## X-force

$$X=-\oint_l p\cdot dy=-\oint_l (p-p_\infty)\cdot dy=-\oint_l\left(\frac{\rho_\infty {U_\infty}^2}{2}-\frac{\rho_\infty U^2}{2}\right)\cdot dy=\frac{\rho_\infty}{2}\oint_l U^2\cdot dy$$

## Y-force

$$Y=\oint_l p\cdot dx=\oint_l (p-p_\infty)\cdot dx=\oint_l\left(\frac{\rho_\infty {U_\infty}^2}{2}-\frac{\rho_\infty U^2}{2}\right)\cdot dx=-\frac{\rho_\infty}{2}\oint_l U^2\cdot dx$$

$$X-iY=-\oint_l p\cdot dy+ipdx=\cdots=i\frac{\rho_\infty}{2}\oint_l U^2\cdot d\bar{z}$$

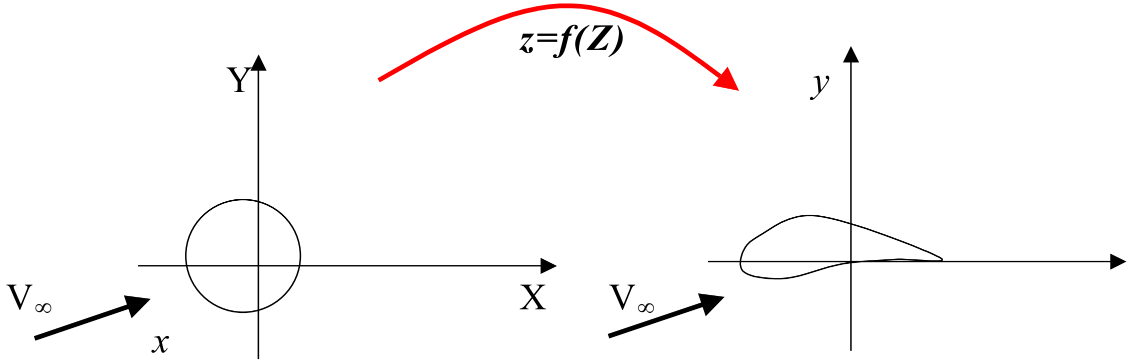
$$M_0=-\oint_l p\cdot(ydy+xdx)=-\oint_l p\cdot Re(zd\bar{z})=\cdots=\frac{\rho_\infty}{2}Re\oint_l U^2\cdot z\,d\bar{z}$$

$$\frac{dw}{dz} = |U|(\cos\Theta - i\sin\Theta) = |U|\frac{d\bar{z}}{ds}$$

$$\left(\frac{dw}{dz}\right)^2 = \left(|U|\frac{d\bar{z}}{ds}\right)^2 = U^2\frac{d\bar{z}\cdot d\bar{z}}{ds\cdot ds} = U^2\frac{d\bar{z}\cdot \boxed{d\bar{z}}}{dz\cdot \boxed{d\bar{z}}} = U^2\frac{d\bar{z}}{dz}$$

$$U^2 d\bar{z} = \left(\frac{dw}{dz}\right)^2 dz \quad ; \quad U^2 z d\bar{z} = \left(\frac{dw}{dz}\right)^2 z dz$$

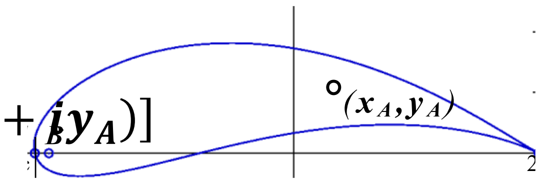
$$X - iY = i\frac{\rho_\infty}{2}\oint_l U^2 \cdot d\bar{z} = i\frac{\rho_\infty}{2}\oint_l \left(\frac{dw}{dz}\right)^2 dz \quad ; \quad M_0 = \frac{\rho_\infty}{2} Re\oint_l \left(\frac{dw}{dz}\right)^2 z dz$$




$$X - iY = i\frac{\rho_\infty}{2}\oint_l \left(\frac{dw}{dz}\right)^2 dz = i\frac{\rho_\infty}{2}\oint_l \left(\frac{dw}{dZ}\right)^2 \frac{dZ}{dz}\frac{dZ}{dz}dz = \; i\frac{\rho_\infty}{2}\oint_l \left(\frac{dw}{dZ}\right)^2 \frac{dZ}{dz}dZ$$

$$M_0 = \frac{\rho_\infty}{2} Re \oint_l \left(\frac{dw}{dz}\right)^2 zdz = \frac{\rho_\infty}{2} Re \oint_l \left(\frac{dw}{dZ}\right)^2 \frac{dZ}{dz}\frac{dZ}{dz} zdz = \frac{\rho_\infty}{2} Re \oint_l \left(\frac{dw}{dZ}\right)^2 \frac{dZ}{dz} zdZ$$

$$M_A = M_0 + Yx_A - Xy_A = Mo + Re[i(X - iY)(x_A + iy_A)]$$

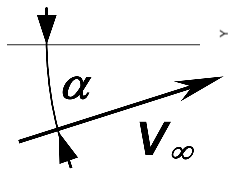


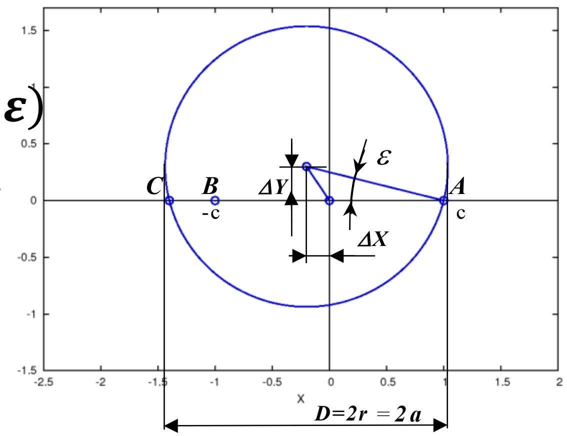
$$z = f(Z) \quad = Z + c_o + c_1 Z^{-1} + c_2 Z^{-2} + c_3 Z^{-3} + \cdots$$

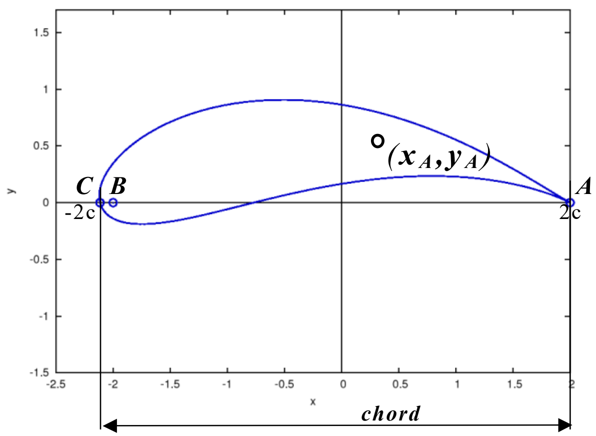
$$\frac{dZ}{dz} = 1 + c_1 Z^{-2} + \cdots \quad ; \qquad \frac{dZ}{dz} z = Z + c_0 + 2c_1 Z^{-1} + \cdots$$

$$W(Z) = U_\infty \left( e^{-i\alpha} Z + \frac{e^{i\alpha}}{Z} \right) + i \frac{\Gamma}{2\pi} ln(Z)$$

