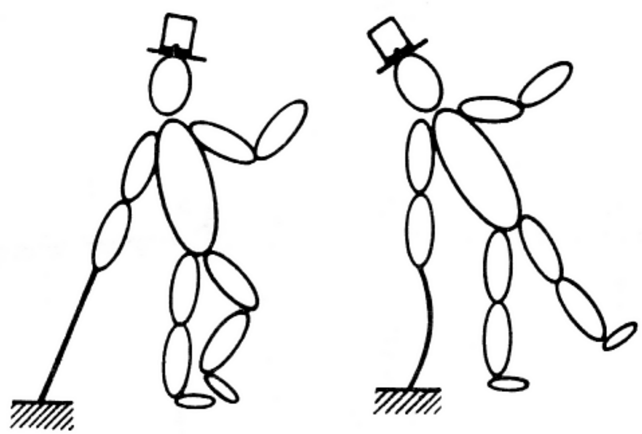
BIOMECHANIKA KRĘGOSŁUPA

Stateczność kręgosłupa



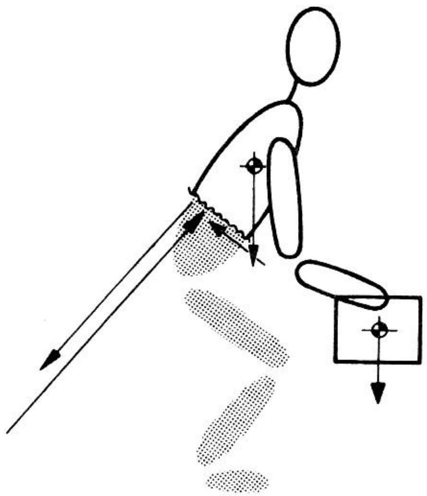
Mechanical Stability of the Human Lumbar Spine

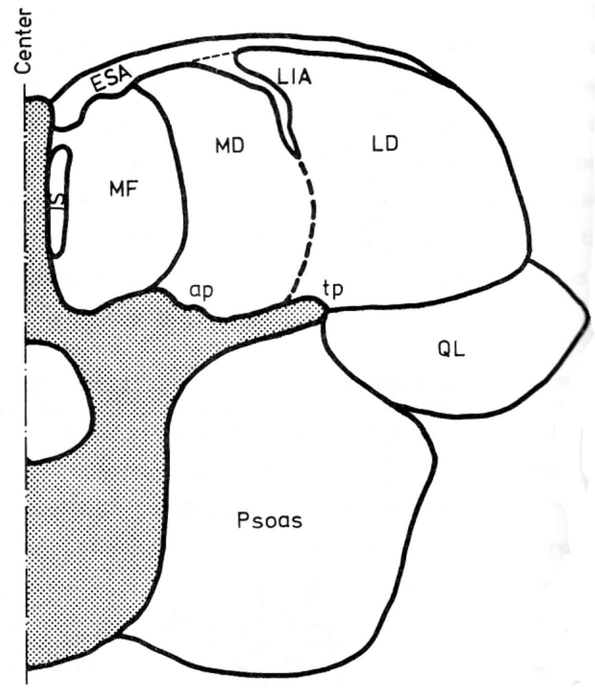
Anders Bergmark

1987

Lund Institute of Technology, Department of Solid Mechanics

Wstęp





Pojęcie stateczności

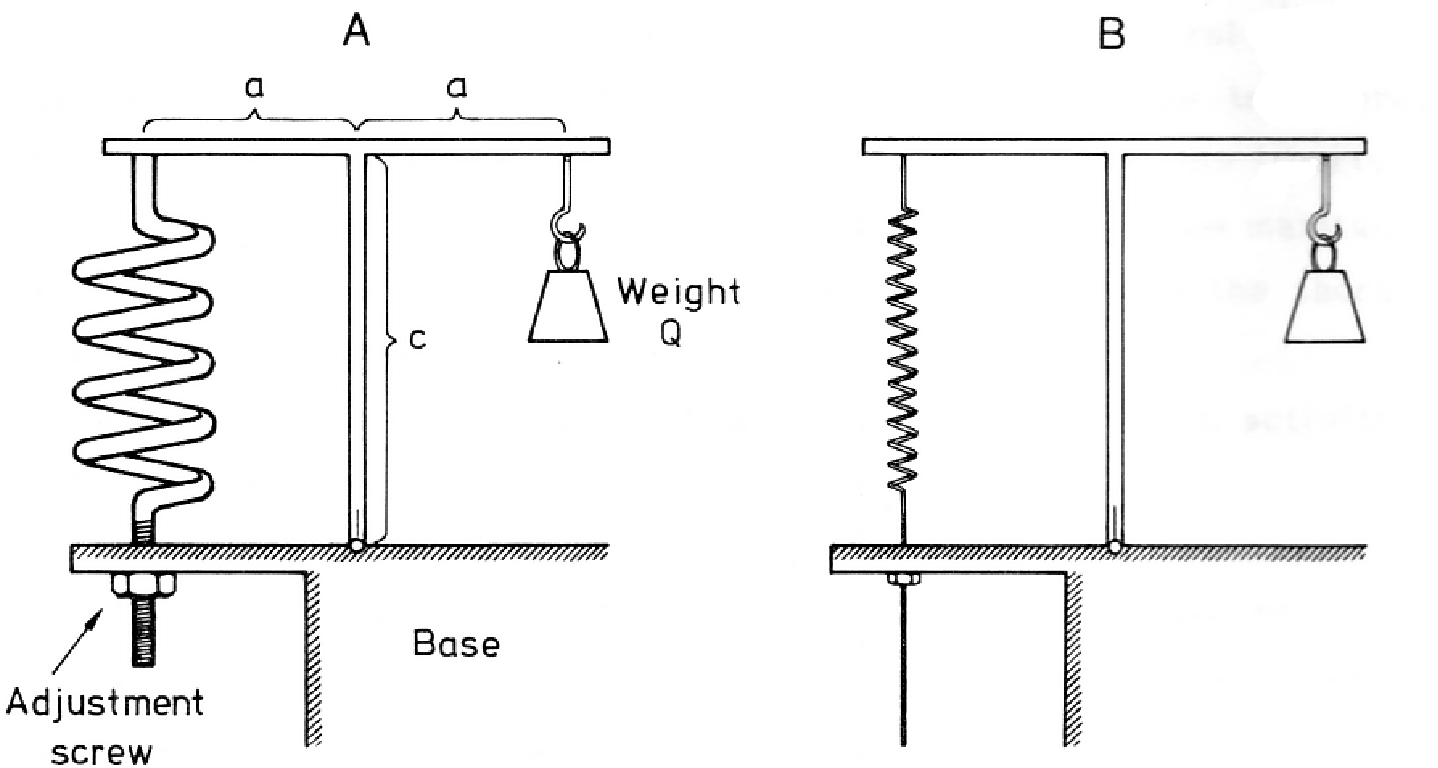
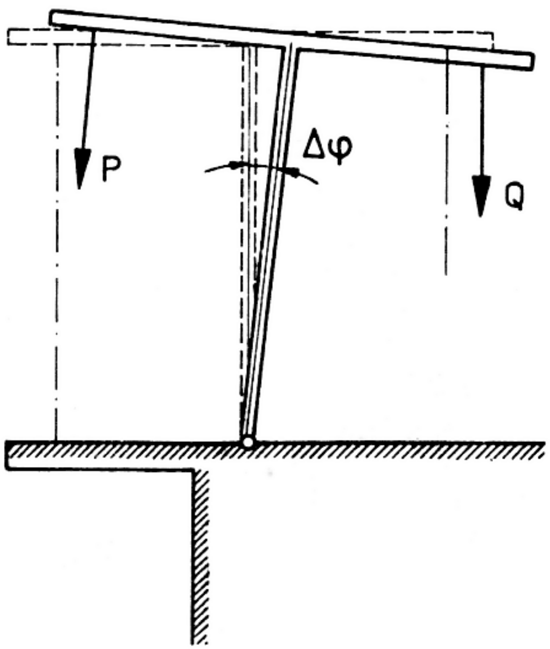


Figure 6-1. The two versions of the system for illustration of the concept of mechanical stability. System A has a much stiffer spring than system B. The lateral stiffness of the springs is assumed to be small enough to be neglected.

Małe zakłócenie kątowe Δφ



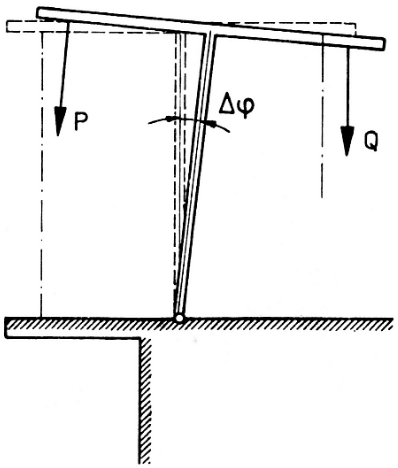
$Q \cdot \Delta$ spadek energii potencjalnej

$Q\delta + \frac{1}{2} \cdot k\delta^2$ przyrost energii w sprężynie

$$\Delta V = Q\delta + \tfrac{1}{2} \cdot k\delta^2 - Q \cdot \Delta$$

Stabilna równowaga występuje
gdy energia potencjalna układu osiąga minimum

Energia potencjalna układu

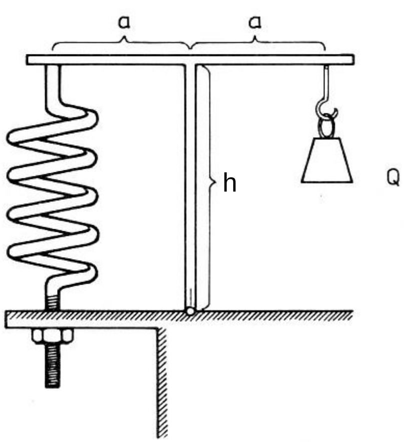


$$\Delta V = Q\delta + \tfrac{1}{2}\cdot k\delta^2 - Q\cdot\Delta$$

$$\delta = a\sin\Delta\varphi \approx a\cdot\Delta\varphi$$

$$\Delta = a\sin\Delta\varphi + h(1-\cos\Delta\varphi) =$$

$$= a\cdot\Delta\varphi + \tfrac{1}{2}\cdot h(\Delta\varphi)^2$$

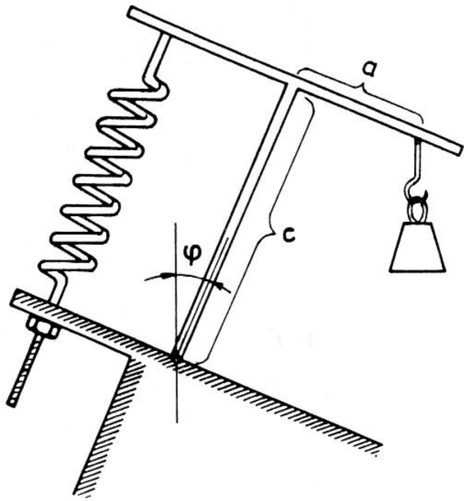


$$\Delta V = -Q\cdot\tfrac{1}{2}\cdot h(\Delta\varphi)^2 + \tfrac{1}{2}\cdot ka^2(\Delta\varphi)^2 =$$

$$= (ka^2 - Q\cdot h)\cdot\tfrac{1}{2}\cdot(\Delta\varphi)^2$$

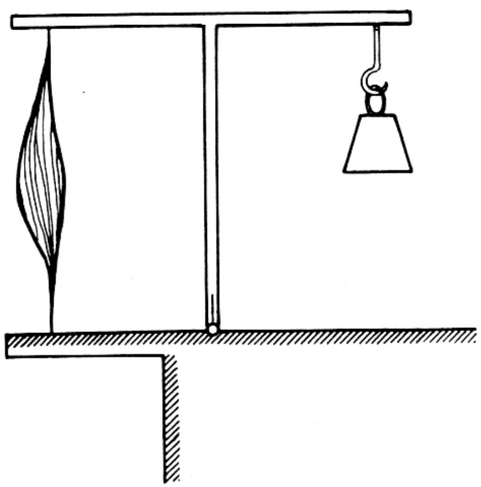
$$Qh \le ka^2 \qquad k_{kryt} = Q\cdot\frac{h}{a^2}$$

Układ obrócony o kąt φ



$$k_{kryt} = Q \cdot \frac{h\cos\varphi - a\sin\varphi}{a^2}$$

Mięsień w rozważanym układzie



$$k_{kryt} = Q \cdot \frac{h}{a^2}$$

sztywność mięśnia

siła w mięśniu

$$k = q \cdot \frac{F}{L}$$

długość mięśnia

q – współczynnik sztywności mięśnia

$q \approx 40$

Stateczność stawu skokowego

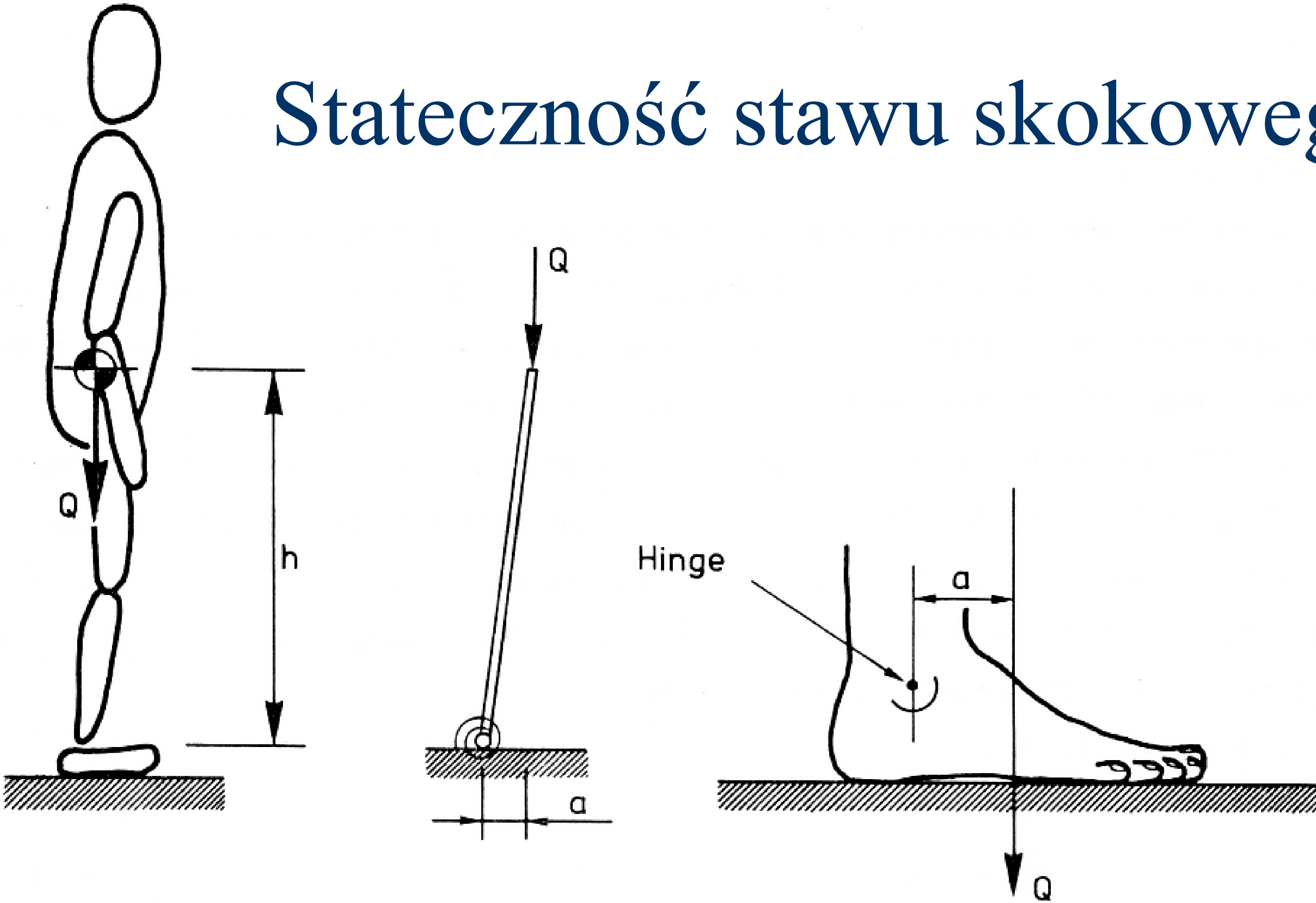


Figure 6-7. The body considered as a rigid inverted pendulum. Q: gravity force, a: distance from the hinge to gravity line, h: vertical height to the center of gravity of the body and λ: torque stiffness of the ankle joint. The numerical figures used are given in the text.

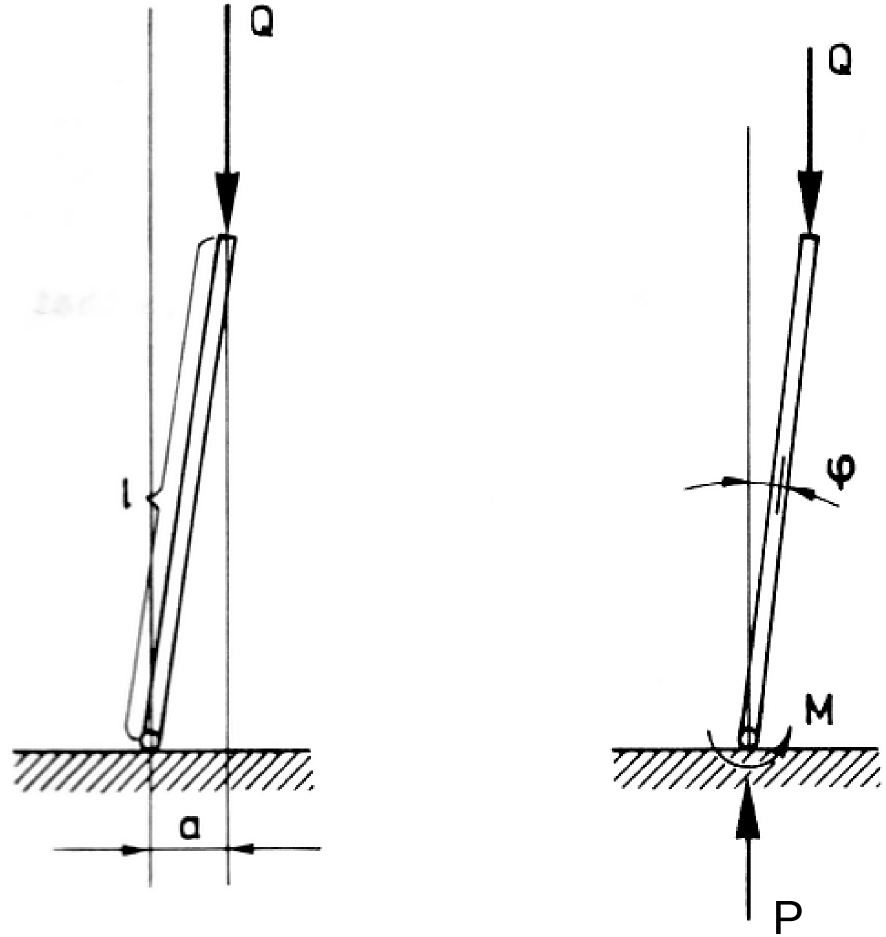
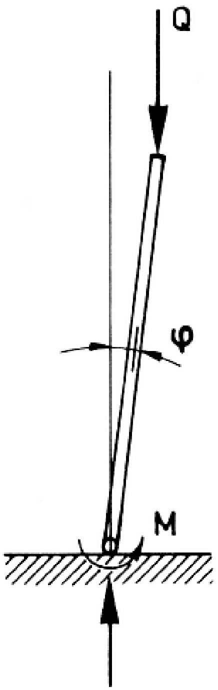
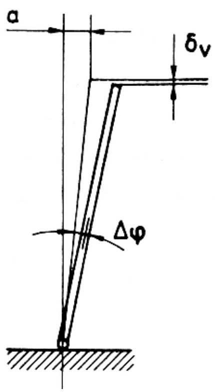


Figure A3-1. An inverted pendulum of length ℓ is subjected to a vertical load Q at the top. The pendulum is attached to the ground by means of a torque spring with stiffness λ. Equilibrium is maintained at the inclination φ (corresponding to a horizontal displacement a at the top) by means of the torque M in the prestressed spring. To the right is the free body diagram of the system. (i.e. the physical devices, the foundation and the torque spring at B, are substituted by the arrow symbols P and M showing their force and moment action.) At equilibrium, the vertical force P at B equals Q.

Zmiana energii potencjalnej





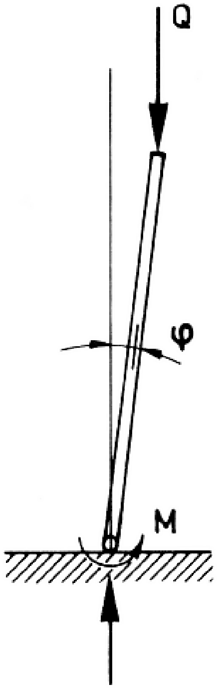
$$\Delta V = M \cdot \Delta\varphi - Q \cdot \delta_V + \tfrac{1}{2} \cdot \lambda (\Delta\varphi)^2$$

$$\delta_V = l[\cos\varphi - \cos(\varphi + \Delta\varphi)] =$$

$$= l[\Delta\varphi \cdot \sin\varphi + \tfrac{1}{2}(\Delta\varphi)^2 \cdot \cos\varphi]$$

$$\cos(a + x) = \cos a - x\sin a - \frac{x^2 \cos a}{2!} + \dots$$

Zmiana energii potencjalnej



$$\Delta V = M \cdot \Delta\varphi - Q \cdot \delta_V + \tfrac{1}{2} \cdot \lambda(\Delta\varphi)^2$$

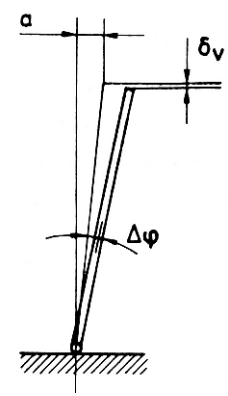
$$\delta_V = l[\Delta\varphi \cdot \sin\varphi + \tfrac{1}{2}(\Delta\varphi)^2 \cdot \cos\varphi]$$

ozn. $a = l \cdot \sin\varphi$ $h = l \cdot \cos\varphi$

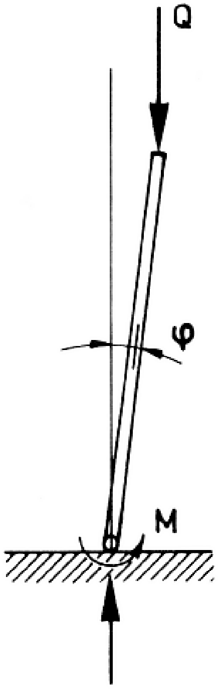
$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V_1 = (M - Q \cdot a)\Delta\varphi$$

$$\Delta V_2 = (-Q \cdot h + \lambda) \cdot \tfrac{1}{2}(\Delta\varphi)^2$$



Warunek równowagi



$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V_1 = (M - Q \cdot a)\Delta\varphi \qquad \Delta V_2 = (-Q \cdot h + \lambda) \cdot \tfrac{1}{2}(\Delta\varphi)^2$$

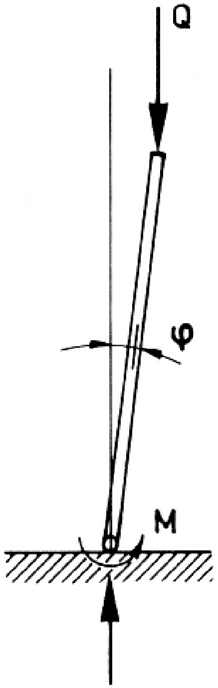
$$\frac{\partial \Delta V}{\partial \Delta\varphi} = 0 \quad \text{dla} \quad \Delta\varphi = 0$$

czyli

$$\frac{\partial \Delta V_1}{\partial \Delta\varphi} = 0 \quad \text{dla} \quad \Delta\varphi = 0$$

zatem $M = Q \cdot a$

Równowaga stateczna



$$\Delta V = \Delta V_1 + \Delta V_2$$

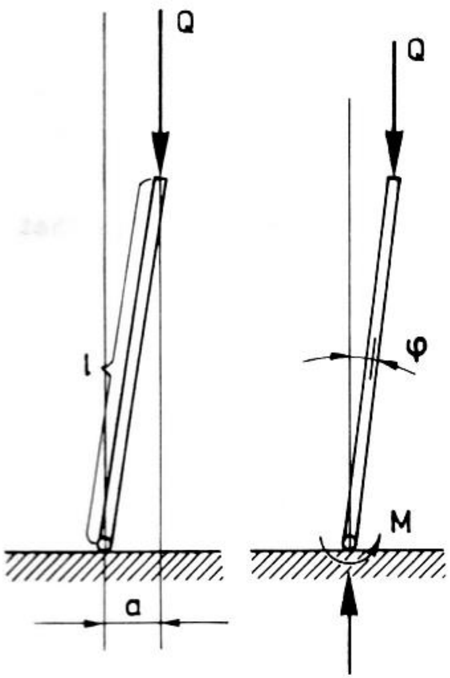
$$\Delta V_1 = (M - Q \cdot a)\Delta\varphi \qquad \Delta V_2 = (-Q \cdot h + \lambda) \cdot \tfrac{1}{2}(\Delta\varphi)^2$$

$$\Delta V > 0 \quad \text{dla} \quad |\Delta\varphi| > 0$$

$$\Delta V_2 > 0 \quad \text{dla} \quad |\Delta\varphi| > 0$$

$$\lambda > \lambda_{kryt} \qquad \lambda_{kryt} = Q \cdot h$$

Stateczność wahadła



Warunek równowagi

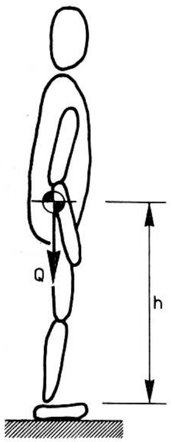
$$M = Q \cdot a = Q \cdot l \cdot \sin \varphi$$

Warunek stateczności

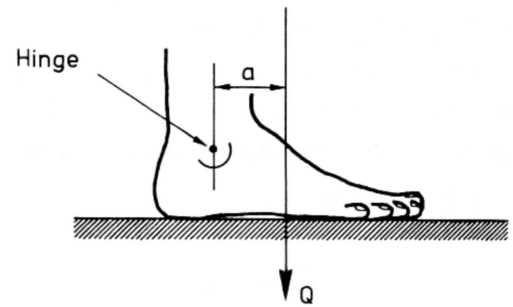
$$\lambda > \lambda_{kryt}$$

$$\lambda_{kryt} = Q \cdot h = Q \cdot l \cdot \cos \varphi$$

Sztywność stawu skokowego



$$\lambda_{kryt} = Q \cdot h$$

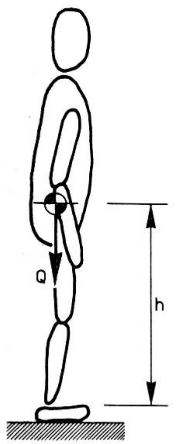


Bendix i in. zbadali 18 kobiet,
średni wzrost 166 cm, średnia waga 55,7 (2Q ≈ 550 N),
a = 63 mm

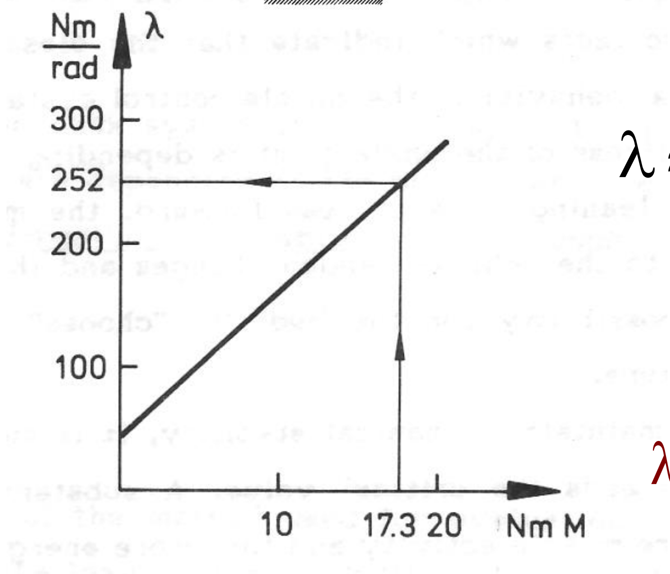
$$\lambda_{kryt} = \mathrm{Q} \cdot h = \tfrac{1}{2} \cdot 550 \cdot 0{,}9 = 248\ Nm / rad$$

$$\mathrm{M} = \mathrm{Q} \cdot a \approx \tfrac{1}{2} 550 \cdot 0{,}063 \approx 17{,}3\ Nm$$

