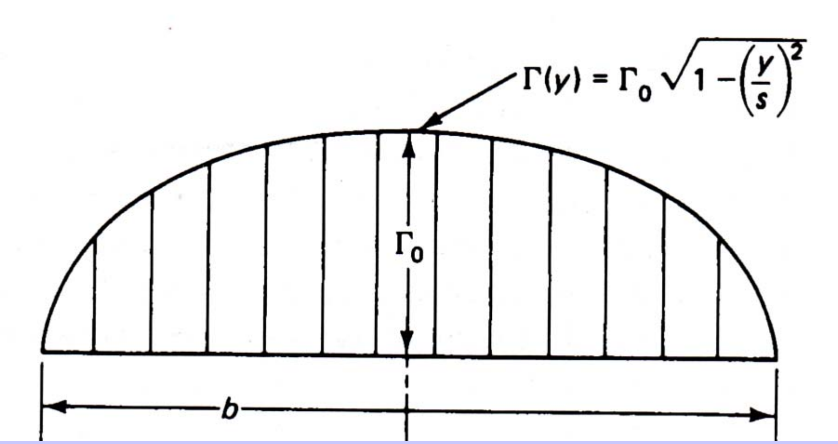
Elliptical lift (circulation) distribution

$$\Gamma(\Theta) = 2 \cdot V_\infty \cdot b \cdot A_1 \cdot \sin(\Theta)$$

$$= 2 \cdot V_\infty \cdot b \cdot A_1 \cdot \sqrt{1 - \left(\frac{\eta}{b/2}\right)^2}$$

$$\eta = -\frac{b}{2} \cdot \cos(\Theta)$$

$$\cos(\Theta) = -\frac{\eta}{b/2}$$



Lifting Line Theory (Prandtl)

Elliptical lift (circulation) distribution

$$\Gamma(\Theta) = 2 \cdot V_\infty \cdot b \cdot A_1 \cdot \sin(\Theta)$$

$$C_L = \pi \cdot \Lambda \cdot A_1$$

$$\alpha_i(\Theta) = \sum_n n \cdot A_n \cdot \frac{\sin(n\Theta)}{\sin(\Theta)} = A_1 = \frac{C_L}{\pi\Lambda} = const$$

Lifting Line Theory (Prandtl)

NON-Elliptical lift (circulation) distribution

$$C_{L\,AIRF} = \left.\frac{dC_L}{d\alpha}\right|_{\infty} \cdot \tilde{\alpha}_{\infty} = a_{\infty} \cdot \tilde{\alpha}_{\infty}$$

$$C_{L\,\Lambda} = \left.\frac{dC_L}{d\alpha}\right|_{\Lambda} \cdot \tilde{\alpha}_{\Lambda} = a_{\Lambda} \cdot \tilde{\alpha}_{\Lambda}$$

MEAN(AVERAGE) WING INDUCED ANGLE :

$$\alpha_i = \alpha_{\Lambda} - \alpha_{\infty} = \frac{C_L}{a_{\Lambda}} - \frac{C_L}{a_{\infty}} = \frac{C_L}{\pi\Lambda} \cdot \underbrace{\left(\frac{\pi\Lambda}{a_{\Lambda}} - \frac{\pi\Lambda}{a_{\infty}}\right)}_{1+\tau}$$

$$\tau \geq 0$$

Lifting Line Theory (Prandtl)

NON-Elliptical lift (circulation) distribution

$$C_{L\,AIRF} = \left.\frac{dC_L}{d\alpha}\right|_{\infty} \cdot \tilde{\alpha}_{\infty} = a_{\infty} \cdot \tilde{\alpha}_{\infty}$$

$$C_{L\,\Lambda} = \left.\frac{dC_L}{d\alpha}\right|_{\Lambda} \cdot \tilde{\alpha}_{\Lambda} = a_{\Lambda} \cdot \tilde{\alpha}_{\Lambda}$$

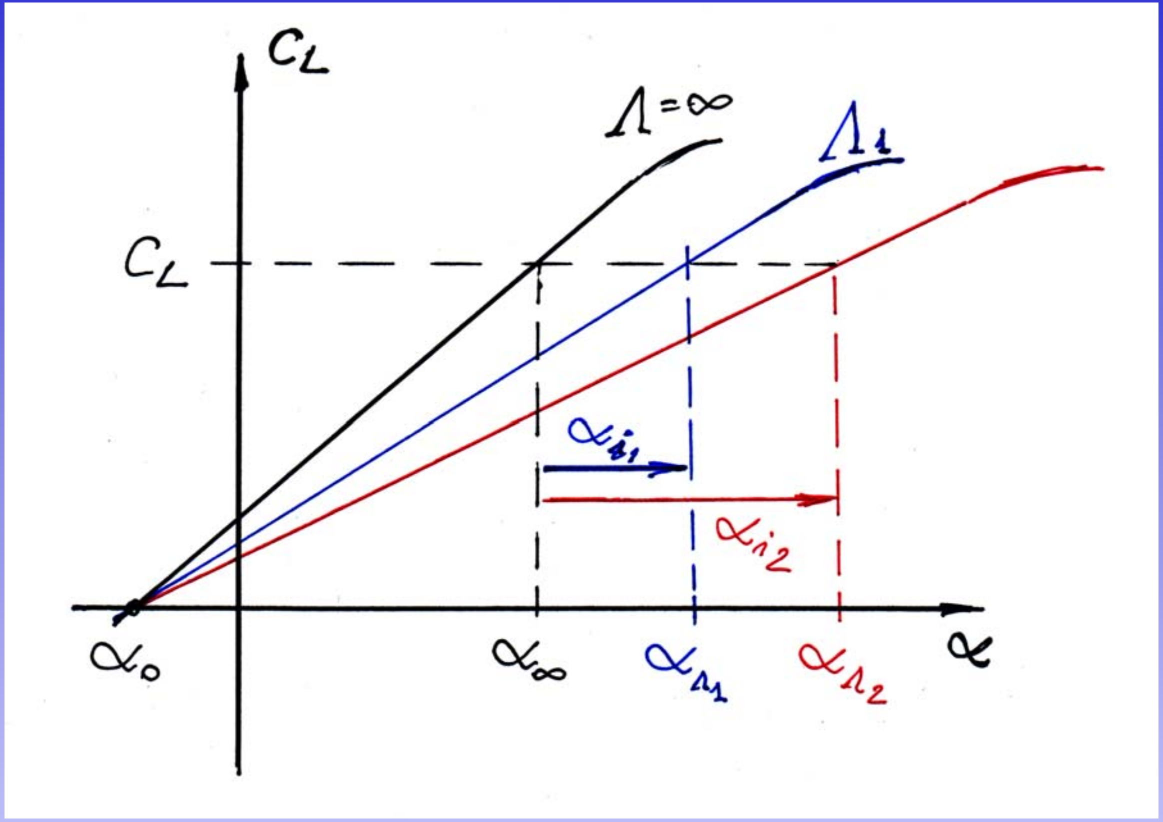
MEAN(AVERAGE) WING INDUCED ANGLE :

$$\alpha_i = \alpha_{\Lambda} - \alpha_{\infty} = \frac{C_L}{a_{\Lambda}} - \frac{C_L}{a_{\infty}} = \frac{C_L}{\pi\Lambda} \cdot \underbrace{\left(\frac{\pi\Lambda}{a_{\Lambda}} - \frac{\pi\Lambda}{a_{\infty}}\right)}_{1+\tau}$$

$$\tau \geq 0$$

$$\alpha_i = \frac{C_L}{\pi\Lambda} \cdot (1+\tau)$$

Lifting Line Theory (Prandtl)



$$\alpha_i = \frac{C_L}{\pi\Lambda}\cdot(1+\tau)$$

Lifting Line Theory (Prandtl)

$$\Delta C_L = a_\Lambda \cdot \Delta\alpha_G \quad = \quad a_\infty \cdot \Delta\alpha_R \quad = \quad a_\infty \cdot \left(\Delta\alpha_G - \Delta\alpha_i\right) =$$

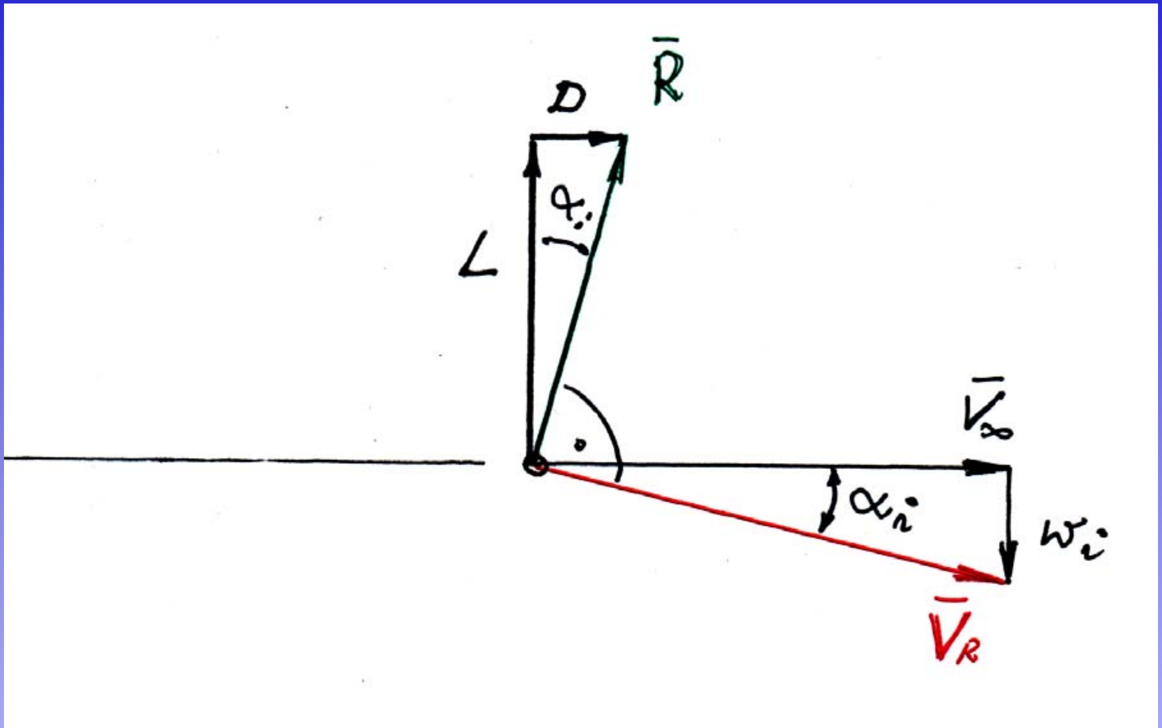
$$a_\infty \cdot \left(\Delta\alpha_G - \frac{\Delta C_L}{\pi\Lambda}\left(1+\tau\right)\right) \quad = \quad a_\infty \cdot \Delta\alpha_G - a_\infty \cdot \frac{\Delta C_L}{\pi\Lambda}\left(1+\tau\right)$$