$$\Gamma = 4\pi a U_\infty \cdot sin(\alpha - \varepsilon)$$



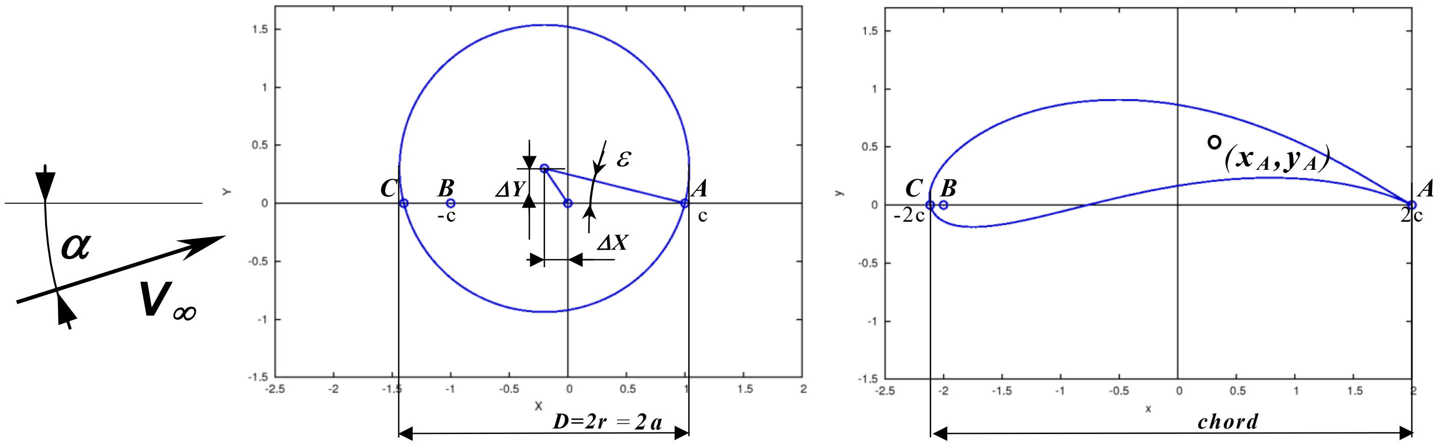



$$M_A = \frac{\rho_\infty}{2} Re \oint_l \left(\frac{dw}{dZ}\right)^2 \frac{dZ}{dz}(z - z_A)dZ = \cdots$$

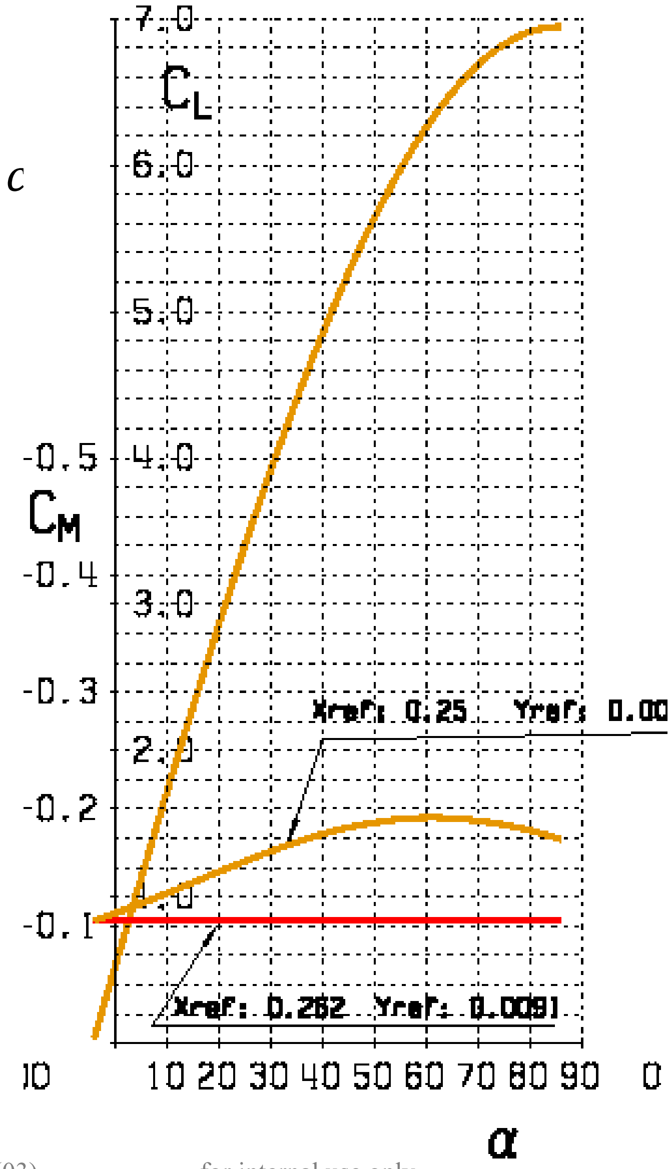
$$= 2\pi\ \rho_\infty U_\infty^2 \quad Re\left\{ie^{-2i\alpha}\left[c_1 + a(z_A + c_0)e^{-i\varepsilon}\right] - ia(z_A + c_0)e^{i\varepsilon}\right\}$$

$$z_A = c_0 - \frac{c_1}{a}e^{i\varepsilon} \quad \Rightarrow \quad M_A = const$$



# AERODYNAMIC CENTER

… $M_A = c$




thin, symmetrical airfoil $\boldsymbol{x_{ac}/c} = \boldsymbol{0}.25$

$\boldsymbol{y_{ac}/c} = 0.00$

thick, symmetrical/

nonsymmetrical airfoil $\boldsymbol{x_{ac}/c} \approx \boldsymbol{0}.25$

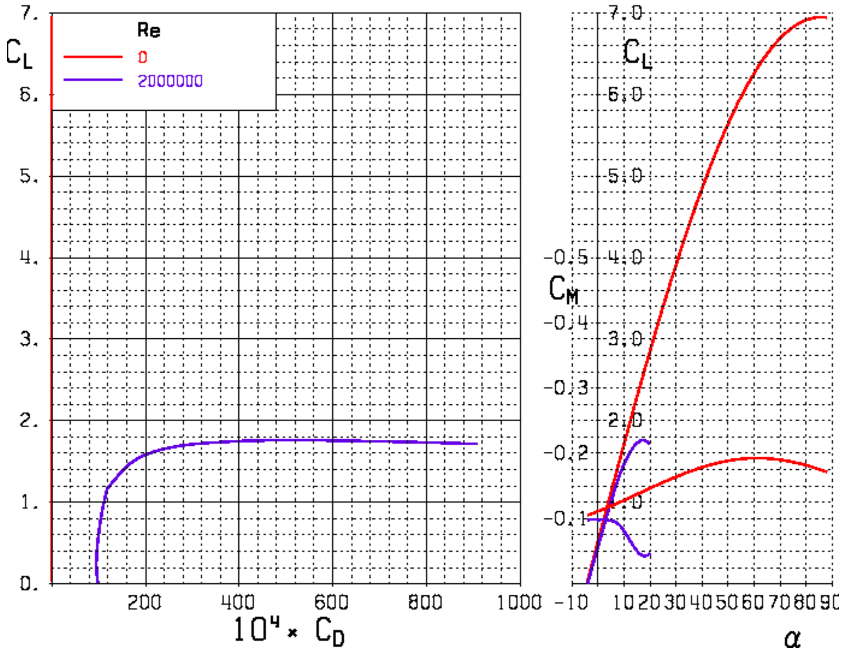
$\boldsymbol{y_{ac}/c} \approx 0.00$

airfoil: NACA 4412 potential flow

$$L = C_L \cdot S \cdot \rho_\infty \frac{V_\infty^2}{2} \qquad C_L = fun(\alpha,\ geom,\ Re,\ \ldots)$$

