Integrated Laboratory

Strength of Materials and Structures

Strain gauge

Before attending the laboratory students should recollect the following topics: plane stress, Hook's law for plane stress, stress concentration, stress concentration factor, Mohr's circle, principal directions

Recommended Bibliography:

- William A. Nash *Strenght of materials*

- Roy R. Craig *Mechanics of Materials*

- *Mechanika Materiałów i Konstrukcji* edited by Marek Bijak-Żochowski

- Own lecture notes

- the Internet (for lazy students)

Measurement of strain by means of strain gauges is based on the phenomenon of changing resistance due to change of the length of a conductor. The resistance is given by formula:

$$R = \rho \frac{l}{A}$$

where:

- R is resistance $[\Omega]$,

- l is length $[m]$,

- A is cross-section area $[m^2]$,

- ρ is specific resistance $[\Omega m]$.

After finding the logarithm and differentiation we obtain:

$$\ln R = ln\rho + \ln l - \ln A$$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l} - \frac{dA}{A}$$

In finite increments it may be written as:

$$\frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + \frac{\Delta l}{l} - \frac{\Delta A}{A}$$

For contemporary materials used in strain gauges specific resistance does not change in function of strain $(\frac{\Delta \rho}{\rho} = 0)$, however

$$\frac{\Delta l}{l} = \epsilon_x$$

$$\frac{\Delta A}{A} = \epsilon_y + \epsilon_z = -2\nu\epsilon_x$$

where ϵ_x is strain in direction along wire, ϵ_y, ϵ_z are strains in transverse direction. Finally we obtain:

$$\frac{\Delta R}{R} \approx \epsilon_x(1 + \nu) = \epsilon_x k_1$$

Strain gauges are usually grids of wires created by etching of metal foil (picture 1). Transverse segments have cross-sections a few hundreds times smaller than active wires, so that their resistance can be neglected. Such construction causes that:

$$\frac{\Delta R}{R} = k\epsilon$$

where k is gauge factor; its value is higher than it results from Poisson ratio and $k = 1.8 \div 2.6$ depending on specific design and material used. The other quantities which characterize strain gauges are:

- active grid length $l = 0.1 \div 100 \ mm$;

- resistance $R = 150 \div 250 \ \Omega$

A wide range of active grid lengths allows us to investigate many different objects. Small l is used when we want to measure strains almost in a point, when strain gradient is high, whereas high l strain gauges are used to investigate structures made of inhomogeneous materials with large grains (e.g. concrete). The value of ϵ measured by means of strain gauges is an average strain on the basis of active grid length along the strain gauge axis (picture 2). Strain gauges are not sensitive to shear strain.
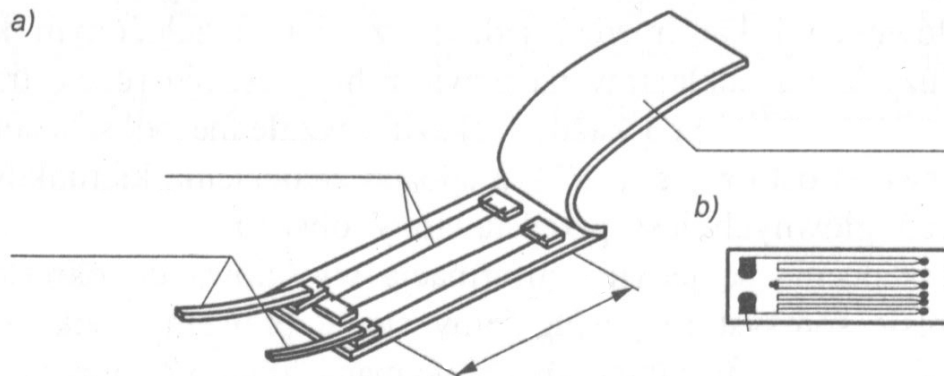
Picture 1: *Strain gauges: (a) made of wires, (b) etched*
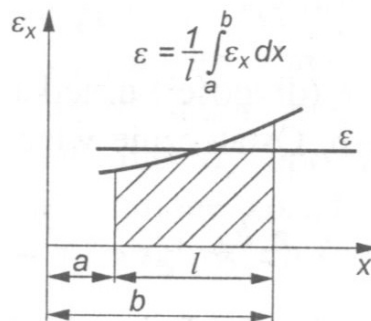
Picture 2: *Average strain on the basis of active grid length*

Strains are measured on the free unloaded surface of the investigated object, i.e. in plane stress state. There are three independent items of data needed to describe this state fully. They are two principal stresses and an angle between one of them and known direction. To get this information, a strain rosette, which consists of 3 strain gauges, is applied on the surface (picture 3). Each strain gauge shows linear strain in its direction (ϵ', ϵ'', ϵ''') and how to find principal stresses and their directions is described elsewhere.

Usually strain state is input information to determine stress state: for elastic materials, Hooke's law is used; for inelastic materials, knowledge of all their properties beyond yield point is necessary; strain gauges have to be specially calibrated as well.

In order to determine strain gauge's relative change of resistance the device is connected into a very sensitive Wheatstone's bridge, whose feature is very high precision (below 10^{-6}). Into
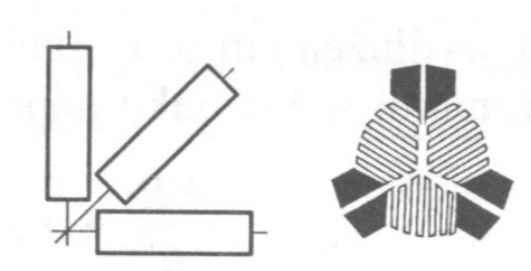
Picture 3: *Strain rosette's layouts: (a) rectangular, (b) equiangular*

the circuit of four resistors two strain gauges are connected. One of them is active (stressed) and the other compensatory (bonded to an object made of the same material as in case of the active gauge but unstressed). What is actually measured is not the active strain gauge's relative change of resistance but change of resistance between active and compensatory strain gauges. This eliminates influence of incidental external factors such as temperature changes, humidity etc.