Sztywność stawu skokowego



$$\lambda_{kryt} = Q \cdot h = 248\ Nm/rad$$



$$\lambda \approx \lambda_0 + \kappa \cdot M$$

λ_0 = 35 Nm/rad,
κ = 12,4 rad^{-1}

$$\lambda \approx \lambda_0 + \kappa \cdot M = 35 + 12{,}4 \cdot 17{,}3 = 252\ Nm/rad$$

Hunter i Kearney

Model przestrzenny

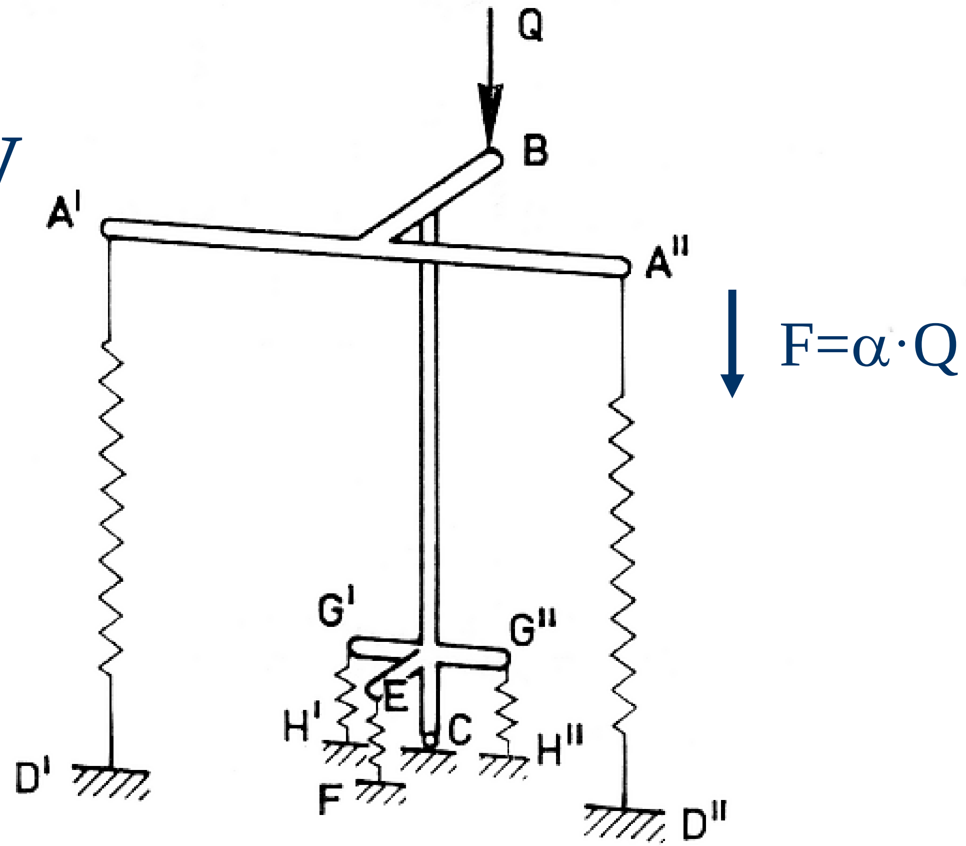
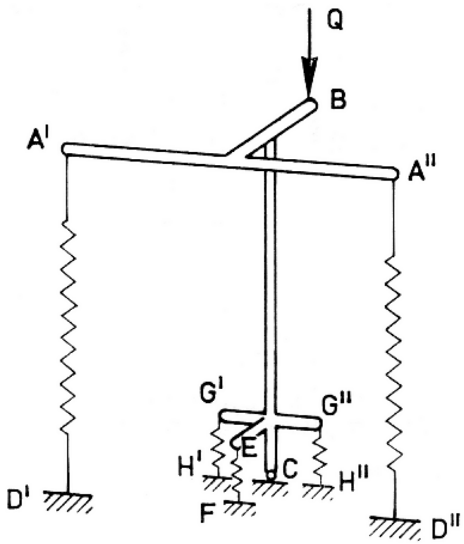


Figure 6-9. Simple example for demonstration of the method for evaluation of the stability of a system containing a local and a global system. A'A"B corresponds to the thoracic cage and G'G"C corresponds to the lumbar spine with a joint at C. A'D' respectively A"D" correspond to the global erector spinae muscle and G'H', G"H" and EF correspond to the local muscles. The stiffness of the joint C (including the contribution from the short bridging muscles) is λ_s in flexion-extension and λ_ℓ in lateral bending. Each part of the global spring, A'D' and A"D", has the stiffness k/2.

Wykres stateczności modelu przestrzennego



F=α·Q

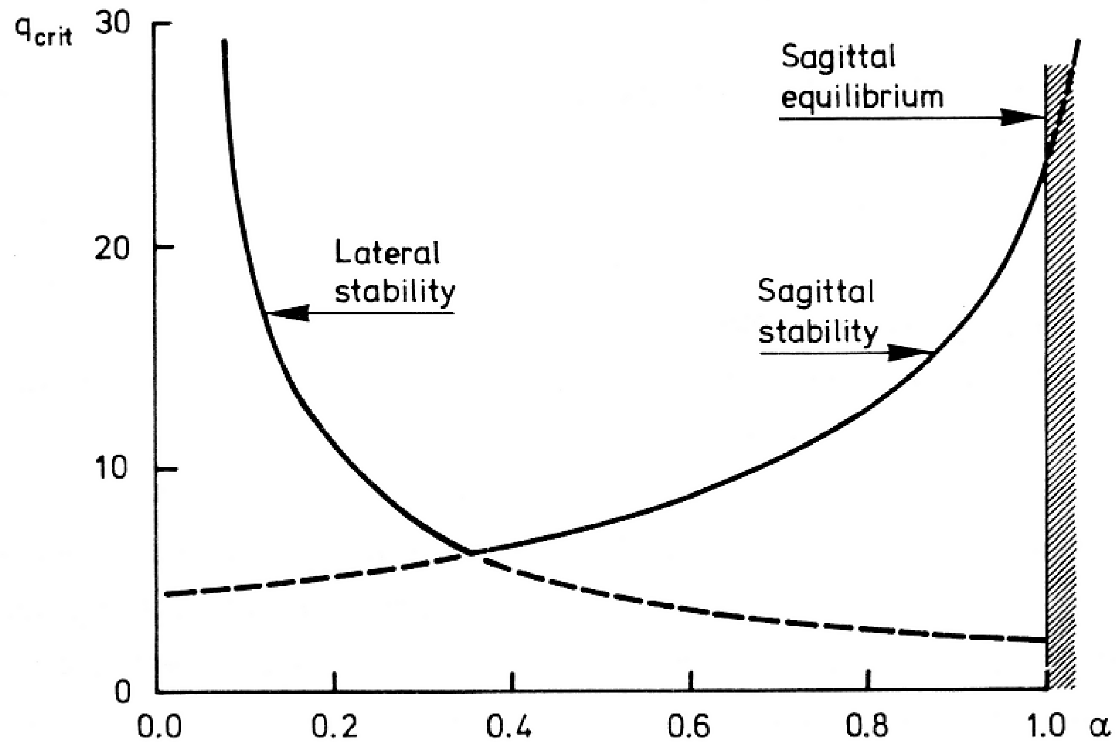
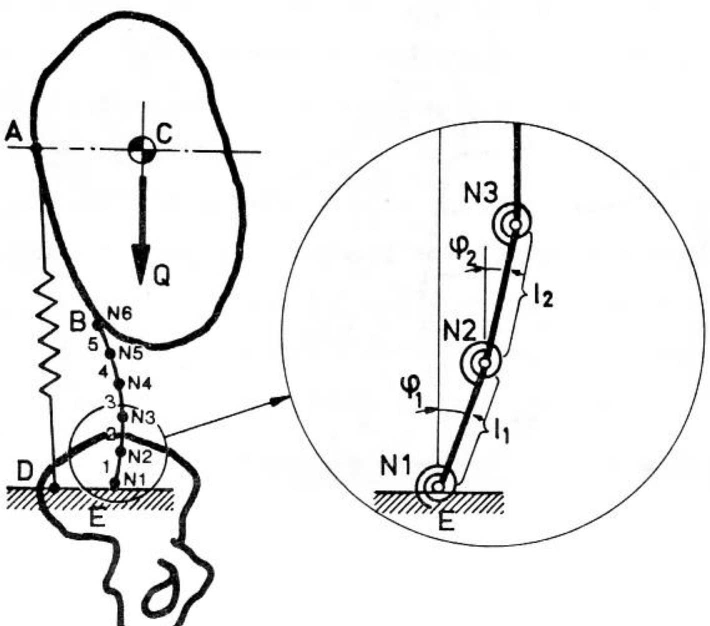


Figure 6-14. The complete stability diagram of the mechanical system in figure 6-9. $Qh/\lambda so = 10$, $Qh/\lambda lo = 10$ and $\beta = 1/10$. q_{crit} is given by the full-drawn line. Stability requires that the muscle stiffness coefficient $q > q_{crit}$. Regardless of q, equilibrium cannot be satisfied for $\alpha > \alpha_c = 1$.

Model kręgosłupa

Figure 6-15. Sagittal projection of the realistic model for evaluation of the mechanical stability of the spinal system. ABC: the stiff thoracic cage, DE: the pelvis, BE: the lumbar spine. The rigid vertebrae are interconnected by the torque springs N1,...,N6 (illustrated by filled circles to the left). The numbering of elements, nodes etc of the model always starts from the pelvis. L5 corresponds to element number 1 and N1 is the L5-S1 interconnection. AD: the global erector spinae muscle and C: the center of gravity of the upper body. l_i and φ_i are the length and the inclination of each element. The local muscles shown in figure 6-16 are also included in the stability model.



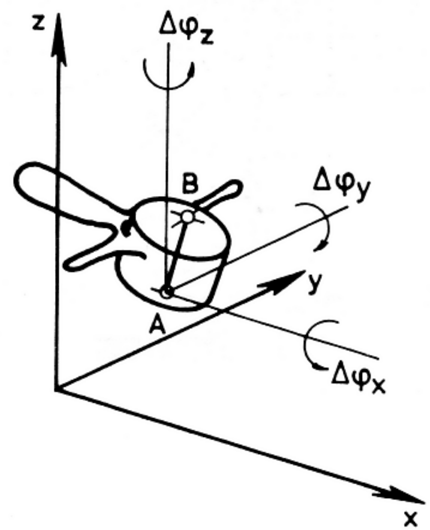


Figure A6-3. The deformations of the system. All spinal elements and the thoracic cage have three degrees of freedom: the rotations $\Delta\varphi_{xi}$, $\Delta\varphi_{yi}$ and $\Delta\varphi_{zi}$ around the global x,y and z axes. Two adjacent elements are connected at the midpoint of the intermediate intervertebral disk. The vertebra shown is connected to the structure below at node N_i and to the structure above at node N_{i+1}.

Siły działające w przekroju przez krążek międzykręgowy

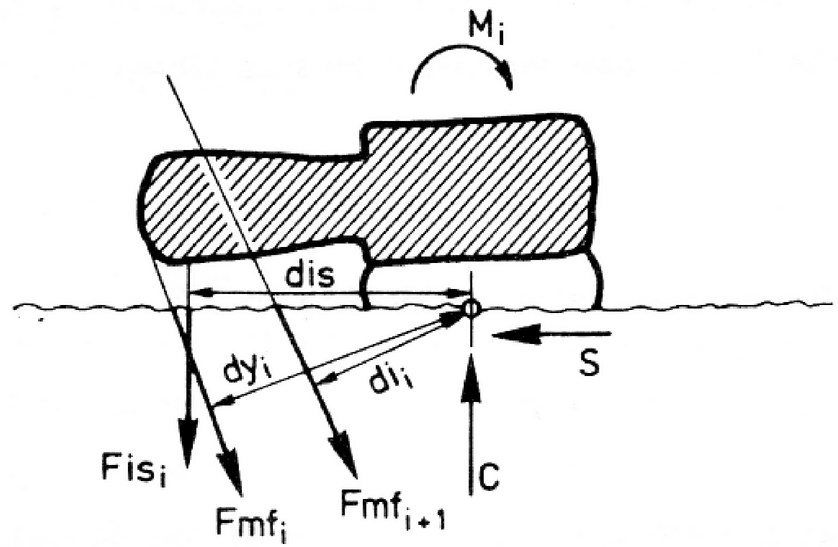


Figure 6-20. Free body diagram for an imaginary cut through level number i. At each level the resulting moment M_i from the gravity load, global erector spinae-, the quadratus lumborum- and the intertransverse muscles is balanced by activity in the interspinal and the multifidi muscles. C: disk compression force (assumed to pass through the disk midpoint), S: disk shear force, Fis_i: inter spinal muscle force at level i and Fmf_i: multifidi muscle force at level i. dy_i and di_i are the multifidi muscle projected lever arms and dis is the lever arm of the interspinal muscle. The multifidi muscle fibers are inclined relative to the sagittal plane, see further text.

Siła w mięśniach prostownika grzbietu

$F = \alpha \cdot Q$

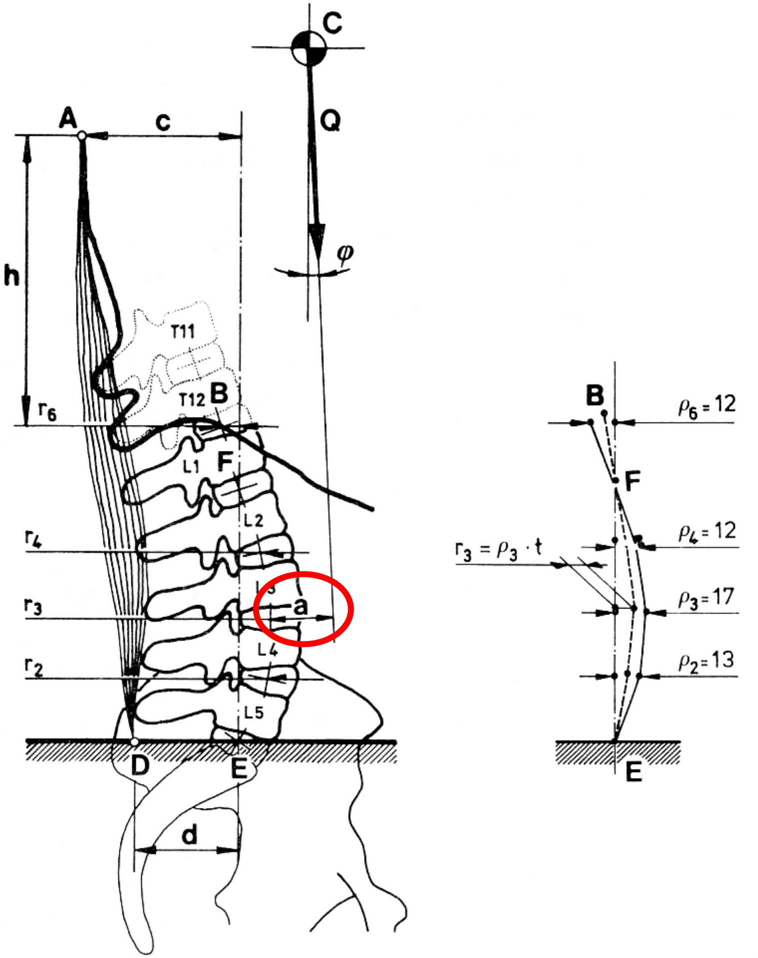


Figure 6-17. The geometrical parameters used to describe posture and position of the gravity line (which is vertical, although it is drawn here at an angle to the sides of the page). A: sagittal projection of the insertion of the global erector spinae muscle, B: T12-L1 disk-midpoint, C: The combined center of gravity of upper-body weight and the weight that constitutes the outer load (Q), D: sagittal projection of the origin of the global muscle, E: L5-S1 disk-midpoint, F: L1-L2 disk-midpoint. The posture of the pelvic-spinal-thoracic cage system relative to the EF-line is defined by the parameters r_i, ϱ_i and t, the relative lordosis. The distances from the EF-line to the nodes (disk-midpoints) are $r_i = \varrho_i \cdot t$. Φ: inclination of the EF-line relative to the gravity line. a: distance from the most anterior disk-midpoint to the gravity line. The numerical figures for ϱ_i in mm are given to the right. The coordinates of the global muscle insertions on the thoracic cage (A relative to B) are given by c and h. c = 59 + 21t mm and h = 100 mm. The lateral positions of the insertions are taken as ± 100 mm. The coordinates of the origin D are given by d = 60 mm with the lateral positions ± 70 mm.

Mięsień wielodzielny

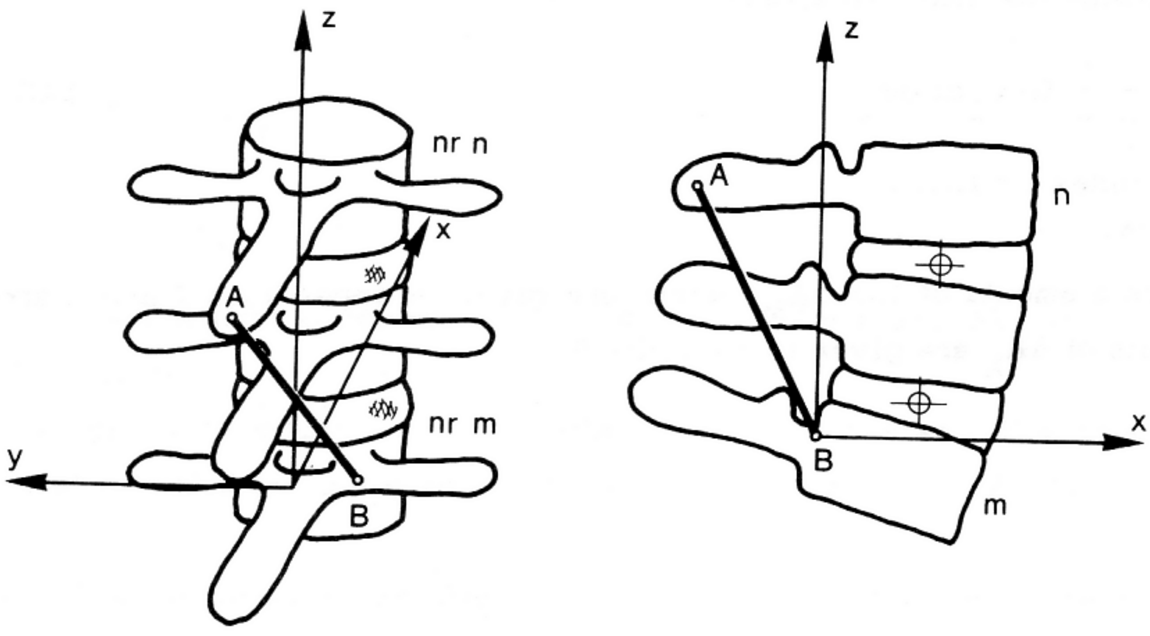


Figure A7-1. A multifidi muscle fiber AB originating at vertebra number m and inserted in element (vertebra or the thoracic cage) number n. The origin of the coordinate system is placed in the sagittal plane at the projection of the muscle origin on this plane. The x- and z-axes lie in the sagittal plane with the z-axis vertical.

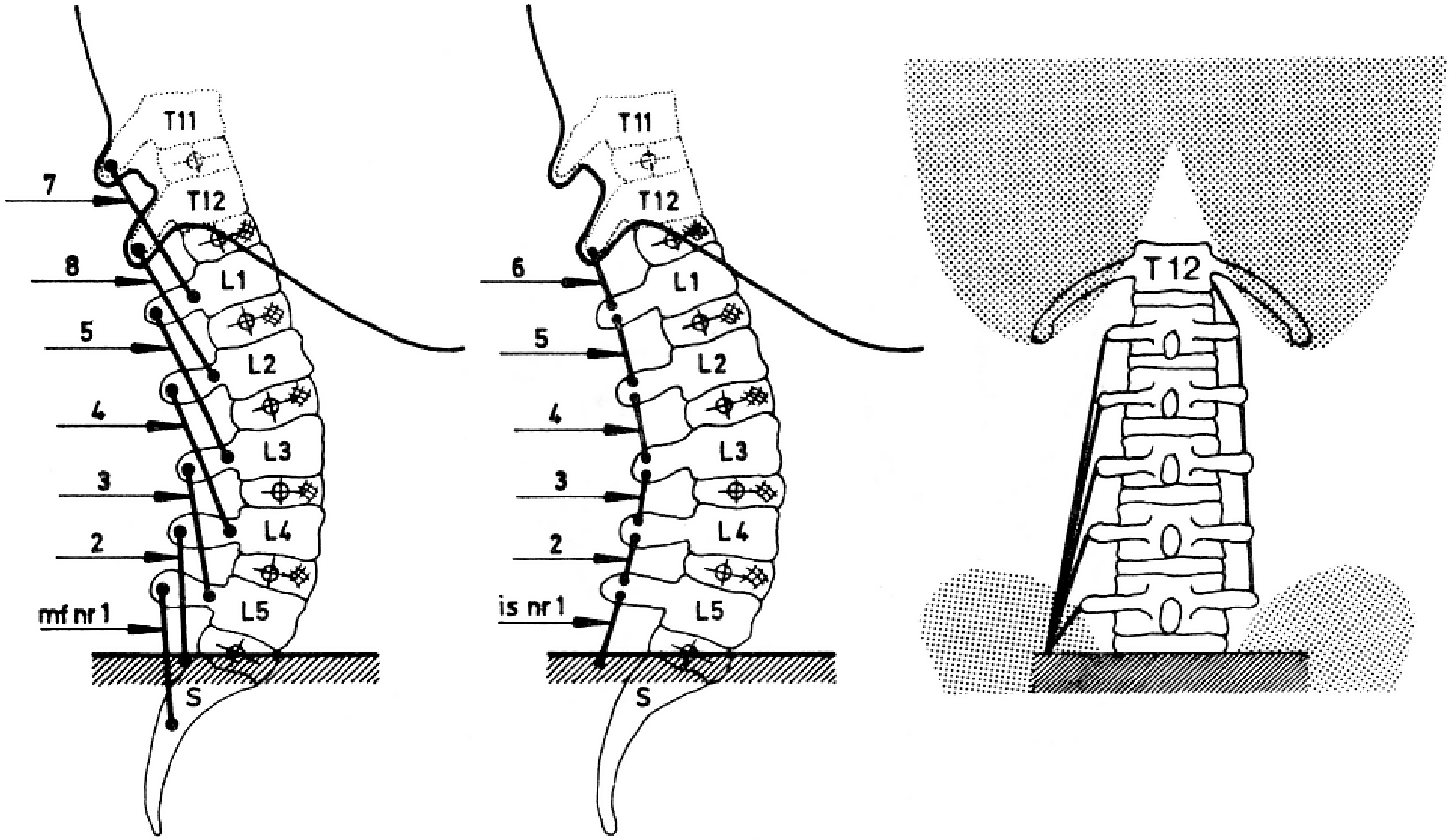


Figure 6-16. The local muscles included in the model. Left: the multifidi fiber pairs numbered from 1 to 7 starting from the pelvis. Each of the 7 fibers has one left and one right part, originating from the mammilar processes and inserted to the spinous processes of the vertebra two levels above. The first and second multifidi-fibers originate from the sacrum. Middle: the interspinal muscles. Right: the left side shows the eight quadratus lumborum fibers with origin on the pelvis and insertions on the outermost part of the transverse processes of L1 to L4. The right side shows the five intertransverse fibers connecting the transverse processes of L5 to L1 and L1 to the thoracic cage. The T12-L1 muscle fibers included in the intertransverse group is introduced according to the diagrams by Langenberg[29]. Note that all thoracic vertebrae are included in the thoracic cage, which is considered as a rigid body.

Wyniki

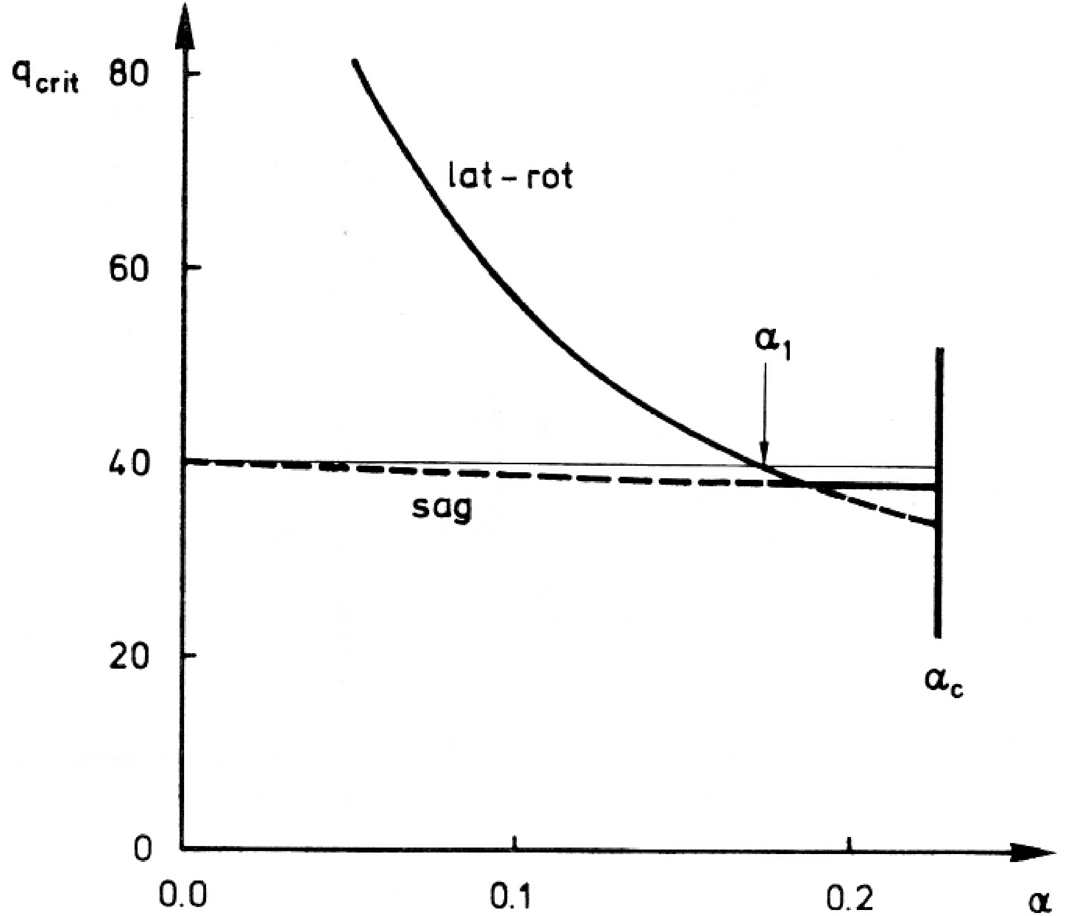


Figure 7-1. The complete stability diagram for loadcase 1, i.e. normal standing posture with no outer load, a = 22.5 mm, t = 1.0, $\Phi = 0$ and cmf = 0.5. q_{crit} is given for $\lambda_{\ell o} = 1.3$, $\lambda_{so} = 1.8$, $\lambda_{ro} = 13.0$, Fit = 10*5.6 N (corresponding to λ = 0.5 Nm/degree) and Fql,etc. = 8*5 N. In the diagram, the condition for lateral - rotational and sagittal stability are shown. Stability prevails if the muscle stiffness coefficient $q > q_{crit}$ according to the full drawn line. Regardless of q, equilibrium cannot be satisfied if $\alpha > \alpha_c$. If, for example, q = 40, the stability can be maintained for $\alpha_1 < \alpha < \alpha_c$. One also observes that stability at the given posture and values of Fit and Fql,etc. cannot be maintained if q is less than about 37.