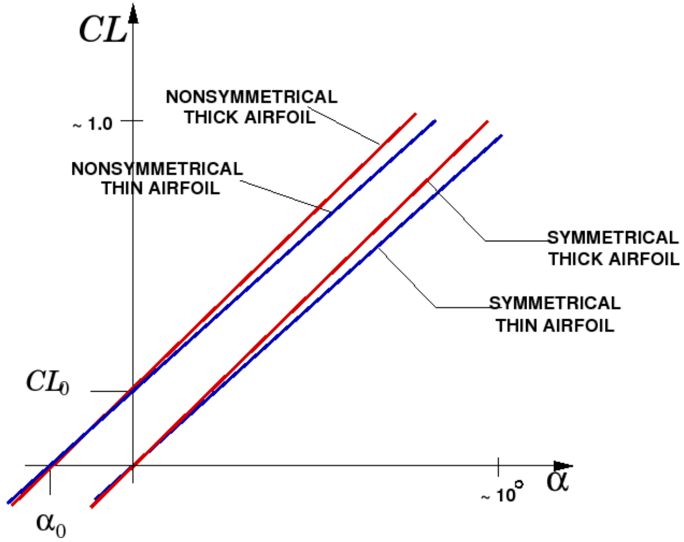
$$D = C_D \cdot S \cdot \rho_\infty \frac{V_\infty^2}{2} \qquad C_D = fun(\alpha,\ geom,\ Re,\ \ldots)[FRICTION]$$

$$My = Cm \cdot c \cdot S \cdot \rho_\infty \frac{V_\infty^2}{2} \qquad Cm = fun(\alpha, geom,\ Re, (x_r, y_r)\ \ldots)$$




$C_L,\ C_D,\ Cm \quad \cdots$

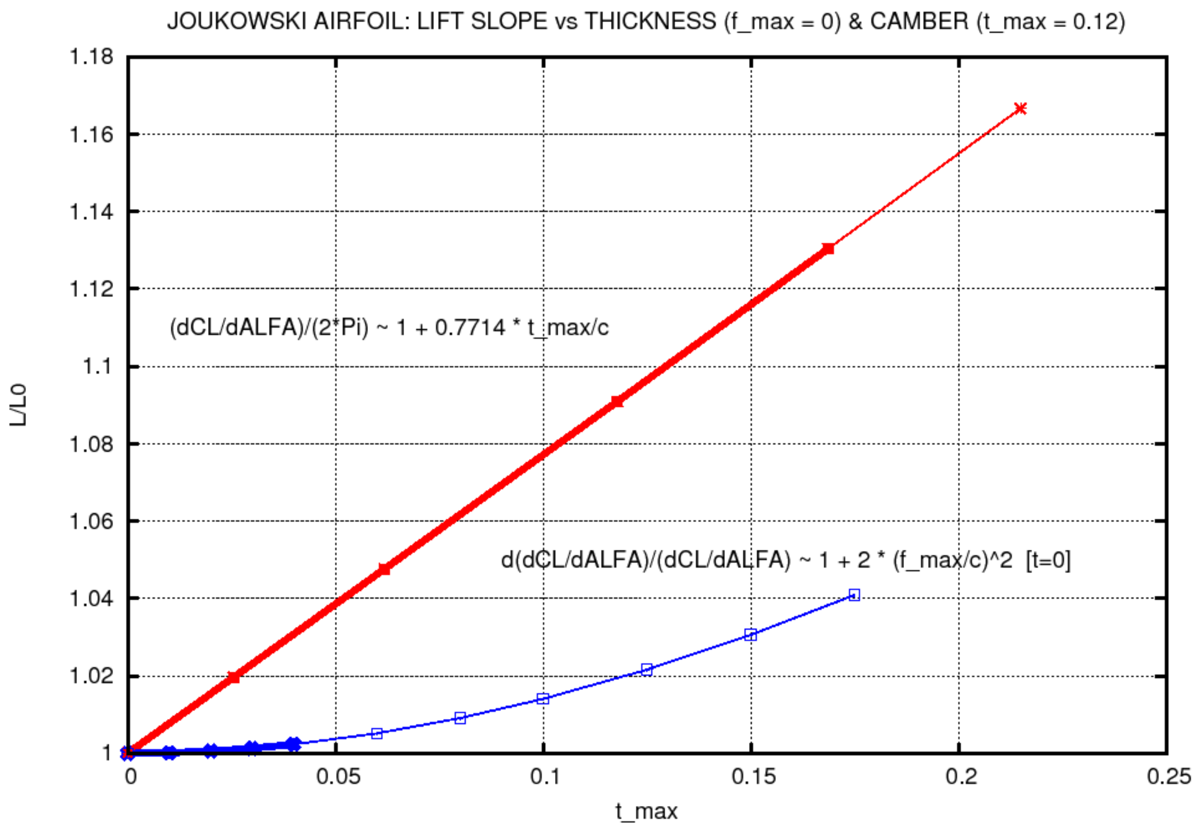
## <u>low angles of attack</u>




$$C_L \approx \frac{dC_L}{d\alpha}\cdot\left(\alpha-\alpha_0\right)=\frac{dC_L}{d\alpha}\cdot\alpha+C_{L0}$$

$$\frac{dC_L}{d\alpha}=fun\left(\bar{t},\cancel{\bar{f}},\mathrm{Re},\ldots\right)\quad\approx\quad 0.1\left[{}^{1}\!/_{\deg}\right];\ 2\pi\left[{}^{1}\!/_{rad}\right]$$

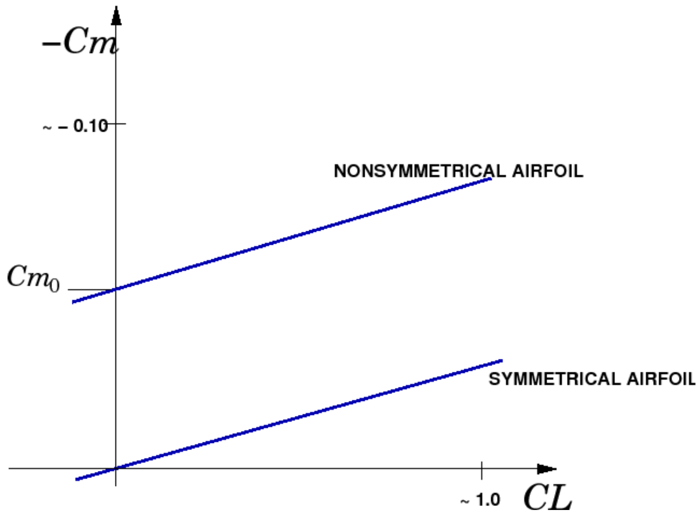
# lift force slope, Joukowski airfoil, inviscid flow




$$\left(\frac{dC_L}{d\alpha}\right)_{\bar{t}} - \left(\frac{dC_L}{d\alpha}\right)_{\bar{t}=0} = fun1(\bar{t})$$

$$\left(\frac{dC_L}{d\alpha}\right)_{\bar{f}} - \left(\frac{dC_L}{d\alpha}\right)_{\bar{f}=0} = fun2(\bar{f})$$