$$\Delta C_L \cdot \left(1 + \cdot \frac{a_\infty}{\pi\Lambda}\left(1+\tau\right)\right) = a_\infty \cdot \Delta\alpha_G$$

$$\Delta C_L \quad = \quad a_\infty \Big/ \left(1 + \frac{a_\infty}{\pi\Lambda}\left(1+\tau\right)\right) \cdot \Delta\alpha_G$$

$$a_\Lambda \quad = \quad \Delta C_L / \Delta\alpha_G \quad = \quad \frac{a_\infty}{1 + \frac{a_\infty}{\pi\Lambda}\left(1+\tau\right)} \quad = \quad \frac{1}{\frac{1}{a_\infty} + \frac{1+\tau}{\pi\Lambda}}$$

Lifting Line Theory (Prandtl)



$$D_i = R \cdot \sin(\alpha_i)$$
$$L = R \cdot \cos(\alpha_i)$$

$$D_i = L \cdot \tan(\alpha_i) \approx L \cdot \alpha_i$$

Lifting Line Theory (Prandtl)

$$L = \rho_\infty \cdot V_\infty \cdot \int_{-b/2}^{b/2} \Gamma(\eta) \cdot d\eta = \rho_\infty \cdot V_\infty \cdot \int_{0}^{\pi} \Gamma(\Theta) \cdot \frac{b}{2} \cdot \sin(\Theta) \cdot d\Theta$$

$$\Gamma(\Theta) = 2V_\infty \cdot b \cdot \sum_n A_n \cdot \sin(n\Theta)$$

$$\alpha_i(\Theta) = \sum_n n \cdot A_n \cdot \frac{\sin(n\Theta)}{\sin(\Theta)}$$

$$\eta = -\frac{b}{2} \cdot \cos(\Theta) \qquad d\eta = \frac{b}{2} \cdot \sin(\Theta) \cdot d\Theta$$

$$D_i = \rho_\infty \cdot V_\infty \cdot \int_{-b/2}^{b/2} \Gamma(\eta) \cdot \alpha_i(\eta) \cdot d\eta$$

$$= \rho_\infty \cdot V_\infty \cdot \int_{0}^{\pi} \Gamma(\Theta) \cdot \alpha_i(\Theta) \cdot \frac{b}{2} \cdot \sin(\Theta) \cdot d\Theta$$

$$= \rho_\infty \cdot V_\infty \cdot \int_{0}^{\pi} \left(2V_\infty \cdot b \cdot \sum_n A_n \cdot \sin(n\Theta) \right) \cdot \left(\sum_m m \cdot A_m \cdot \frac{\sin(m\Theta)}{\sin(\Theta)} \right) \cdot \frac{b}{2} \cdot \sin(\Theta) \cdot d\Theta$$

Lifting Line Theory (Prandtl)

$$D_i = \rho_\infty \cdot V_\infty \cdot \int_0^\pi \left(2V_\infty \cdot b \cdot \sum_n A_n \cdot \sin(n\Theta) \right) \cdot \left(\sum_m m \cdot A_m \cdot \frac{\sin(m\Theta)}{\sin(\Theta)} \right) \cdot \frac{b}{2} \cdot \sin(\Theta) \cdot d\Theta$$

$$= \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \int_0^\pi \left(\sum_n A_n \cdot \sin(n\Theta) \right) \cdot \left(\sum_m m \cdot A_m \cdot \sin(m\Theta) \right) \cdot d\Theta$$

$$= \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \int_0^\pi \sum_n \sum_m m \cdot A_m \cdot A_n \cdot \sin(n\Theta) \cdot \sin(m\Theta) \cdot d\Theta$$

$$= \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \sum_n \sum_m \int_0^\pi m \cdot A_m A_n \cdot \sin(n\Theta) \cdot \sin(m\Theta) d\Theta$$

Lifting Line Theory (Prandtl)

$$D_i = \rho_\infty \cdot V_\infty \cdot \int_0^\pi \left(2V_\infty \cdot b \cdot \sum_n A_n \cdot \sin(n\Theta) \right) \cdot$$

$$\int_0^\pi \sin(n\Theta) \cdot \sin(m\Theta) \cdot d\Theta \quad = \begin{cases} \frac{\pi}{2} & n = m \\ 0 & n \neq m \end{cases}$$

$$= \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \int_0^\pi \left(\sum_n A_n \cdot \sin(n\Theta) \right) \cdot \left(\sum_m m \cdot A_m \cdot \sin(m\Theta) \right) \cdot d\Theta$$

$$= \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \int_0^\pi \sum_n \sum_m m \cdot A_m \cdot A_n \cdot \sin(n\Theta) \cdot \sin(m\Theta) \cdot d\Theta$$

$$= \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \sum_n \sum_m \int_0^\pi m \cdot A_m A_n \cdot \sin(n\Theta) \cdot \sin(m\Theta) d\Theta$$

Lifting Line Theory (Prandtl)

$$D_i = \rho_\infty \cdot V_\infty \cdot \int_0^\pi \left(2V_\infty \cdot b \cdot \sum_n A_n \cdot \sin(n\Theta) \right) \cdot$$

$$\int_0^\pi \sin(n\Theta) \cdot \sin(m\Theta) \cdot d\Theta = \begin{cases} \frac{\pi}{2} & n = m \\ 0 & n \neq m \end{cases}$$

$$= \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \int_0^\pi \left(\sum_n A_n \cdot \sin(n\Theta) \right) \cdot \left(\sum_m m \cdot A_m \cdot \sin(m\Theta) \right) \cdot d\Theta$$

$$= \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \int_0^\pi \sum_n \sum_m m \cdot A_m \cdot A_n \cdot \sin(n\Theta) \cdot \sin(m\Theta) \cdot d\Theta$$

$$= \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \sum_n \sum_m \int_0^\pi m \cdot A_m A_n \cdot \sin(n\Theta) \cdot \sin(m\Theta) d\Theta$$

$$= \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \frac{\pi}{2} \cdot \sum_n n \cdot A_n^2$$

Lifting Line Theory (Prandtl)

$$D_i = \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \frac{\pi}{2} \cdot \sum_n n \cdot A_n^2$$

$$C_{D_i} = \frac{D_i}{S \cdot \rho_\infty \cdot \frac{V_\infty^2}{2}} = \frac{\rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \frac{\pi}{2} \cdot \sum_n n \cdot A_n^2}{S \cdot \rho_\infty \cdot \frac{V_\infty^2}{2}} =$$

$$= \frac{\pi \cdot b^2}{S} \sum_{n=1}^{\infty} n \cdot A_n^2 = \frac{\pi \cdot b^2}{S} \left(A_1^2 + \sum_{n=2}^{\infty} n \cdot A_n^2 \right) = \pi \Lambda \cdot A_1^2 \left(1 + \sum_{n=2}^{\infty} n \cdot \frac{A_n^2}{A_1^2} \right)$$

Lifting Line Theory (Prandtl)

$$D_i = \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \frac{\pi}{2} \cdot \sum_n n \cdot A_n^2$$

$$C_{D_i} = \frac{D_i}{S \cdot \rho_\infty \cdot \frac{V_\infty^2}{2}} = \frac{\rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \frac{\pi}{2} \cdot \sum_n n \cdot A_n^2}{S \cdot \rho_\infty \cdot \frac{V_\infty^2}{2}} =$$

$$= \frac{\pi \cdot b^2}{S} \sum_{n=1}^{\infty} n \cdot A_n^2 = \frac{\pi \cdot b^2}{S} \left(A_1^2 + \sum_{n=2}^{\infty} n \cdot A_n^2 \right) = \pi\Lambda \cdot A_1^2 \left(1 + \sum_{n=2}^{\infty} n \cdot \frac{A_n^2}{A_1^2} \right)$$

$$C_L = \pi \cdot \Lambda \cdot A_1$$

$$A_1 = \frac{C_L}{\pi \cdot \Lambda}$$

Lifting Line Theory (Prandtl)

$$C_L = \pi \cdot \Lambda \cdot A_1$$

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$$D_i = \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \frac{\pi}{2} \cdot \sum_n n \cdot A_n^2$$

$$C_{D_i} = \frac{D_i}{S \cdot \rho_\infty \cdot \frac{V_\infty^2}{2}} = \frac{\rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \frac{\pi}{2} \cdot \sum_n n \cdot A_n^2}{S \cdot \rho_\infty \cdot \frac{V_\infty^2}{2}} =$$

$$= \frac{\pi \cdot b^2}{S} \sum_{n=1}^{\infty} n \cdot A_n^2 = \frac{\pi \cdot b^2}{S} \left(A_1^2 + \sum_{n=2}^{\infty} n \cdot A_n^2 \right) = \pi\Lambda \cdot A_1^2 \left(1 + \sum_{n=2}^{\infty} n \cdot \frac{A_n^2}{A_1^2} \right)$$

$$= \frac{C_L^2}{\pi\Lambda} \cdot \left(1 + \sum_{n=2}^{\infty} n \cdot \frac{A_n^2}{A_1^2} \right)$$

Lifting Line Theory (Prandtl)

$$D_i = \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \frac{\pi}{2} \cdot \sum_n n \cdot A_n^2$$

$$C_{D_i} = \frac{C_L^2}{\pi \Lambda} \cdot \left(1 + \sum_{n=2}^{\infty} n \cdot \frac{A_n^2}{A_1^2} \right) = \frac{C_L^2}{\pi \Lambda} \cdot (1 + \delta)$$