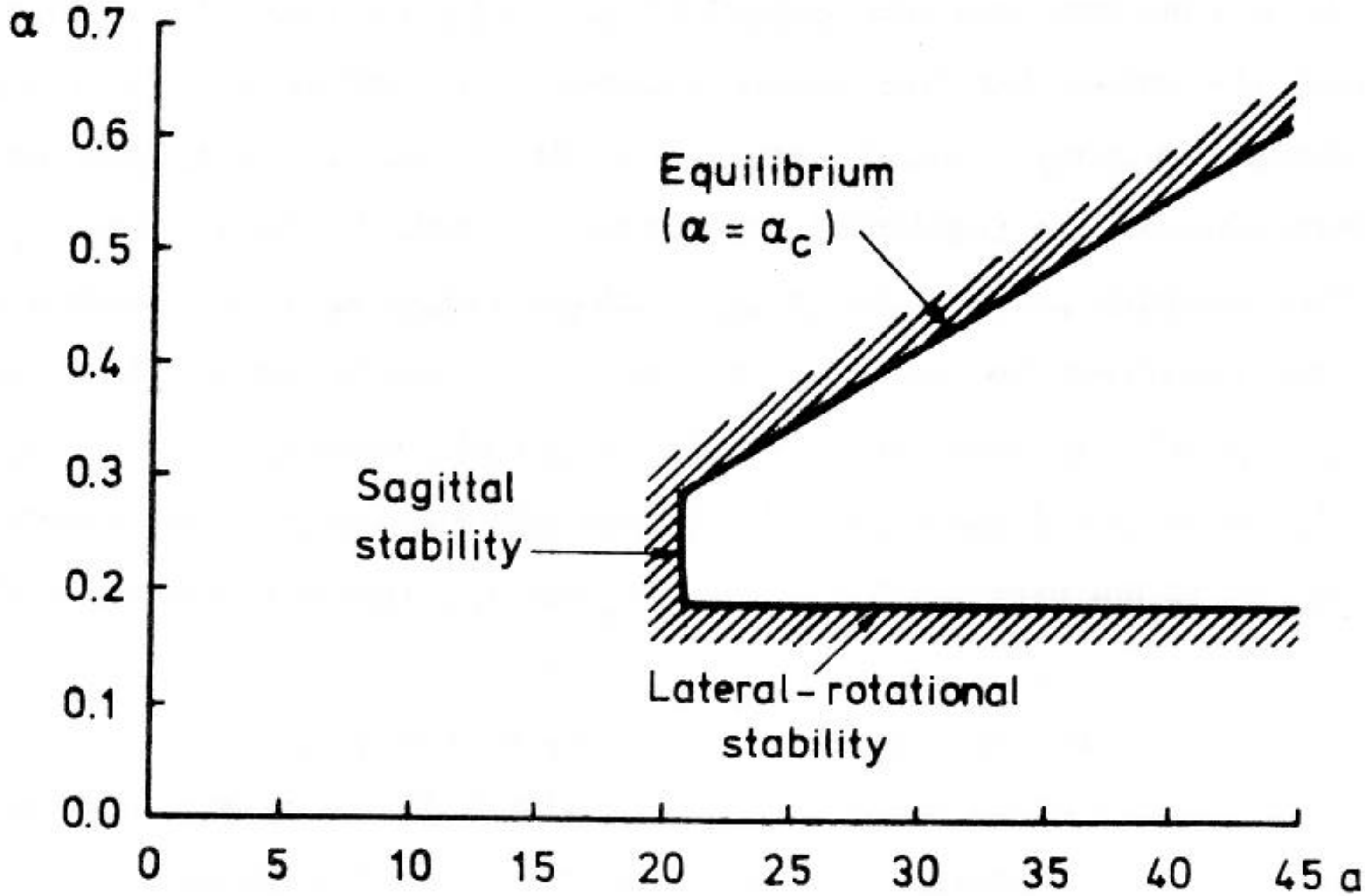
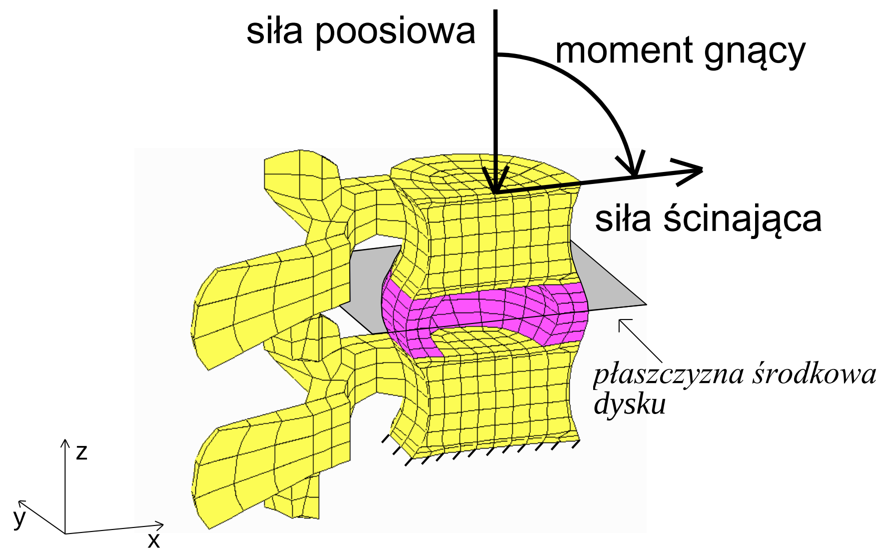


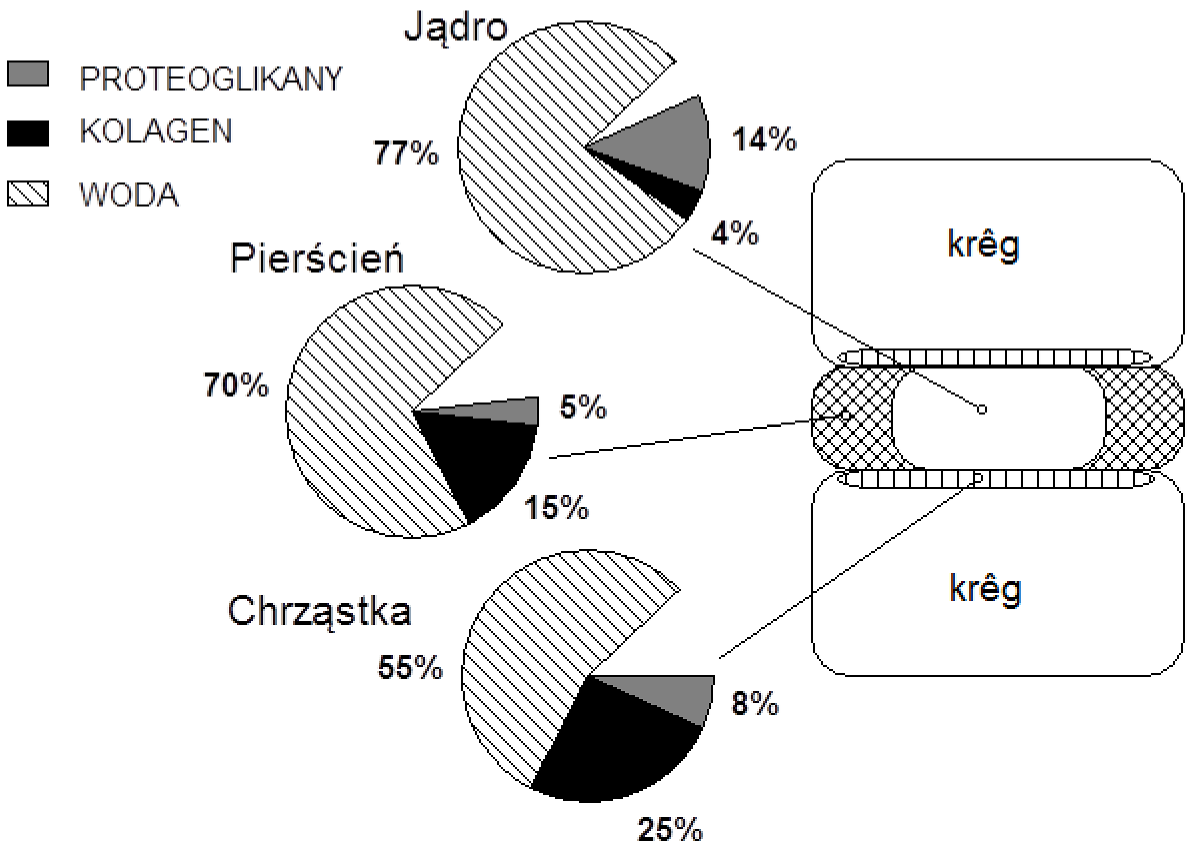
Figure 7-2. Posture-activity diagram for loadcase 1, q = 40. The diagram shows the three limits for allowed combinations of the global muscle activity α and the posture parameter a: the lateral - rotational and sagittal stability limits and the equilibrium limit. $\lambda_{\ell o} = 1.3$, $\lambda_{so} = 1.8$ and $\lambda_{ro} = 13.0$, Fit = 10*5.6 N (corresponding to $\lambda = 0.5$ Nm/degree) and Fql,etc. = 8*5 N.

BIOMECHANIKA KRĘGOSŁUPA

Poro-sprężysty model segmentu ruchowgo kręgosłupa

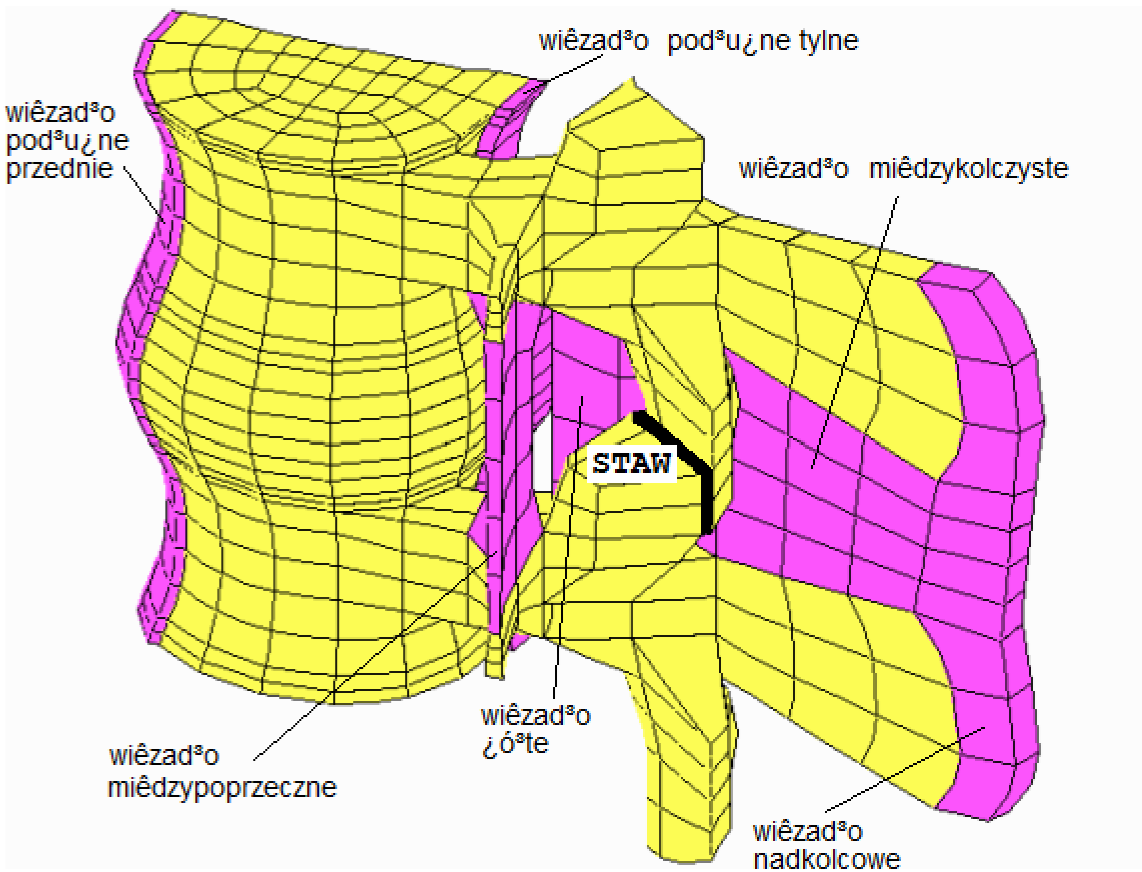


Skład krążka międzykręgowego

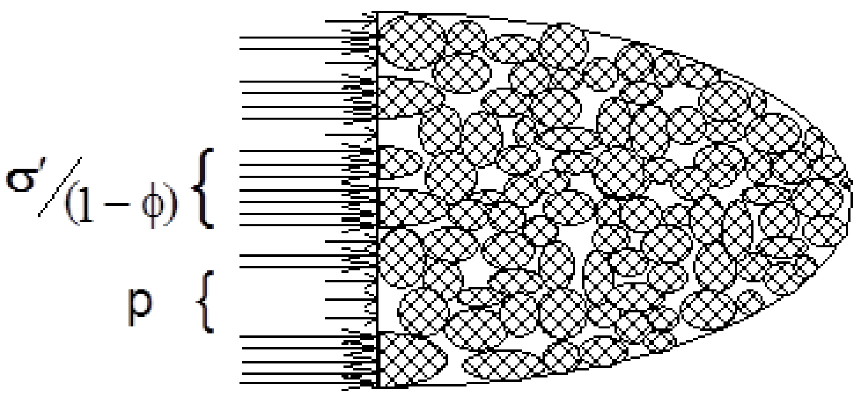


Względna zawartość trzech głównych składników w: jądrze miażdżystym, pierścieniu włóknistym oraz płytce chrzęstnej końcowej

Model segmentu ruchowego



Ciało poro-sprężyste

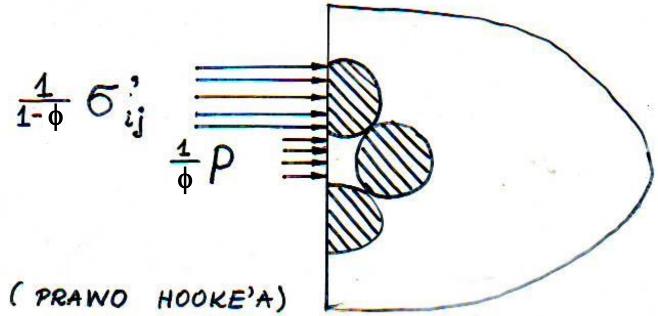


$$\phi = \frac{V_p}{V} = 1 - \frac{V_s}{V}$$ porowatość

V_p - objętości płynu
V - całkowita objętość ciała
V_s - objętość fazy stałej

$$\mathbf{u} = \lfloor u_1, u_2, u_3 \rfloor$$

$$\mathbf{w} = \lfloor w_1, w_2, w_3 \rfloor$$



$$\sigma_{ij} = \sigma'_{ij} - \delta_{ij} p$$

$$\sigma'_{ij} = \delta_{ij}\lambda\varepsilon + 2\mu e_{ij}$$ (PRAWO HOOKE'A)

$$p = -\alpha M \varepsilon + M\xi$$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

RÓWNANIA RÓWNOWAGI $\quad \dfrac{\partial \sigma_{ij}}{\partial x_j} = 0 \quad (i,j = 1,2,3)$

$$\mu \nabla^2 \mathbf{u} + (\mu + \lambda_c)\,\mathbf{grad}\,\varepsilon - \alpha M\,\mathbf{grad}\,\xi = 0$$

$$\lambda_c = \lambda + \alpha^2 M$$

$$\xi = -\mathrm{div}\,\mathbf{w}, \quad \varepsilon = \mathrm{div}\,\mathbf{u}$$

$$\frac{\partial \mathbf{w}}{\partial t} = -k\,\mathbf{grad}\,p$$ (PRAWO DARCY'EGO)

$$\frac{\partial \mathbf{w}}{\partial t} = k\alpha M\,\mathbf{grad}\,\varepsilon - kM\,\mathbf{grad}\,\xi$$

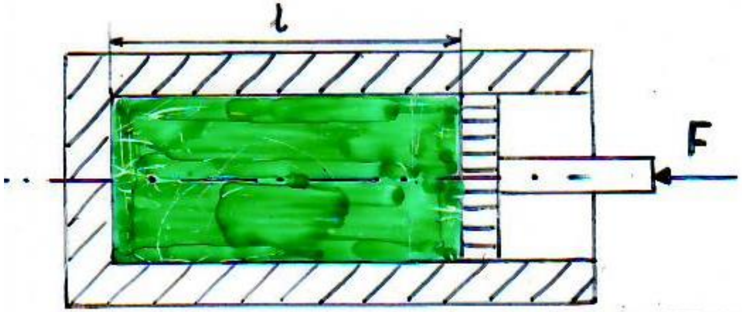
STAŁE MATERIAŁOWE

$\mu \quad \lambda$

$\alpha \quad M$

k

Konsolidacja próbki ziemi



$$\sigma_0 = \frac{F}{A}$$

initial conditions:

$t = 0 \qquad u(x, 0) = 0$

$\qquad\quad\;\; p(x, 0) = \sigma_0$

$$u_B = \frac{8}{\pi^2} a l \sigma_0 \sum_{0}^{\infty} \frac{1}{(2n+1)^2} \left\{ \exp\left[-\left(\frac{(2n+1)\pi}{2l} \right)^2 \frac{k}{a} t \right] - 1 \right\}$$

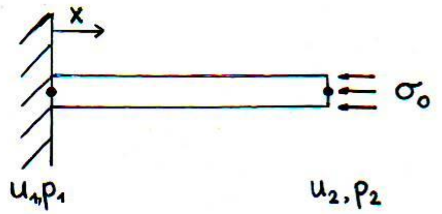
$$a = \frac{1 - 2\nu}{2G(1-\nu)}$$

boundary conditions:

$x = 0 \qquad u(0, t) = 0 \qquad \frac{\partial p}{\partial x} = 0$

$x = 1 \qquad \sigma_x = \sigma_0 \qquad p = 0$

Model MES 1D



$u_1 = 0$ $\quad u_2(t=0) = 0$

$p_2 = 0$

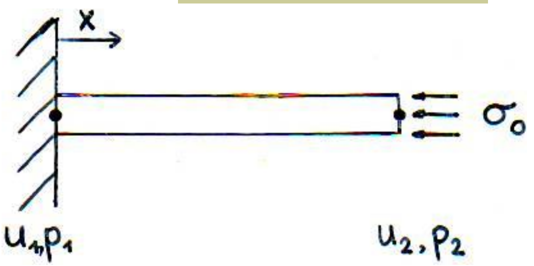
$$\frac{1}{al}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} - \frac{1}{2}\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}\begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} = \begin{Bmatrix} \sigma_R \\ -\sigma_0 \end{Bmatrix}$$

$$-\frac{1}{2}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} - \frac{k}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -Q_R \end{Bmatrix}$$

$$\begin{cases} \dfrac{1}{al} u_2 - \dfrac{1}{2} p_1 = -\sigma_0 \\[2mm] -\dfrac{1}{2}\dot{u}_2 - \dfrac{k}{l} p_1 = 0 \end{cases}$$

$$\dot{u}_2 + \frac{4k}{l^2 a} u_2 = -\frac{4k}{l}\sigma_0$$

Rozwiązanie



ROZWIĄZANIE

$$u_2 = a l \sigma_0 \left(\exp\left[-\frac{4k}{l^2 a} t \right] - 1 \right)$$

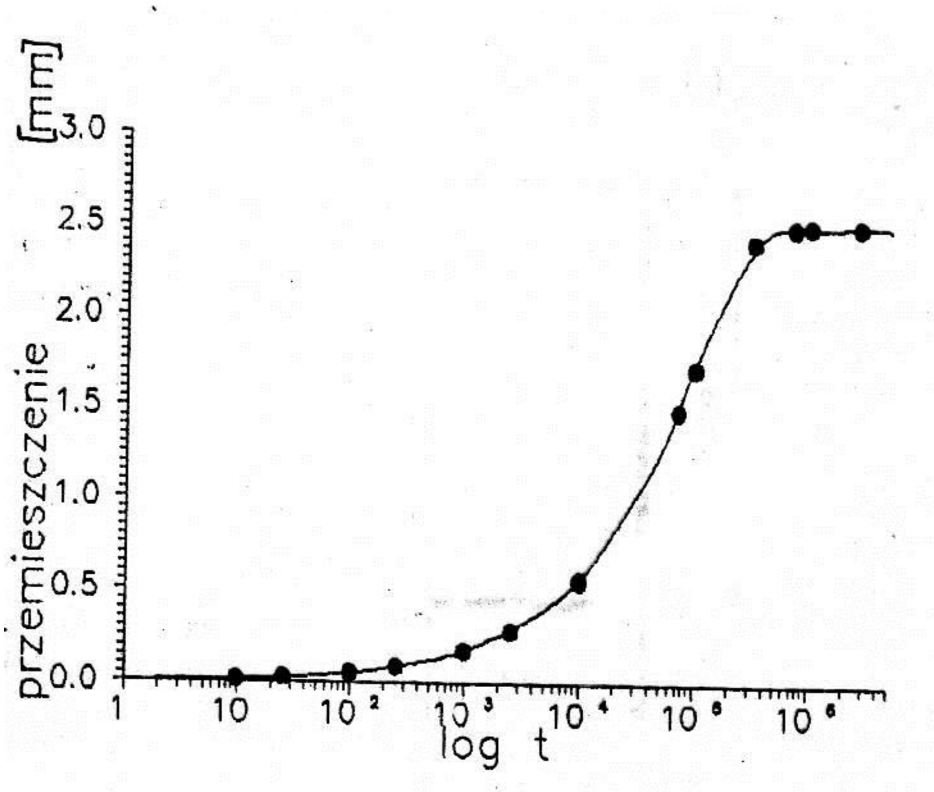
$$p_1 = 2 \sigma_0 \exp\left[-\frac{4k}{l^2 a} t \right]$$

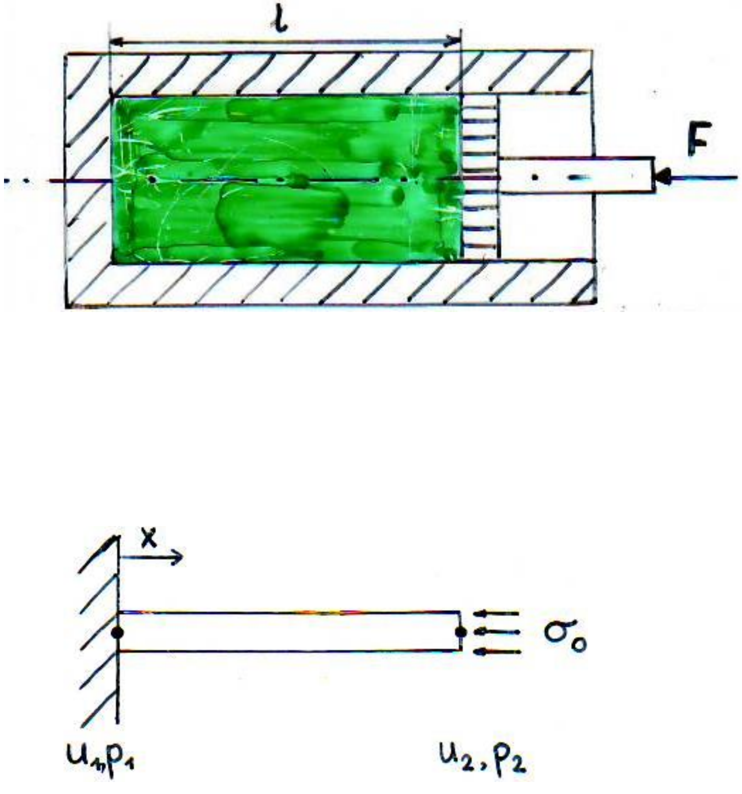
$$Q_R = \frac{4k\sigma_0}{l} \exp\left[-\frac{4k}{l^2 a} t \right]$$

$$V_{pT} = A \int_0^\infty Q_R \, dt = A \int_0^\infty \frac{4k\sigma_0}{l} \exp\left[-\frac{4k}{l^2 a} t \right] dt = -A a l \sigma_0 \exp\left[-\frac{4k}{l^2 a} t \right] \Big|_0^\infty = A a l \sigma_0$$

$$A u_B = A a l \sigma_0$$

Konsolidacja próbki





Model matematyczny

$$\sigma_{ij,j} = 0$$

$$m v_{i,i} = -u_{i,i} \quad (\dot{u}_{i,i} + m \dot{v}_{i,i} = 0)$$

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} + m\pi\,\delta_{ij}$$

$$\dot{v}_i = k_{ij}\,\pi_{,j}$$

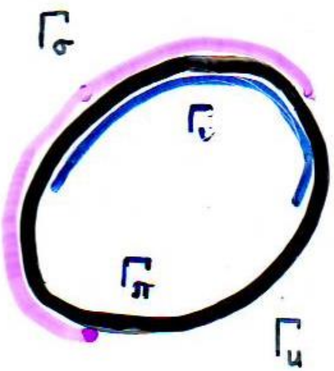
BOUNDARY CONDITIONS:

a) $\sigma_{ij} n_j = T_i$ na Γ_σ

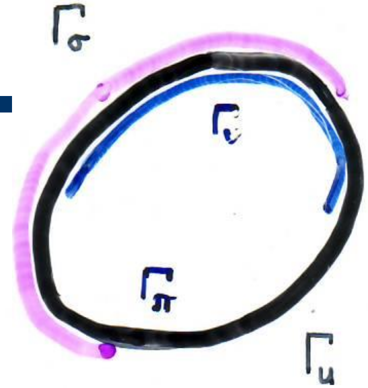
b) $u_i = 0$ na Γ_u

c) $\pi = 0$ na Γ_π

d) $v_i n_i = 0$ na Γ_v



Forma słaba zagadnienia



$$-\int_\Omega \dot{w}_i \sigma_{ij,j}\, d\Omega + \int_\Omega q\left(\dot{u}_{i,i} + m\dot{v}_{i,i}\right) d\Omega = 0$$

$$\int_\Omega \dot{w}_{i,j}\, \sigma_{ij}\, d\Omega - \int_\Gamma \dot{w}_i\, \sigma_{ij}\, n_j\, d\Gamma + \int_\Omega q\, \dot{u}_{i,i}\, d\Omega - \int_\Omega q_{,i}\, m\dot{v}_i\, d\Omega +$$
$$+ \int_\Gamma q\, m\dot{v}_i\, n_i\, d\Gamma = 0$$

$$\int_\Omega \dot{w}_{i,j}\, c_{ijkl}\, u_{(k,l)}\, d\Omega + \int_\Omega \dot{w}_{i,j}\, m\pi\, \delta_{ij}\, d\Omega - \int_{\Gamma_\sigma} \dot{w}_i T_i\, d\Gamma - \int_{\Gamma_u} \dot{w}_i R_i\, d\Gamma +$$
$$+ \int_\Omega q\, \dot{u}_{i,i}\, d\Omega - \int_\Omega q_{,i}\, k_{ij}\, m\pi_{,j}\, d\Omega + \int_{\Gamma_\pi} q\, Q\, d\Gamma = 0$$

$$A(\dot{w}, u) + (\mathrm{div}\, \dot{w},\, m\pi) - \bar{A}(q, m\pi) + (q, \mathrm{div}\, \dot{u}) = (\dot{w}, T)_{\Gamma_\sigma} + (\dot{w}, R)_{\Gamma_u} - (q, Q)_{\Gamma_\pi}$$

Równania MES

$$\bar{\dot{w}}^T\left(K_c\bar{u}_c + G_c\bar{p}_c - F_c - R_c\right) + \bar{q}^T\left(G_c^T\bar{\dot{u}}_c - M_c\bar{p}_c - Q_c^R\right) = 0$$

$$K_c\bar{u}_c + G_c\bar{p}_c = F_c + R_c$$
$$G_c^T\bar{\dot{u}}_c - M_c\bar{p}_c = Q_c^R$$

$$K\bar{u} + G\bar{p} = F \qquad\qquad K_R\bar{u}_c + G_R\bar{p}_c = R$$
$$G^T\bar{\dot{u}} - M\bar{p} = 0 \qquad\qquad G_R^T\bar{\dot{u}}_c - M_R\bar{p}_c = Q^R$$