## low angles of attack




$$Cm \simeq Cm_0 + \frac{dCm}{dC_L} \cdot C_L$$

$$Cm_{(x_r,y_r)} = Cm_{(x_0,y_0)} + C_Y \cdot (x_r - x_0)/c - C_X \cdot (y_r - y_0)/c \simeq$$

$$\simeq Cm_{(x_0,y_0)} + C_L \cdot (x_r - x_0)/c$$

$$\left(\frac{dCm}{dC_L}\right)_{(x_r,y_r)} \simeq \left(\frac{dCm}{dC_L}\right)_{(x_0,y_0)} + (x_r - x_0)/c$$

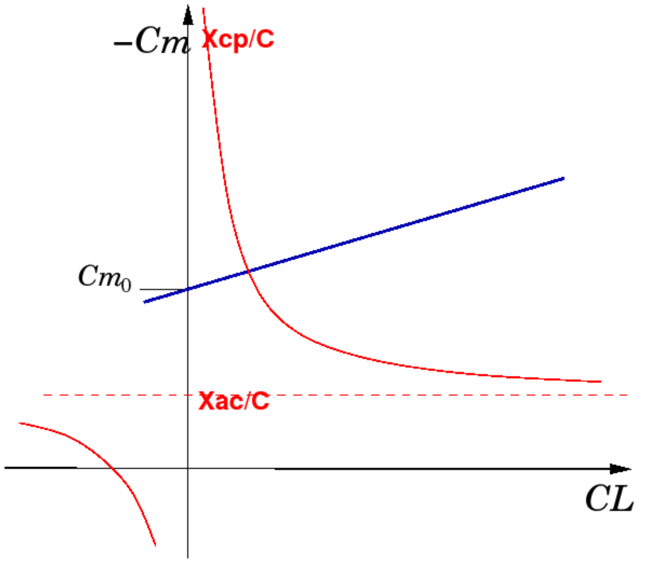
$$\left(\frac{dCm}{dC_L}\right)_{(x_{sa},y_{sa})} = 0 \quad \Rightarrow \quad (x_{ac}/c) = (x_0/c) - \left(\frac{dCm}{dC_L}\right)_{(x_0,y_0)}$$

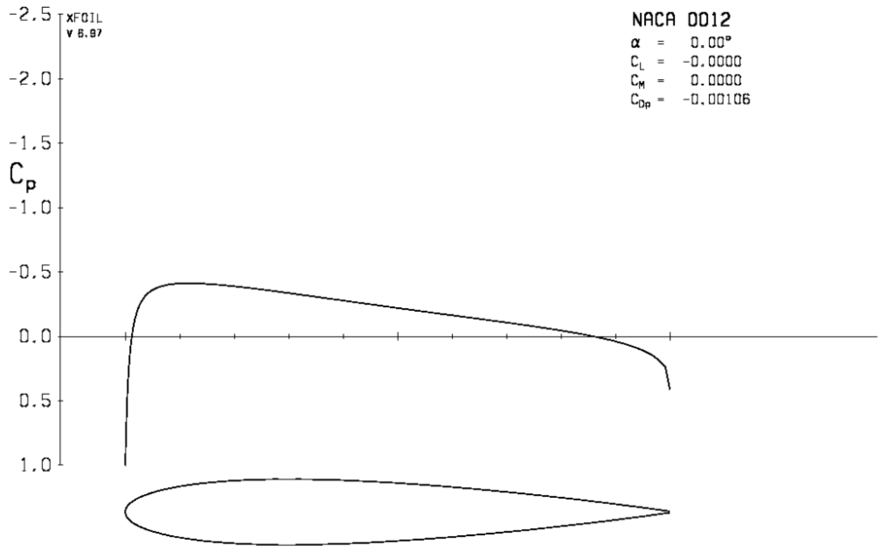
$$Cm_{(x_r,y_r)} = Cm_{(x_0,y_0)} + C_Y \cdot (x_r - x_0)/c - C_X \cdot (y_r - y_0)/c \approx$$

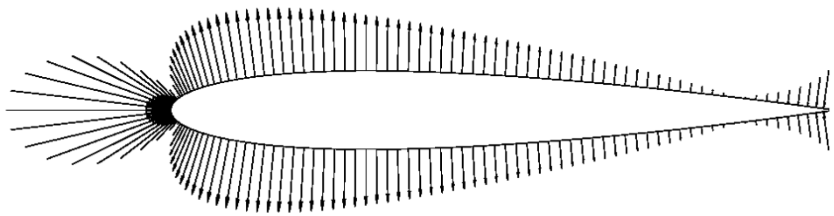
$$\approx Cm_{(x_0,y_0)} + C_L \cdot (x_r - x_0)/c$$

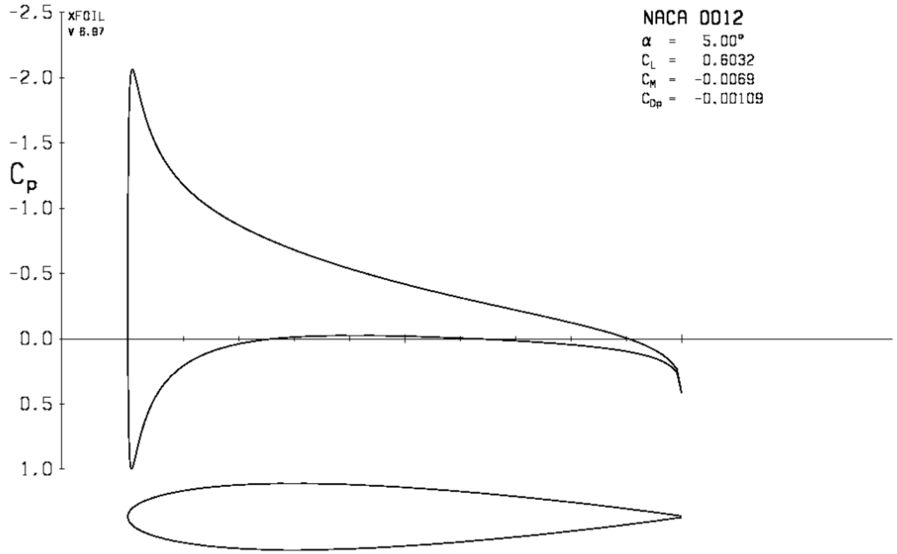
$$Cm_{(x_r,y_r)} = 0 \quad \Rightarrow \quad (x_r, y_r) = \left(x_{cp}, y_{cp}\right)$$