Lifting Line Theory (Prandtl)

$$D_i = \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \frac{\pi}{2} \cdot \sum_n n \cdot A_n^2$$

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$$\textbf{\textit{Elliptical}} \quad \Gamma \quad \textbf{\textit{distribution}}: \quad A_2, A_3, A_4, \ldots = 0 \quad \rightarrow \quad \delta = 0$$

Lifting Line Theory (Prandtl)

$$D_i = \rho_\infty \cdot V_\infty^2 \cdot b^2 \cdot \frac{\pi}{2} \cdot \sum_n n \cdot A_n^2$$

$$C_{D_i} = \frac{C_L^2}{\pi\Lambda} \cdot \left(1 + \sum_{n=2}^{\infty} n \cdot \frac{A_n^2}{A_1^2}\right) = \frac{C_L^2}{\pi\Lambda} \cdot (1+\delta)$$

$$\textbf{Non-Elliptical} \quad \Gamma \quad \textbf{distribution}: \quad A_2, A_3, A_4, \ldots \neq 0 \quad \rightarrow \quad \delta > 0$$

Lifting Line Theory (Prandtl)

$$\Gamma(y) = \frac{1}{2} V_\infty \cdot c(y) \cdot a_\infty \cdot \left(\left(\alpha_G(y) - \alpha_0(y) \right) - \frac{1}{4\pi} \int_{-b/2}^{b/2} \frac{d\Gamma/d\eta}{y-\eta} \cdot d\eta \right)$$

$$GIVEN: \quad c(y),\ \alpha_G(y),\ \alpha_0(y)$$

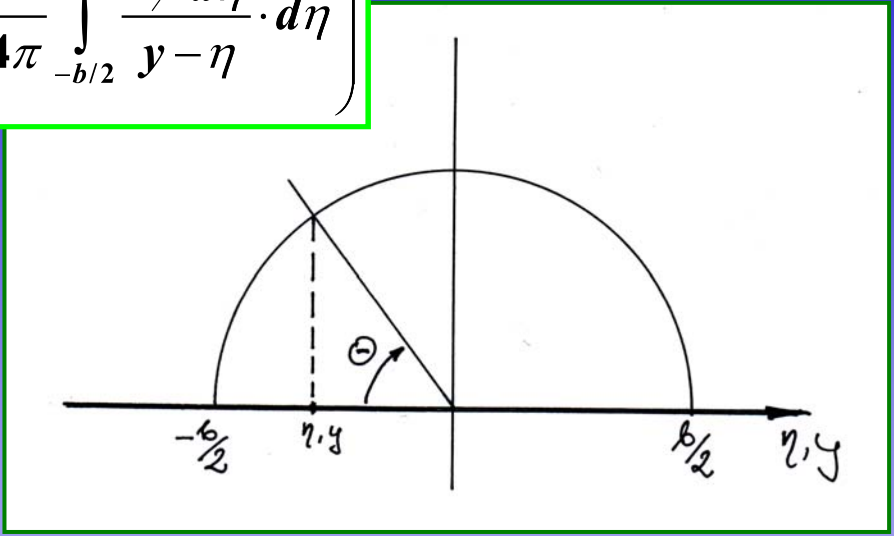
$$TO\ FIND: \quad \Gamma(y)\ \left(c_L(y)\right)$$

Lifting Line Theory (Prandtl)

$$\Gamma(y) = \frac{1}{2} V_\infty \cdot c(y) \cdot a_\infty \cdot \left(\left(\alpha_G(y) - \alpha_0(y)\right) - \frac{1}{4\pi} \int_{-b/2}^{b/2} \frac{d\Gamma / d\eta}{y - \eta} \cdot d\eta \right)$$

$$\eta = -\frac{b}{2} \cdot \cos(\Theta) \qquad y = -\frac{b}{2} \cdot \cos(\Theta_0)$$

$$\Gamma(y) = 2V_\infty \cdot b \cdot \sum_n A_n \cdot \sin(n\Theta)$$



$$2V_\infty \cdot b \cdot \sum_{n=1}^{\infty} A_n \cdot \sin(n\Theta_0) = \frac{1}{2} V_\infty \cdot c(\Theta_0) \cdot a_\infty \cdot \left(\left(\alpha_G(\Theta_0) - \alpha_0(\Theta_0)\right) - \sum_{n=1}^{\infty} n \cdot A_n \cdot \frac{\sin(n\Theta_0)}{\sin(\Theta_0)} \right)$$

$GIVEN: \quad c(y),\ \alpha_G(y),\ \alpha_0(y)$

$TO\ FIND: \quad \Gamma(y)\ \left(c_L(y)\right)$

Lifting Line Theory (Prandtl)

$$\textbf{GIVEN:} \qquad c(y),\ \alpha_G(y),\ \alpha_0(y)$$

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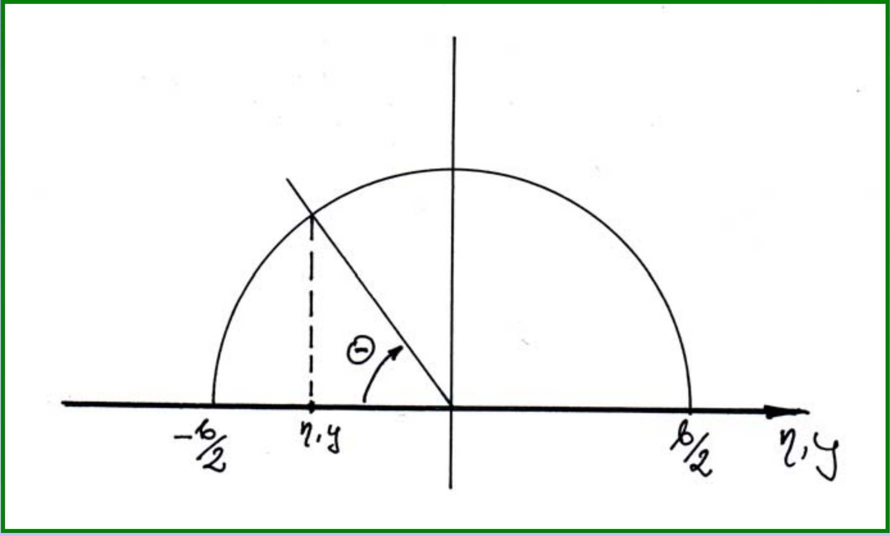
$$2V_\infty \cdot b \cdot \sum_{n=1}^{\infty} A_n \cdot \sin(n\Theta_0) = \frac{1}{2} V_\infty \cdot c(\Theta_0) \cdot a_\infty \cdot \left(\left(\alpha_G(\Theta_0) - \alpha_0(\Theta_0)\right) - \sum_{n=1}^{\infty} n \cdot A_n \cdot \frac{\sin(n\Theta_0)}{\sin(\Theta_0)} \right)$$

$$\eta = -\frac{b}{2} \cdot \cos(\Theta) \qquad y = -\frac{b}{2} \cdot \cos(\Theta_0)$$

$$\Gamma(y) = 2V_\infty \cdot b \cdot \sum_n A_n \cdot \sin(n\Theta)$$



Lifting Line Theory (Prandtl)

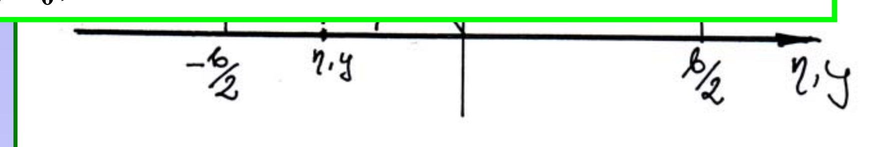
GIVEN : $c(\Theta_0),\ \alpha_G(\Theta_0),\ \alpha_0(\Theta_0)$

TO FIND : $\Gamma(\Theta_0)\ \left(c_L(\Theta_0)\right)$

$$2V_\infty \cdot b \cdot \sum_{n=1}^{\infty} A_n \cdot \sin(n\Theta_0) = \frac{1}{2} V_\infty \cdot c(\Theta_0) \cdot a_\infty \cdot \left(\left(\alpha_G(\Theta_0) - \alpha_0(\Theta_0)\right) - \sum_{n=1}^{\infty} n \cdot A_n \cdot \frac{\sin(n\Theta_0)}{\sin(\Theta_0)} \right)$$

$$\sum_{n=1}^{\infty} A_n \cdot \sin(n\Theta_0) = \frac{c(\Theta_0)}{4b} \cdot a_\infty \cdot \left(\left(\alpha_G(\Theta_0) - \alpha_0(\Theta_0)\right) - \sum_{n=1}^{\infty} n \cdot A_n \cdot \frac{\sin(n\Theta_0)}{\sin(\Theta_0)} \right)$$

$$\sum_{n=1}^{\infty} A_n \cdot \sin(n\Theta_0) + \frac{c(\Theta_0)}{4b} \cdot a_\infty \cdot \sum_{n=1}^{\infty} n \cdot A_n \cdot \frac{\sin(n\Theta_0)}{\sin(\Theta_0)} = -\frac{c(\Theta_0)}{4b} \cdot a_\infty \cdot \left(\alpha_G(\Theta_0) - \alpha_0(\Theta_0)\right)$$



