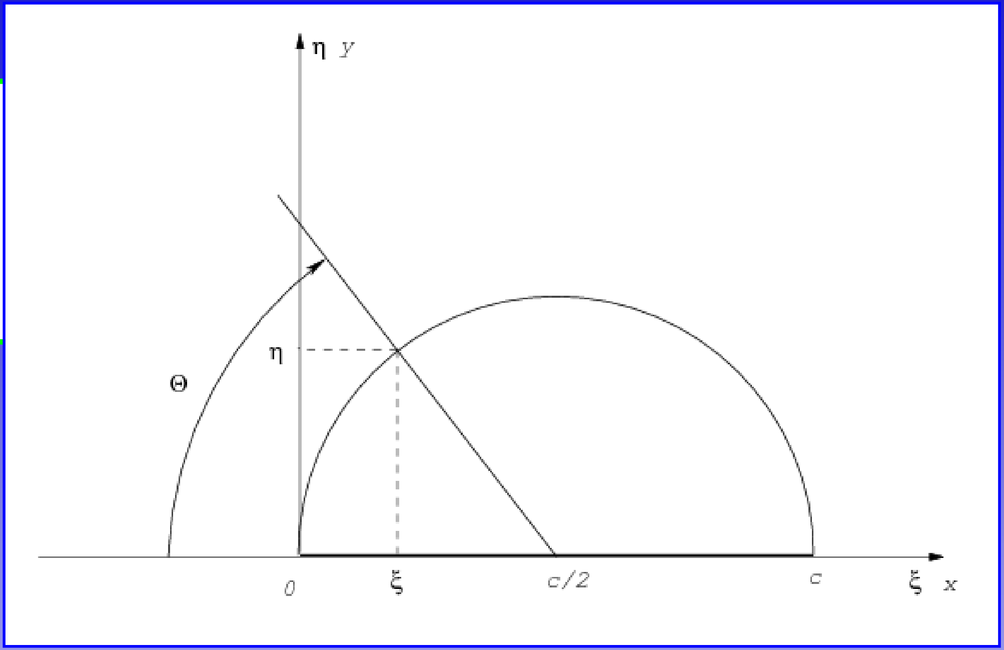
Thin Airfoil Theory (Glauert)

$$\int_0^c \frac{\gamma_{(\xi)} \cdot d\xi}{2\pi(\xi - x)} = V_\infty\left(\frac{dy}{dx} - \alpha\right)\Bigg|_x$$

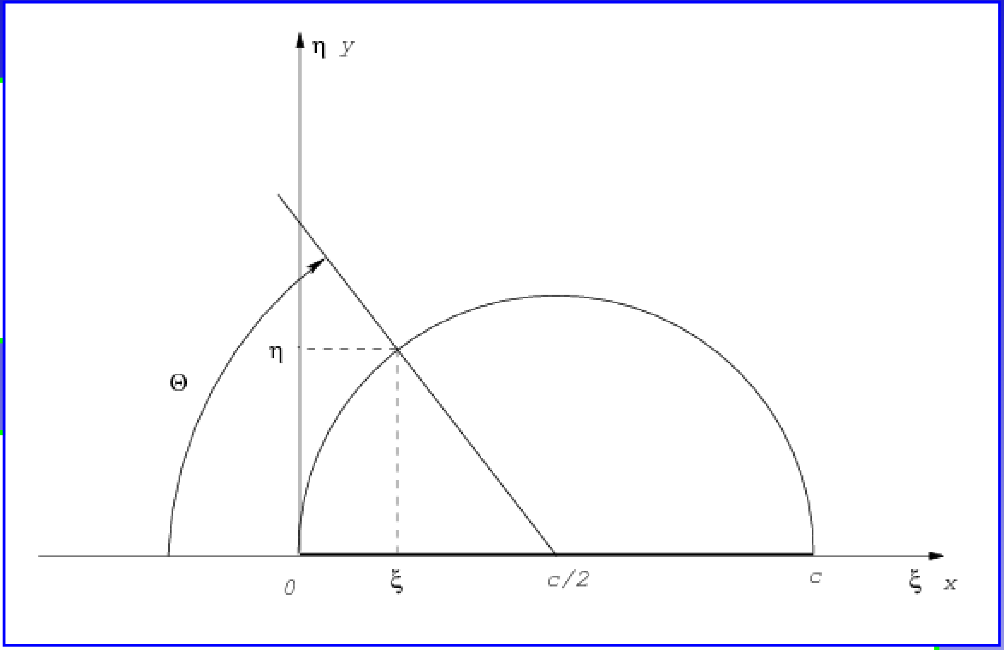
Thin Airfoil Theory (Glauert)

$$\int_0^c \frac{\gamma_{(\xi)} \cdot d\xi}{2\pi(\xi - x)}$$



Thin Airfoil Theory (Glauert)

$$\int_0^c \frac{\gamma_{(\xi)} \cdot d\xi}{2\pi(\xi - x)}$$



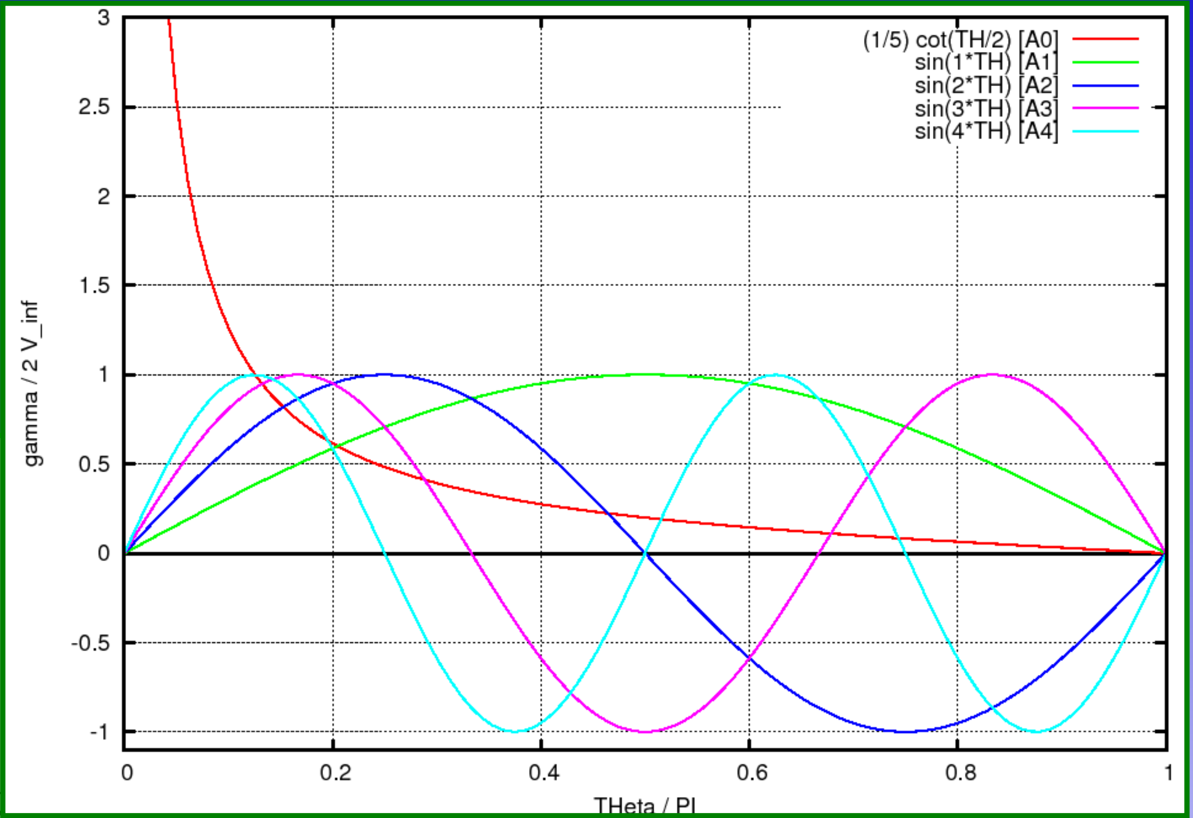
$$x = \frac{c}{2}\left(1 - \cos\left(\Theta_0\right)\right)$$

$$\xi = \frac{c}{2}\left(1 - \cos\left(\Theta\right)\right) \qquad \Theta_0, \Theta = 0 \ldots \pi$$

$$\gamma_{(\Theta)} = 2V_\infty\left(A_0 \cdot \cot\left(\frac{\Theta}{2}\right) + A_1 \cdot \sin\left(\Theta\right) + A_2 \cdot \sin\left(2\Theta\right) + A_3 \cdot \sin\left(3\Theta\right) + \ldots\right)$$

$$d\xi = \frac{c}{2}\sin\left(\Theta\right) \cdot d\Theta$$

Thin Airfoil Theory (Glauert)