$$\left(x_{cp}/c\right) = (x_0/c) - \frac{Cm_{(x_0,y_0)}}{C_L}$$

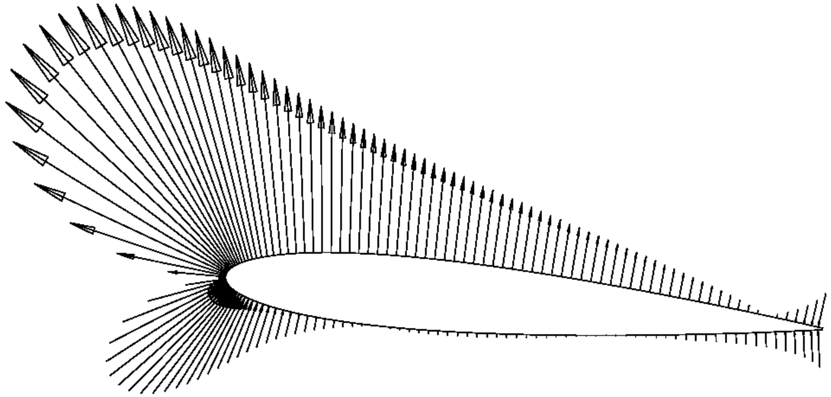


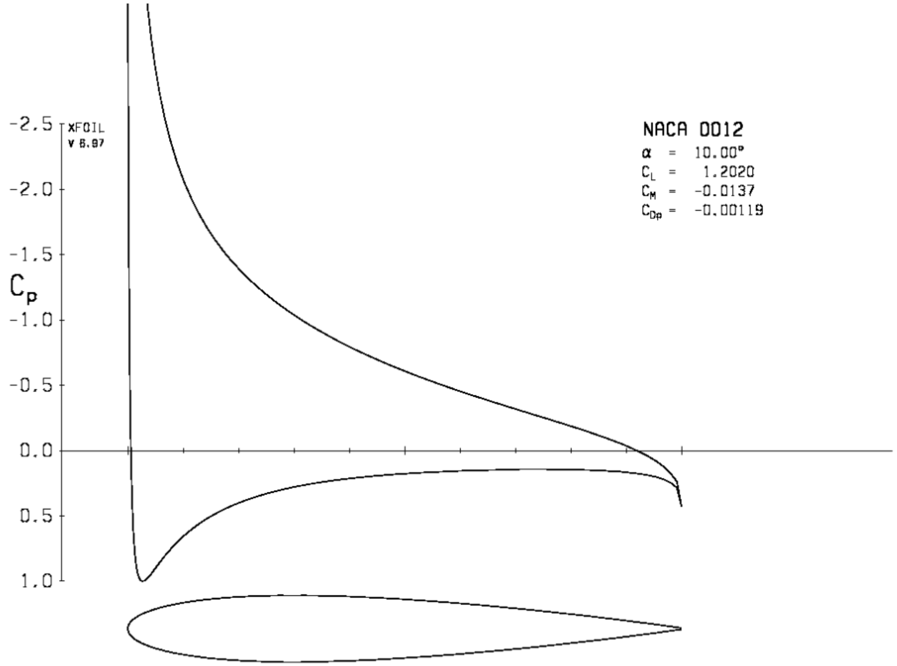

-2.5
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
Cp
XFOIL
V 6.97
NACA 0012
α = 0.00°
CL = -0.0000
CM = 0.0000
CDp = -0.00106

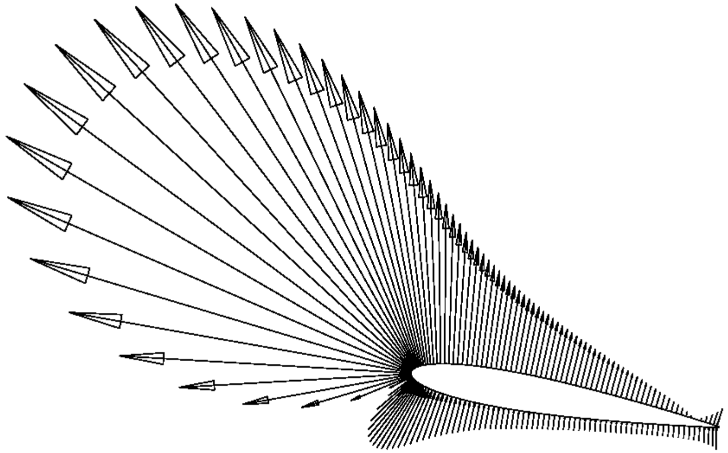


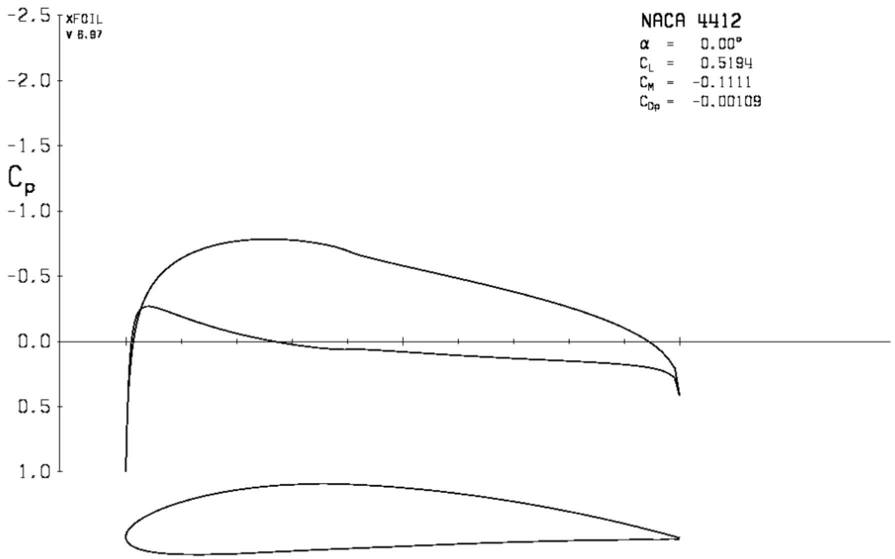


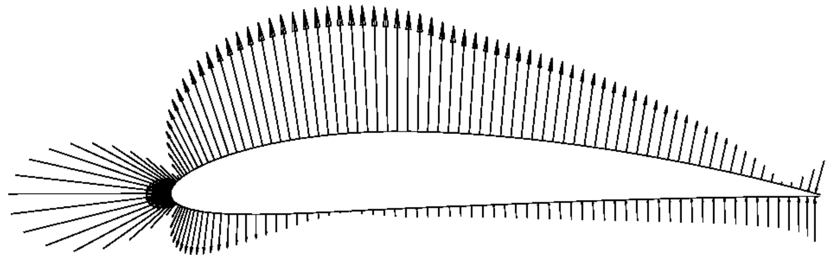
-2.5
-2.0
-1.5
Cp
-1.0
-0.5
0.0
0.5
1.0
XFOIL
V 6.97
NACA 0012
α = 5.00°
CL = 0.6032
CM = -0.0069
CDp = -0.00109