Lifting Line Theory (Prandtl)

$$GIVEN: \quad c(\Theta_0),\ \alpha_G(\Theta_0),\ \alpha_0(\Theta_0)$$

$$TO\ FIND: \quad \Gamma(\Theta_0)\ \left(c_L(\Theta_0)\right)$$

$$2V_\infty \cdot b \cdot \sum_{n=1}^{\infty} A_n \cdot \sin(n\Theta_0) = \frac{1}{2} V_\infty \cdot c(\Theta_0) \cdot a_\infty \cdot \left(\left(\alpha_G(\Theta_0) - \alpha_0(\Theta_0)\right) - \sum_{n=1}^{\infty} n \cdot A_n \cdot \frac{\sin(n\Theta_0)}{\sin(\Theta_0)} \right)$$

$$\sum_{n=1}^{\infty} A_n \cdot \sin(n\Theta_0) = \frac{c(\Theta_0)}{4b} \cdot a_\infty \cdot \left(\left(\alpha_G(\Theta_0) - \alpha_0(\Theta_0)\right) - \sum_{n=1}^{\infty} n \cdot A_n \cdot \frac{\sin(n\Theta_0)}{\sin(\Theta_0)} \right)$$

$$\sum_{n=1}^{\infty} A_n \cdot \sin(n\Theta_0) + \frac{c(\Theta_0)}{4b} \cdot a_\infty \cdot \sum_{n=1}^{\infty} n \cdot A_n \cdot \frac{\sin(n\Theta_0)}{\sin(\Theta_0)} = -\frac{c(\Theta_0)}{4b} \cdot a_\infty \cdot \left(\alpha_G(\Theta_0) - \alpha_0(\Theta_0)\right)$$

Glauert's Method

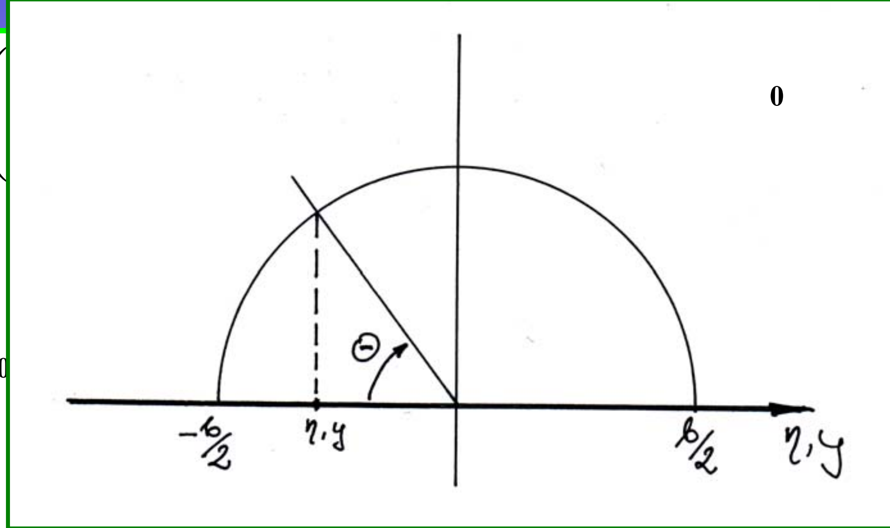
$$\sum_{n=1}^{N} \left(\sin(n\Theta_0) + \left(a_\infty \cdot \frac{c(\Theta_0)}{4b} \right) \cdot n \cdot \frac{\sin(n\Theta_0)}{\sin(\Theta_0)} \right) \cdot A_n = -\frac{c(\Theta_0)}{4b} \cdot a_\infty \cdot \left(\alpha_G(\Theta_0) - \alpha_0(\Theta_0)\right)$$

Lifting Line Theory (Prandtl)

GIVEN : $c(\Theta_0),\ \alpha_G(\Theta_0),\ \alpha_0(\Theta_0)$

TO FIND : $\Gamma(\Theta_0)\ \left(c_L(\Theta_0)\right)$

$$2V_\infty \cdot b \cdot \sum_{n=1}^{\infty} A_n \cdot \sin(n\Theta_0) = \frac{1}{2} V_\infty \cdot c(\Theta_0) \cdot a_\infty \cdot \Big(\ldots$$



η …

$\Gamma($ …

$$\sum_{n=1}^{\infty} A_n \cdot \sin(n\Theta_0) = \frac{c(\Theta_0)}{4b} \cdot a_\infty \cdot \Big(\big(\alpha_G(\Theta_0) - \alpha_0 \ldots$$

$$\sum_{n=1}^{\infty} A_n \cdot \sin(n\Theta_0) + \frac{c(\Theta_0)}{4b} \cdot a_\infty \cdot \sum_{n=1}^{\infty} n \cdot A_n \cdot \frac{\sin(n\Theta_0)}{\sin(\Theta_0)} = -\frac{c(\Theta_0)}{4b} \cdot a_\infty \cdot \left(\alpha_G(\Theta_0) - \alpha_0(\Theta_0)\right)$$

Glauert's Method

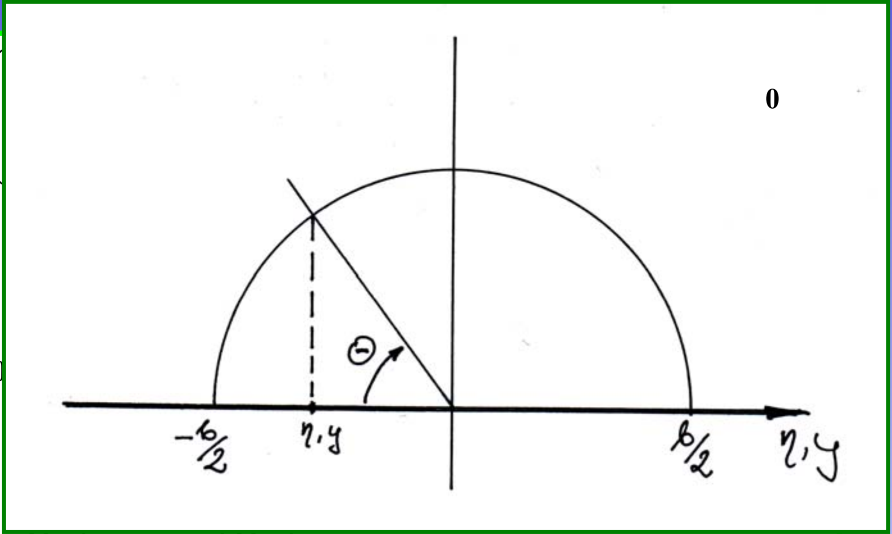
$$\sum_{n=1}^{N} \left(\sin(n\Theta_0) + \left(a_\infty \cdot \frac{c(\Theta_0)}{4b} \right) \cdot n \cdot \frac{\sin(n\Theta_0)}{\sin(\Theta_0)} \right) \cdot A_n = -\frac{c(\Theta_0)}{4b} \cdot a_\infty \cdot \left(\alpha_G(\Theta_0) - \alpha_0(\Theta_0)\right)$$

Lifting Line Theory (Prandtl)

GIVEN : $c(\Theta_0),\ \alpha_G(\Theta_0),\ \alpha_0(\Theta_0)$

TO FIND : $\Gamma(\Theta_0)\ \left(c_L(\Theta_0)\right)$

$$2V_\infty \cdot b \cdot \sum_{n=1}^{\infty} A_n \cdot \sin(n\Theta_0) = \frac{1}{2} V_\infty \cdot c(\Theta_0) \cdot a_\infty \cdot \Big(\ldots$$



$$\sum_{n=1}^{\infty} A_n \cdot \sin(n\Theta_0) = \frac{c(\Theta_0)}{4b} \cdot a_\infty \cdot \Big(\big(\alpha_G(\Theta_0) - \alpha_0 \ldots$$

$$\sum_{n=1}^{\infty} A_n \cdot \sin(n\Theta_0) + \frac{c(\Theta_0)}{4b} \cdot a_\infty \cdot \sum_{n=1}^{\infty} n \cdot A_n \cdot \frac{\sin(n\Theta_0)}{\ldots} = -\frac{c(\Theta_0)}{4b} \cdot a_\infty \cdot \left(\alpha_G(\Theta_0) - \alpha_0(\Theta_0)\right)$$

$$n = 1, 3, 5, 7 \ldots$$

Glauert's Method

$$\sum_{n=1}^{N} \left(\sin(n\Theta_0) + \left(a_\infty \cdot \frac{c(\Theta_0)}{4b} \right) \cdot n \cdot \frac{\sin(n\Theta_0)}{\sin(\Theta_0)} \right) \cdot A_n = -\frac{c(\Theta_0)}{4b} \cdot a_\infty \cdot \left(\alpha_G(\Theta_0) - \alpha_0(\Theta_0)\right)$$

Lifting Line Theory (Prandtl)

$$\text{GIVEN}: \quad c(\Theta_0),\ \alpha_G(\Theta_0),\ \alpha_0(\Theta_0)$$

$$\text{TO FIND}: \quad \Gamma(\Theta_0)\ \left(c_L(\Theta_0)\right)$$

$$\sum_{n=1}^{N}\left(\sin(n\Theta_0)+\left(a_\infty\cdot\frac{c(\Theta_0)}{4b}\right)\cdot n\cdot\frac{\sin(n\Theta_0)}{\sin(\Theta_0)}\right)\cdot A_n=-\frac{c(\Theta_0)}{4b}\cdot a_\infty\cdot\left(\underbrace{\alpha_G(\Theta_0)}_{\alpha_\infty+\alpha_T(\Theta_0)}-\alpha_0(\Theta_0)\right)$$

$$\sum_{n=1}^{N}\left(\sin(n\Theta_0)+\left(a_\infty\cdot\frac{c(\Theta_0)}{4b}\right)\cdot n\cdot\frac{\sin(n\Theta_0)}{\sin(\Theta_0)}\right)\cdot\left(\Delta A_n^{\alpha_\infty}+A_n^0+\right)=-\frac{c(\Theta_0)}{4b}\cdot a_\infty\cdot\left(\alpha_\infty+\overbrace{\alpha_T(\Theta_0)-\alpha_0(\Theta_0)}^{\alpha_{Wo}(\Theta_0)}\right)$$

$$\sum_{n=1}^{N}\left(\sin(n\Theta_0)+\left(a_\infty\cdot\frac{c(\Theta_0)}{4b}\right)\cdot n\cdot\frac{\sin(n\Theta_0)}{\sin(\Theta_0)}\right)\cdot A_n^0=\frac{c(\Theta_0)}{4b}\cdot a_\infty\cdot\alpha_{Wo}(\Theta_0)\quad\rightarrow\quad CL_0$$

$$\sum_{n=1}^{N}\left(\sin(n\Theta_0)+\left(a_\infty\cdot\frac{c(\Theta_0)}{4b}\right)\cdot n\cdot\frac{\sin(n\Theta_0)}{\sin(\Theta_0)}\right)\cdot\Delta A_n^{\alpha_G}=-\frac{c(\Theta_0)}{4b}\cdot a_\infty\cdot\alpha_\infty\quad\rightarrow\quad\Delta CL_{\alpha_\infty}$$