$$x = \frac{c}{2}\left(1 - \cos\left(\Theta_0\right)\right)$$

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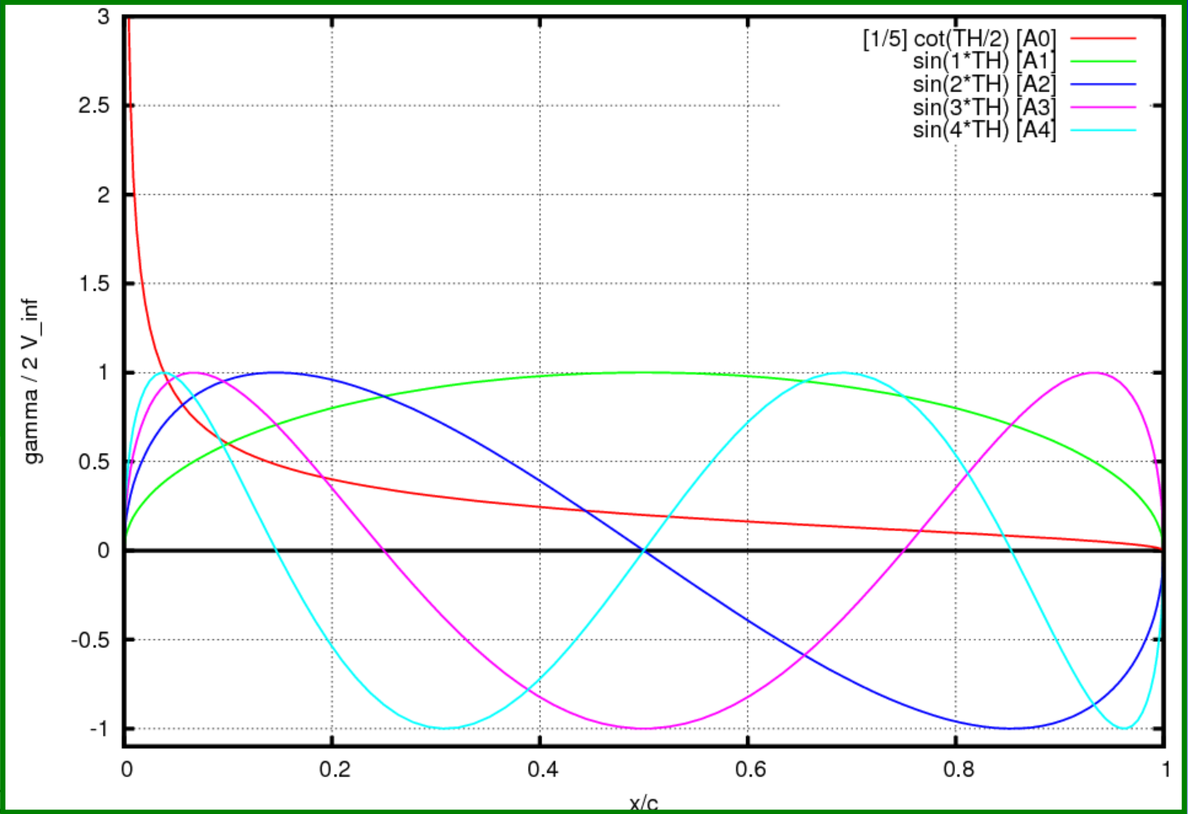
Thin Airfoil Theory (Glauert)



$$x = \frac{c}{2}\left(1-\cos\left(\Theta_0\right)\right)$$

$$\xi = \frac{c}{2}\left(1-\cos\left(\Theta\right)\right)$$

$$\gamma_{(\Theta)} = 2V_\infty\left(A_0 \cdot \cot\left(\frac{\Theta}{2}\right) + A_1 \cdot \sin\left(\Theta\right) + A_2 \cdot \sin\left(2\Theta\right) + A_3 \cdot \sin\left(3\Theta\right) + \ldots\right)$$

$$d\xi = \frac{c}{2}\sin\left(\Theta\right)\cdot d\Theta$$

Thin Airfoil Theory (Glauert)

$$\int_0^c \frac{\gamma_{(\xi)} \cdot d\xi}{2\pi(\xi - x)} = V_\infty \left(\frac{dy}{dx} - \alpha\right)\Bigg|_x$$

$$x = \frac{c}{2}\left(1 - \cos(\Theta_0)\right)$$

$$\xi = \frac{c}{2}\left(1 - \cos(\Theta)\right) \qquad \Theta_0, \Theta = 0 \ldots \pi$$

$$\gamma_{(\Theta)} = 2V_\infty\left(A_0 \cdot \cot\left(\frac{\Theta}{2}\right) + A_1 \cdot \sin(\Theta) + A_2 \cdot \sin(2\Theta) + A_3 \cdot \sin(3\Theta) + \ldots\right)$$

$$d\xi = \frac{c}{2}\sin(\Theta) \cdot d\Theta$$

$$\int_0^\pi \frac{2V_\infty\left(A_0 \cdot \cot\left(\frac{\Theta}{2}\right) + A_1 \cdot \sin(\Theta) + A_2 \cdot \sin(2\Theta) + A_3 \cdot \sin(3\Theta) + \ldots\right) \cdot \frac{c}{2}\sin(\Theta) \cdot d\Theta}{2\pi \frac{c}{2}\left(\cos(\Theta_0) - \cos(\Theta)\right)} = V_\infty \left(\frac{dy}{dx} - \alpha\right)\Bigg|_{\Theta_0}$$

Thin Airfoil Theory (Glauert)

$$\int_0^c \frac{\gamma_{(\xi)} \cdot d\xi}{2\pi(\xi - x)} = V_\infty \left(\frac{dy}{dx} - \alpha \right) \Bigg|_x$$

$$\frac{1}{\pi} \int_0^\pi \frac{\left(A_0 \cdot \cot\left(\frac{\Theta}{2}\right) + A_1 \cdot \sin(\Theta) + A_2 \cdot \sin(2\Theta) + A_3 \cdot \sin(3\Theta) + \ldots \right) \cdot \sin(\Theta) \cdot d\Theta}{\cos(\Theta_0) - \cos(\Theta)} = \frac{dy}{dx}\Bigg|_{\Theta_0} - \alpha$$

Thin Airfoil Theory (Glauert)

$$\int_0^c \frac{\gamma_{(\xi)} \cdot d\xi}{2\pi(\xi - x)} = V_\infty\left(\frac{dy}{dx} - \alpha\right)\Bigg|_x$$

$$\frac{1}{\pi}\int_0^\pi \frac{A_0 \cdot (\cos(\Theta)+1)\cdot d\Theta}{\cos(\Theta_0)-\cos(\Theta)} + \frac{1}{\pi}\int_0^\pi \frac{\left(\sum_{i=1}^{\infty} A_i \cdot \sin(i\Theta)\right)\cdot \sin(\Theta)\cdot d\Theta}{\cos(\Theta_0)-\cos(\Theta)} = \frac{dy}{dx}\Bigg|_{\Theta_0} - \alpha$$

Thin Airfoil Theory (Glauert)

$$\int_0^c \frac{\gamma_{(\xi)} \cdot d\xi}{2\pi(\xi - x)} = V_\infty \left(\frac{dy}{dx} - \alpha \right)\Bigg|_x$$

$$A_0 \cdot \frac{1}{\pi} \int_0^\pi \frac{(\cos(\Theta)+1) \cdot d\Theta}{\cos(\Theta_0) - \cos(\Theta)} + \sum_{i=1}^{\infty} A_i \cdot \frac{1}{\pi} \int_0^\pi \frac{\sin(i\Theta) \cdot \sin(\Theta) \cdot d\Theta}{\cos(\Theta_0) - \cos(\Theta)} = \frac{dy}{dx}\Bigg|_{\Theta_0} - \alpha$$

Thin Airfoil Theory (Glauert)

$$\int_0^{2\pi} \frac{d\Theta}{\cos(\Theta_0)-\cos(\Theta)} = 0$$

$$\int_0^{2\pi} \frac{\cos(\Theta)\cdot d\Theta}{\cos(\Theta_0)-\cos(\Theta)} = -\pi$$

$$\int_0^{2\pi} \frac{\cos(i\Theta)\cdot d\Theta}{\cos(\Theta_0)-\cos(\Theta)} = -\pi\frac{\sin(i\Theta_0)}{\sin(\Theta_0)}$$

$$A_0\cdot\frac{1}{\pi}\int_0^{\pi}\frac{(\cos(\Theta)+1)\cdot d\Theta}{\cos(\Theta_0)-\cos(\Theta)} + \sum_{i=1}^{\infty} A_i\cdot\frac{1}{\pi}\int_0^{\pi}\frac{\sin(i\Theta)\cdot\sin(\Theta)\cdot d\Theta}{\cos(\Theta_0)-\cos(\Theta)} = \left.\frac{dy}{dx}\right|_{\Theta_0}-\alpha$$

Thin Airfoil Theory (Glauert)

$$\int_0^{2\pi} \frac{d\Theta}{\cos(\Theta_0) - \cos(\Theta)} = 0$$

$$\int_0^{2\pi} \frac{\cos(\Theta) \cdot d\Theta}{\cos(\Theta_0) - \cos(\Theta)} = -\pi$$

$$\int_0^{2\pi} \frac{\cos(i\Theta) \cdot d\Theta}{\cos(\Theta_0) - \cos(\Theta)} = -\pi \frac{\sin(i\Theta_0)}{\sin(\Theta_0)}$$

$$A_0 \cdot \frac{1}{\pi} \int_0^{\pi} \frac{(\cos(\Theta) + 1) \cdot d\Theta}{\cos(\Theta_0) - \cos(\Theta)} + \sum_{i=1}^{\infty} A_i \cdot \frac{1}{\pi} \int_0^{\pi} \frac{\cos((i-1)\Theta) - \cos((i+1)\Theta) \cdot d\Theta}{\cos(\Theta_0) - \cos(\Theta)} = \left.\frac{dy}{dx}\right|_{\Theta_0} - \alpha$$

Thin Airfoil Theory (Glauert)

$$-A_0 + \sum_{i=1}^{\infty} A_i \cdot \cos\left(i\Theta_0\right) = \left.\frac{dy}{dx}\right|_{\Theta_0} - \alpha$$

Thin Airfoil Theory (Glauert)

$$\int_0^\pi \cos(i\Theta)\cdot\cos(n\Theta)\cdot d\Theta = \begin{cases} \pi & i = n = 0 \\ \dfrac{\pi}{2} & i = n \neq 0 \\ 0 & i \neq n \end{cases}$$

$$-A_0 + \sum_{i=1}^{\infty} A_i \cdot \cos(i\Theta_0) = \left.\frac{dy}{dx}\right|_{\Theta_0} - \alpha$$

Thin Airfoil Theory (Glauert)

$$\int_0^{\pi} \cos(i\Theta)\cdot\cos(n\Theta)\cdot d\Theta \quad = \quad \begin{cases} \pi & i = n = 0 \\ \dfrac{\pi}{2} & i = n \neq 0 \\ 0 & i \neq n \end{cases}$$

$$\int_0^{\pi} \cos(n\Theta)\cdot\left(-A_0 + \sum_{i=1}^{\infty} A_i \cdot\cos(i\Theta)\right)\cdot d\Theta \quad = \quad \int_0^{\pi}\left(\left.\frac{dy}{dx}\right|_{\Theta} - \alpha\right)\cdot\cos(n\Theta)\cdot d\Theta$$