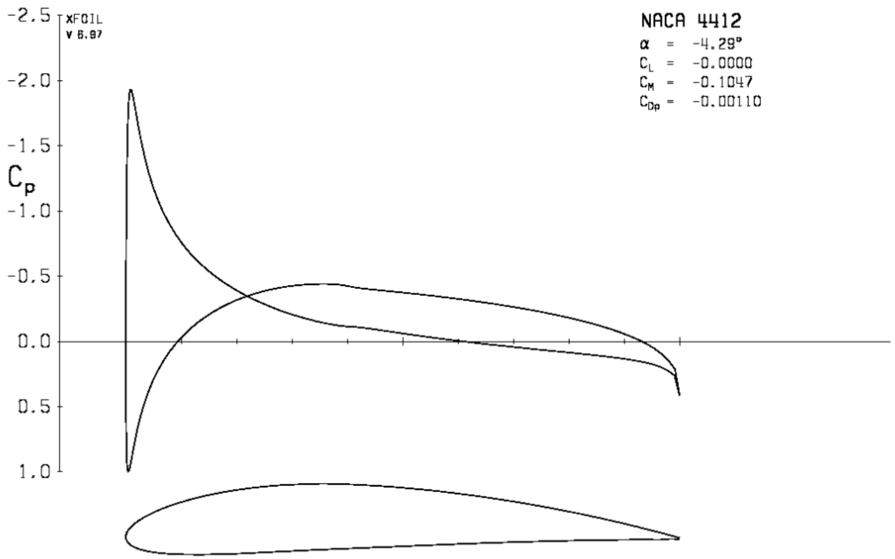



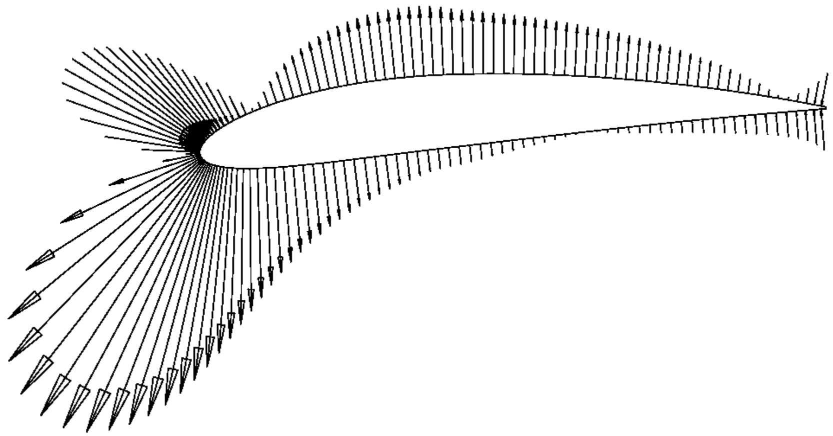

XFOIL
V 6.97
-2.5
-2.0
-1.5
$C_p$
-1.0
-0.5
0.0
0.5
1.0
NACA 0012
α = 10.00°
$C_L$ = 1.2020
$C_M$ = -0.0137
$C_{Dp}$ = -0.00119

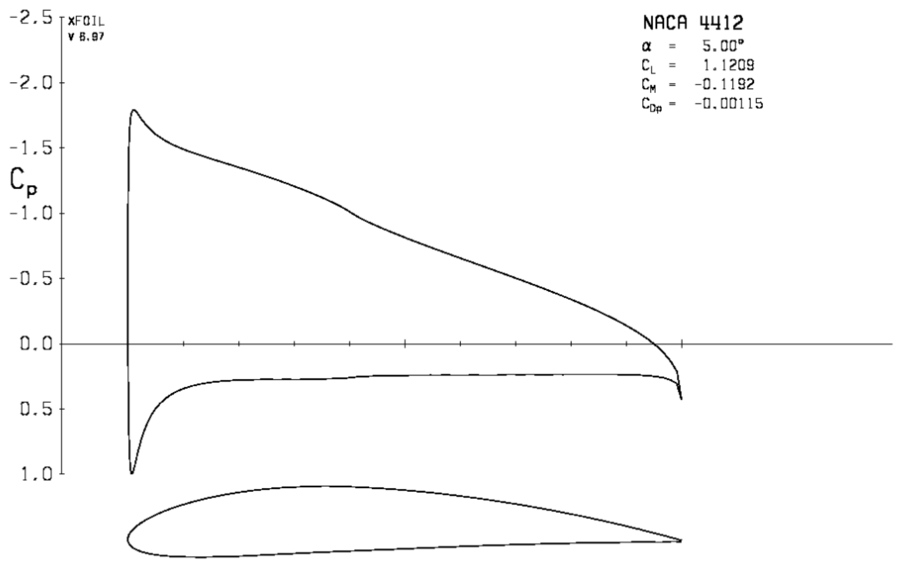



-2.5
XFOIL
V 6.97
-2.0
-1.5
Cp
-1.0
-0.5
0.0
0.5
1.0
NACA 4412
α = 0.00°
CL = 0.5194
CM = -0.1111
CDp = -0.00109

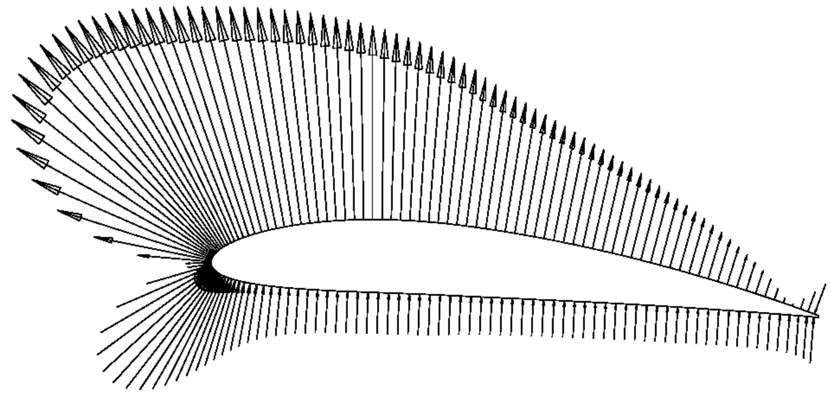


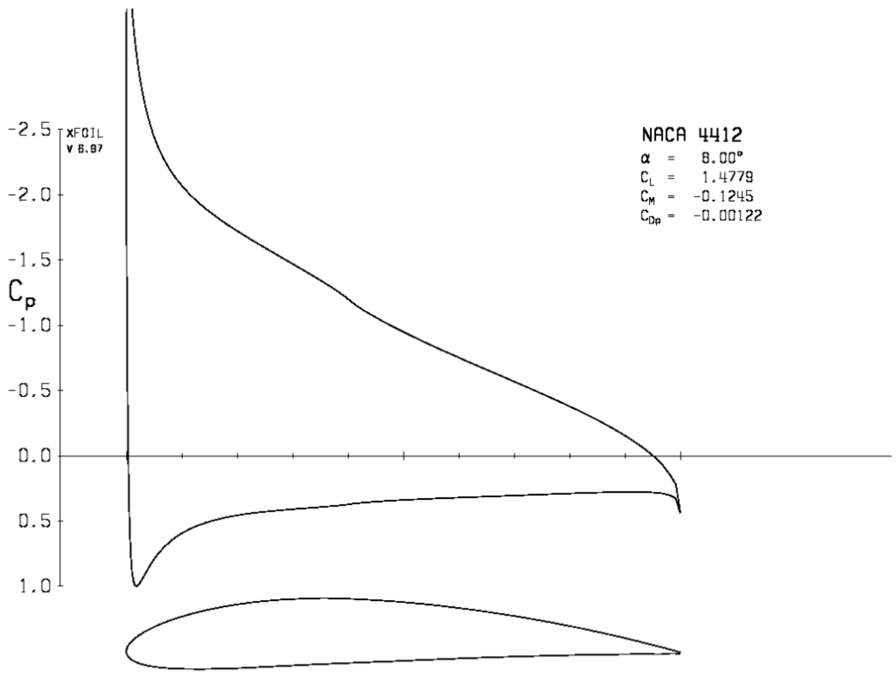

-2.5
-2.0
-1.5
Cp
-1.0
-0.5
0.0
0.5
1.0
XFOIL
V 6.97
NACA 4412
α = -4.29°
$C_L$ = -0.0000
$C_M$ = -0.1047
$C_{Dp}$ = -0.00110