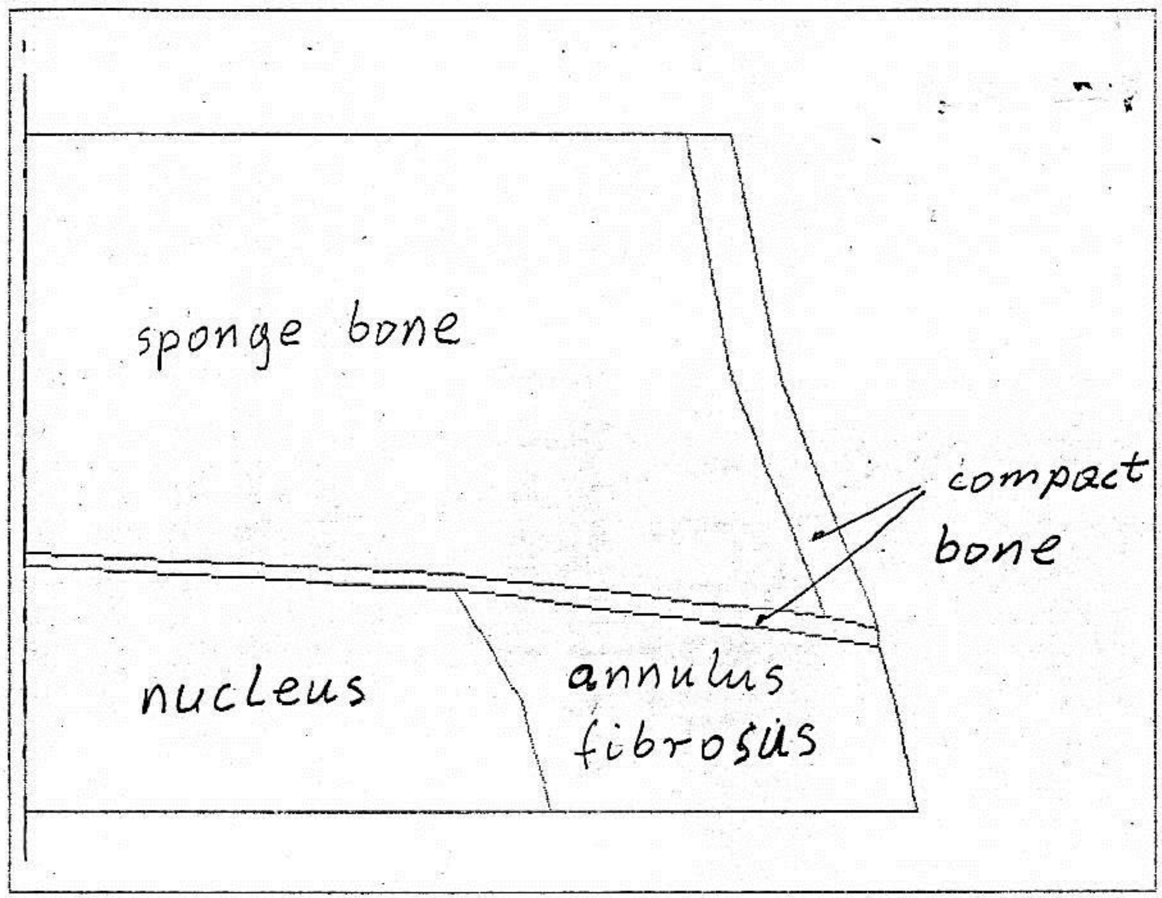
$$K\bar{u}_{n+1} + G\bar{p}_{n+1} = F$$

$$G^T\frac{\bar{u}_{n+1} - \bar{u}_n}{\Delta t} - M\left((1-\beta)\bar{p}_{n+1} + \beta\bar{p}_n\right) = 0$$

Metoda Newmarka

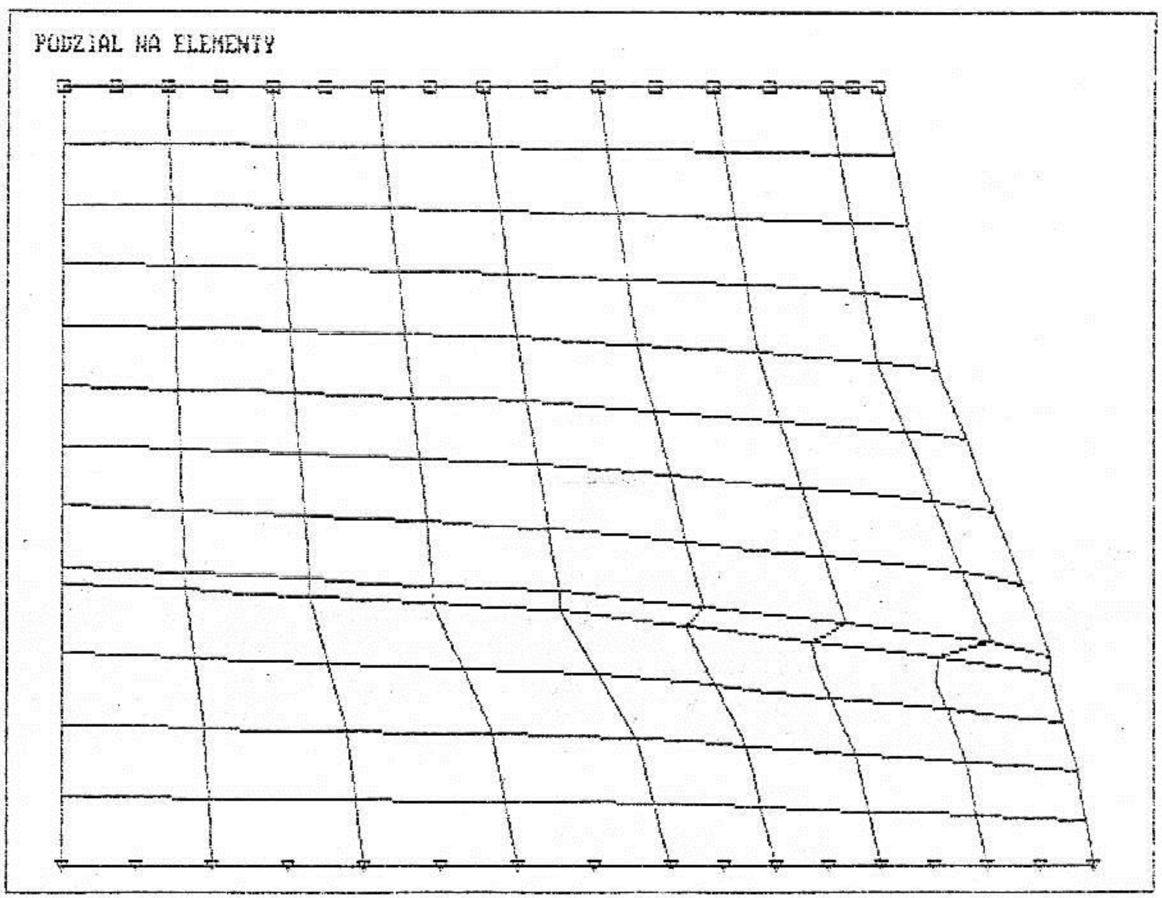
$$\begin{bmatrix} K & G \\ G^T & -\Delta t(1-\beta)M \end{bmatrix}\begin{Bmatrix} \bar{u}_{n+1} \\ \bar{p}_{n+1} \end{Bmatrix} = \begin{Bmatrix} F \\ G^T\bar{u}_n + \Delta t\beta M\bar{p}_n \end{Bmatrix}$$

Model osiowo-symetryczny

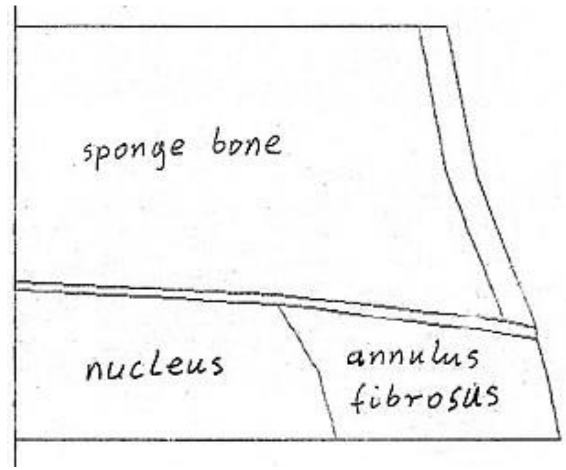


Axi-symmetric motion segment model

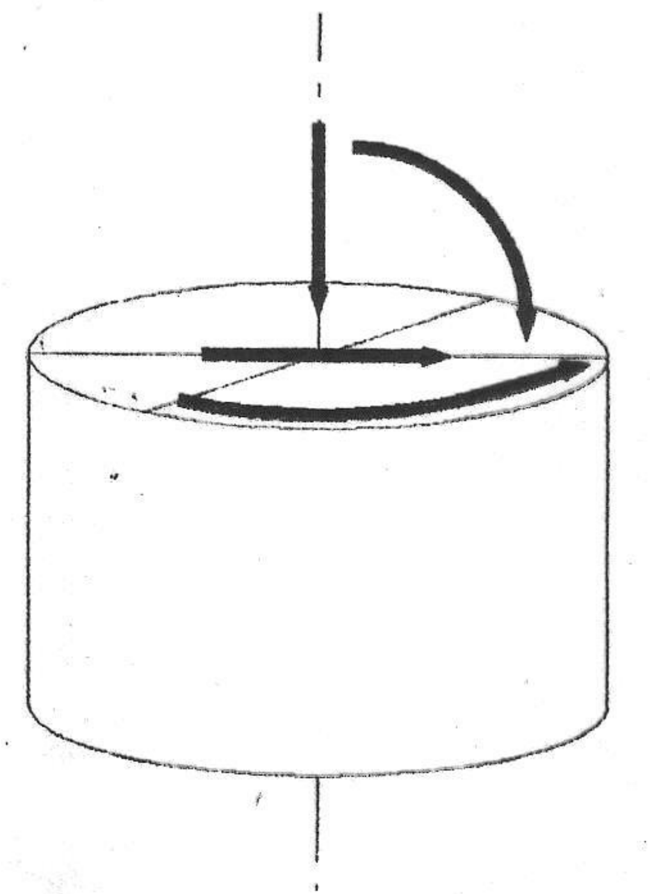
Model MES 2D

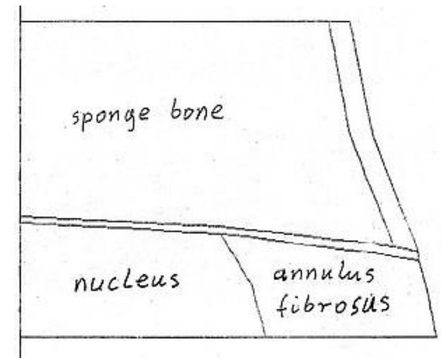


FEM Mesh



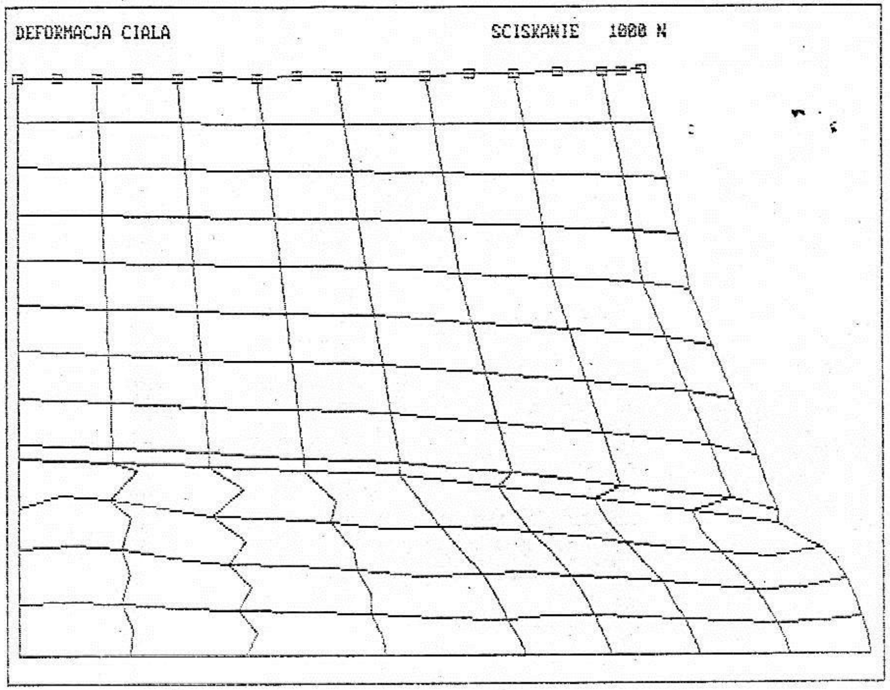
Obciążenie dowolne



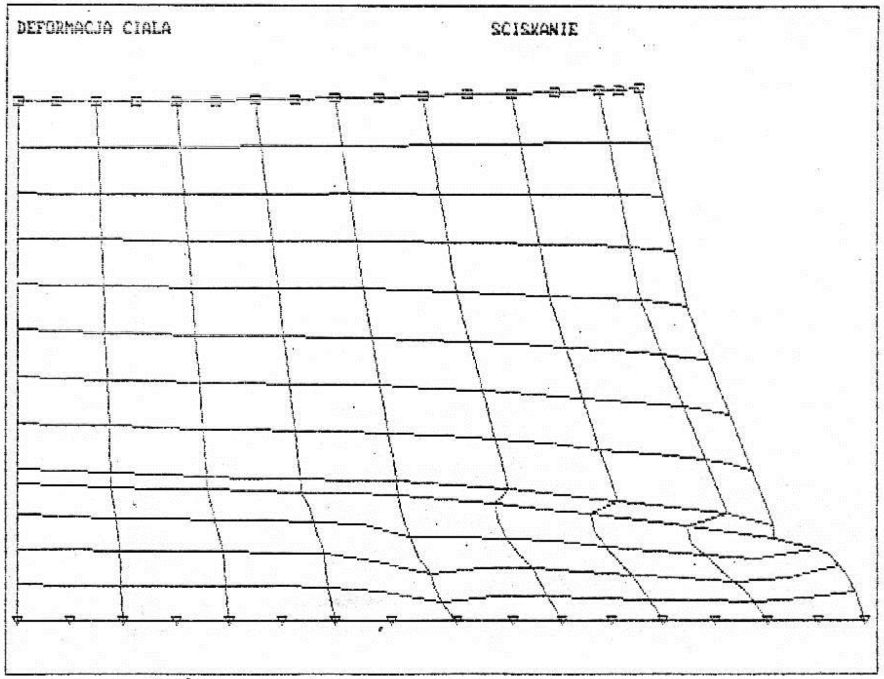


Model osiowo-symetryczny, obciążenia rozwijane w szeregi Fouriera

Wyniki dla ściskania 1000 N

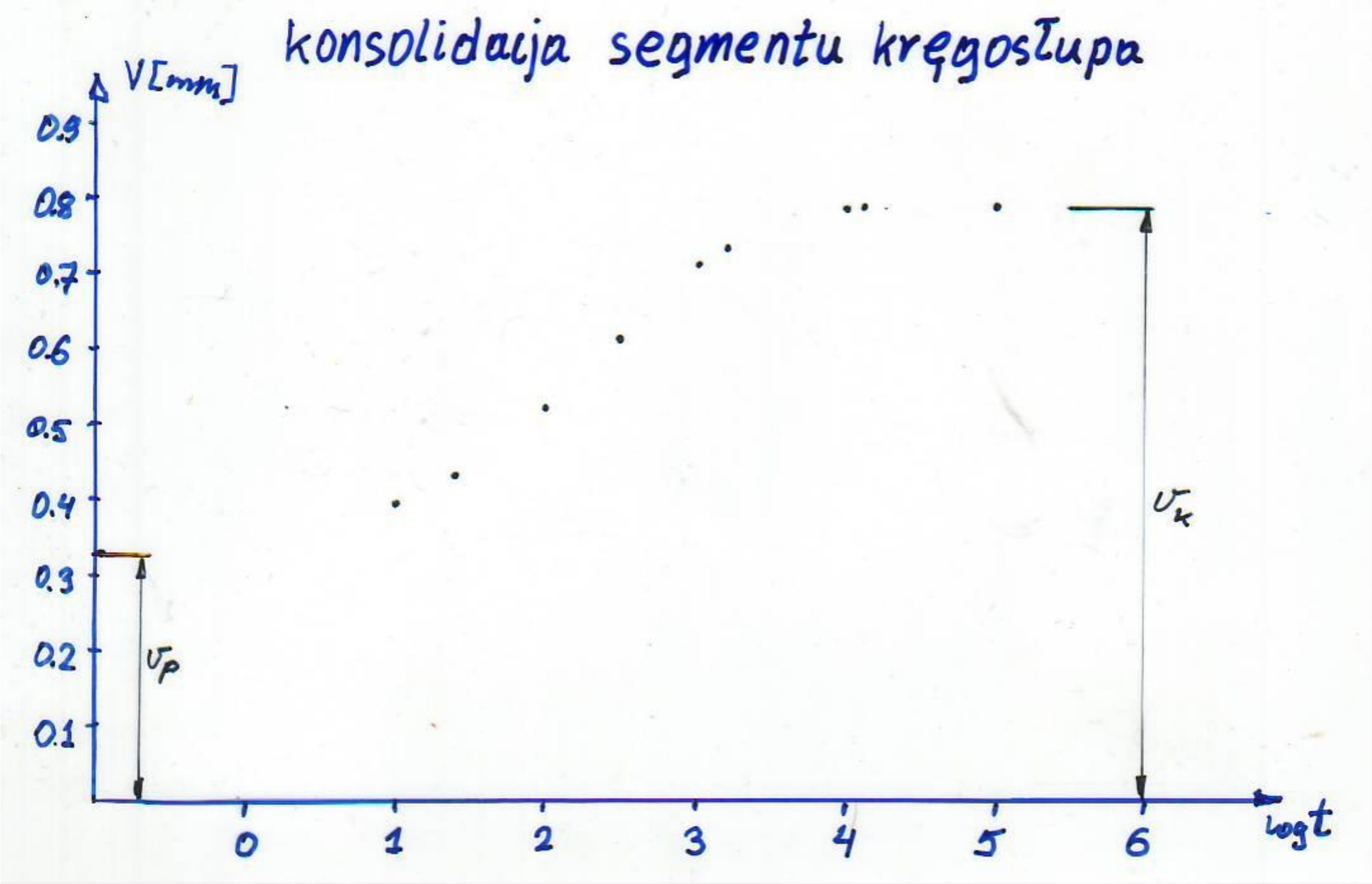


Body deformation t=0



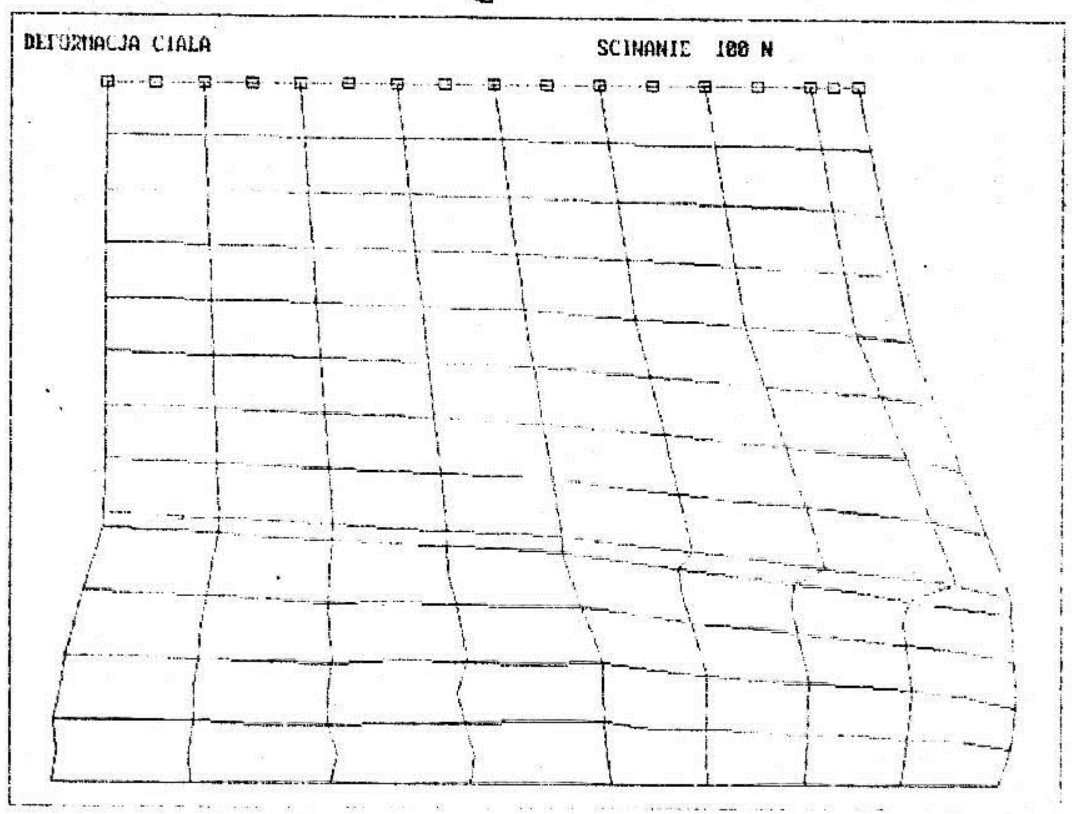
Body deformation $t=\infty$

Zmiana wysokości w czasie



Ścinanie siłą 100 N

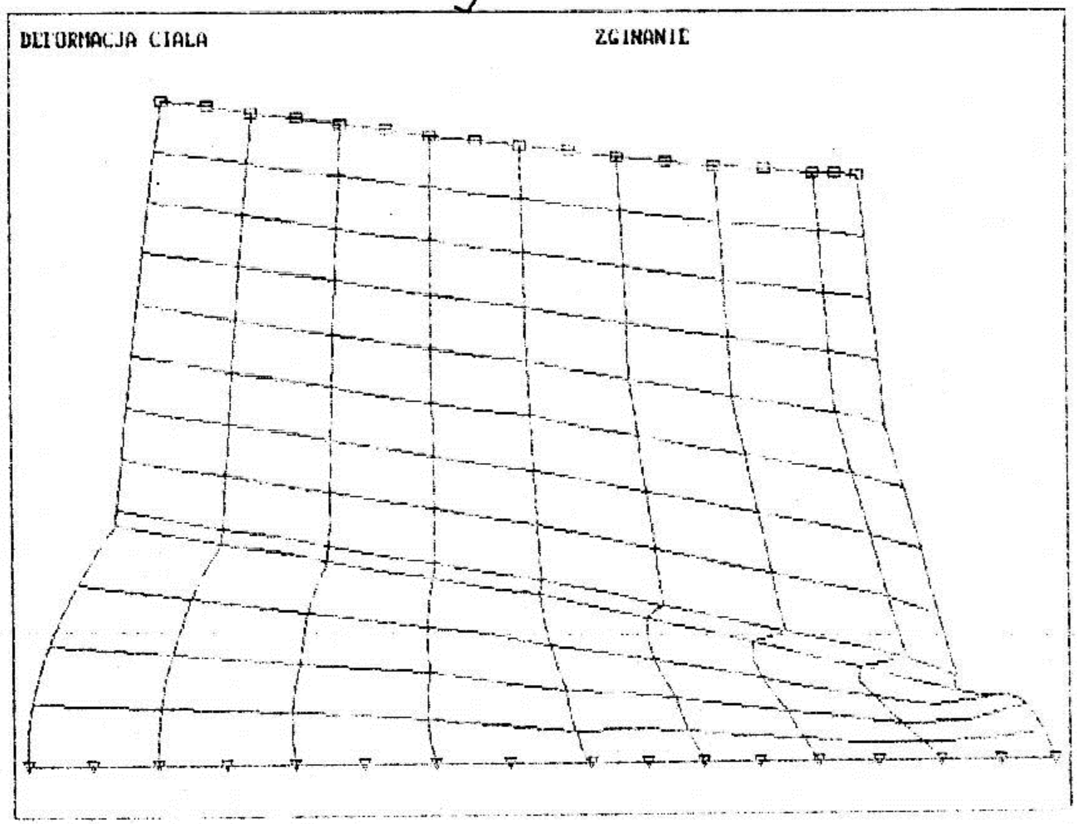
shearing load



Body deformation, t = 0

Zginanie t = ∞

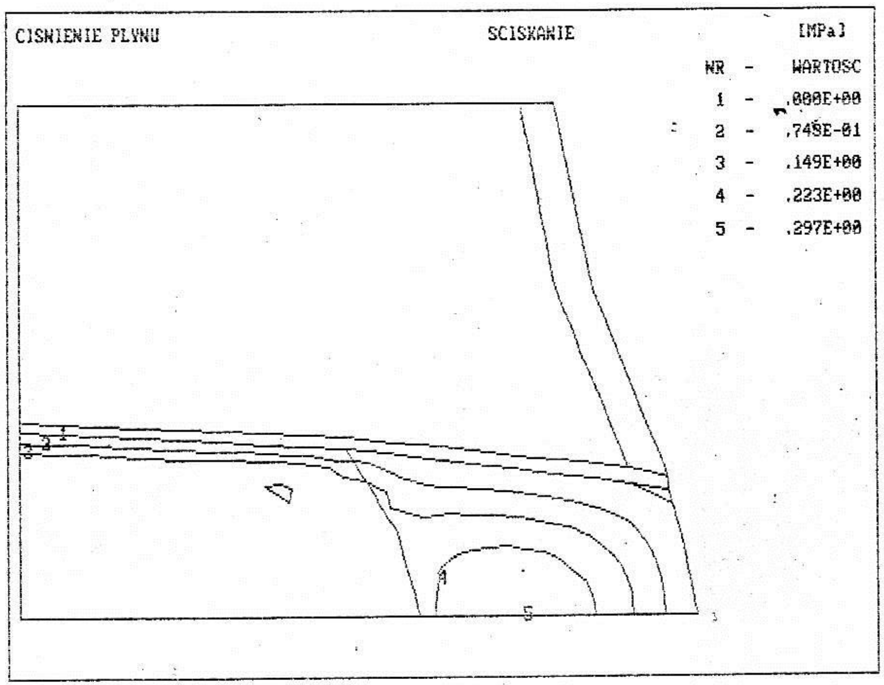
bending load



Body deformation. t = ∞

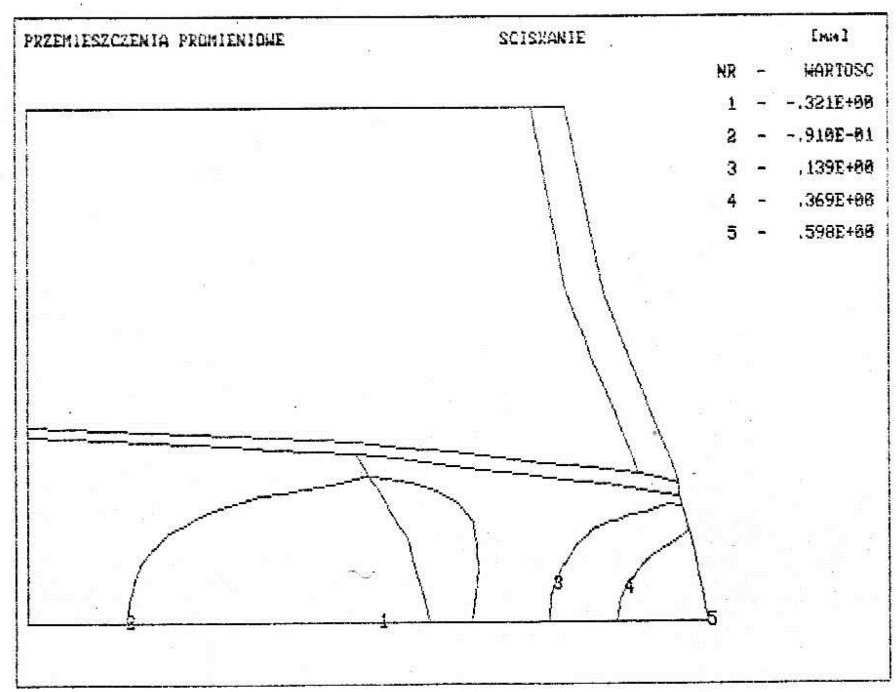
Ściskanie dla czasu t = 1000 s

Rozkład ciśnienia płynu



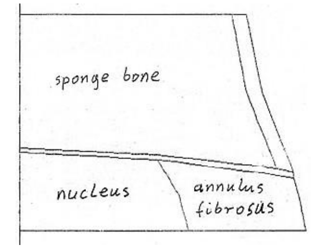
Fluid pressure distribution t = 1000 s

Przemieszczenia promieniowe



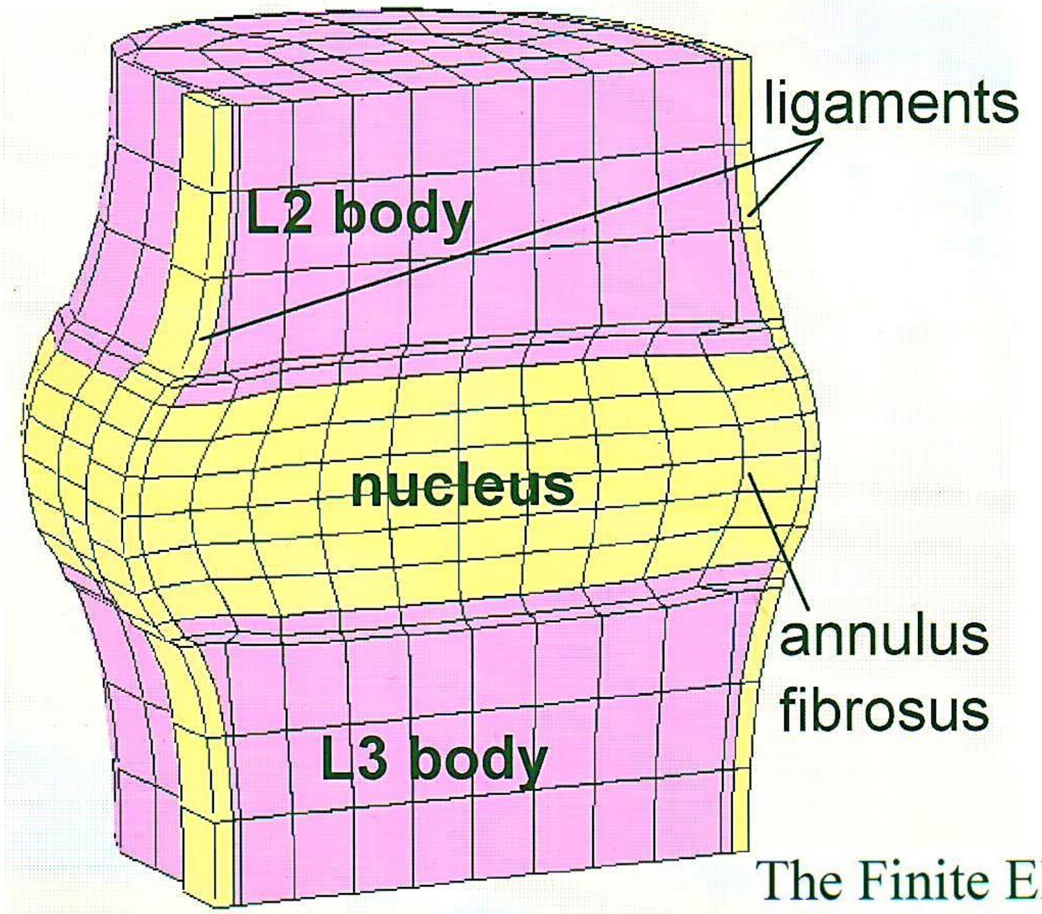
Radial displacements.