Thin Airfoil Theory (Glauert)

$$\int_0^{\pi} \cos(i\Theta)\cdot\cos(n\Theta)\cdot d\Theta = \begin{cases} \pi & i = n = 0 \\ \dfrac{\pi}{2} & i = n \neq 0 \\ 0 & i \neq n \end{cases}$$

$$\int_0^{\pi} \cos(0\Theta)\cdot\left(-A_0 + \sum_{i=1}^{\infty} A_i \cdot \cos(i\Theta)\right)\cdot d\Theta = \int_0^{\pi}\left(\left.\frac{dy}{dx}\right|_{\Theta} - \alpha\right)\cdot\cos(0\Theta)\cdot d\Theta$$

$$n = 0 \quad \rightarrow \quad -\pi A_0 = \int_0^{\pi}\left(\left.\frac{dy}{dx}\right|_{\Theta} - \alpha\right)\cdot d\Theta = \int_0^{\pi}\left(\left.\frac{dy}{dx}\right|_{\Theta}\right)\cdot d\Theta - \pi\alpha$$

$$A_0 = \alpha - \frac{1}{\pi}\int_0^{\pi}\left.\frac{dy}{dx}\right|_{\Theta}\cdot d\Theta$$

Thin Airfoil Theory (Glauert)

$$\int_0^{\pi} \cos(i\Theta)\cdot\cos(n\Theta)\cdot d\Theta \quad = \quad \begin{cases} \pi & i=n=0 \\ \dfrac{\pi}{2} & i=n\neq 0 \\ 0 & i\neq n \end{cases}$$

$$\int_0^{\pi} \cos(n\Theta)\cdot\left(-A_0+\sum_{i=1}^{\infty} A_i\cdot\cos(i\Theta)\right)\cdot d\Theta \quad = \quad \int_0^{\pi}\left(\left.\frac{dy}{dx}\right|_{\Theta}-\alpha\right)\cdot\cos(n\Theta)\cdot d\Theta$$

$$n>0 \quad \rightarrow \quad \int_0^{\pi}\left(\sum_{i=1}^{\infty} A_i\cdot\cos(i\Theta)\right)\cdot\cos(n\Theta)\cdot d\Theta \quad = \quad \frac{\pi}{2}A_n \quad = \quad \int_0^{\pi}\left.\frac{dy}{dx}\right|_{\Theta}\cdot\cos(n\Theta)\cdot d\Theta$$

$$A_n \quad = \quad \frac{2}{\pi}\int_0^{\pi}\left.\frac{dy}{dx}\right|_{\Theta}\cdot\cos(n\Theta)\cdot d\Theta$$

Thin Airfoil Theory (Glauert)

$$A_0 = \alpha - \frac{1}{\pi}\int_0^{\pi} \left.\frac{dy}{dx}\right|_{\Theta} \cdot d\Theta$$

$$A_n = \frac{2}{\pi}\int_0^{\pi} \left.\frac{dy}{dx}\right|_{\Theta} \cdot \cos(n\Theta) \cdot d\Theta \quad n = 1,2,\ldots$$

Thin Airfoil Theory (Glauert)

$$dL = \rho_\infty V_\infty \cdot d\Gamma \cdot [b] = \rho_\infty V_\infty \cdot [b] \cdot \gamma_{(x)} \cdot dx$$

$$L \quad = \quad C_L \cdot c \cdot [b] \cdot \frac{\rho_\infty V_\infty{}^2}{2} \quad = \quad \rho_\infty V_\infty \cdot [b] \cdot \int_0^c \gamma_{(x)} \cdot dx$$

$$C_L = \frac{2}{c \cdot V_\infty} \cdot \int_0^c \gamma_{(x)} \cdot dx = \ldots =$$

$$\frac{2}{c \cdot V_\infty} \cdot \int_0^\pi 2 \cdot V_\infty \cdot \left(A_0 \cdot \cot\left(\frac{\Theta}{2}\right) + A_1 \cdot \sin(\Theta) + A_2 \cdot \sin(2\Theta) + A_3 \cdot \sin(3\Theta) + \ldots \right) \cdot \frac{c}{2} \sin(\Theta) \cdot d\Theta$$

Thin Airfoil Theory (Glauert)

$$\int_0^{\pi} \sin(i\Theta)\cdot\sin(n\Theta)\cdot d\Theta \quad = \begin{cases} \dfrac{\pi}{2} & i = n \\ 0 & i \neq n \end{cases}$$

$$dL = \rho_\infty V_\infty \cdot d\Gamma \cdot [b] = \rho_\infty V_\infty \cdot [b] \cdot \gamma_{(x)} \cdot dx$$

$$L \quad = \quad C_L \cdot c \cdot [b] \cdot \frac{\rho_\infty V_\infty^{\ 2}}{2} \quad = \quad \rho_\infty V_\infty \cdot [b] \cdot \int_0^c \gamma_{(x)} \cdot dx$$

$$C_L = \frac{2}{c \cdot V_\infty} \cdot \int_0^c \gamma_{(x)} \cdot dx = \ldots =$$

$$\frac{2}{c \cdot V_\infty} \cdot \int_0^{\pi} 2 \cdot V_\infty \cdot \left(A_0 \cdot \cot\left(\frac{\Theta}{2}\right) + A_1 \cdot \sin(\Theta) + A_2 \cdot \sin(2\Theta) + A_3 \cdot \sin(3\Theta) + \ldots \right) \cdot \frac{c}{2} \sin(\Theta) \cdot d\Theta$$

Thin Airfoil Theory (Glauert)

$$\int_0^{\pi} \sin(i\Theta)\cdot\sin(n\Theta)\cdot d\Theta = \begin{cases} \dfrac{\pi}{2} & i=n \\ 0 & i\neq n \end{cases}$$

$$dL = \rho_\infty V_\infty \cdot d\Gamma \cdot [b] = \rho_\infty V_\infty \cdot [b]\cdot \gamma_{(x)} \cdot dx$$

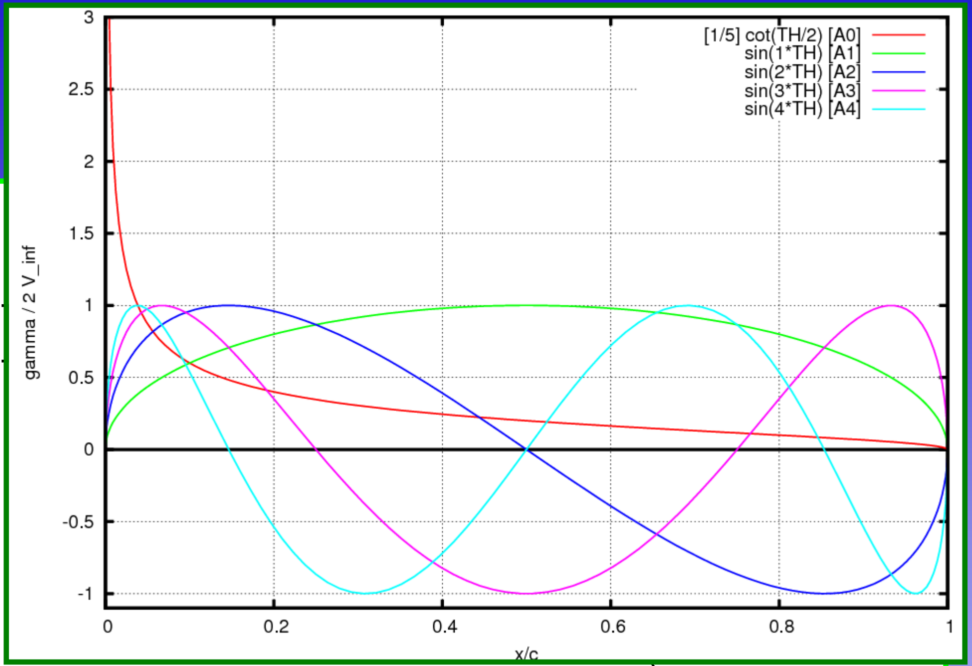
$$L = C_L \cdot c \cdot [b] \cdot \frac{\rho_\infty V_\infty{}^2}{2} = \rho_\infty V_\infty \cdot [b] \cdot \int_0^c \gamma_{(x)} \cdot dx$$

$$C_L = \frac{2}{c\cdot V_\infty}\cdot \int_0^c \gamma_{(x)}\cdot dx = \ldots =$$

$$\frac{2}{c\cdot V_\infty}\cdot\int_0^{\pi} 2\cdot V_\infty \cdot \left(A_0 \cdot \cot\left(\frac{\Theta}{2}\right) + A_1\cdot\sin(\Theta) + A_2\cdot\sin(2\Theta) + A_3\cdot\sin(3\Theta) + \ldots\right)\cdot \frac{c}{2}\sin(\Theta)\cdot d\Theta$$

$$= \ldots = 2\pi\cdot\left(A_0 + \frac{1}{2}A_1\right)$$

Thin Airfoil Theory (Glauert)



$$dL = \rho_\infty V_\infty \cdot d\Gamma \cdot [b] = \rho_\infty V_\infty \cdot [b] \cdot \gamma_{(x)} \cdot dx$$

$$L \quad = \quad C_L \cdot c \cdot [b] \cdot \frac{\rho_\infty V_\infty{}^2}{2} \quad = \quad \rho_\infty V_\infty \cdot [b$$

$$C_L = \frac{2}{c \cdot V_\infty} \cdot \int_0^c \gamma_{(x)} \cdot dx = \ldots =$$

$$\frac{2}{c \cdot V_\infty} \cdot \int_0^\pi 2 \cdot V_\infty \cdot \left(A_0 \cdot \cot\left(\frac{\Theta}{2}\right) + A_1 \cdot \sin(\Theta) + A_2 \cdot \sin(2\Theta) + A_3 \cdot \sin(3\Theta) + \ldots \right) \cdot \frac{c}{2} \sin(\Theta) \cdot d\Theta$$

$$= \ldots = 2\pi \cdot \left(A_0 + \frac{1}{2} A_1 \right)$$

Thin Airfoil Theory (Glauert)

$$C_L = 2\pi \cdot \left(A_0 + \frac{1}{2} A_1 \right) =$$

$$2\pi \cdot \alpha \quad \underbrace{- \quad 2\int_0^{\pi} \frac{dy}{dx}\bigg|_{\Theta} \cdot d\Theta + 2\int_0^{\pi} \frac{dy}{dx}\bigg|_{\Theta} \cdot \cos(n\Theta) \cdot d\Theta}_{C_{L0}}$$

$$2\pi \cdot \left(\alpha \quad \underbrace{- \quad \frac{1}{\pi}\int_0^{\pi} \frac{dy}{dx}\bigg|_{\Theta} \cdot d\Theta + \frac{1}{\pi}\int_0^{\pi} \frac{dy}{dx}\bigg|_{\Theta} \cdot \cos(n\Theta) \cdot d\Theta}_{\alpha_0} \right)$$