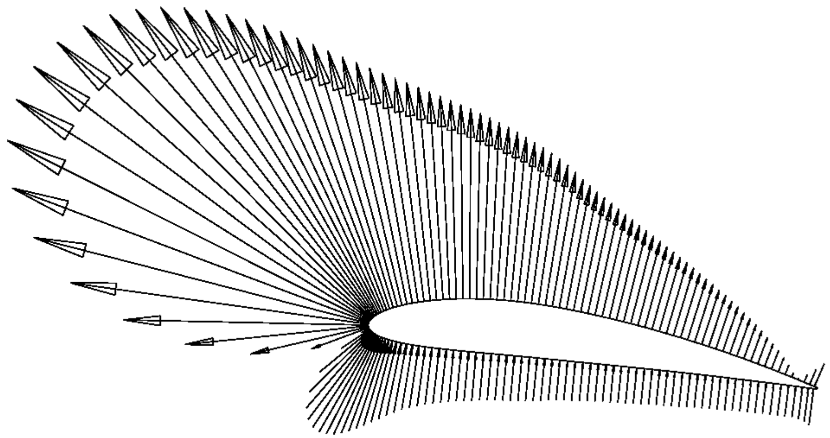


XFOIL
V 6.97
NACA 4412
α = 5.00°
C_L = 1.1209
C_M = -0.1192
C_Dp = -0.00115
C_p
-2.5
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0



-2.5
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
Cp
XFOIL
V 6.97
NACA 4412
α = 8.00°
CL = 1.4779
CM = -0.1245
CDp = -0.00122

## BOUNDARY LAYER:

**x-momentum eq.:** $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}\right) - \frac{\partial\left(\overline{u'v'}\right)}{\partial y}$$

**y-momentum eq.:** $$\frac{\partial p}{\partial y} = 0\ , \quad p(y) = const$$

**continuity eq.:** $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Near the surface: ***u,v << U***$_\infty$ ***(~0)***, so:

Turbulent boundary layer, no pressure gradient:

$$\frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y} - \overline{u'v'}\right) = 0 \qquad \textbf{\textit{lub}}: \qquad \nu\frac{\partial u}{\partial y} - \overline{u'v'} = const = \frac{\tau_w}{\rho}$$

**def:**

$$\frac{\tau_w}{\rho} = (V^*)^2, \qquad \frac{u}{V^*} = u^+, \qquad \frac{y}{\nu/V^*} = y^+$$

near the surface:

$$\overline{u'v'} \ll \nu \frac{\partial u}{\partial y}$$

so:

$$\nu \frac{\partial u}{\partial y} = \frac{\tau_w}{\rho} = (V^*)^2$$

$$u = \frac{(V^*)^2}{\nu} \cdot y \quad \rightarrow \quad u^+ = y^+$$

***LAMINAR (VISCOUS) SUBLAYER***

**above laminar sublayer:**

$$\overline{u'v'} \gg \nu \frac{\partial u}{\partial y}$$

**so:**

$$\overline{u'v'} = (V^*)^2$$

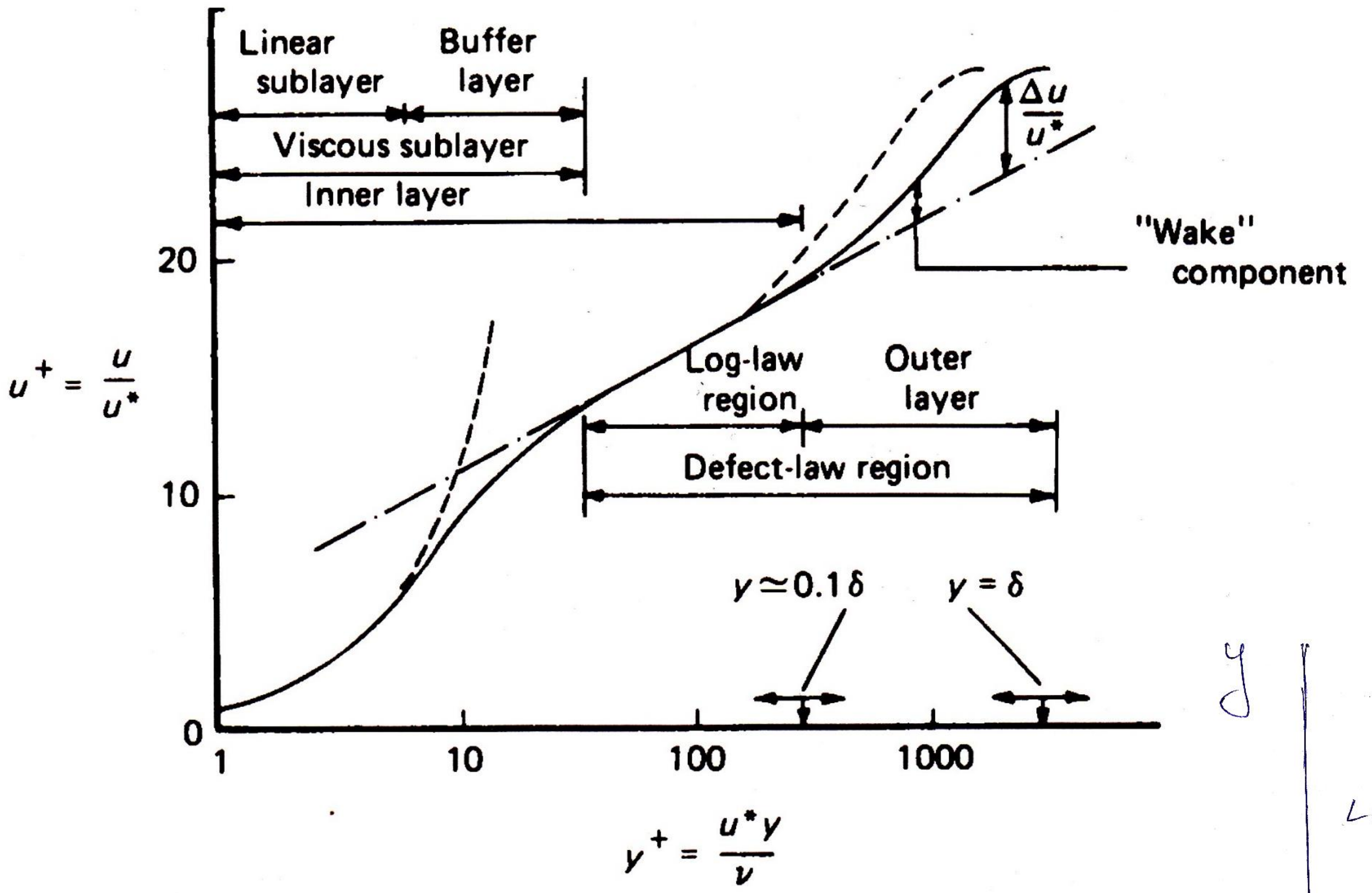
**turbulence model (mixing length):**

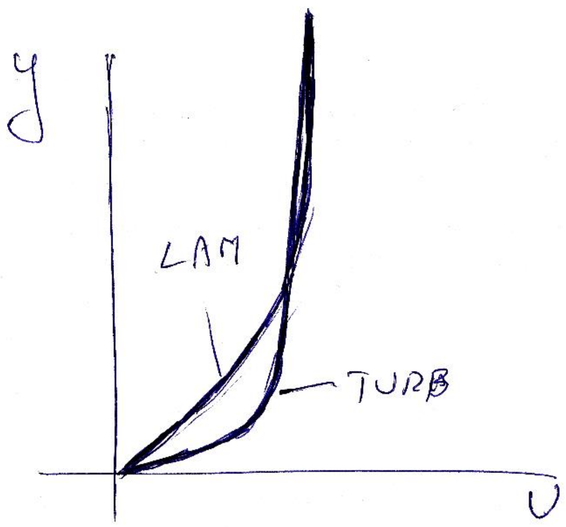
$$\nu_T = l_m^2 \left|\frac{\partial u}{\partial y}\right|, \qquad l_m = \kappa \cdot y$$

$$l_m^2 \left(\frac{\partial u}{\partial y}\right)^2 = \frac{\tau_w}{\rho} = (V^*)^2, \quad l_m \left(\frac{\partial u}{\partial y}\right) = V^*, \quad \frac{\partial u}{\partial y} = \frac{V^*}{\kappa \cdot y} \quad \rightarrow \quad u = \frac{V^*}{\kappa} ln\, y + C_1$$

$$u^+ = \frac{ln\, y^+}{\kappa} + C \qquad \kappa = 0.41, \quad C = 6.1$$

J.J.Bertin, M.L.Smith – Aerodynamics for Engineers, Printice-Hall International, Inc., 1998