Lifting Line Theory (Prandtl)

$$\textbf{GIVEN}: \quad c(\Theta_0),\ \alpha_G(\Theta_0),\ \alpha_0(\Theta_0)$$

$$\textbf{TO FIND}: \quad \Gamma(\Theta_0)\ \left(c_L(\Theta_0)\right)$$

$$\sum_{n=1}^{N}\left(\sin(n\Theta_0)+\left(a_\infty\cdot\frac{c(\Theta_0)}{4b}\right)\cdot n\cdot\frac{\sin(n\Theta_0)}{\sin(\Theta_0)}\right)\cdot A_n=-\frac{c(\Theta_0)}{4b}\cdot a_\infty\cdot\left(\underbrace{\alpha_G(\Theta_0)}_{\alpha_\infty+\alpha_T(\Theta_0)}-\alpha_0(\Theta_0)\right)$$

0

$$\sum_{n=1}^{N}\left(\sin(n\Theta_0)+\left(a_\infty\cdot\frac{c}{\ldots}\right.\right. \ldots \cdot a_\infty\cdot\left(\alpha_\infty+\overbrace{\alpha_T(\Theta_0)-\alpha_0(\Theta_0)}^{\alpha_{Wo}(\Theta_0)}\right)$$

$$a_\Lambda = \frac{a_\infty}{1+\frac{a_\infty}{\pi\Lambda}(1+\tau)} = \frac{1}{\frac{1}{a_\infty}+\frac{1+\tau}{\pi\Lambda}}$$

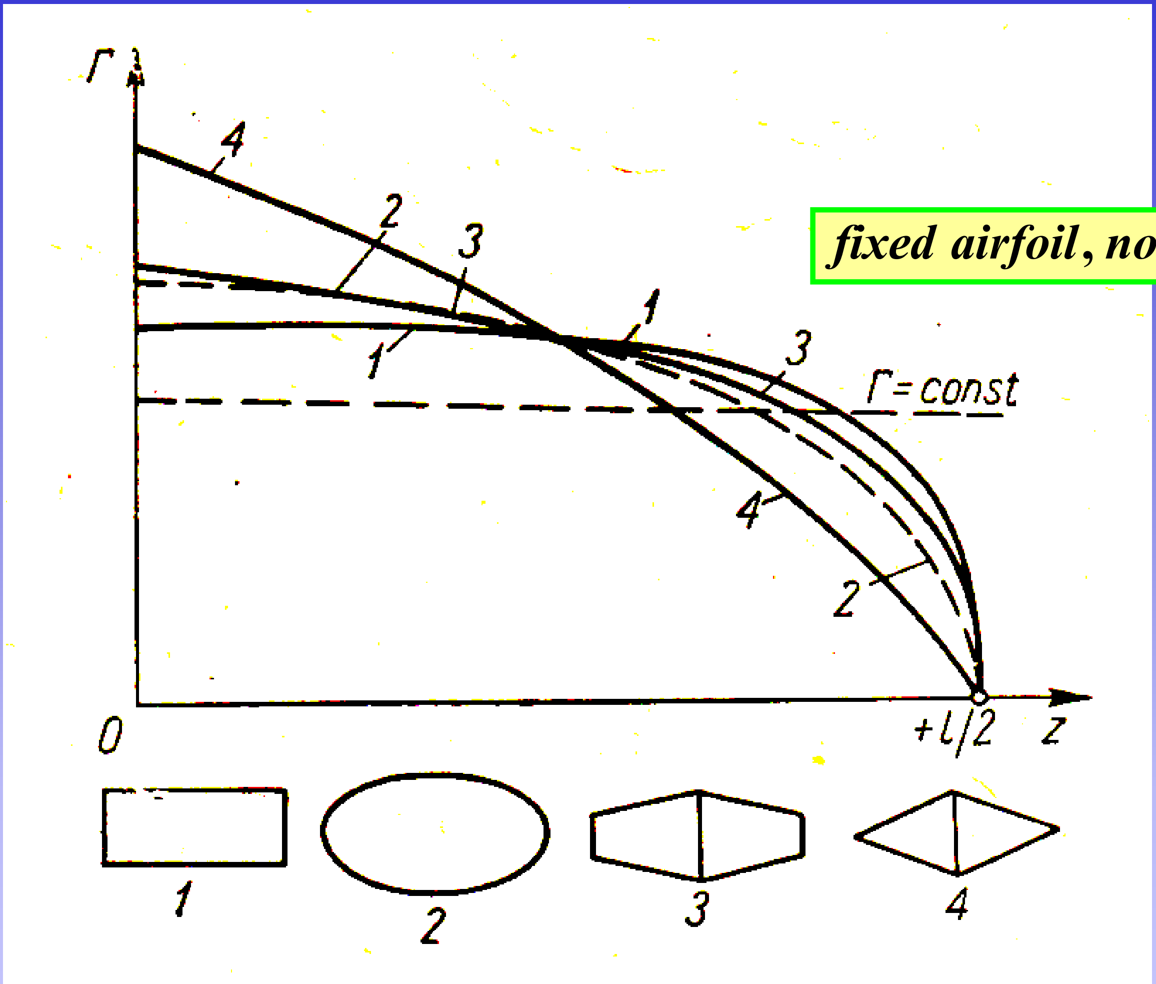
$$\sum_{n=1}^{N}\left(\sin(n\Theta_0)+\left(a_\infty\cdot\frac{c(\Theta_0)}{4b}\right)\cdot n\cdot\frac{\sin(n\Theta_0)}{\sin(\Theta_0)}\right)\cdot A_n^0 = \ldots \rightarrow CL_0$$

$$C_{X_i}=\frac{C_Z^2}{\pi\Lambda}\cdot(1+\delta)$$

$$\sum_{n=1}^{N}\left(\sin(n\Theta_0)+\left(a_\infty\cdot\frac{c(\Theta_0)}{4b}\right)\cdot n\cdot\frac{\sin(n\Theta_0)}{\sin(\Theta_0)}\right)\cdot \Delta A_n^{\alpha_G}=-\frac{c(\Theta_0)}{4b}\cdot a_\infty\cdot\alpha_\infty \rightarrow \Delta CL_{\alpha_\infty}$$

...od

Lifting Line Theory (Prandtl)

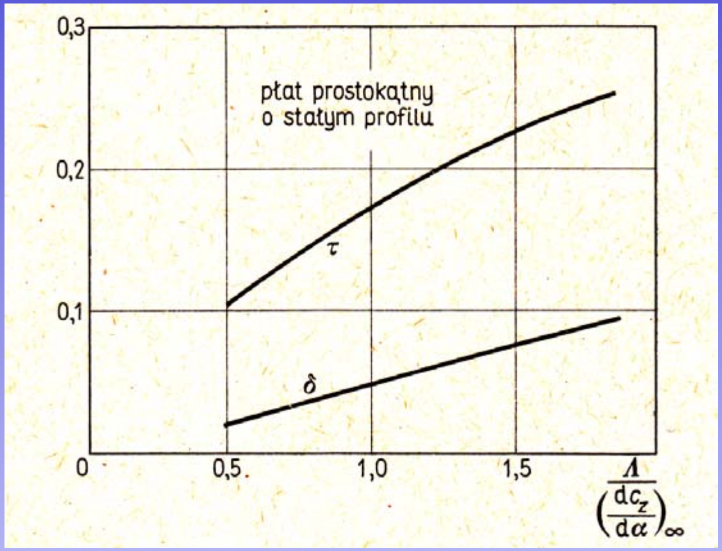


fixed airfoil, no twist

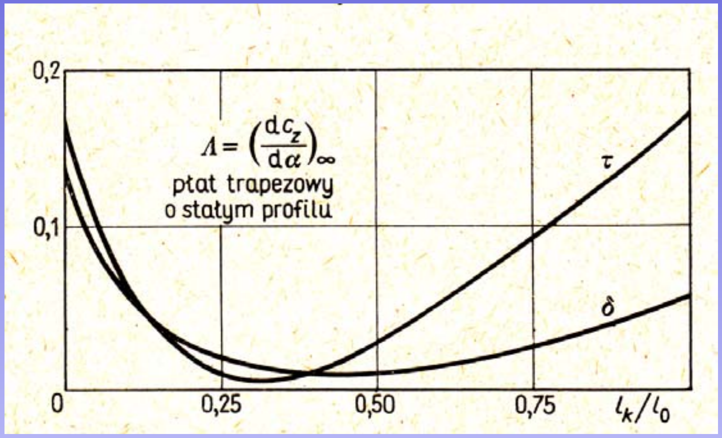
$$\Gamma = \frac{c \cdot c_L}{2} \; V$$

$$c_L = \frac{2\Gamma}{cV}$$

Lifting Line Theory (Prandtl)