Thin Airfoil Theory (Glauert)

$$dM = x \cdot \rho_\infty V_\infty \cdot d\Gamma \cdot [b] = x \cdot \rho_\infty V_\infty \cdot [b] \cdot \gamma_{(x)} \cdot dx$$

$$M = C_M \cdot c^2 \cdot [b] \cdot \frac{\rho_\infty V_\infty{}^2}{2} = \rho_\infty V_\infty \cdot [b] \cdot \int_0^c x \cdot \gamma_{(x)} \cdot dx$$

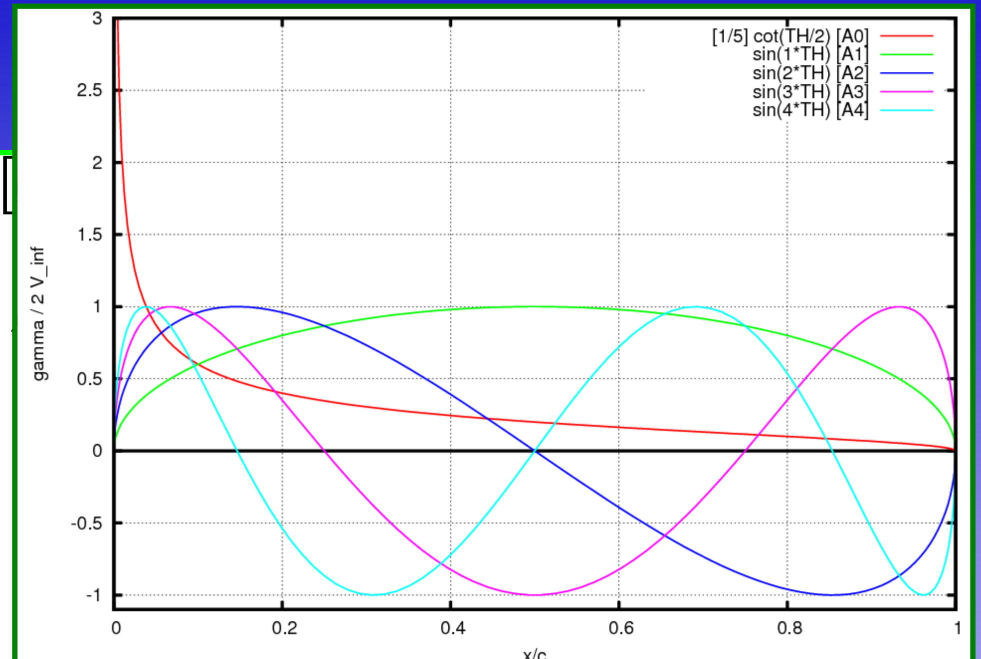
$$C_M = \frac{2}{c^2 \cdot V_\infty} \cdot \int_0^c x \cdot \gamma_{(x)} \cdot dx = \ldots =$$

$$\frac{2}{c^2 \cdot V_\infty} \cdot \int_0^\pi 2 \cdot V_\infty \cdot \left(A_0 \cdot \cot\left(\frac{\Theta}{2}\right) + A_1 \cdot \sin(\Theta) + A_2 \cdot \sin(2\Theta) + A_3 \cdot \sin(3\Theta) + \ldots \right) \cdot$$

$$\cdot \frac{c}{2} \sin(\Theta) \cdot \frac{c}{2} (1 - \cos(\Theta)) \cdot d\Theta$$

$$= \ldots = -\frac{\pi}{2} \cdot \left(A_0 + A_1 - \frac{1}{2} A_2 \right)$$

Thin Airfoil Theory (Glauert)



$$dM = x \cdot \rho_\infty V_\infty \cdot d\Gamma \cdot [b] = x \cdot \rho_\infty V_\infty \cdot [$$

$$M \quad = \quad C_M \cdot c^2 \cdot [b] \cdot \frac{\rho_\infty V_\infty{}^2}{2} \quad =$$

$$C_M = \frac{2}{c^2 \cdot V_\infty} \cdot \int_0^c x \cdot \gamma_{(x)} \cdot dx = \ldots =$$

$$\frac{2}{c^2 \cdot V_\infty} \cdot \int_0^\pi 2 \cdot V_\infty \cdot \left(A_0 \cdot \cot\left(\frac{\Theta}{2}\right) + A_1 \cdot \sin(\Theta) + A_2 \cdot \sin(2\Theta) + A_3 \cdot \sin(3\Theta) + \ldots \right) \cdot$$

$$\cdot \frac{c}{2} \sin(\Theta) \cdot \frac{c}{2} (1 - \cos(\Theta)) \cdot d\Theta$$

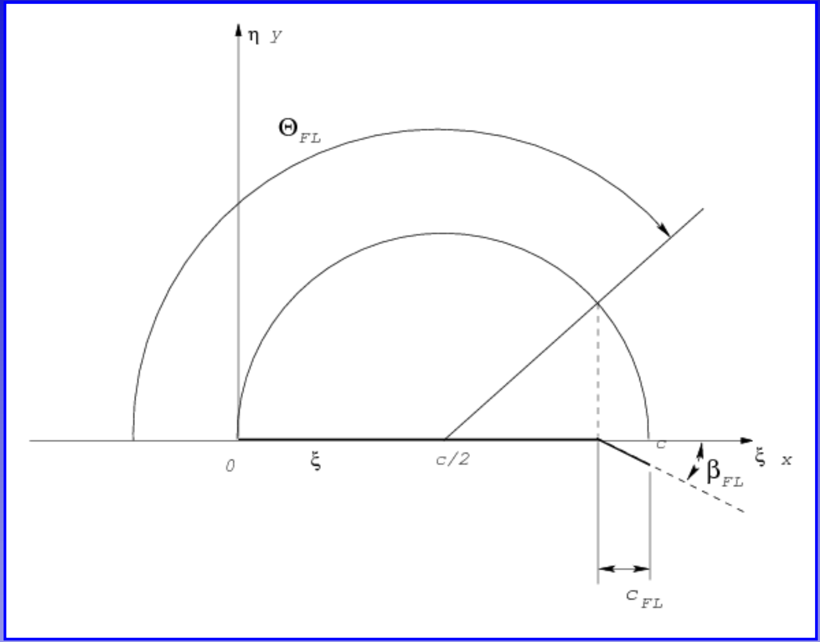
$$= \ldots = -\frac{\pi}{2} \cdot \left(A_0 + A_1 - \frac{1}{2} A_2 \right)$$

Thin Airfoil Theory (Glauert)

$$C_M = -\frac{\pi}{2}\cdot\left(A_0 + A_1 - \frac{1}{2}A_2\right) = -\frac{\pi}{2}\cdot\left(A_0 + \frac{1}{2}A_1 + \frac{1}{2}A_1 - \frac{1}{2}A_2\right)$$

$$-\frac{1}{4}C_L \underbrace{- \frac{\pi}{4}\cdot\left(A_1 - A_2\right)}_{C_{M0}}$$

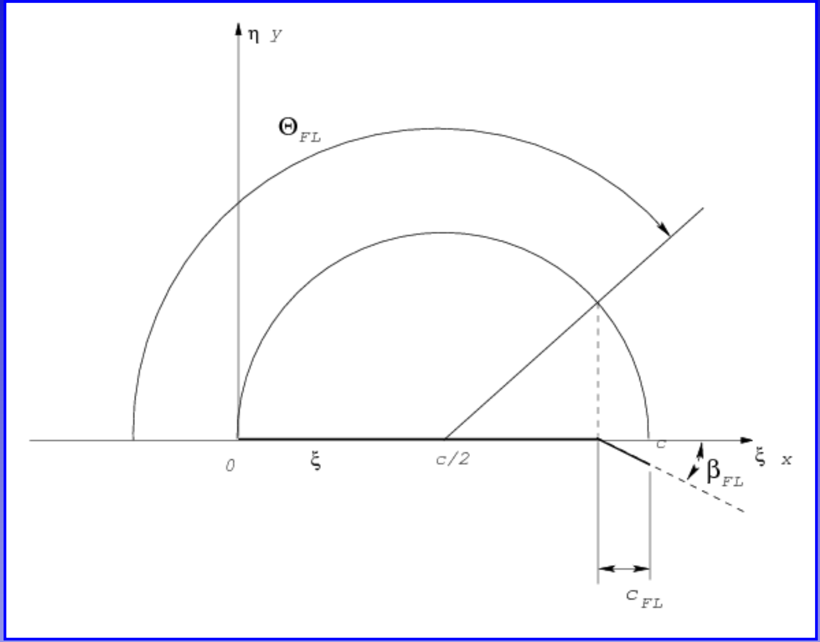
Thin Airfoil Theory (Glauert)



$$\left.\frac{dy}{dx}\right|_x = \begin{cases} 0 & x < x_{FL} \\ -\beta_{FL} & x > x_{FL} \end{cases}$$

$$\left.\frac{dy}{dx}\right|_\Theta = \begin{cases} 0 & \Theta < \Theta_{FL} \\ -\beta_{FL} & \Theta > \Theta_{FL} \end{cases}$$

Thin Airfoil Theory (Glauert)