Weryfikacja wyników z danymi eksperymentalnymi



type of load	results FEM	stiffness FEM	1	2
compression	$u_z^b = -0.251$ $u_z^e = -0.799$ $u_r^b = 0.621$ $u_r^e = 0.598$	2000 N/mm 630	556 2100-3500 (bulge) 0.5 - 0.9	667
bending	$\phi^b = 1.6^\circ$ $\phi^e = 2.2^\circ$	3.0 Nm/deg 2.3	2.8 - 5.2	0.7 - 1.4
shearing	$u_r^b = 0.462$ $u_r^e = 0.517$	108 N/mm 97	31 - 82	85
torsion	$\theta^b = \theta^e = 3.9^\circ$	1.3 Nm/deg	2	2.5

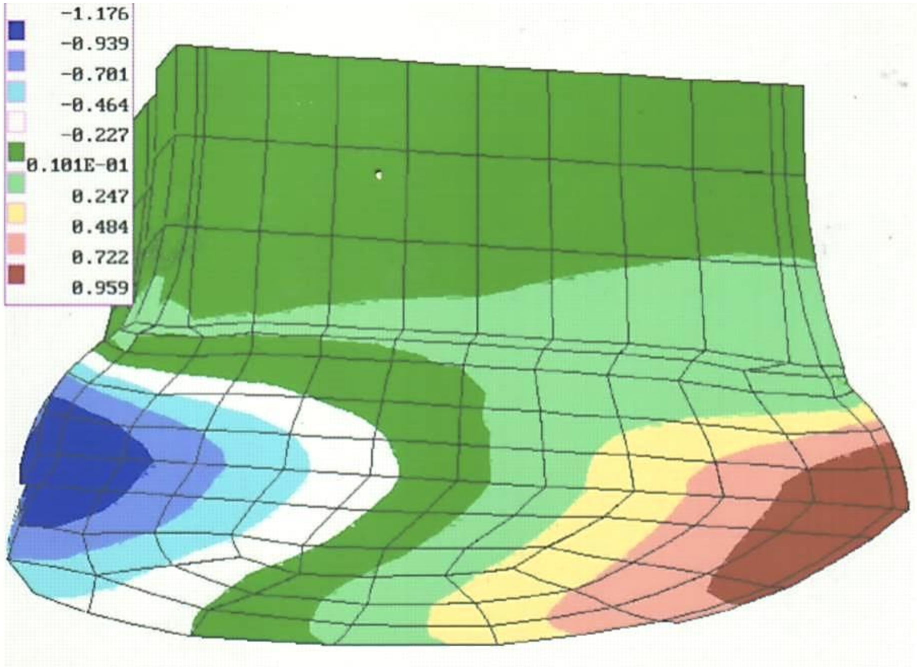
Model MES dysku 3D



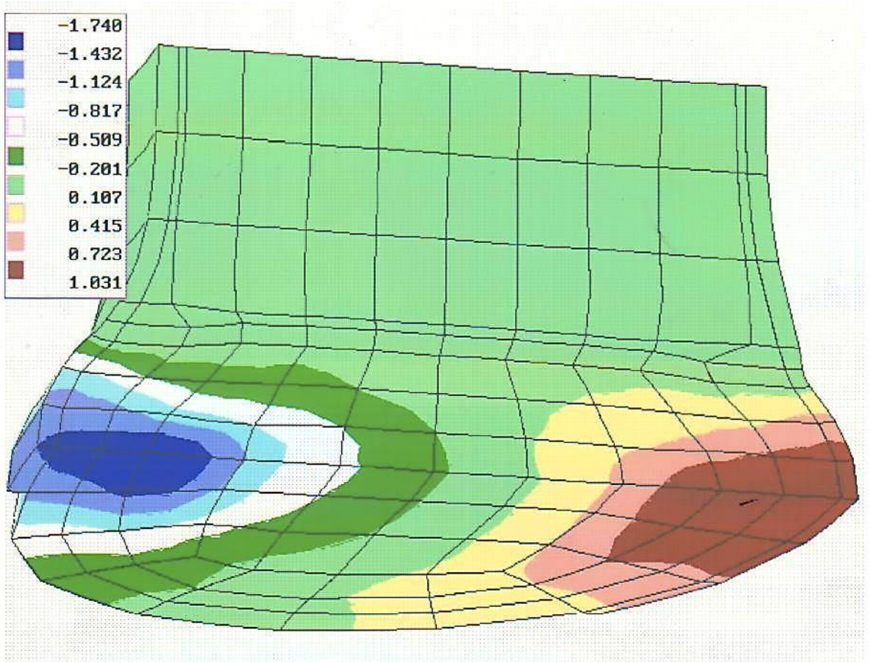
The Finite Element Method model of the disc
(688 twenty-node elements)

Przemieszczenia w płaszczyźnie strzałkowej

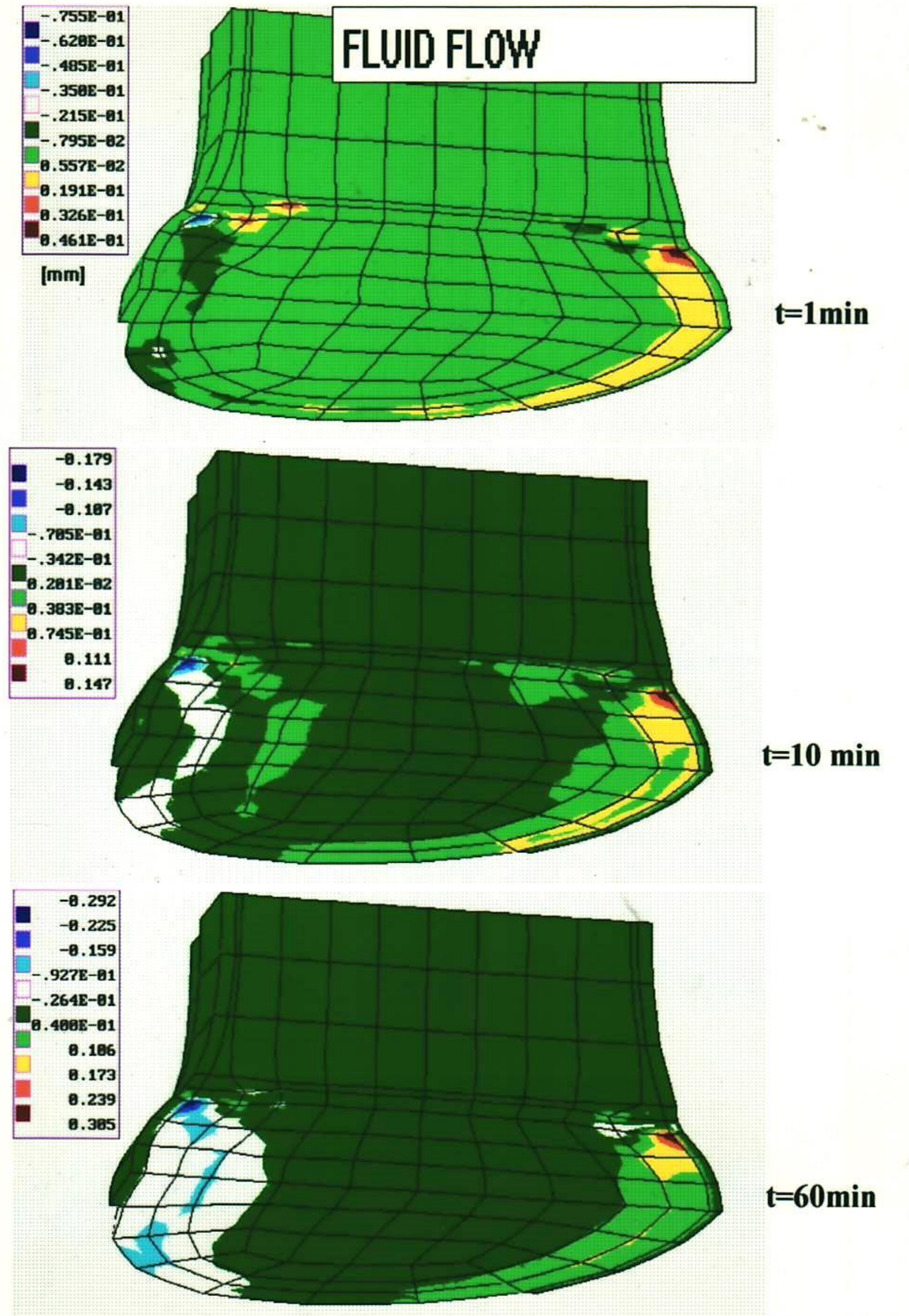
$t = 0$



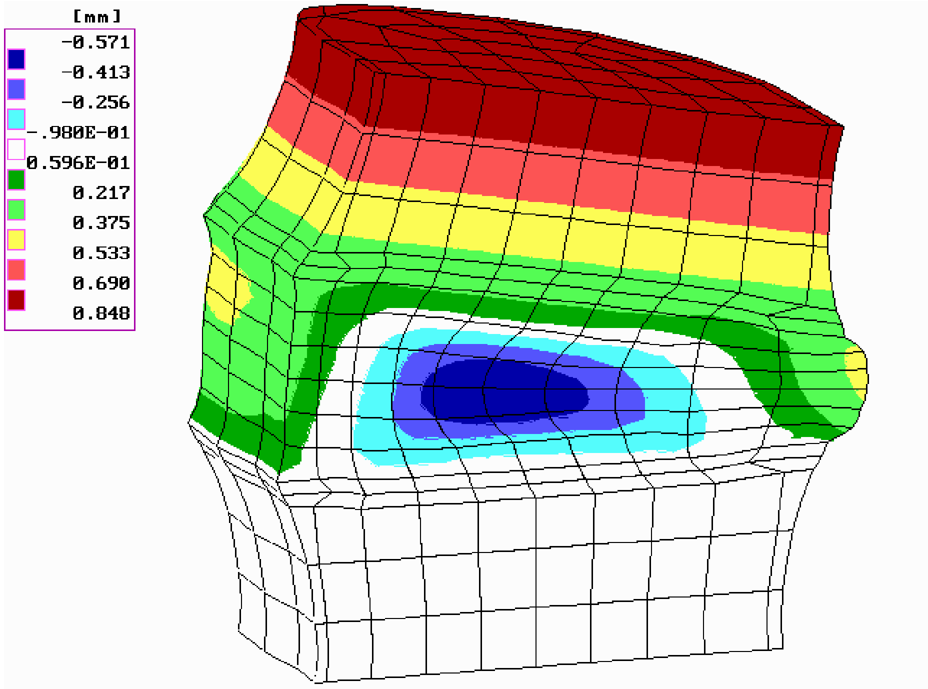
$t = \infty$



Anterior-posterior displacement

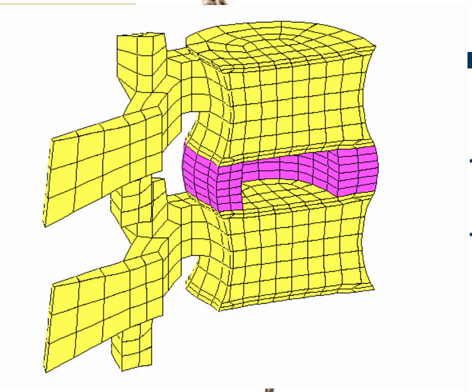

FLUID FLOW
-.755E-01
-.620E-01
-.485E-01
-.350E-01
-.215E-01
-.795E-02
0.557E-02
0.191E-01
0.326E-01
0.461E-01
[mm]
t=1min
-0.179
-0.143
-0.107
-.705E-01
-.342E-01
0.201E-02
0.383E-01
0.745E-01
0.111
0.147
t=10 min
-0.292
-0.225
-0.159
-.927E-01
-.264E-01
0.400E-01
0.106
0.173
0.239
0.305
t=60min

Przemieszczenia dysku w skłonie do przodu



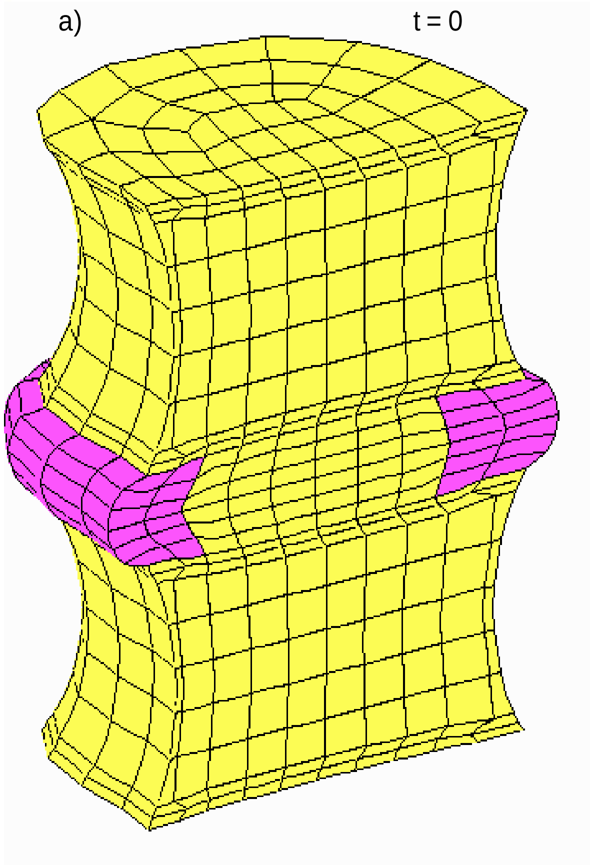
deformacja x 5

Przemieszczenia w kierunku osi strzałkowej (przednio - tylne)

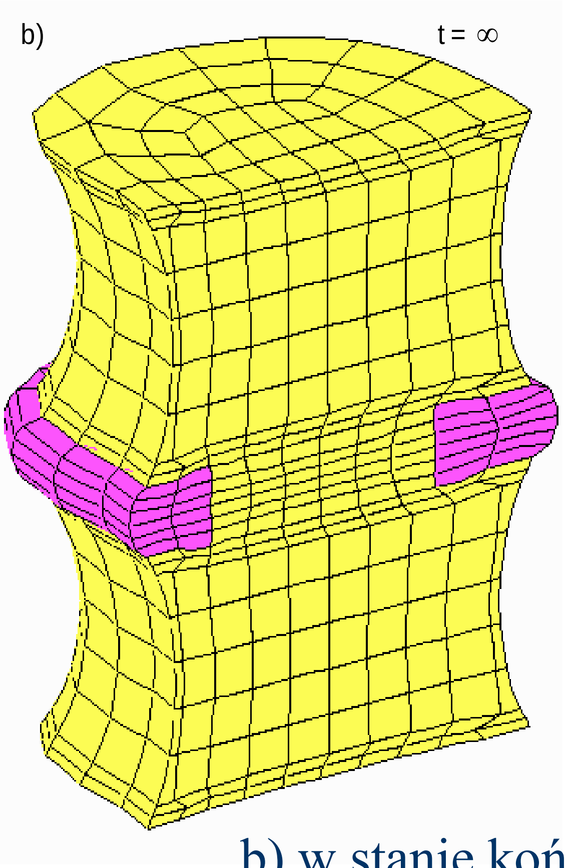


Deformacja dysku L4-L5

(powiększona 5 razy)



a) w chwili przyłożenia siły

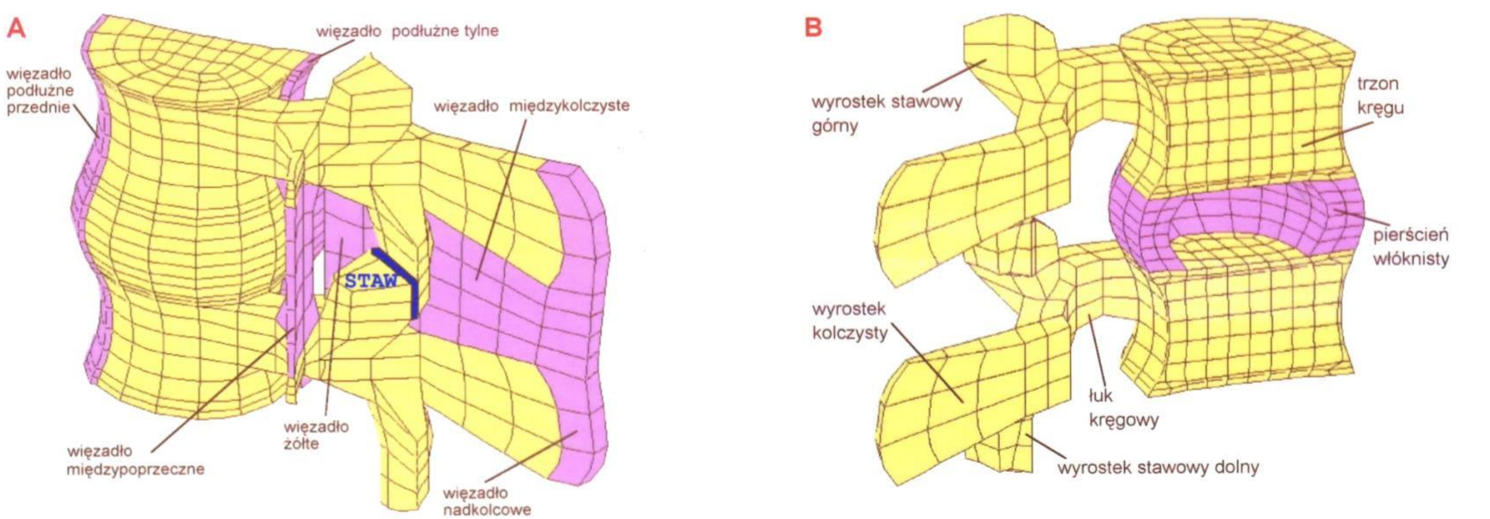


b) w stanie końcowym (bez ciśnienia płynu)

MODELOWANIE I BADANIE SEGMENTU RUCHOWEGO KRĘGOSŁUPA Z UWZGLĘDNIENIEM POROSPRĘŻYSTOŚCI TKANEK

Najważniejsze zagadnienia uwzględnione w modelu

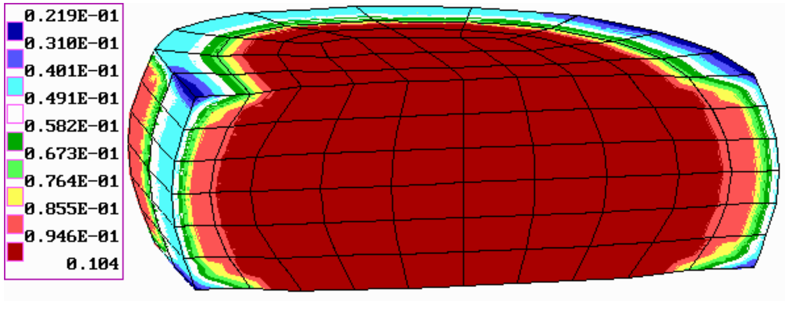
1. ciśnienie w płynie
2. anizotropowa struktura pierścienia włóknistego
3. ciśnienie wchłaniania (osmotyczne)



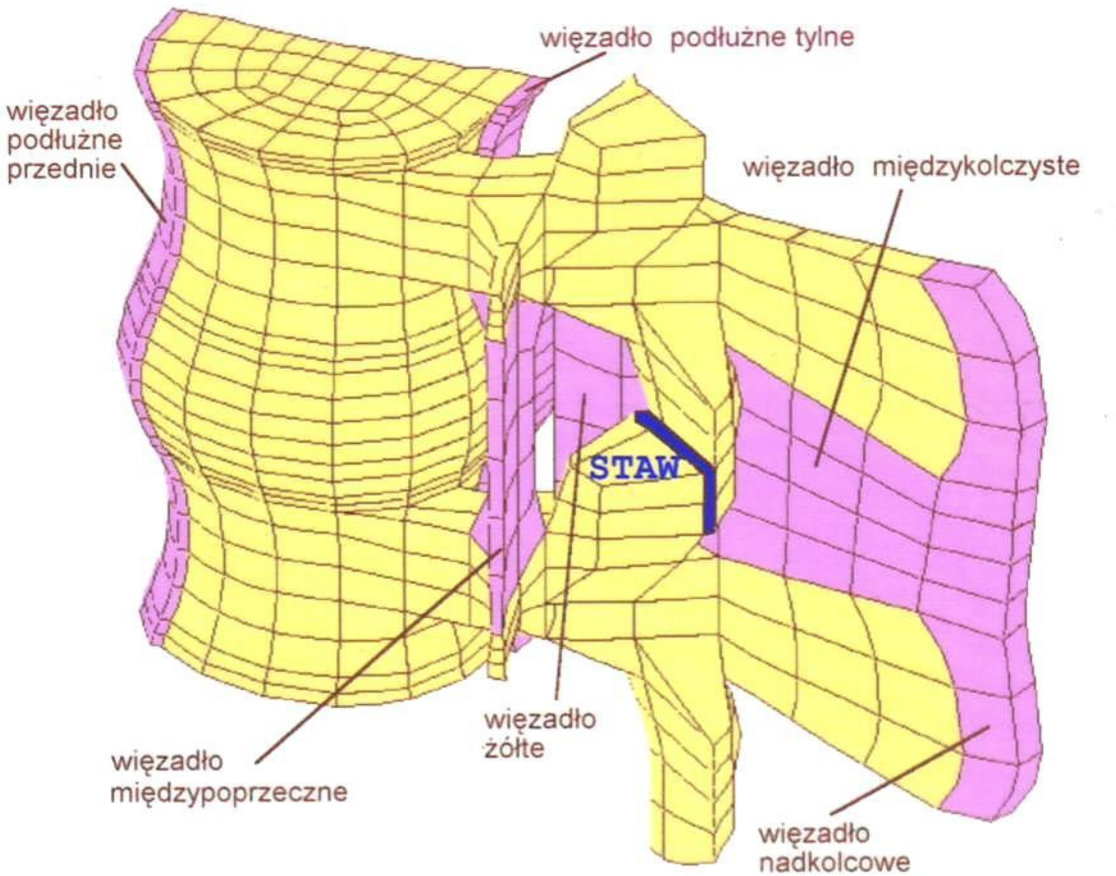
Model dysku

Założenia dotyczące materiału poro-sprężystego:

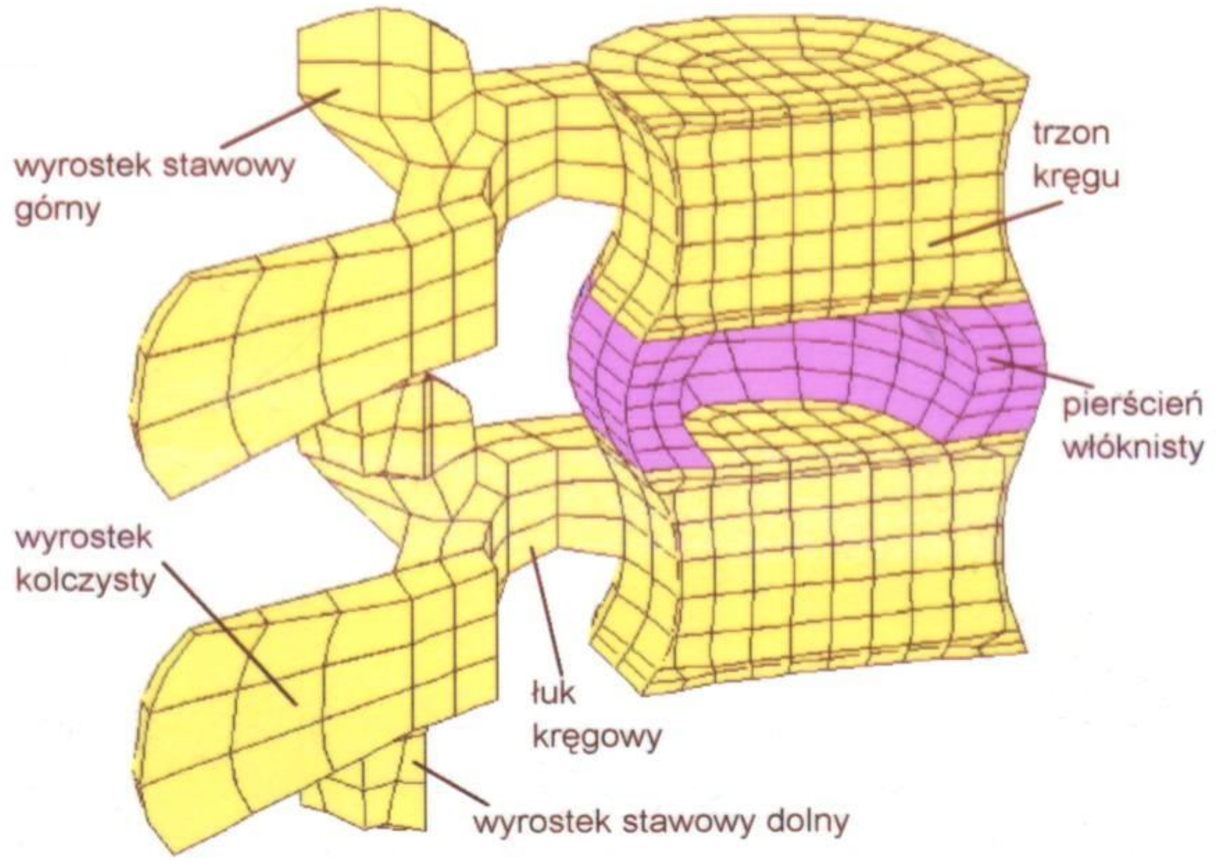
a) jednorodność;
b) skończone odkształcenia;
c) anizotropię, materiał podlega prawu Hooke'a;
d) że przepływem płynu rządzi liniowe prawo Darcy'ego;
e) zależność przepuszczalności ośrodka od porowatości (deformacji);
f) nielepkość płynu;
g) nieściśliwość faz;
h) równowagę wolnych jonów w płynie.