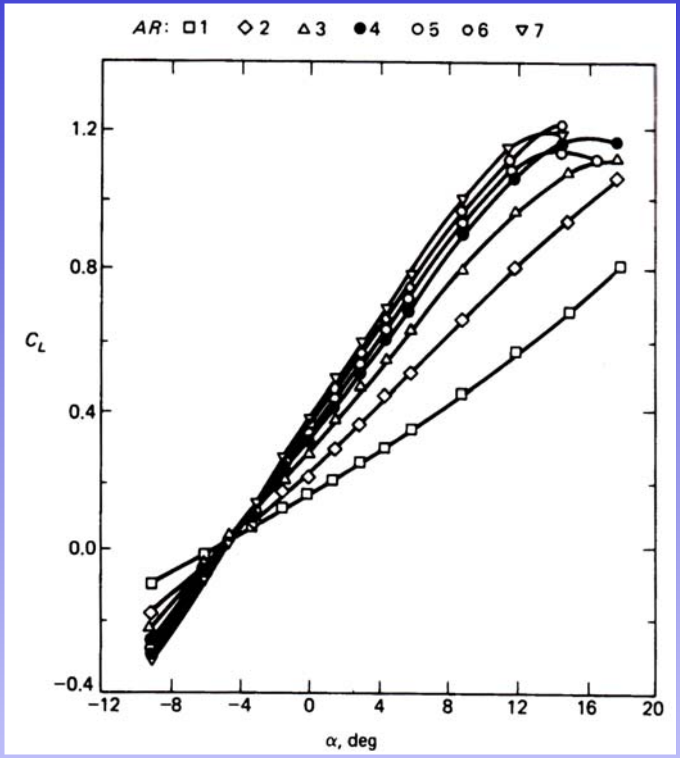


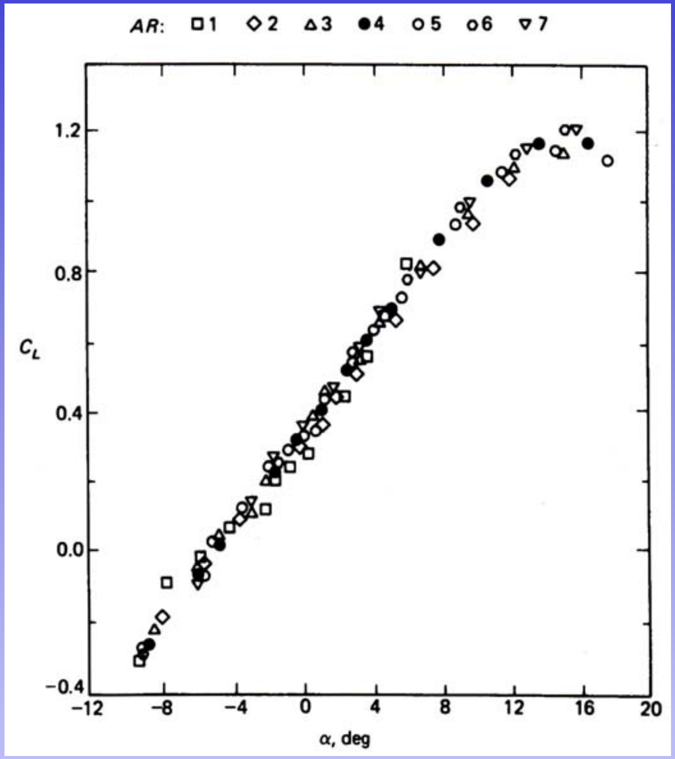
fixed airfoil, no twist



Lifting Line Theory (Prandtl)

eff: AR = 5

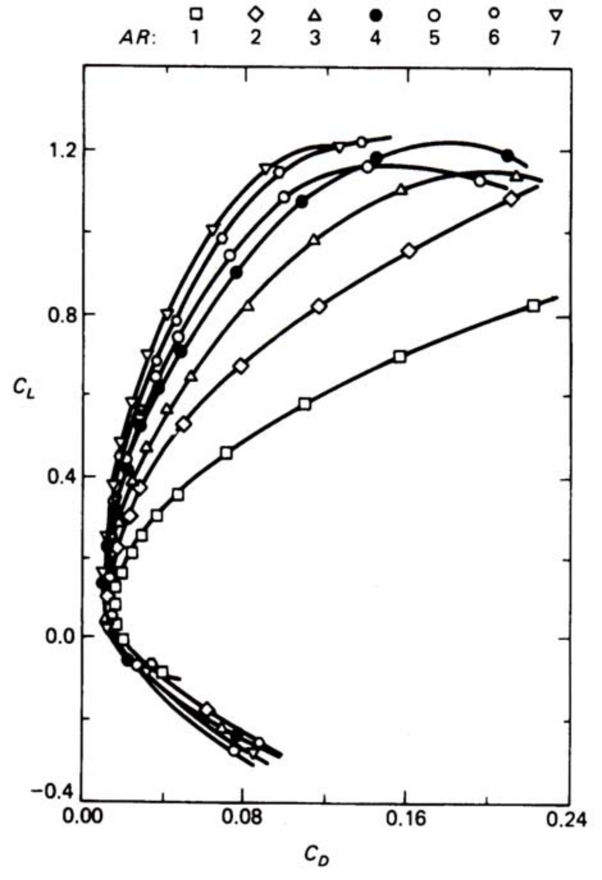




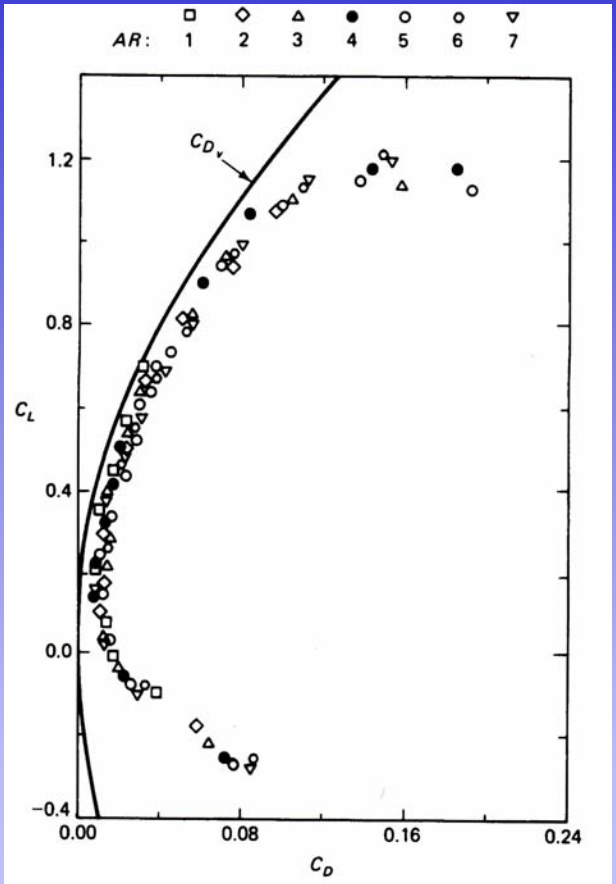
$$\alpha_i = \frac{C_L}{\pi\Lambda}\cdot\left(1+\tau\right)$$

$$a_\Lambda = \frac{a_\infty}{1+\frac{a_\infty}{\pi\Lambda}\left(1+\tau\right)} = \frac{1}{\frac{1}{a_\infty}+\frac{1+\tau}{\pi\Lambda}}$$

Lifting Line Theory (Prandtl)

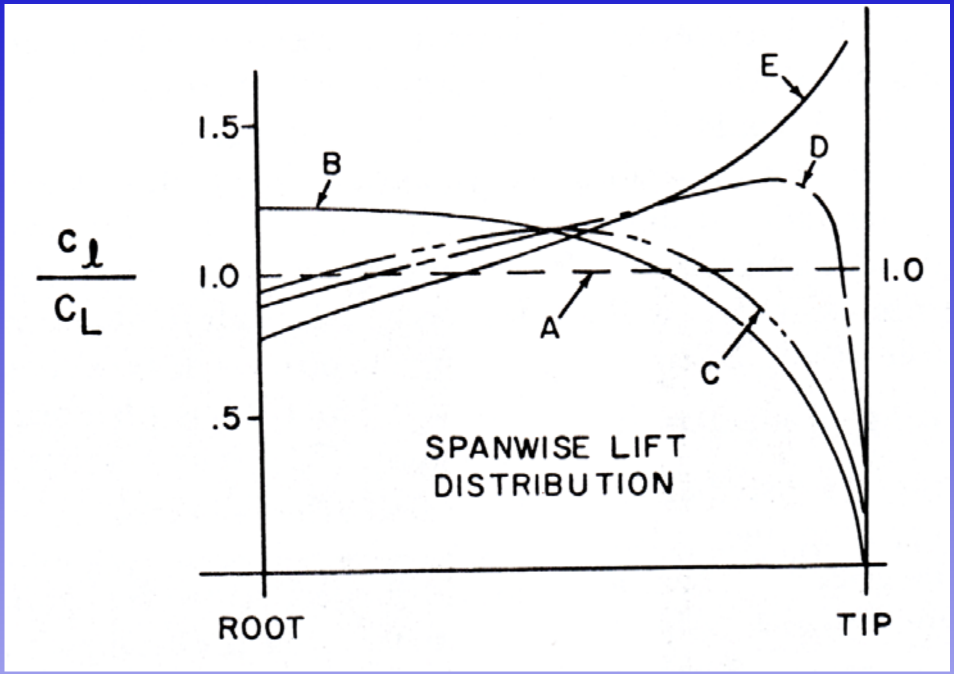


eff: AR = 5



$$C_{X_i} = \frac{C_Z^2}{\pi \Lambda} \cdot (1 + \delta)$$

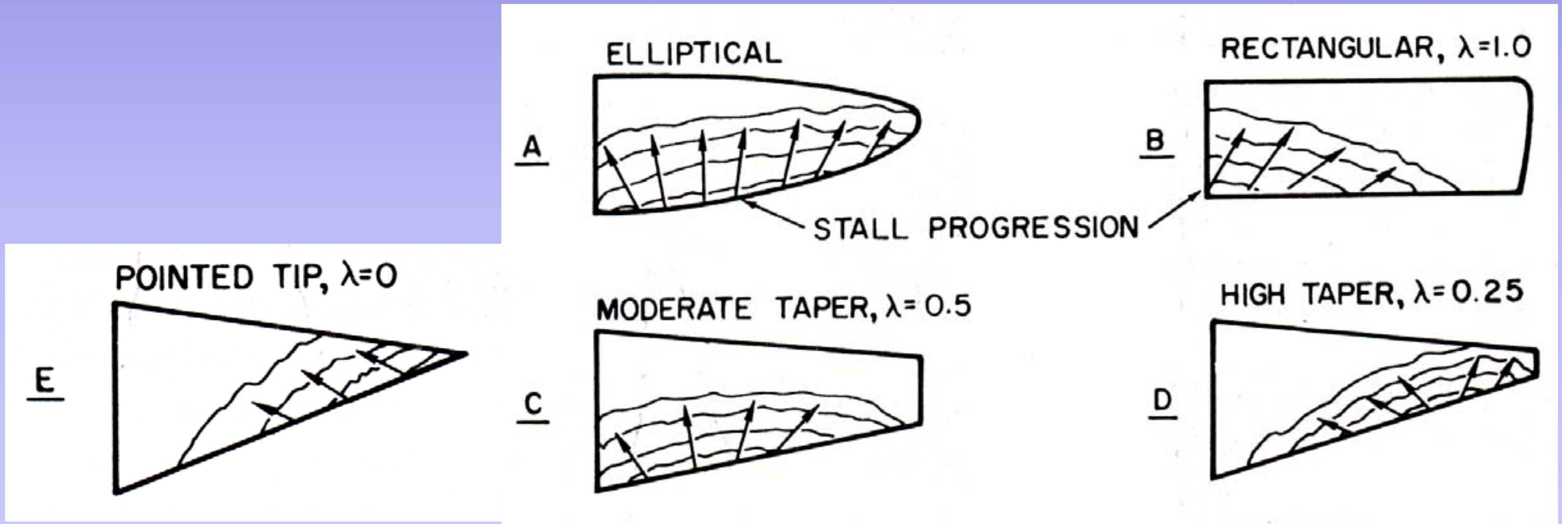
Lifting Line Theory (Prandtl)