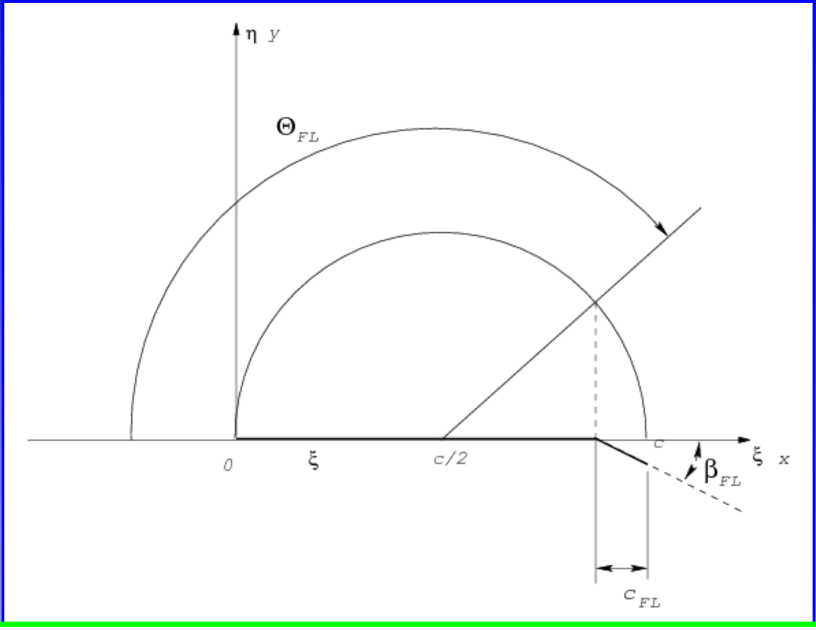
$$\left.\frac{dy}{dx}\right|_x = \begin{cases} 0 & x < x_{FL} \\ -\beta_{FL} & x > x_{FL} \end{cases}$$

$$\left.\frac{dy}{dx}\right|_\Theta = \begin{cases} 0 & \Theta < \Theta_{FL} \\ -\beta_{FL} & \Theta > \Theta_{FL} \end{cases}$$

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \left.\frac{dy}{dx}\right|_\Theta \cdot d\Theta$$

$$A_n = \frac{2}{\pi}\int_0^\pi \left.\frac{dy}{dx}\right|_\Theta \cdot \cos(n\Theta)\cdot d\Theta \qquad n = 1,2,\ldots$$

Thin Airfoil Theory (Glauert)



$$\left.\frac{dy}{dx}\right|_x = \begin{cases} 0 & x < x_{FL} \\ -\beta_{FL} & x > x_{FL} \end{cases}$$

$$\left.\frac{dy}{dx}\right|_\Theta = \begin{cases} 0 & \Theta < \Theta_{FL} \\ -\beta_{FL} & \Theta > \Theta_{FL} \end{cases}$$

$$A_0 = \alpha - \frac{1}{\pi}\int_{\Theta_{FL}}^{\pi}(-)\beta_{FL}\cdot d\Theta = \alpha + \frac{\beta_{FL}}{\pi}(\pi - \Theta_{FL}) = \alpha + \beta_{FL}\left(1 - \frac{\Theta_{FL}}{\pi}\right)$$

$$A_1 = \frac{2}{\pi}\int_{\Theta_{FL}}^{\pi}(-)\beta_{FL}\cdot\cos(\Theta)\cdot d\Theta = -2\frac{\beta_{FL}}{\pi}(\sin(\pi) - \sin(\Theta_{FL})) = 2\beta_{FL}\frac{\sin(\Theta_{FL})}{\pi}$$

$$A_2 = \frac{2}{\pi}\int_{\Theta_{FL}}^{\pi}(-)\beta_{FL}\cdot\cos(2\Theta)\cdot d\Theta = -\frac{\beta_{FL}}{\pi}(\sin(2\pi) - \sin(2\Theta_{FL})) = \beta_{FL}\frac{\sin(2\Theta_{FL})}{\pi}$$

Thin Airfoil Theory (Glauert)

$$C_L = 2\pi \cdot \left(A_0 + \frac{1}{2} A_1 \right) =$$

$$2\pi \cdot \left(\alpha + \underbrace{\left(1 - \frac{\Theta_{FL} - \sin(\Theta_{FL})}{\pi} \right) \cdot \beta_{FL}}_{-\alpha_0} \right)$$

$$2\pi \cdot \alpha + \underbrace{2\pi \left(1 - \frac{\Theta_{FL} - \sin(\Theta_{FL})}{\pi} \right) \cdot \beta_{FL}}_{C_{L_0}}$$

Thin Airfoil Theory (Glauert)

$$C_M = -\frac{\pi}{2}\cdot\left(A_0 + A_1 - \frac{1}{2}A_2\right) \quad = \quad -\frac{1}{4}C_L \quad - \quad \frac{\pi}{4}\cdot\left(A_1 - A_2\right)$$

$$-\frac{1}{4}C_L \quad - \quad \underbrace{\frac{1}{4}\cdot\left(2\sin\left(\Theta_{FL}\right) - \sin\left(2\Theta_{FL}\right)\right)\cdot \beta_{FL}}_{C_{M0}}$$

Thin Airfoil Theory (Glauert)

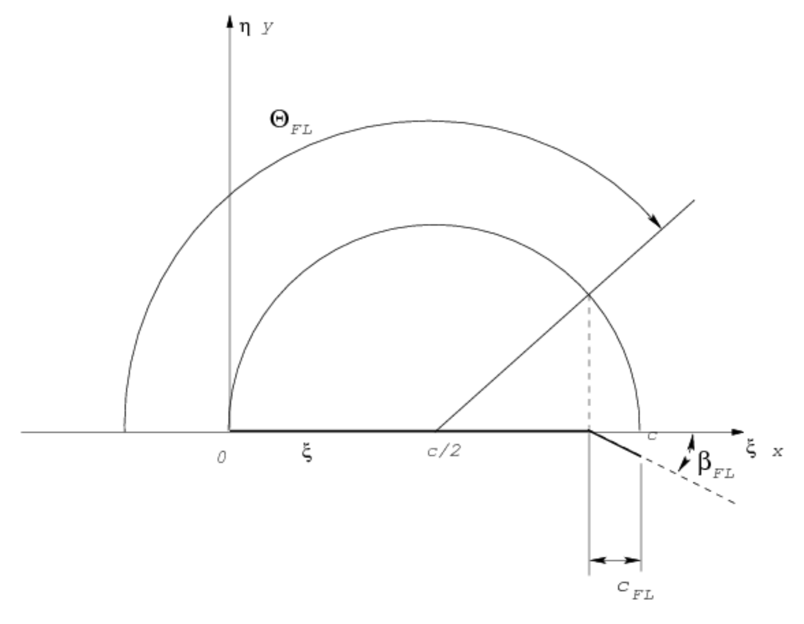
$$c_{FL} / c = \bar{c}_{FL} << 1$$

$$x_{FL} = \frac{c}{2}\left(1-\cos\left(\Theta_{FL}\right)\right)$$

$$\cos\left(\Theta_{FL}\right) = 1 - 2\frac{x_{FL}}{c} = -\left(1 - 2\bar{c}_{FL}\right)$$

$$\sin\left(\Theta_{FL}\right) = \sqrt{1-\cos^2\left(\Theta_{FL}\right)} = \ldots = 2\sqrt{\bar{c}_{FL} - \bar{c}^{\,2}_{FL}} \approx 2\sqrt{\bar{c}_{FL}}$$

$$\Theta_{FL} \approx \pi - 2\sqrt{\bar{c}_{FL}}$$



Thin Airfoil Theory (Glauert)

$$c_{FL} / c = \bar{c}_{FL} << 1$$

$$x_{FL} = \frac{c}{2}\left(1 - \cos\left(\Theta_{FL}\right)\right)$$

$$\cos\left(\Theta_{FL}\right) = 1 - 2\frac{x_{FL}}{c} = -\left(1 - 2\bar{c}_{FL}\right)$$

$$\sin\left(\Theta_{FL}\right) = \sqrt{1 - \cos^2\left(\Theta_{FL}\right)} =$$

$$\Theta_{FL} \approx \pi - 2\sqrt{\bar{c}_{FL}}$$

$$C_L = 2\pi \cdot \left(A_0 + \frac{1}{2}A_1\right) =$$

$$2\pi \cdot \left(\alpha + \underbrace{\left(1 - \frac{\Theta_{FL} - \sin\left(\Theta_{FL}\right)}{\pi}\right) \cdot \beta_{FL}}_{-\alpha_0}\right)$$

$$2\pi \cdot \alpha + \underbrace{2\pi\left(1 - \frac{\Theta_{FL} - \sin\left(\Theta_{FL}\right)}{\pi}\right) \cdot \beta_{FL}}_{C_{L0}}$$

Thin Airfoil Theory (Glauert)

$$c_{FL}/c = \bar{c}_{FL} << 1$$

$$x_{FL} = \frac{c}{2}\left(1 - \cos(\Theta_{FL})\right)$$

$$\cos(\Theta_{FL}) = 1 - 2\frac{x_{FL}}{c} = -\left(1 - 2\bar{c}_{FL}\right)$$

$$\sin(\Theta_{FL}) = \sqrt{1 - \cos^2(\Theta_{FL})}$$

$$\Theta_{FL} \approx \pi - 2\sqrt{\bar{c}_{FL}}$$

$$C_L = 2\pi \cdot \left(A_0 + \frac{1}{2}A_1\right) =$$

$$2\pi \cdot \left(\alpha + \left(1 - \frac{\Theta_{FL} - \sin(\Theta_{FL})}{}\right) \cdot \beta_{FL}\right)$$

$$(\Theta_{FL})) \cdot \beta_{FL}$$

$$C_L = 2\pi \cdot \left(A_0 + \frac{1}{2}A_1\right) =$$

$$2\pi \cdot \left(\alpha + \underbrace{\frac{4\sqrt{\bar{c}_{FL}}}{\pi} \cdot \beta_{FL}}_{-\alpha_0}\right) = 2\pi \cdot \alpha + \underbrace{8\sqrt{\bar{c}_{FL}} \cdot \beta_{FL}}_{C_{L0}}$$