Model segmentu ruchowego



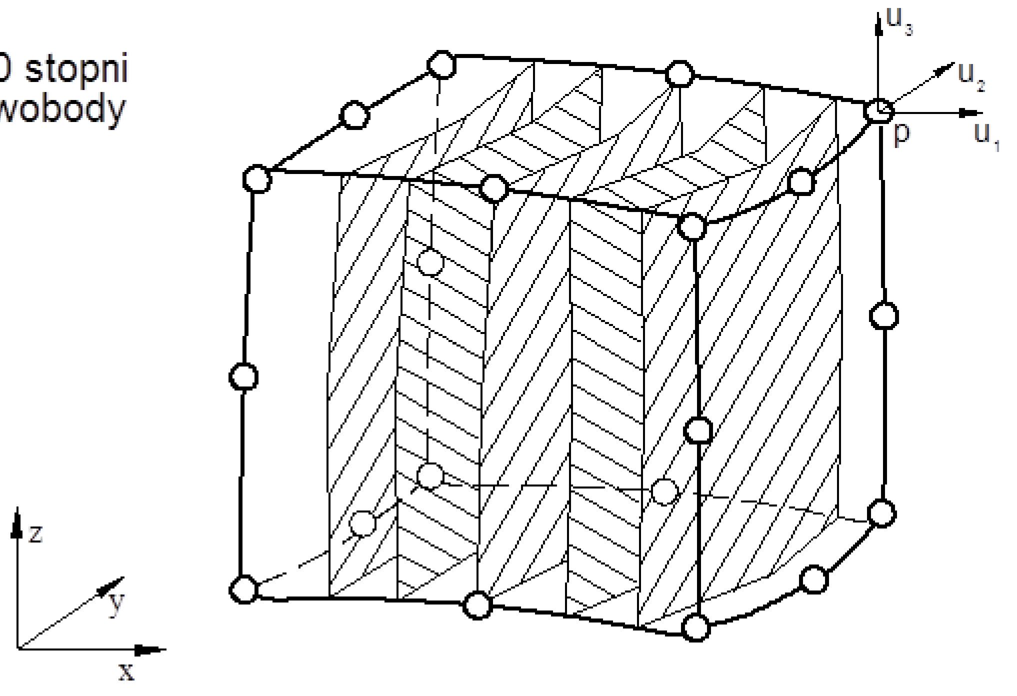
Model segmentu ruchowego



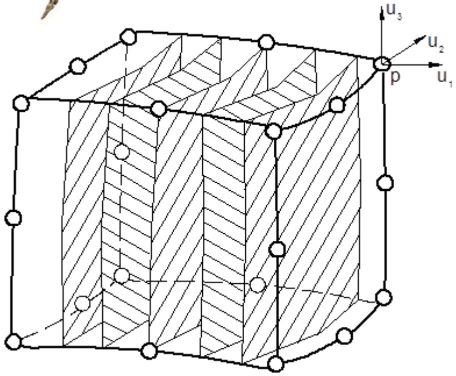
Na rys. pominięto więzadła i jądro miażdżyste

Włókna - specjalny element kompozytowy

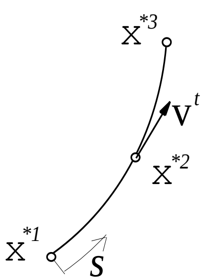
80 stopni
swobody

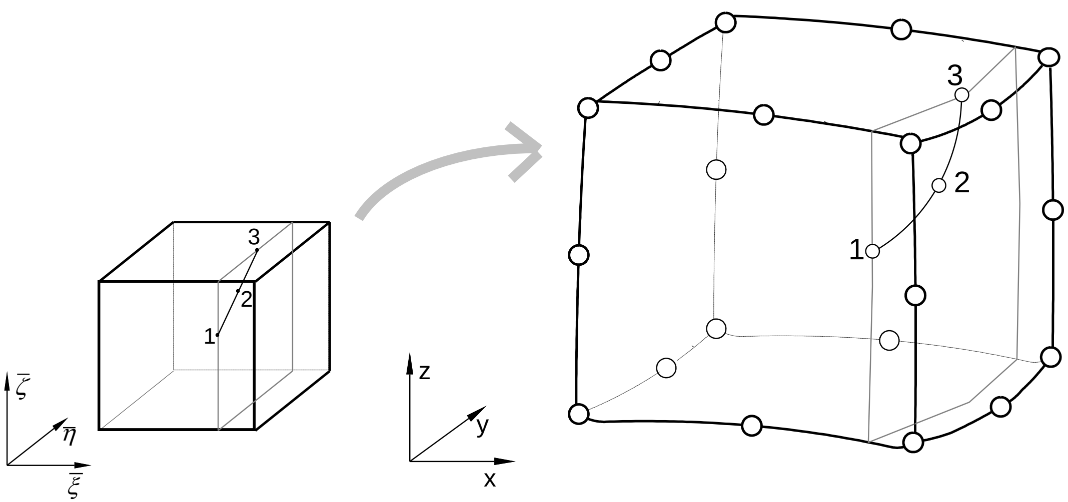


Opracowanie elementu



Wszystkie zależności wyznaczone zostały analitycznie, bez stosowania całkowania numerycznego.



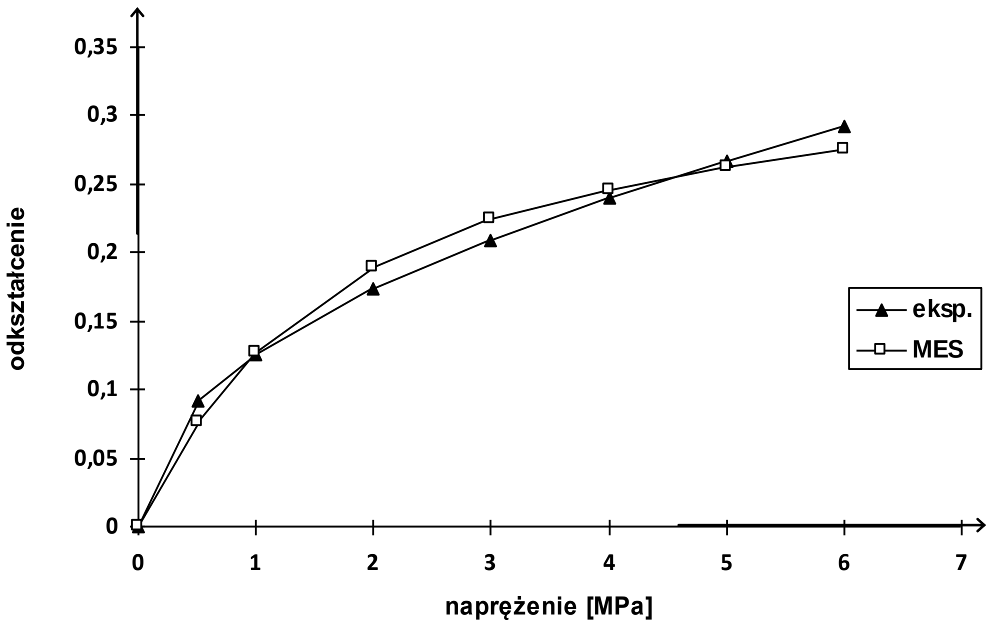


Element rodzimy

Element dziedziczny

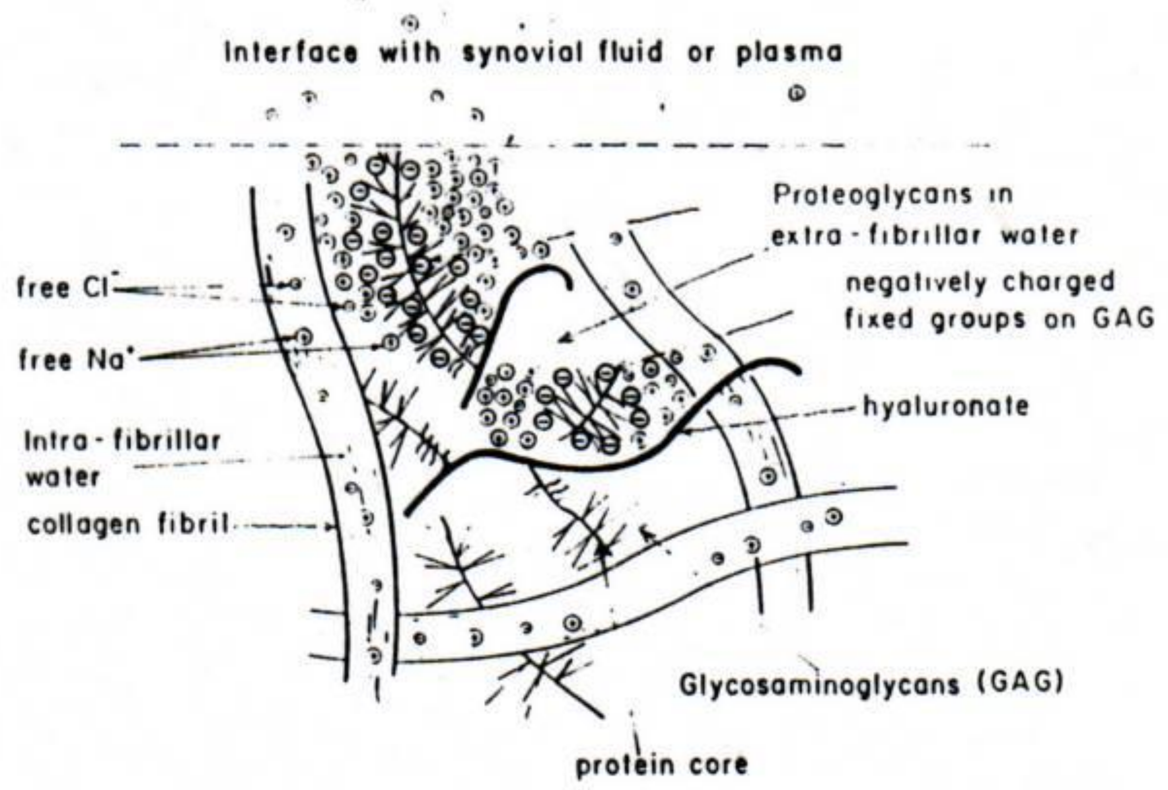
Weryfikacja elementu zbrojonego włóknami

Wyniki doświadczalne rozciągania fragmentów pierścienia włóknistego



E_g = 1 MPa - moduł Younga materiału osnowy,
ν_g = 0,2 - stała Poissona materiału osnowy,
EA = 500 N - parametr określający sztywność włókien,
α_0 = 42° - początkowy kąt nachylenia włókien.

Model chrząstki stawowej

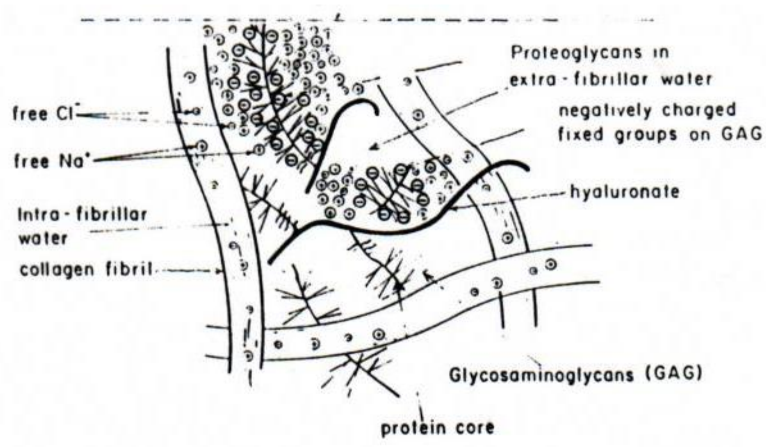


53.1 Schematic representation of cartilage matrix.

Ciśnienie osmotyczne

$$\mathrm{p}^{c} = B\left(FCD_{efekt.}\right)^{2}$$ - ciśnienie osmotyczne

FCD - gęstość ładunków związanych



$$FCD_{efekt.} = FCD_0 \frac{W_0}{W_{EF}}$$

$$FCD_{efekt.} = FCD_0 \frac{W_0}{W_{H_2O} - W_0 \times C_{kol} \times w_{(IF)}}$$

FCD_0 - gęstość ładunków związanych odniesiona do całej masy tkanki W_0;

W_{H_2O} - całkowita masa wody w tkance;

W_{EF} - masa wody poza włóknami kolagenowymi;

W_0 - całkowita, początkowa masa tkanki;

$w_{(IF)}$ - masa wody we włóknach kolagenowych (niedostępna dla PG) na gram kolagenu;

C_{kol} - masa kolagenu na gram całkowitej, początkowej masy tkanki.

Urban - 1979

Ciśnienie osmotyczne

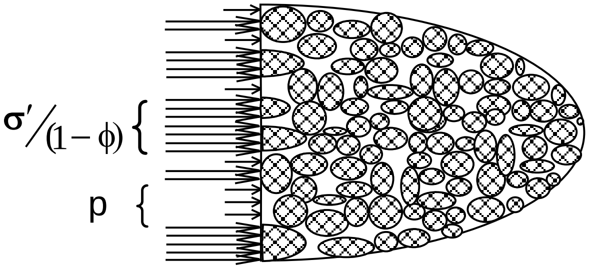
$$w_{(IF)} = 0{,}76 + 0{,}85\exp\left(-3{,}8 \times \mathrm{p}^{c}\right)$$

$$FCD_{efekt.} = FCD_0 \frac{1}{\phi J - C_{kol} \times w_{(IF)}}$$

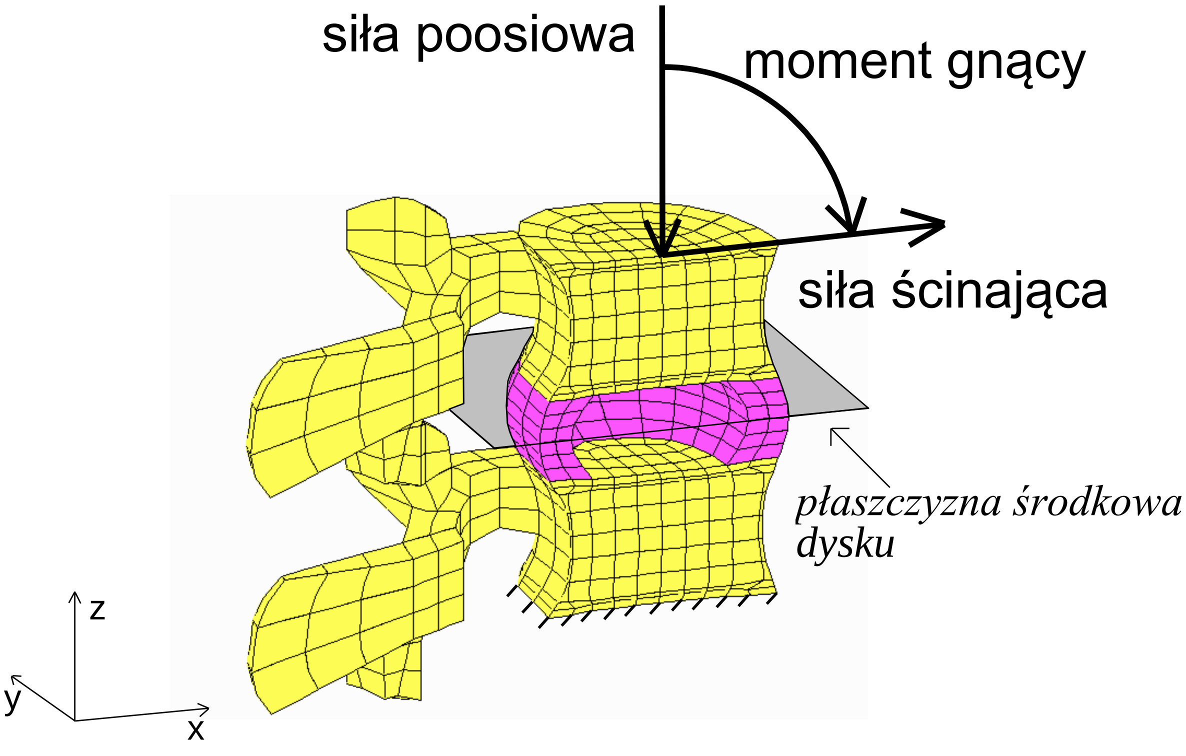
W modelu MES ciśnienie osmotyczne uwzględniono

$$\mathrm{p}' = \phi\left(\mathrm{p} + \mathrm{p}^{c}\right)$$

Warunek dodatkowy $\quad \mathrm{p}' \geq \phi \mathrm{p}^{c}$

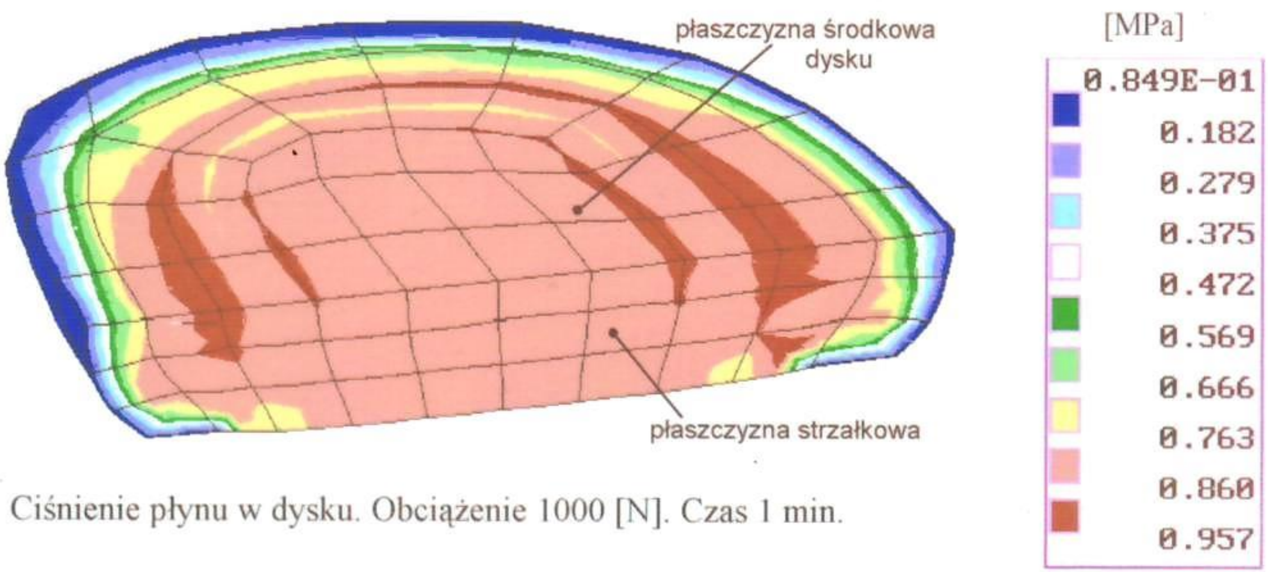


Obciążenie



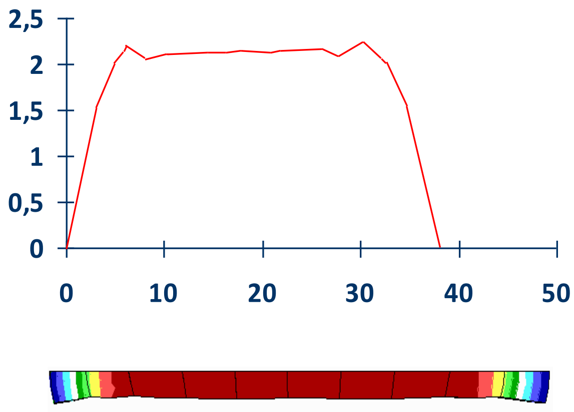
siła poosiowa
moment gnący
siła ścinająca
płaszczyzna środkowa dysku
z
y
x

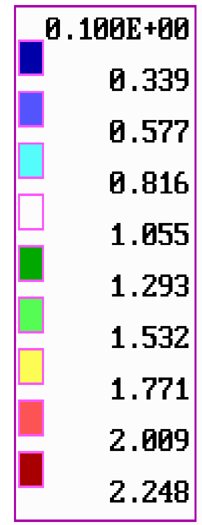
Ciśnienie płynu

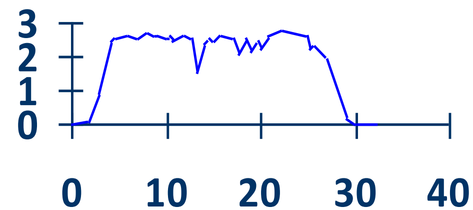


Ciśnienie płynu w dysku. Obciążenie 1000 [N]. Czas 1 min.

Profilogram ciśnienia płynu



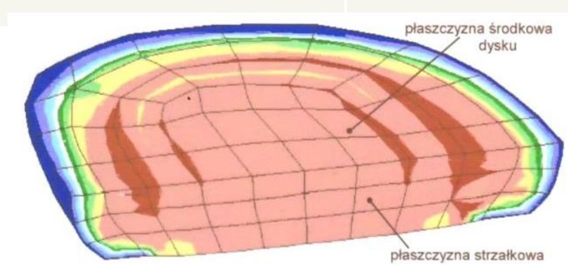




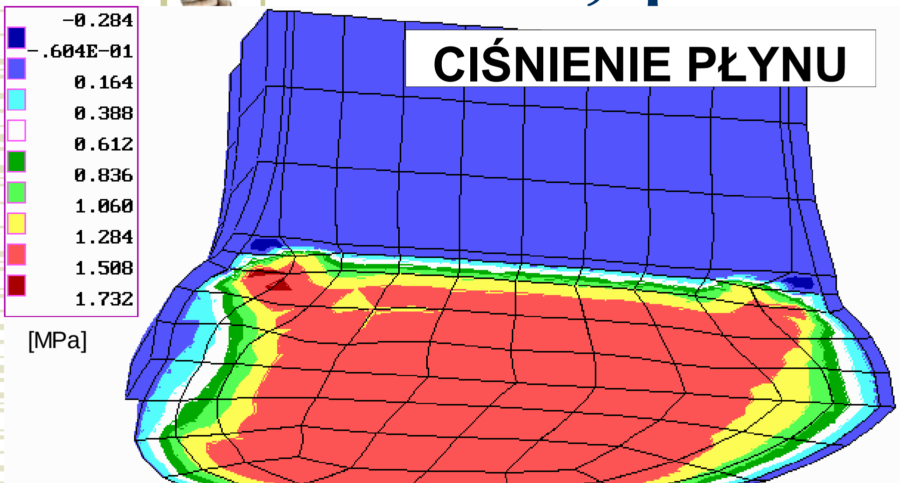
a) wynik obliczeń modelu. Czas 2 min.

[MPa]

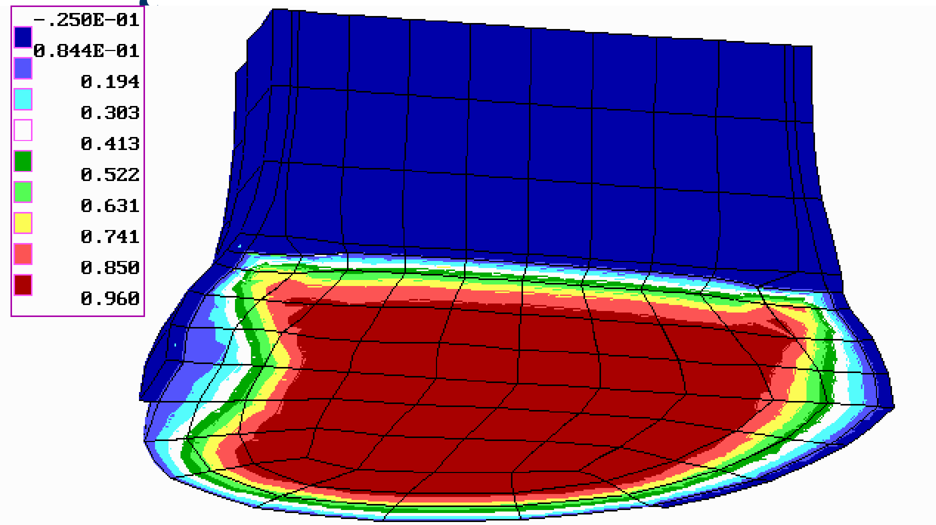
b) przykładowy wynik badań eksperymentalnych [1], dla horyzontalnego położenia miernika.



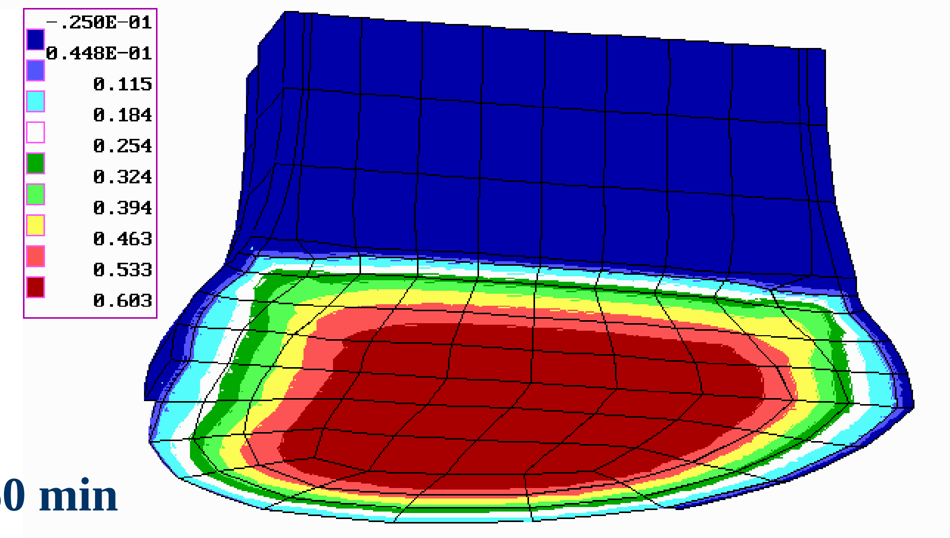
Zmiana ciśnienia płynu w czasie, pod obciążeniem 1000 N



t = 1 min



t = 10 min

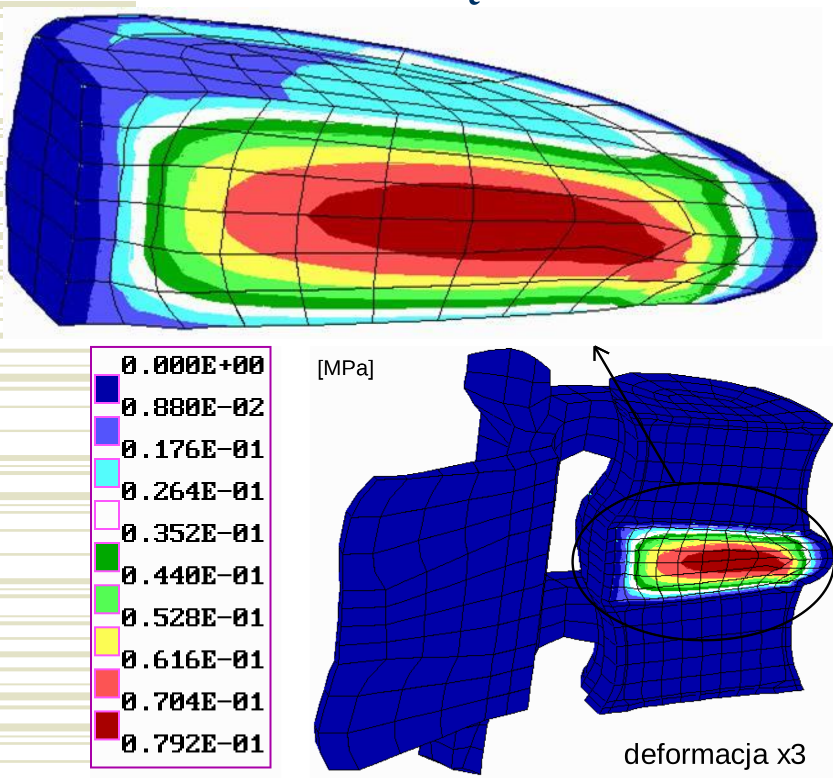


t = 60 min

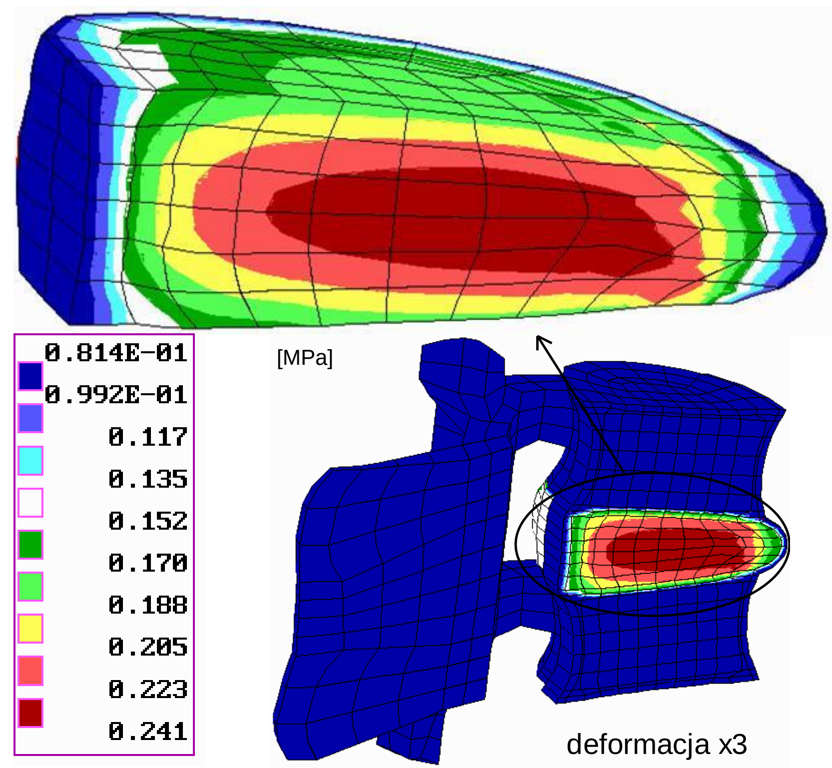
deformacja x 3

Ciśnienie płynu w dysku segmentu ruchowego

Obciążenie momentem siły o wartości 10 Nm



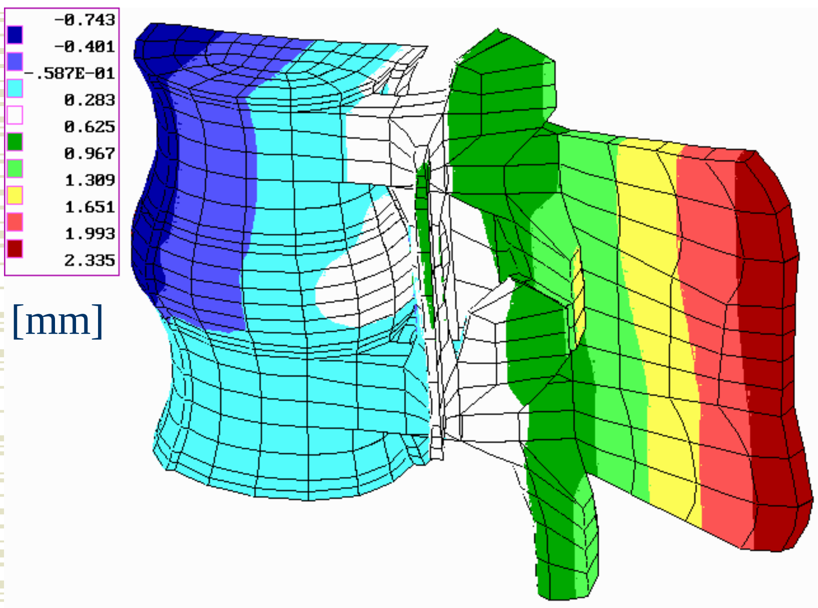
Bez uwzględnienia ciśnienia osmotycznego