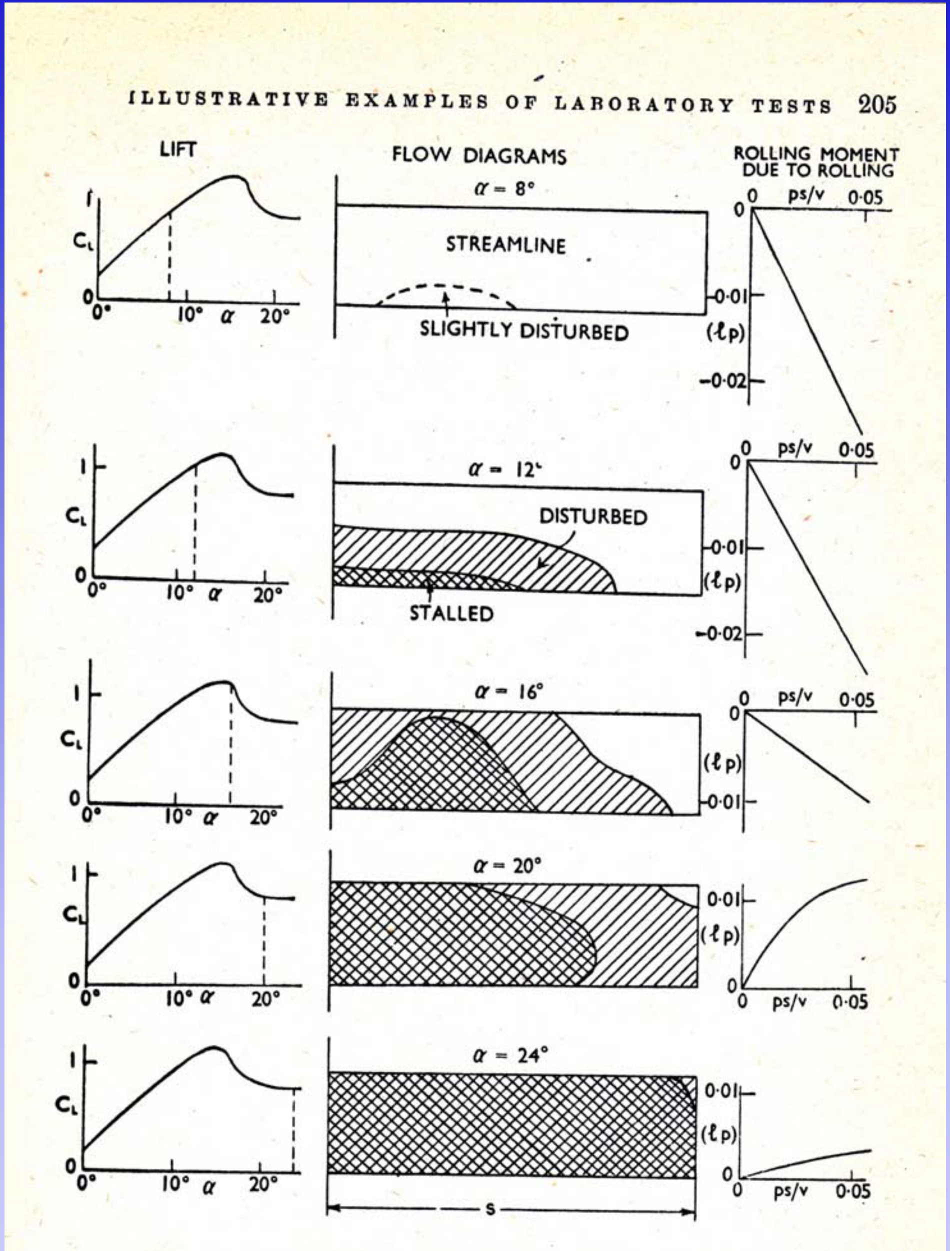
fixed airfoil, no twist

$$\Gamma = \frac{c \cdot c_L}{2} V$$

$$c_L = \frac{2\Gamma}{cV}$$



Lifting Line Theory (Prandtl)



Lifting Line Theory (Prandtl)

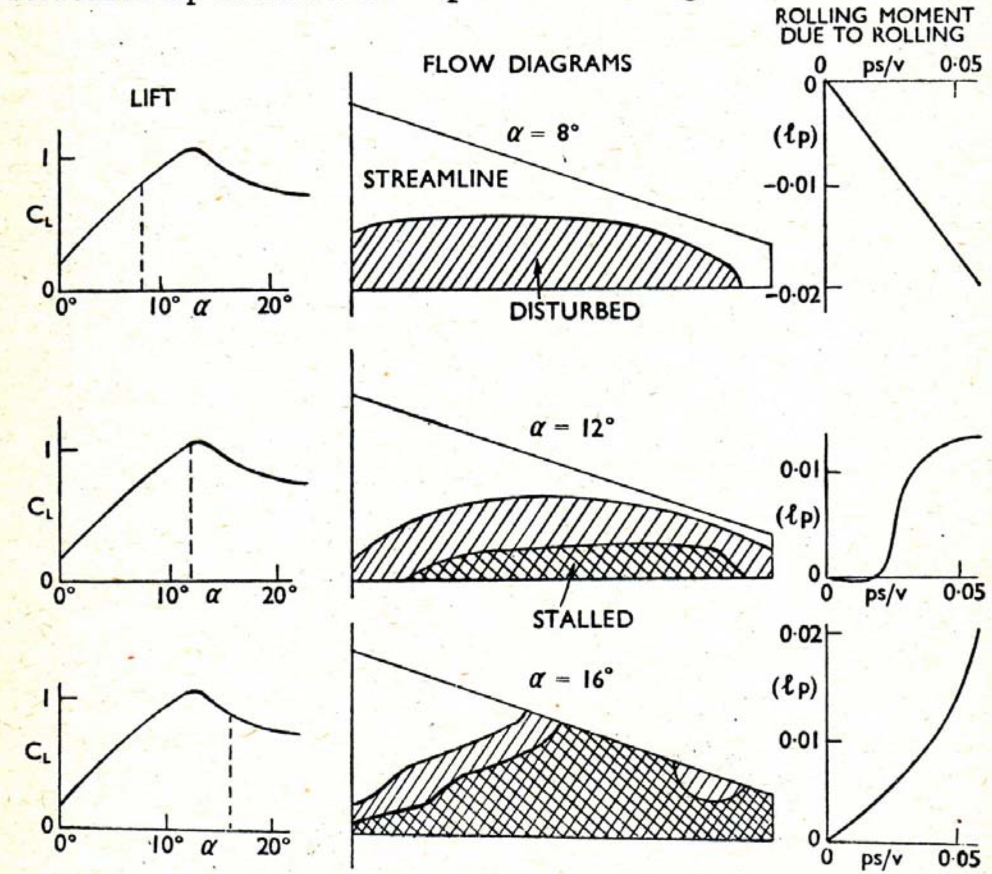
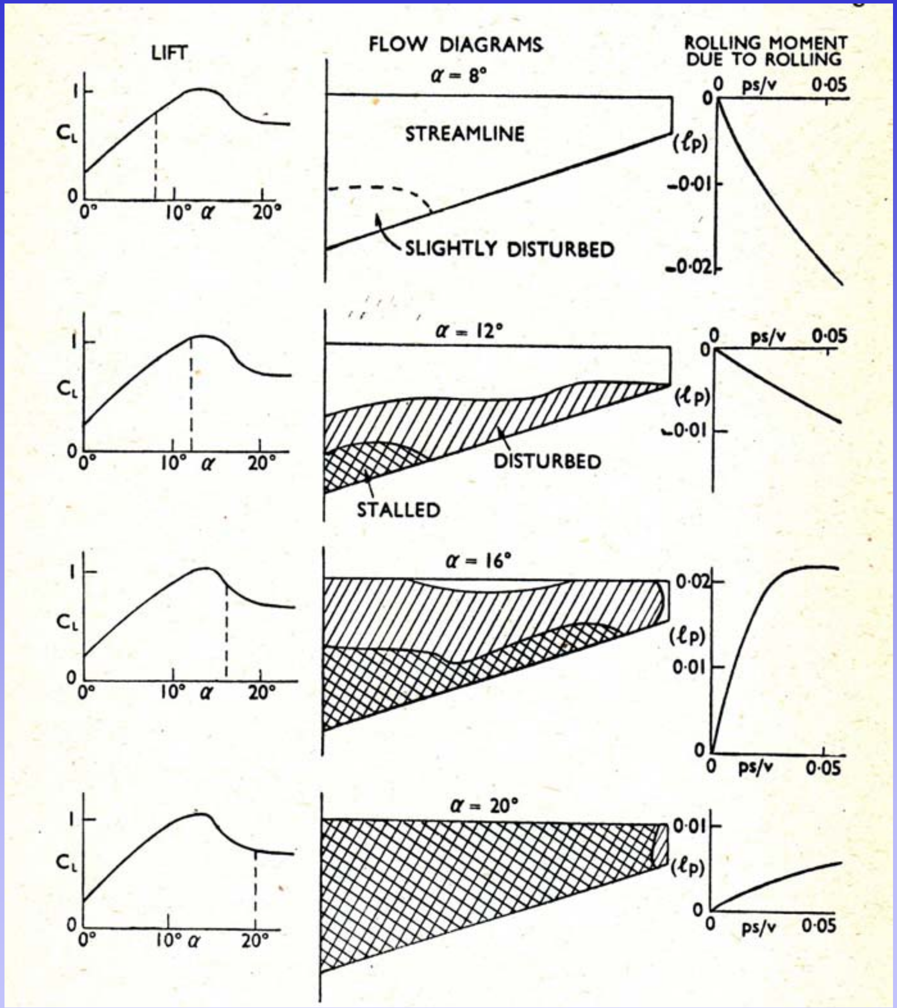