Thin Airfoil Theory (Glauert)

$$c_{FL} / c = \bar{c}_{FL} << 1$$

$$x_{FL} = \frac{c}{2}\left(1 - \cos\left(\Theta_{FL}\right)\right)$$

$$\cos\left(\Theta_{FL}\right) = 1 - 2\frac{x_{FL}}{c} = -\left(1 - 2\bar{c}_{FL}\right)$$

$$\sin\left(\Theta_{FL}\right) = \sqrt{1 - \cos^2\left(\Theta_{FL}\right)} = \ldots = 2\sqrt{\bar{c}_{FL} - \bar{c}^{\,2}{}_{FL}} \approx 2\sqrt{\bar{c}_{FL}}$$

$$\sin\left(2\Theta_{FL}\right) = 2\sin\left(\Theta_{FL}\right)\cos\left(\Theta_{FL}\right) = \ldots \approx -4\sqrt{\bar{c}_{FL}}$$

Thin Airfoil Theory (Glauert)

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$$\sin\left(2\Theta_{FL}\right) = 2\sin\left(\Theta_{FL}\right)\cos\left(\Theta_{FL}\right) = \ldots \approx -4\sqrt{\bar{c}_{FL}}$$

$$C_M = -\frac{\pi}{2}\cdot\left(A_0 + A_1 - \frac{1}{2}A_2\right) = -\frac{1}{4}C_L - \frac{\pi}{4}\cdot\left(A_1 - A_2\right)$$

$$-\frac{1}{4}C_L \underbrace{- \frac{1}{4}\cdot\left(2\sin\left(\Theta_{FL}\right) - \sin\left(2\Theta_{FL}\right)\right)\cdot\beta_{FL}}_{C_{M0}}$$

Thin Airfoil Theory (Glauert)

$$c_{FL}/c = \bar{c}_{FL} << 1$$

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$$\sin\left(\Theta_{FL}\right) = \sqrt{1-\cos^2\left(\Theta_{FL}\right)} = \ldots = 2\sqrt{\bar{c}_{FL} - \bar{c}^2_{FL}} \approx 2\sqrt{\bar{c}_{FL}}$$

$$\sin\left(2\Theta_{FL}\right) = 2\sin\left(\Theta_{FL}\right)\cos\left(\Theta_{FL}\right) = \ldots \approx -4\sqrt{\bar{c}_{FL}}$$

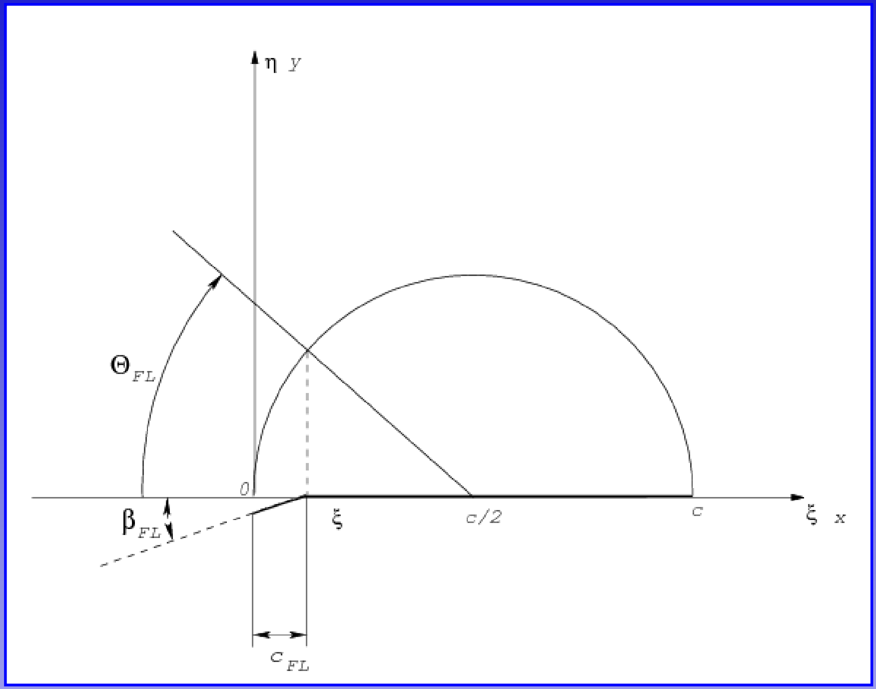
$$C_M = -\frac{\pi}{2}\cdot\left(A_0 + A_1 - \frac{1}{2}A_2\right) = -\frac{1}{4}C_L - \frac{\pi}{4}\cdot\left(A_1 - A_2\right)$$

$$2\Theta_{FL}\right)\right)\cdot\beta_{FL}$$

$$C_M = -\frac{\pi}{2}\cdot\left(A_0 + A_1 - \frac{1}{2}A_2\right) = -\frac{1}{4}C_L - \frac{\pi}{4}\cdot\left(A_1 - A_2\right)$$

$$-\frac{1}{4}C_L - \underbrace{2\sqrt{\bar{c}_{FL}}\cdot\beta_{FL}}_{C_{M_0}}$$

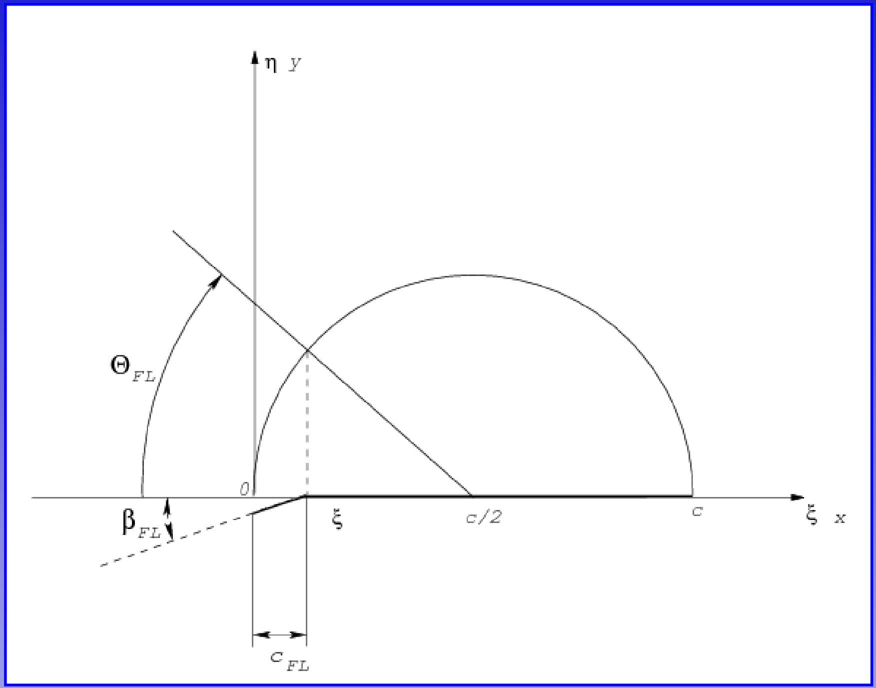
Thin Airfoil Theory (Glauert)



$$\left.\frac{dy}{dx}\right|_x = \begin{cases} \beta_{FL} & x < x_{FL} \\ 0 & x > x_{FL} \end{cases}$$

$$\left.\frac{dy}{dx}\right|_\Theta = \begin{cases} \beta_{FL} & \Theta < \Theta_{FL} \\ 0 & \Theta > \Theta_{FL} \end{cases}$$

Thin Airfoil Theory (Glauert)



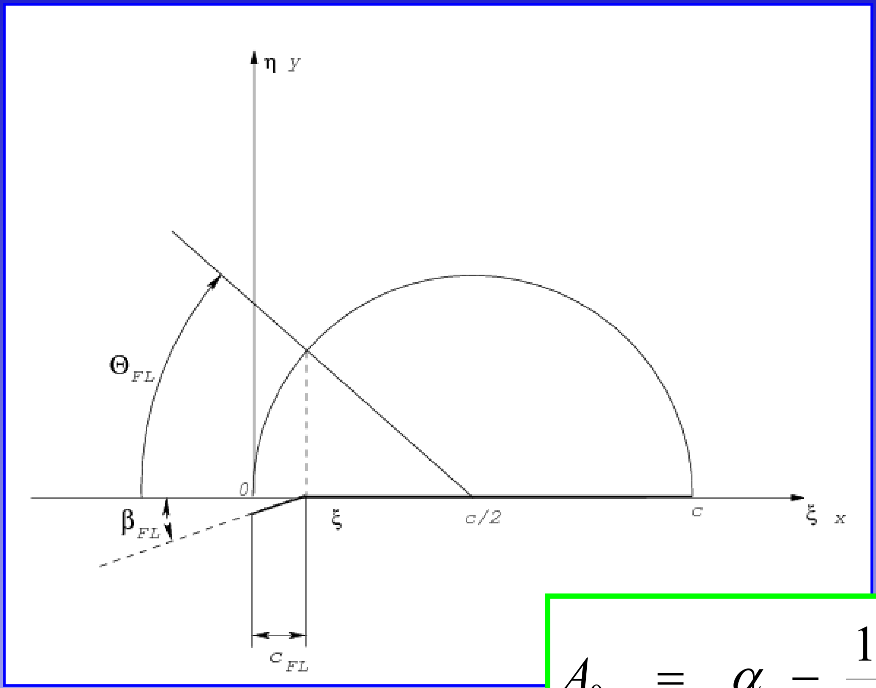
$$\left.\frac{dy}{dx}\right|_x = \begin{cases} \beta_{FL} & x < x_{FL} \\ 0 & x > x_{FL} \end{cases}$$

$$\left.\frac{dy}{dx}\right|_\Theta = \begin{cases} \beta_{FL} & \Theta < \Theta_{FL} \\ 0 & \Theta > \Theta_{FL} \end{cases}$$

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \left.\frac{dy}{dx}\right|_\Theta \cdot d\Theta$$

$$A_n = \frac{2}{\pi}\int_0^\pi \left.\frac{dy}{dx}\right|_\Theta \cdot \cos(n\Theta)\cdot d\Theta \quad n = 1,2,\ldots$$