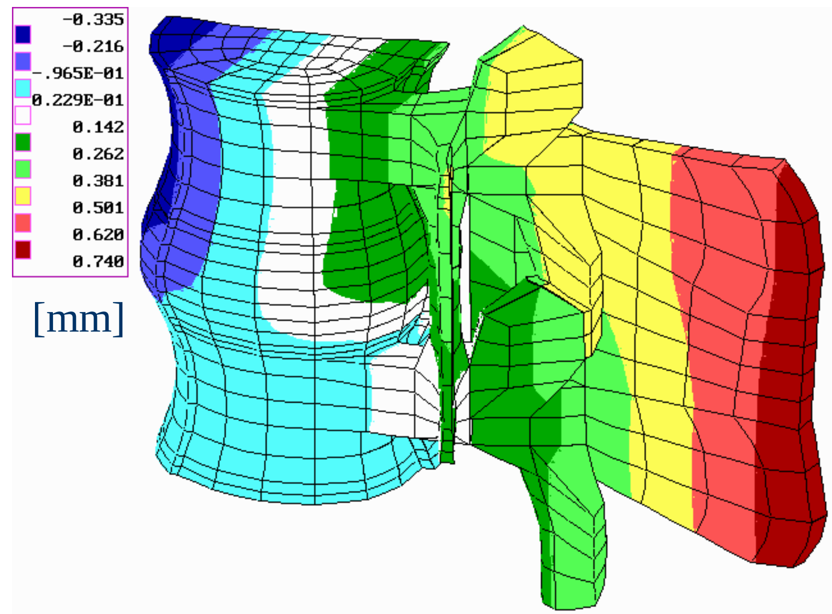


Z uwzględnieniem ciśnienia osmotycznego

Ścinanie w płaszczyźnie strzałkowej

Do górnej powierzchni trzonu kręgu L3 przyłożona została siła pozioma 100 N skierowana do przodu





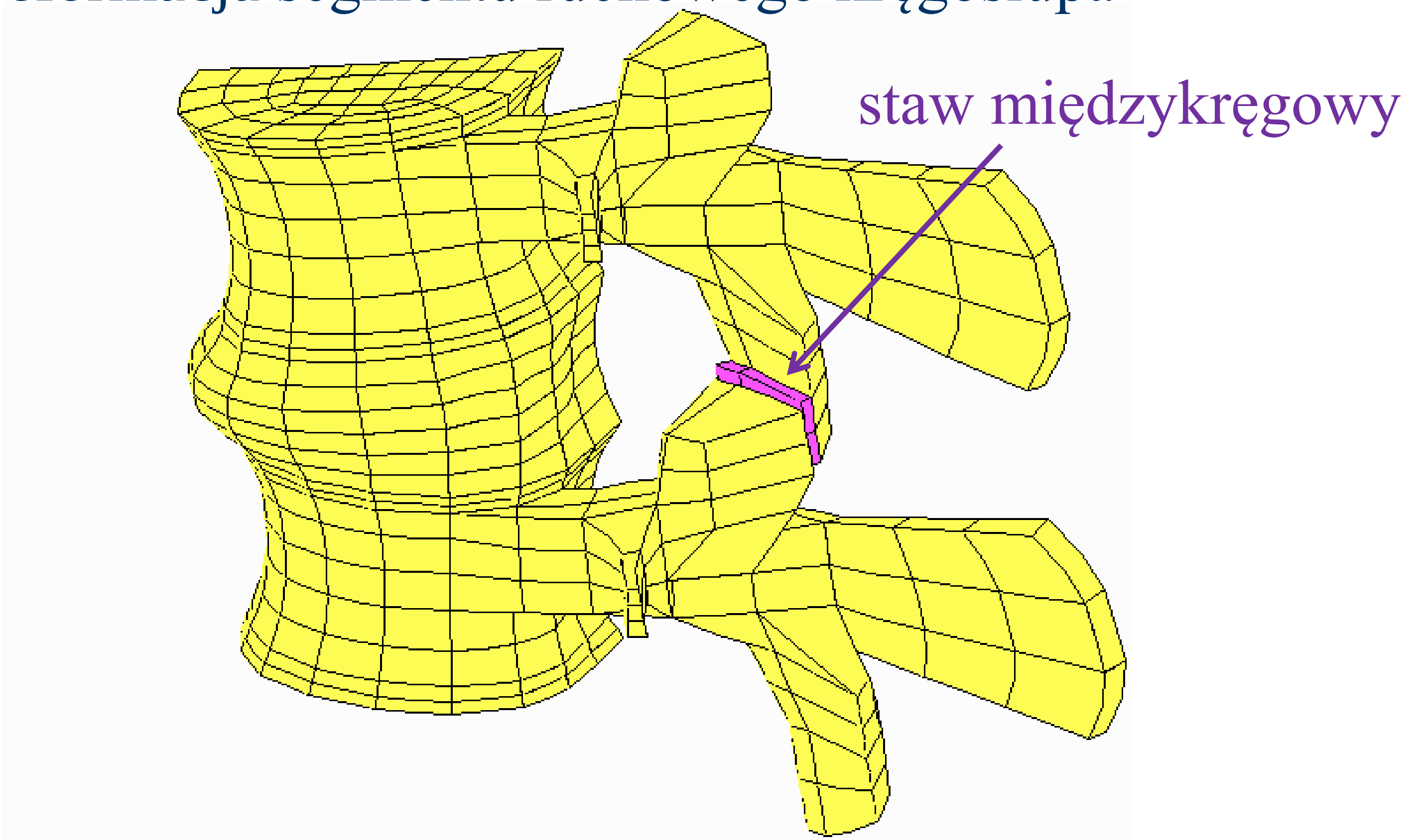
model bez włókien

model z włóknami

Przemieszczenia pionowe

Ścinanie w płaszczyźnie strzałkowej

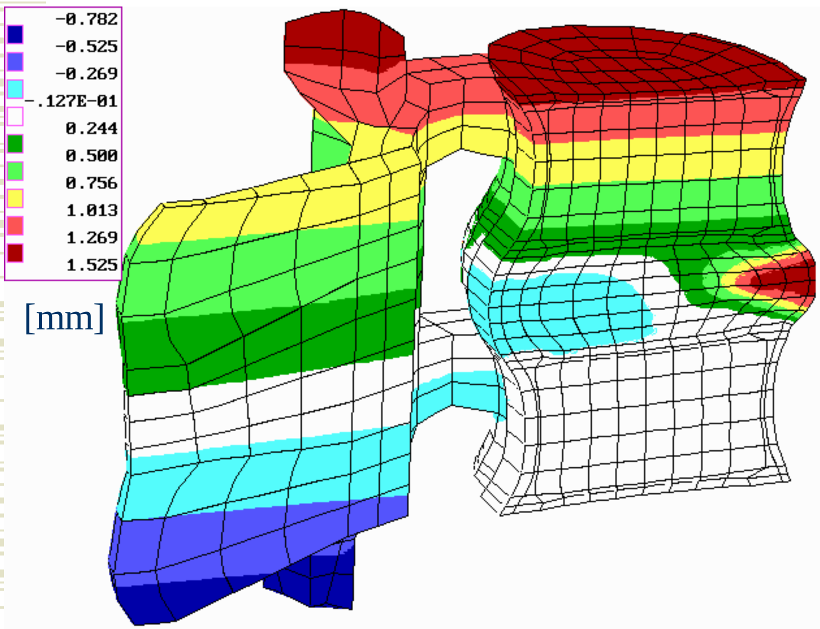
Deformacja segmentu ruchowego kręgosłupa