Fig. IV.3.3

Lifting Line Theory (Prandtl)



Lifting Line Theory (Prandtl)

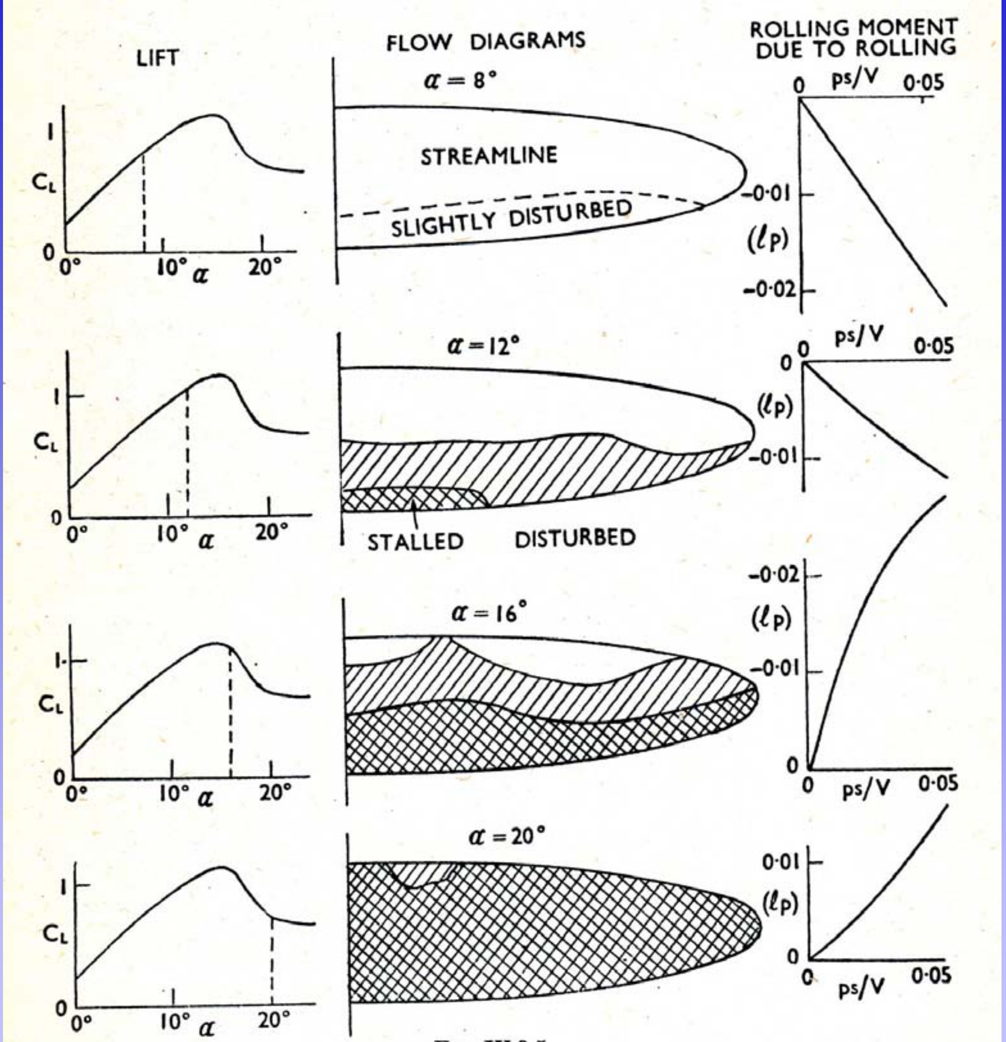


Fig. IV.3.5

Lifting Line Theory (Prandtl)

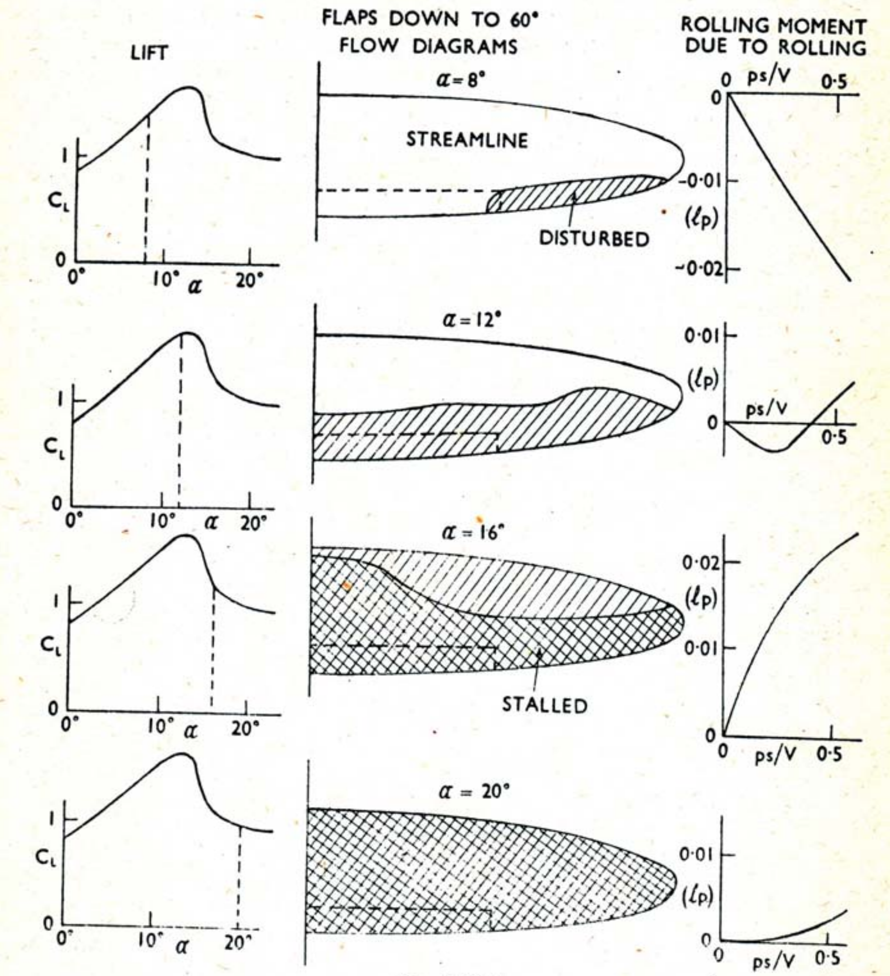


FIG. IV.3.6

Lifting Line Theory (Prandtl)

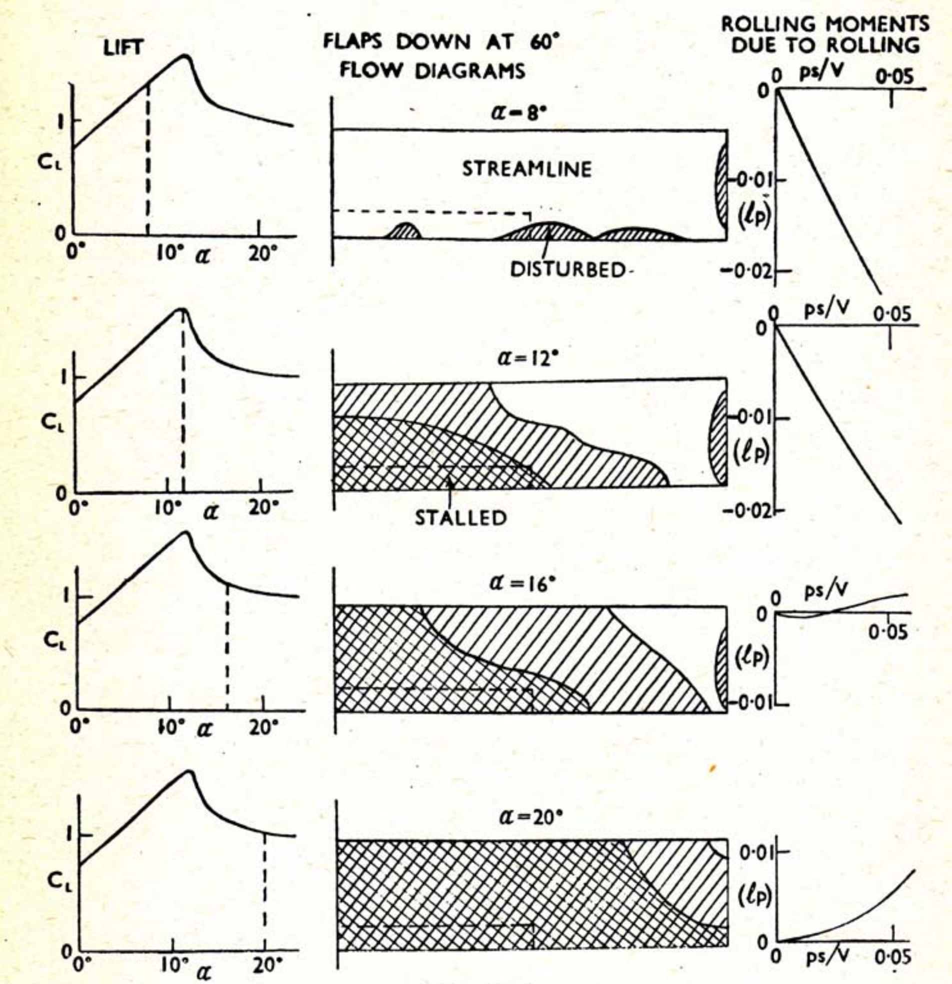


FIG. IV.3.7