Thin Airfoil Theory (Glauert)



$$\left.\frac{dy}{dx}\right|_x = \begin{cases} \beta_{FL} & x < x_{FL} \\ 0 & x > x_{FL} \end{cases}$$

$$\left.\frac{dy}{dx}\right|_\Theta = \begin{cases} \beta_{FL} & \Theta < \Theta_{FL} \\ 0 & \Theta > \Theta_{FL} \end{cases}$$

$$A_0 = \alpha - \frac{1}{\pi}\int_0^{\Theta_{FL}} \beta_{FL} \cdot d\Theta = \alpha - \frac{\Theta_{FL}}{\pi}\ \beta_{FL}$$

$$A_1 = \frac{2}{\pi}\int_0^{\Theta_{FL}} \beta_{FL} \cdot \cos(\Theta) \cdot d\Theta = 2\ \beta_{FL}\frac{\sin(\Theta_{FL})}{\pi}$$

$$A_2 = \frac{2}{\pi}\int_0^{\Theta_{FL}} \beta_{FL} \cdot \cos(2\Theta) \cdot d\Theta = \beta_{FL}\frac{\sin(2\Theta_{FL})}{\pi}$$

Thin Airfoil Theory (Glauert)

$$C_L = 2\pi \cdot \left(A_0 + \frac{1}{2} A_1 \right) =$$

$$2\pi \cdot \left(\alpha \underbrace{- \frac{\Theta_{FL} - \sin(\Theta_{FL})}{\pi} \cdot \beta_{FL}}_{\alpha_0} \right)$$

$$2\pi \cdot \alpha \underbrace{- 2(\Theta_{FL} - \sin(\Theta_{FL})) \cdot \beta_{FL}}_{C_{L0}}$$

Thin Airfoil Theory (Glauert)

$$c_{FL}/c = \bar{c}_{FL} << 1$$

$$x_{FL} = \frac{c}{2}\left(1 - \cos(\Theta_{FL})\right)$$

$$\cos(\Theta_{FL}) = 1 - 2\frac{x_{FL}}{c} = \left(1 - 2\bar{c}_{FL}\right)$$

$$\sin(\Theta_{FL}) = \sqrt{1 - \cos^2(\Theta_{FL})} = \ldots = 2\sqrt{\bar{c}_{FL} - \bar{c}^2_{FL}} \approx 2\sqrt{\bar{c}_{FL}}$$

$$\Theta_{FL} \approx 2\sqrt{\bar{c}_{FL}}$$

$$\ldots \frac{\sin(\Theta_{FL})}{\pi} \cdot \beta_{FL} \Big)$$

$$\underbrace{\qquad\qquad\qquad}_{\alpha_0}$$

$$\underbrace{2\pi \cdot \alpha - 2\left(\Theta_{FL} - \sin(\Theta_{FL})\right) \cdot \beta_{FL}}_{C_{L_0}}$$

Thin Airfoil Theory (Glauert)

$$c_{FL}/c = \bar{c}_{FL} << 1$$

$$x_{FL} = \frac{c}{2}\left(1-\cos\left(\Theta_{FL}\right)\right)$$

$$\cos\left(\Theta_{FL}\right) = 1 - 2\frac{x_{FL}}{c} =$$

$$\sin\left(\Theta_{FL}\right) = \sqrt{1-\cos^2(\Theta}$$

$$\Theta_{FL} \approx 2\sqrt{\bar{c}_{FL}}$$

$$C_L = 2\pi\cdot\left(A_0 + \frac{1}{2}A_1\right) =$$

$$2\pi\cdot\left(\alpha \underbrace{- \frac{\Theta_{FL} - \sin\left(\Theta_{FL}\right)}{\pi}\cdot\beta_{FL}}_{\alpha_0}\right)$$

$$C_L = 2\pi\cdot\left(A_0 + \frac{1}{2}A_1\right) =$$

$$\approx 2\pi\cdot\left(\alpha + \frac{2\sqrt{\bar{c}_{FL}}}{\pi}\cdot\beta_{FL} - \frac{2\sqrt{\bar{c}_{FL}}}{\pi}\cdot\beta_{FL} - \left(\frac{4}{3\pi}\cdot\bar{c}^{3/2}{}_{FL} - \ldots\right)\cdot\beta_{FL}\right) =$$

$$2\pi\cdot\alpha - \left(\frac{8}{3}\cdot\bar{c}^{3/2}{}_{FL} - \ldots\right)\cdot\beta_{FL} \approx 2\pi\cdot\alpha$$

Thin Airfoil Theory (Glauert)

$$C_M = -\frac{\pi}{2}\cdot\left(A_0 + A_1 - \frac{1}{2}A_2\right) = -\frac{1}{4}C_L - \frac{\pi}{4}\cdot\left(A_1 - A_2\right)$$

$$-\frac{1}{4}C_L \underbrace{- \frac{1}{4}\cdot\left(2\sin\left(\Theta_{FL}\right) - \sin\left(2\Theta_{FL}\right)\right)\cdot\beta_{FL}}_{C_{M_0}}$$

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$$-\frac{1}{4}C_L - \underbrace{\frac{1}{4}\cdot\left(2\sin\left(\Theta_{FL}\right) - \sin\left(2\Theta_{FL}\right)\right)\cdot\beta_{FL}}_{C_{M0}}$$

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$$\sin\left(2\Theta_{FL}\right) = 2\sin\left(\Theta_{FL}\right)\cos\left(\Theta_{FL}\right) = \ldots \approx 4\sqrt{\bar{c}_{FL}}$$

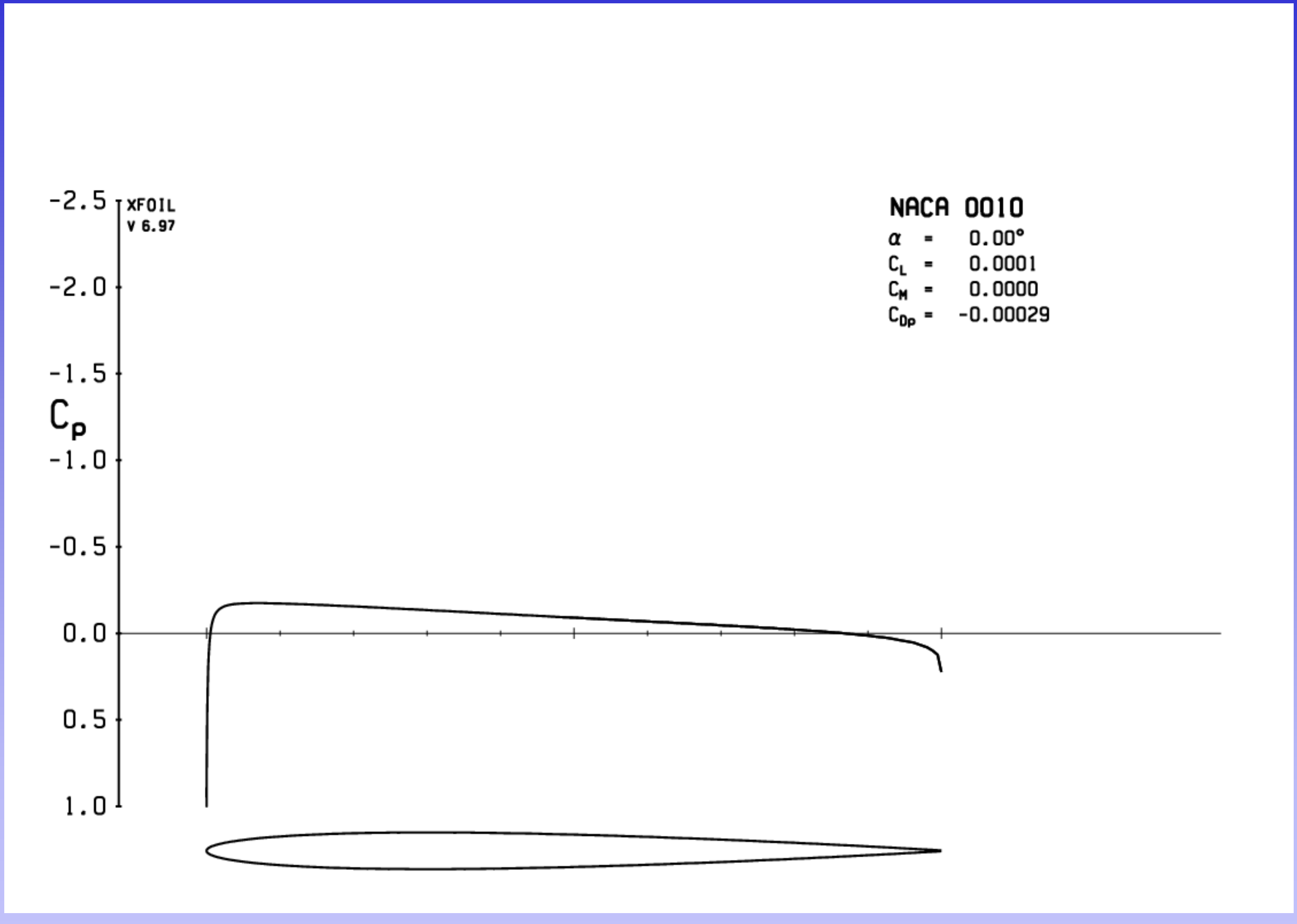
$$C_M = -\frac{\pi}{2}\cdot\left(A_0 + A_1 - \frac{1}{2}A_2\right) = -\frac{1}{4}C_L - \frac{\pi}{4}\cdot\left(A_1 - A_2\right)$$