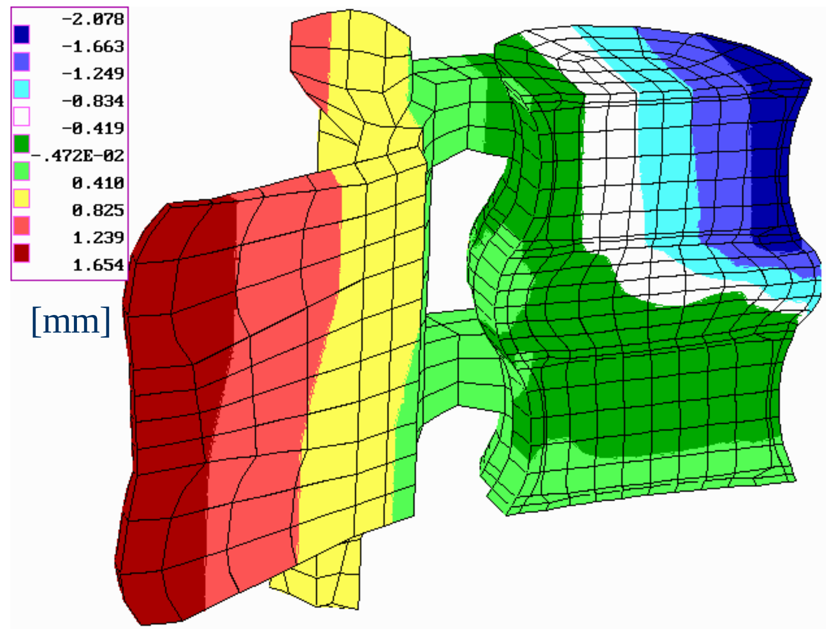
więzadeł nie pokazano

Obciążenie siłą osiową 400 N i momentem gnącym 8 Nm

Przemieszczenia poziome przednio-tylne

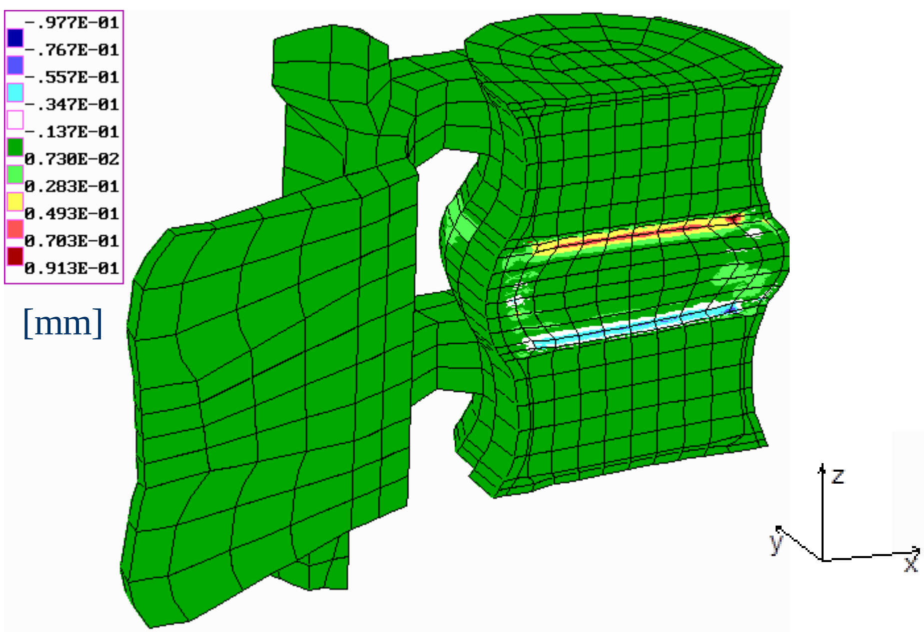


Przemieszczenia pionowe

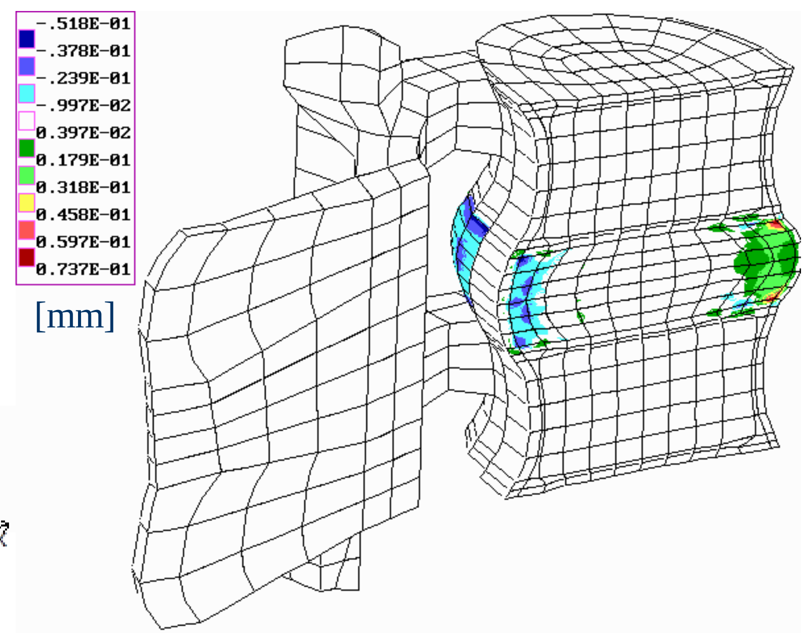


Przemieszczenia płynu. Czas 10 minut

w kierunku osi z



w kierunku osi x

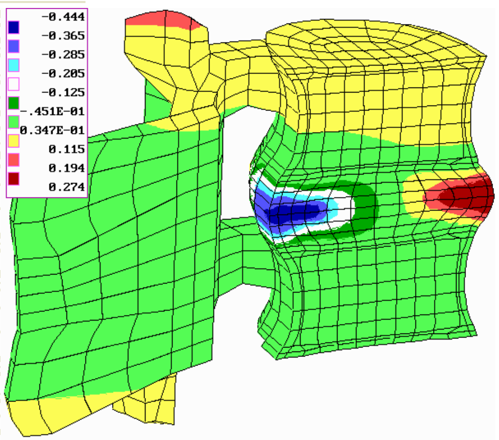


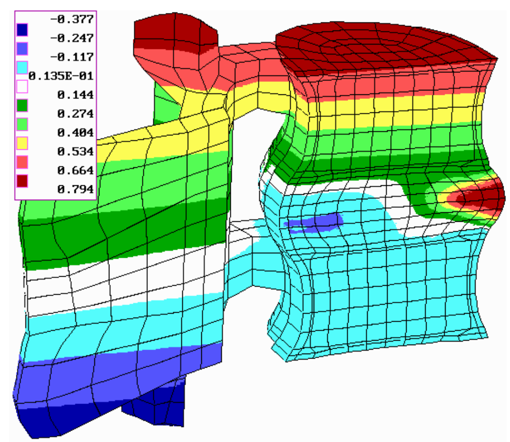
Stan „fizjologiczny” – dysk napęczniał

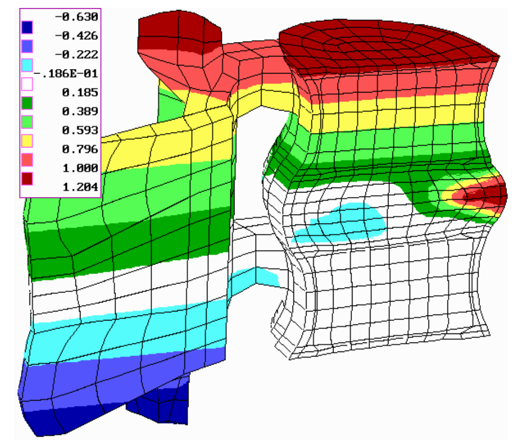
Obciążenie w kolejnych krokach

Obciążenie siłą osiową 400 N

**Obciążenie złożone:
400 N i 6 kN**

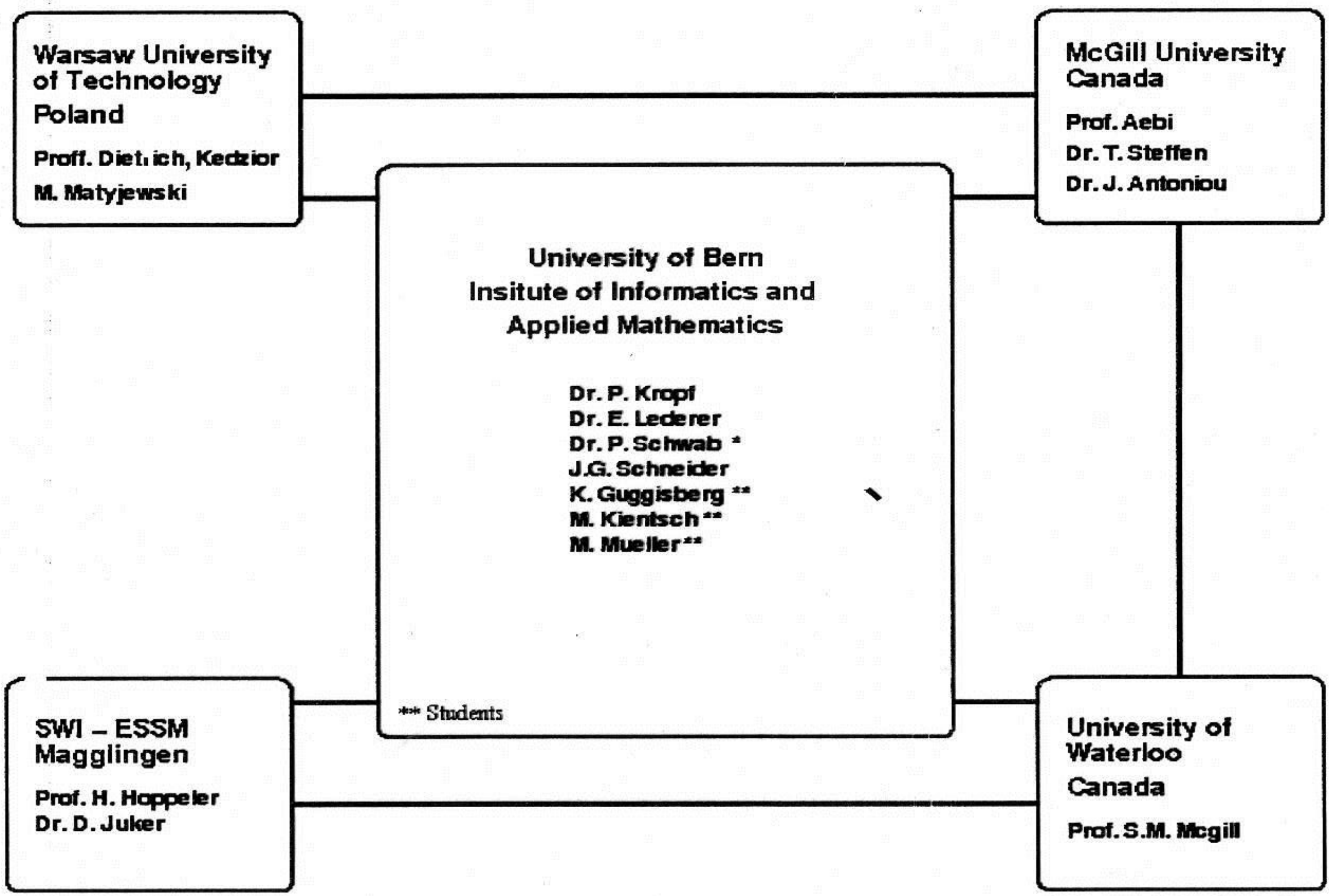




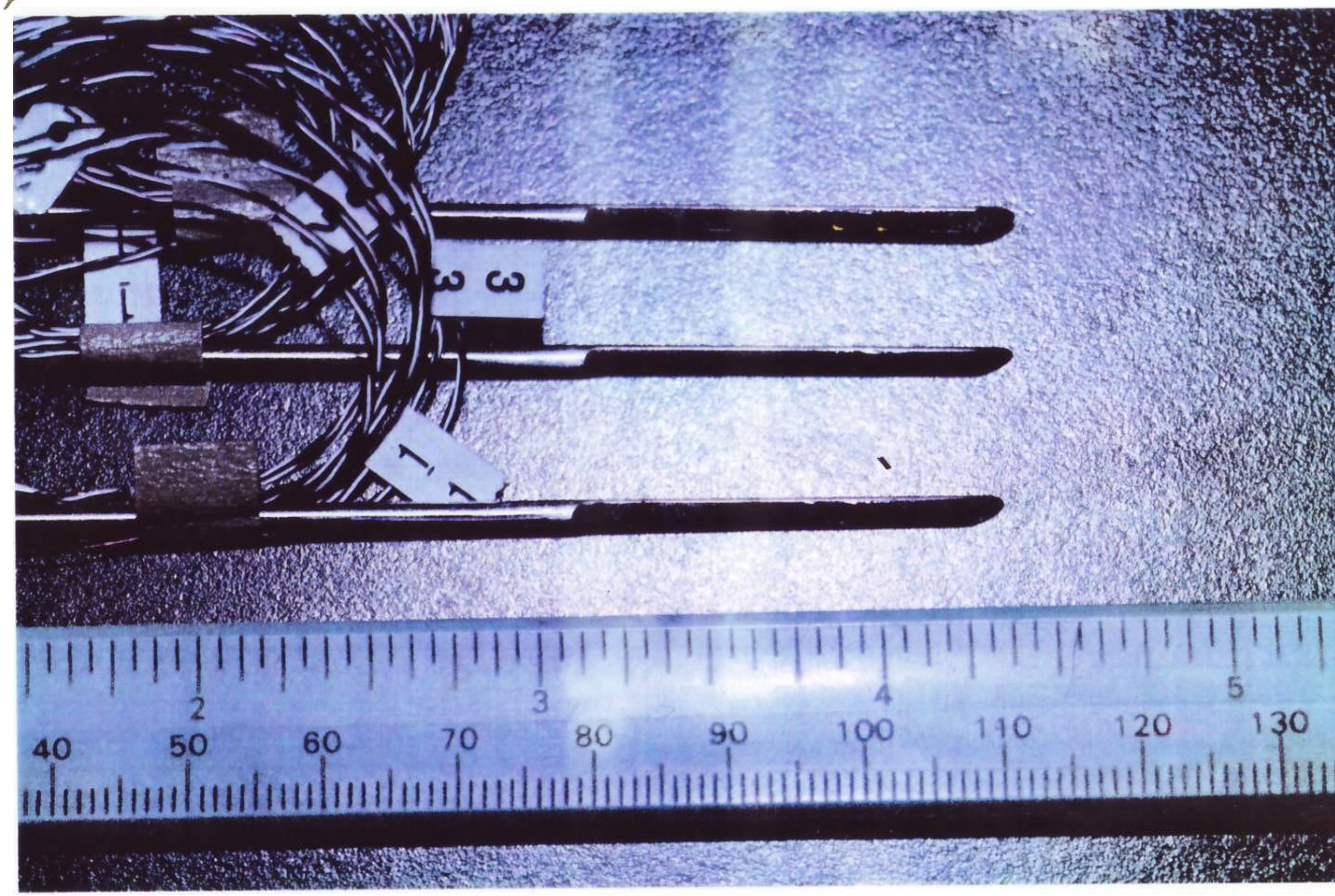


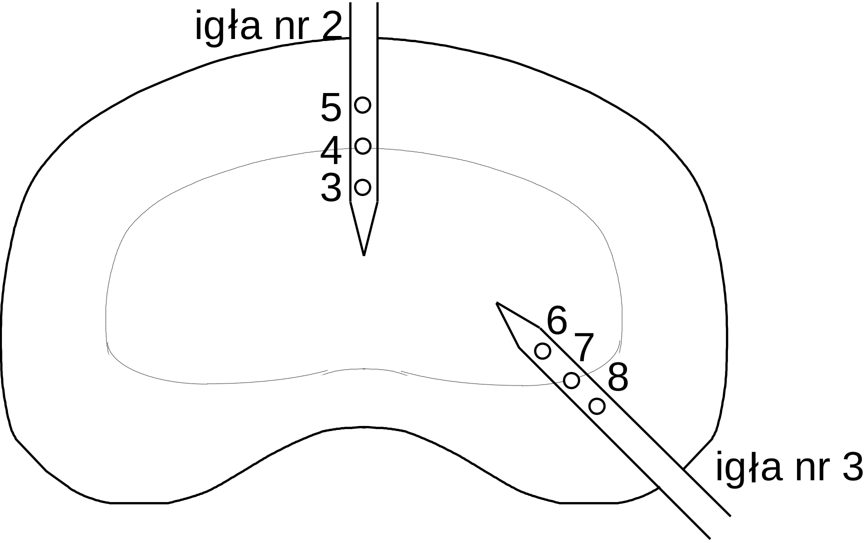
**Obciążenie złożone:
400 N i 3 kN**

Program SPINET

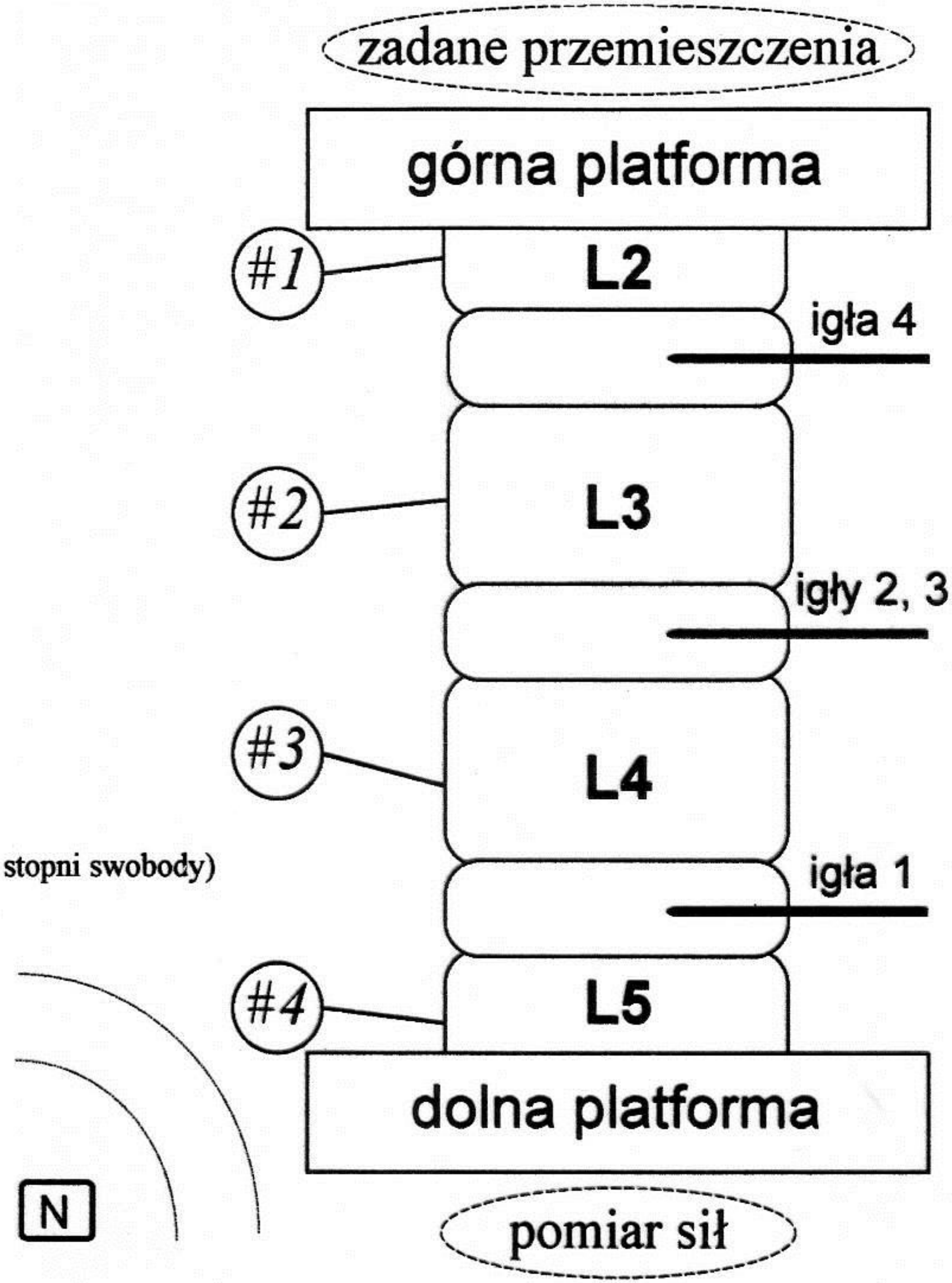


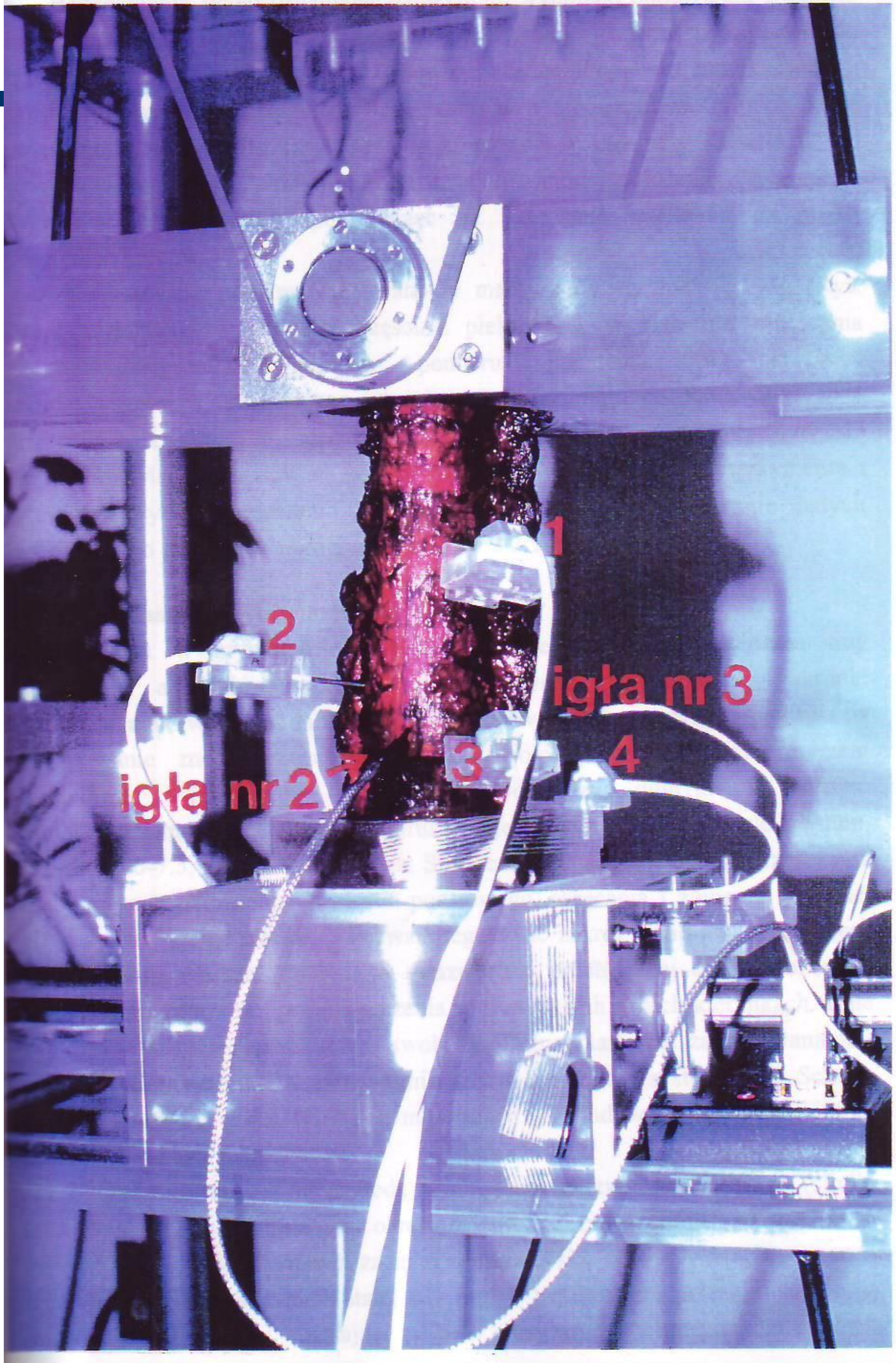
Igły pomiarowe





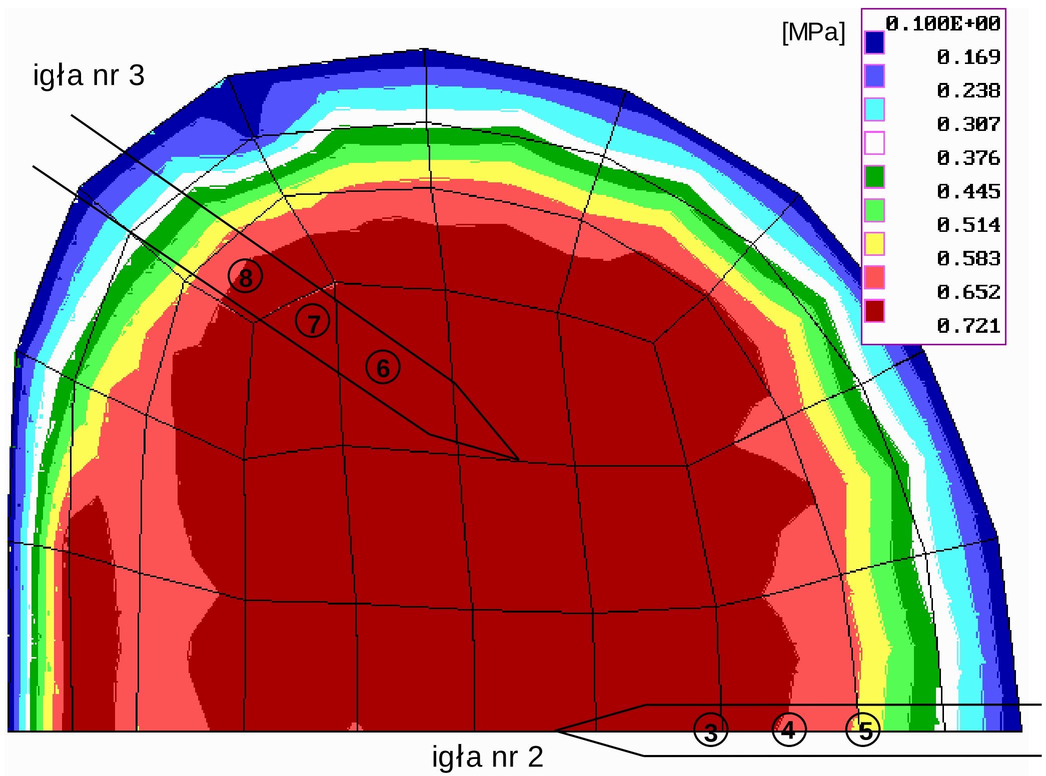
Elektromagnetyczny system śledzenia położenia
N - nadajnik, # - czujniki przemieszczeń i obrotów (6 stopni swobody)




1
2
igła nr 3
3
4
igła nr 2

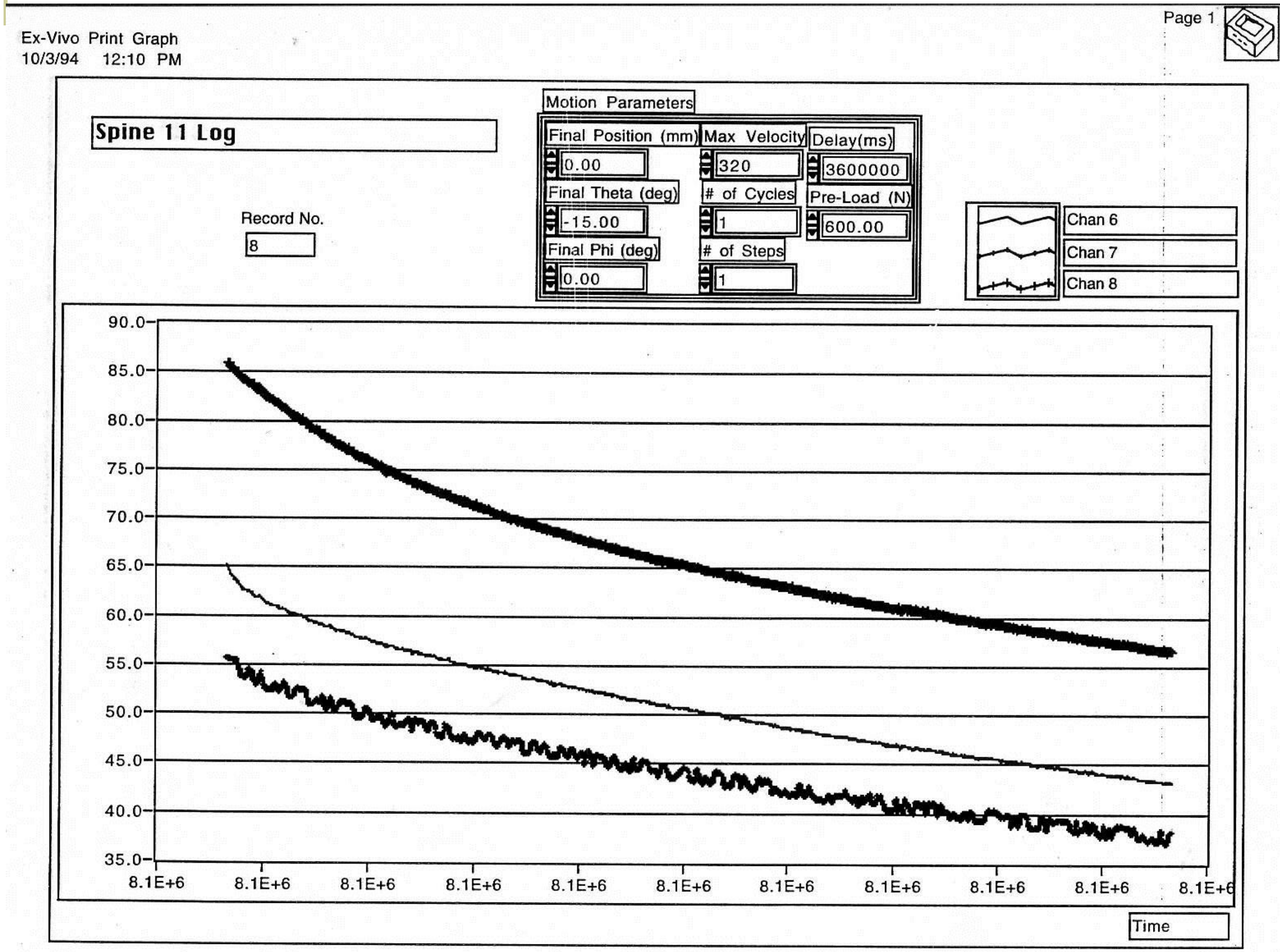
Ciśnienie płynu w płaszczyźnie środkowej dysku

Rozmieszczenie igieł i numery przetworników ciśnienia

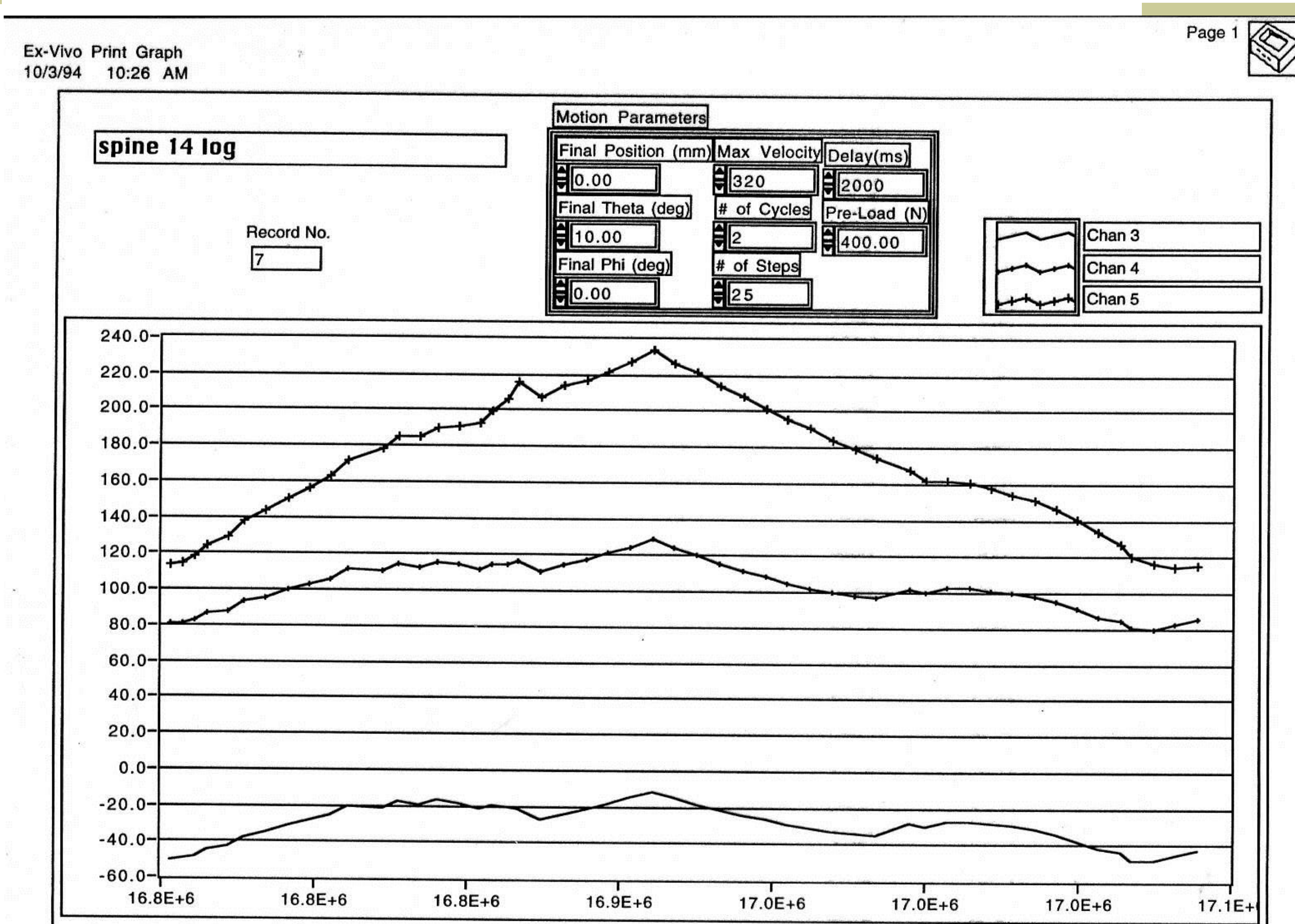


Obciążenie siłą osiową 400 [N] i momentem gnącym 8 [Nm]

Wyniki eksperymentu

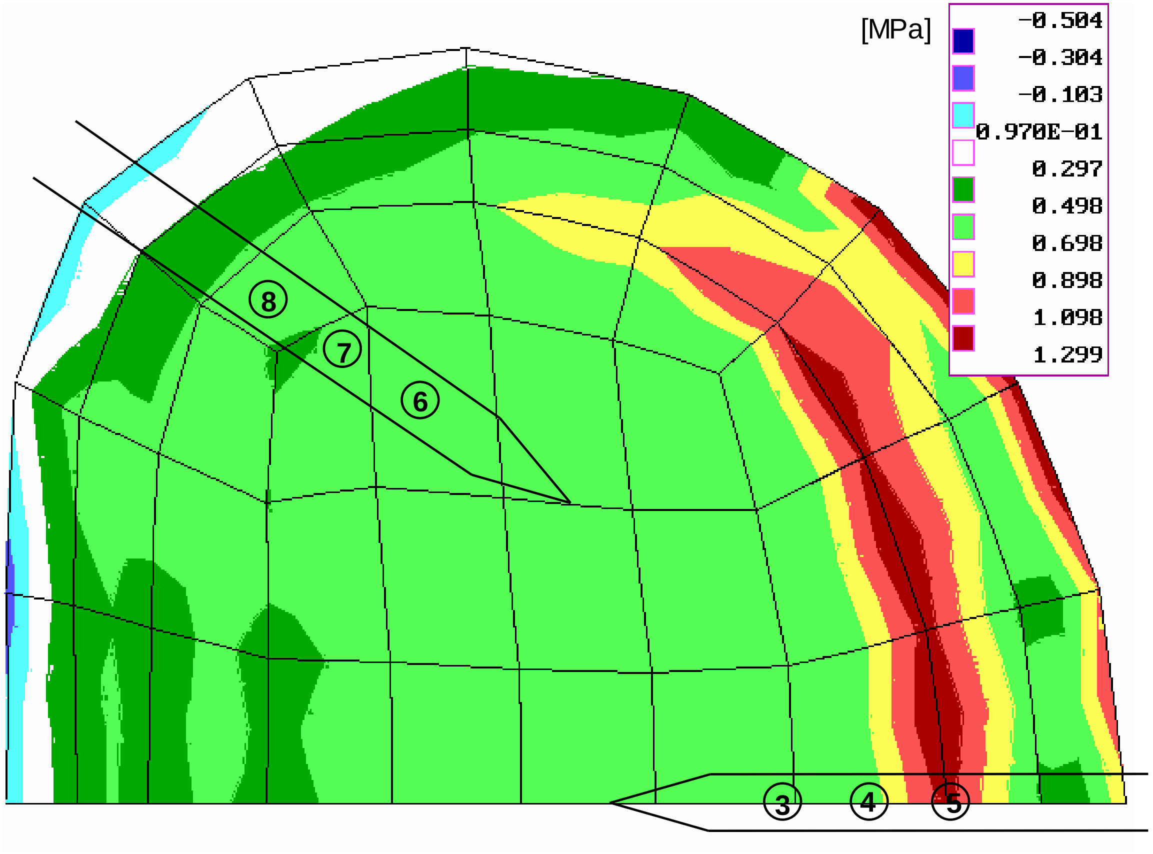


Wyniki eksperymentu

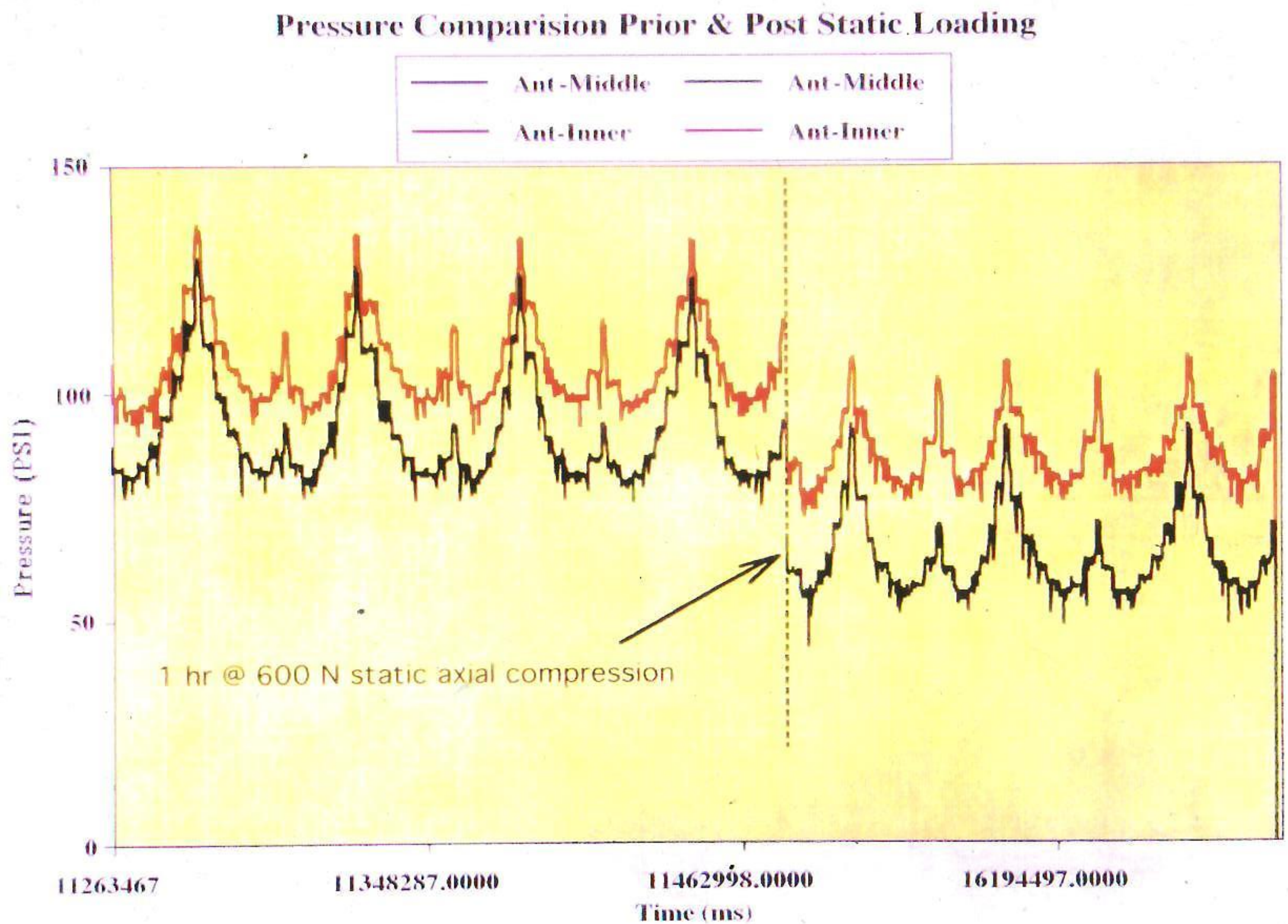


Rys. D.3.7.

Naprężenia całkowite (σ' - ϕp) w płaszczyźnie środkowej dysku



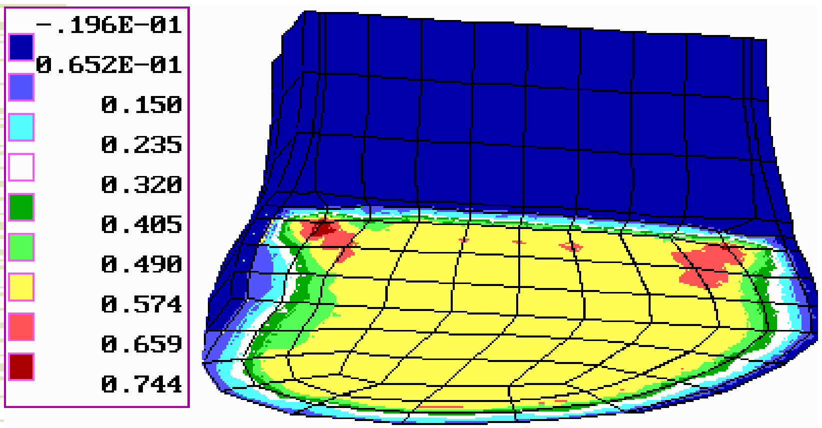
Zmiana ciśnienia w dysku



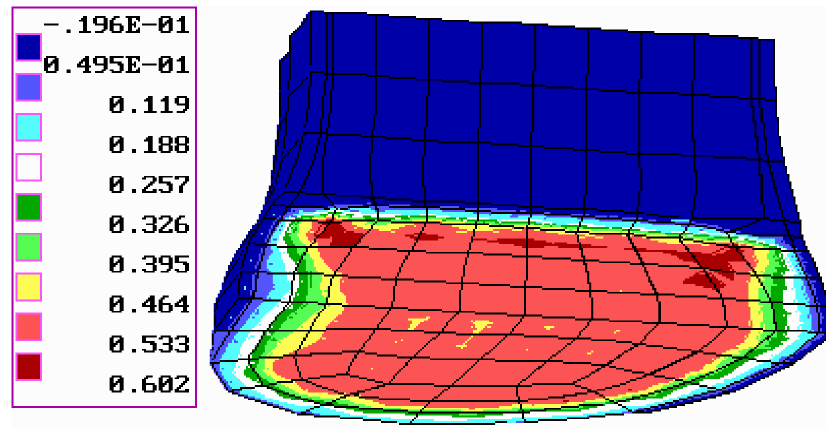
Obciążenie siłą osiową 300 N

Zmiana ciśnienia płynu [MPa] w czasie

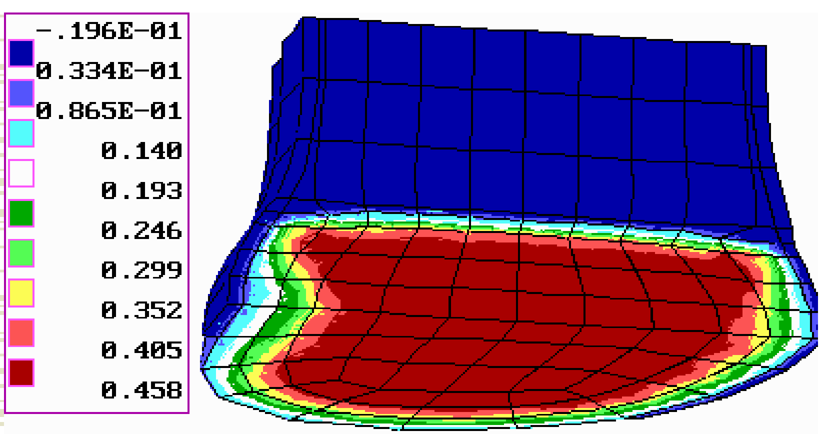
t = 1 min



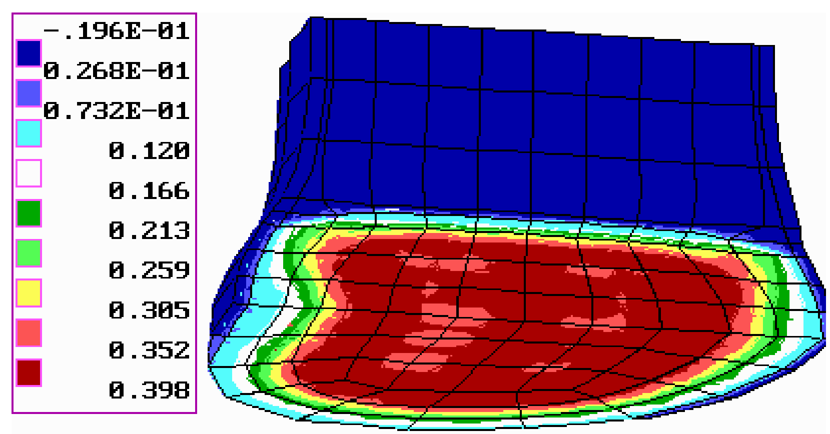
t = 2 min



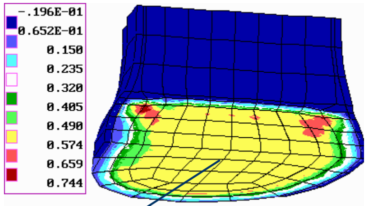
t = 5 min

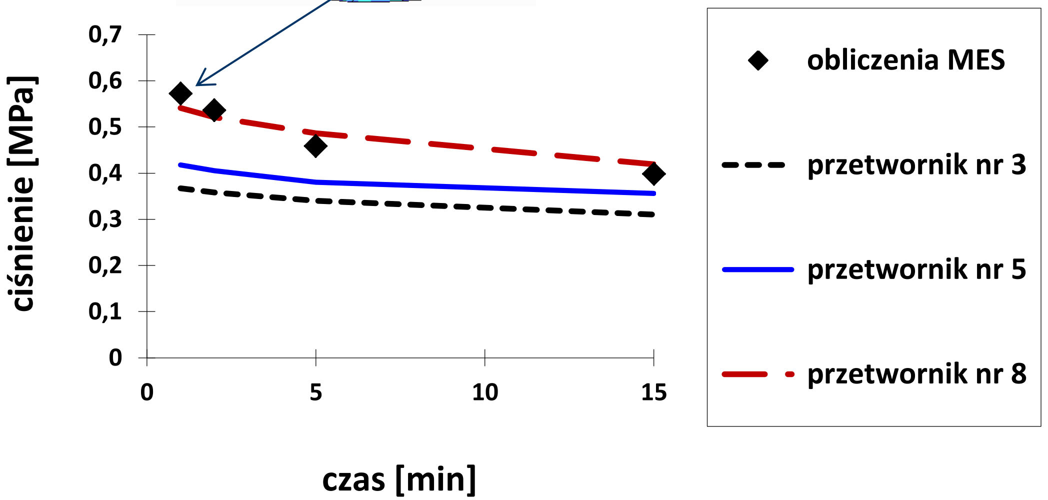


t = 15 min



Porównanie wyników obliczeń MES z pomiarami ciśnienia





Obciążenie stałą siłą ściskającą 300 N