$$\Theta_{FL}))\cdot\beta_{FL}$$

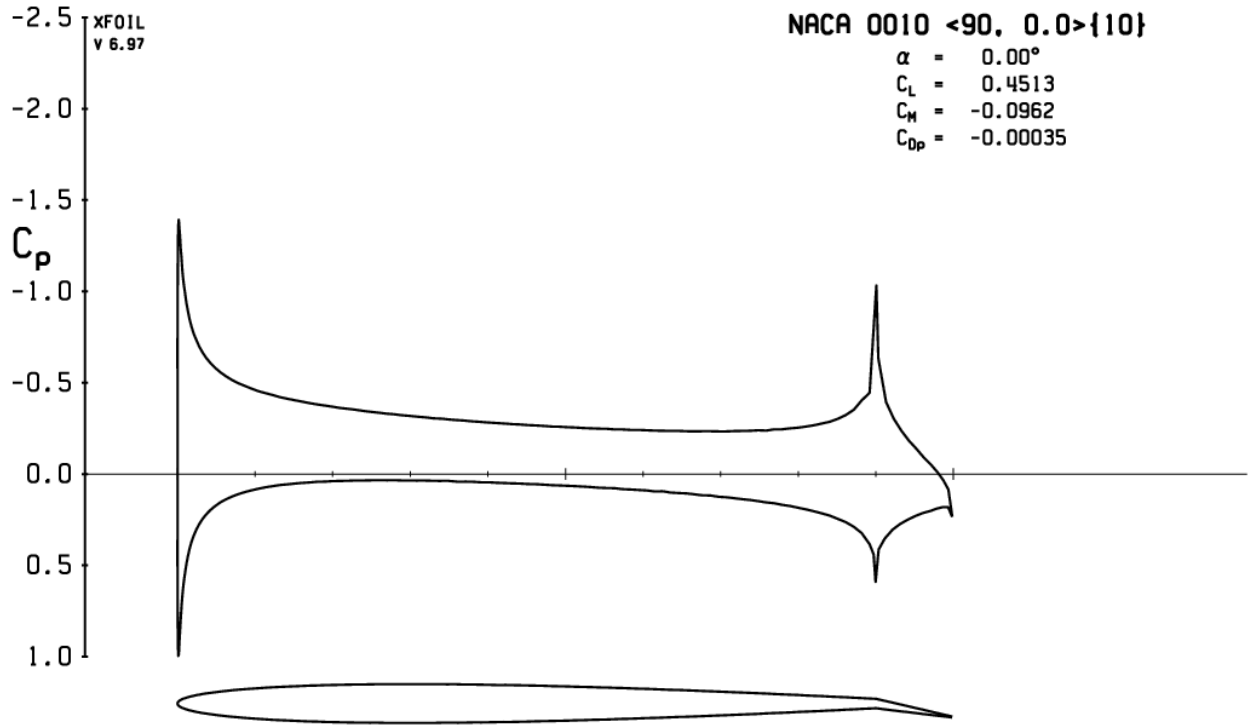
$$\boldsymbol{C_M = -\frac{\pi}{2}\cdot\left(A_0 + A_1 - \frac{1}{2}A_2\right) = -\frac{1}{4}C_L - \frac{\pi}{4}\cdot\left(A_1 - A_2\right)}$$

$$\boldsymbol{= -\frac{1}{4}C_Z \underbrace{- \left(2\bar{c}^{3/2}_{FL} - \ldots\right)\cdot\beta_{FL}}_{C_{M_0}} \approx -\frac{1}{4}C_L}$$

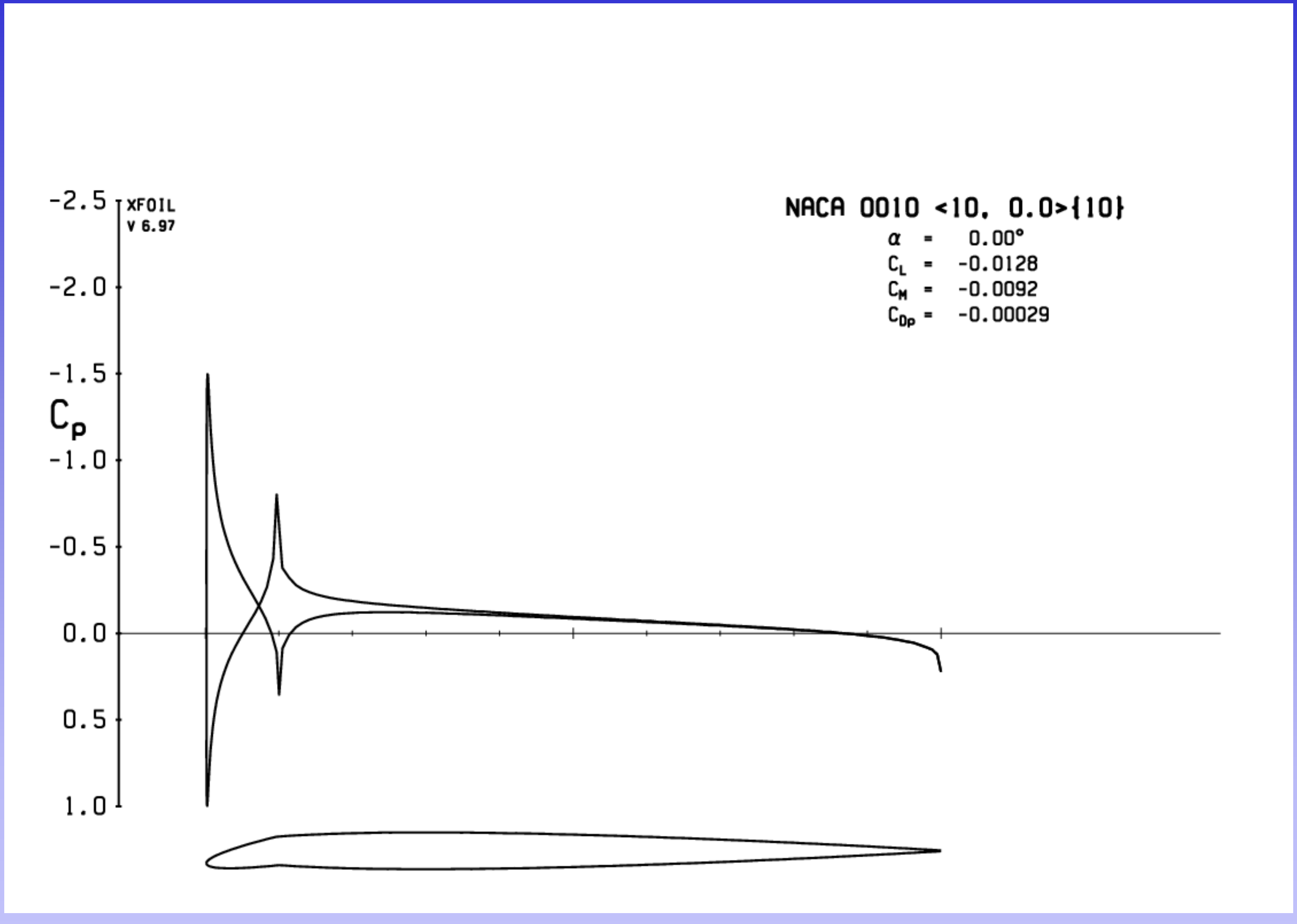
Thin Airfoil Theory (Glauert)



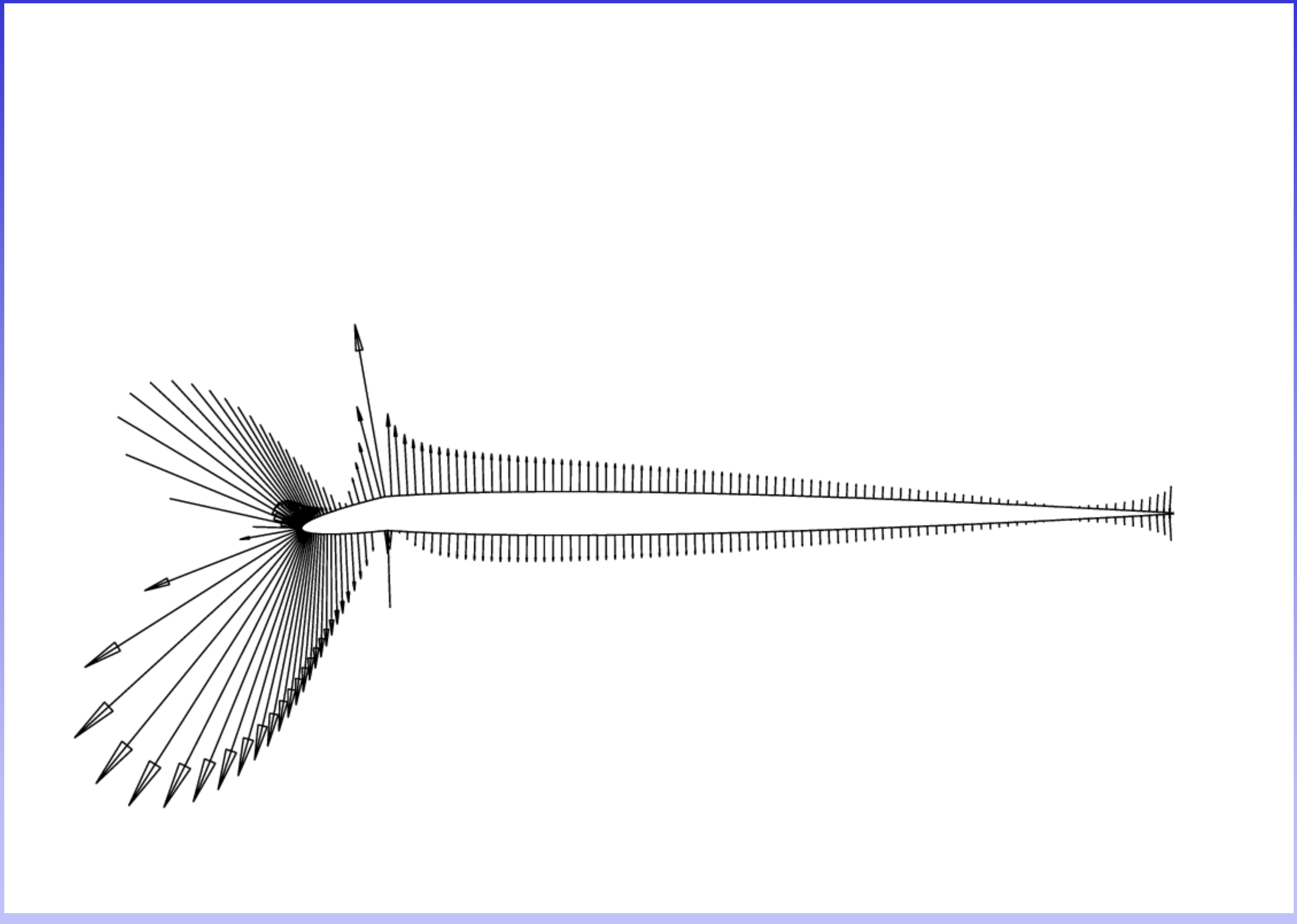
Thin Airfoil Theory (Glauert)



Thin Airfoil Theory (Glauert)



Thin Airfoil Theory (Glauert)



Thin Airfoil Theory (Glauert)