INTEGRAL RELATIONS FOR BOUNDARY LAYER (ORD

$$\frac{\partial(uu)}{\partial x}+\frac{\partial(uv)}{\partial y}=u_e\frac{du_e}{dx}+\nu\frac{\partial^2 u}{\partial y^2} \qquad (1)$$

$$\frac{\partial(u_e u)}{\partial x}+\frac{\partial(u_e v)}{\partial y}=u\frac{du_e}{dx} \qquad (2)$$

***(2)-(1)***

$$\frac{\partial\big((u_e-u)u\big)}{\partial x}+\frac{\partial\big((u_e-u)v\big)}{\partial y}=(u_e-u)\frac{du_e}{dx}-\nu\frac{\partial^2 u}{\partial y^2}$$

$$\int_0^\delta \frac{\partial\left(\left(u_e-u\right)u\right)}{\partial x}dy+\int_0^\delta \frac{\partial\left(\left(u_e-u\right)v\right)}{\partial y}dy+\int_0^\delta \left(u_e-u\right)\frac{du_e}{dx}dy=-\nu\int_0^\delta \frac{\partial^2 u}{\partial y^2}dy$$

$$\frac{d}{dx}\int_0^\delta (u_e-u)u\,dy-[(u_e-u)u]_{y=\delta}\frac{d\delta}{dx}+$$

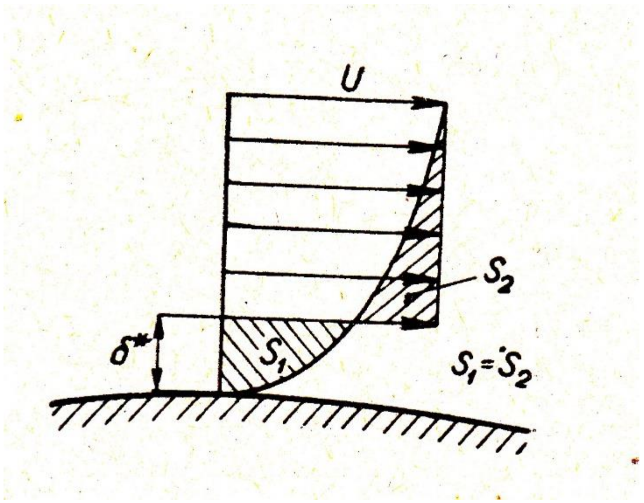
$$\left((u_e-u)v\right)\Big|_0^\delta+\frac{du_e}{dx}\int_0^\delta (u_e-u)dy=-\nu\frac{\partial u}{\partial y}\Big|_0^\delta$$

$$\frac{d}{dx}u_e^2\int_0^\delta \frac{u}{u_e}\left(1-\frac{u}{u_e}\right)dy+\frac{du_e}{dx}u_e\int_0^\delta \left(1-\frac{u}{u_e}\right)dy-\frac{\tau_w}{\rho}=0$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_e}\right) dy$$

$$\theta = \int_0^\delta \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right) dy$$

$$C_f = \frac{\tau_w}{\rho_e u_e^2/2}$$



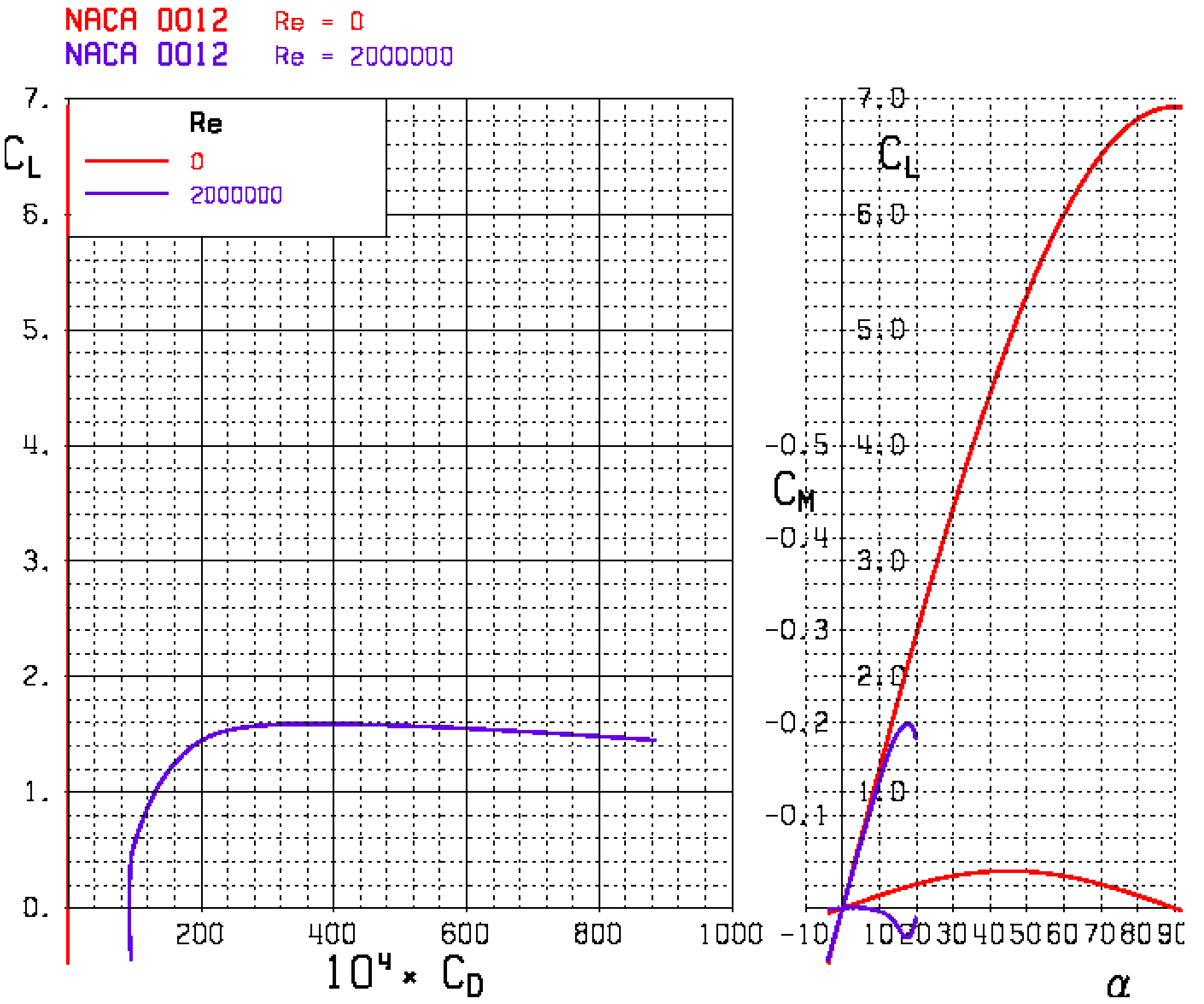

$$H = \frac{\delta^*}{\theta}$$

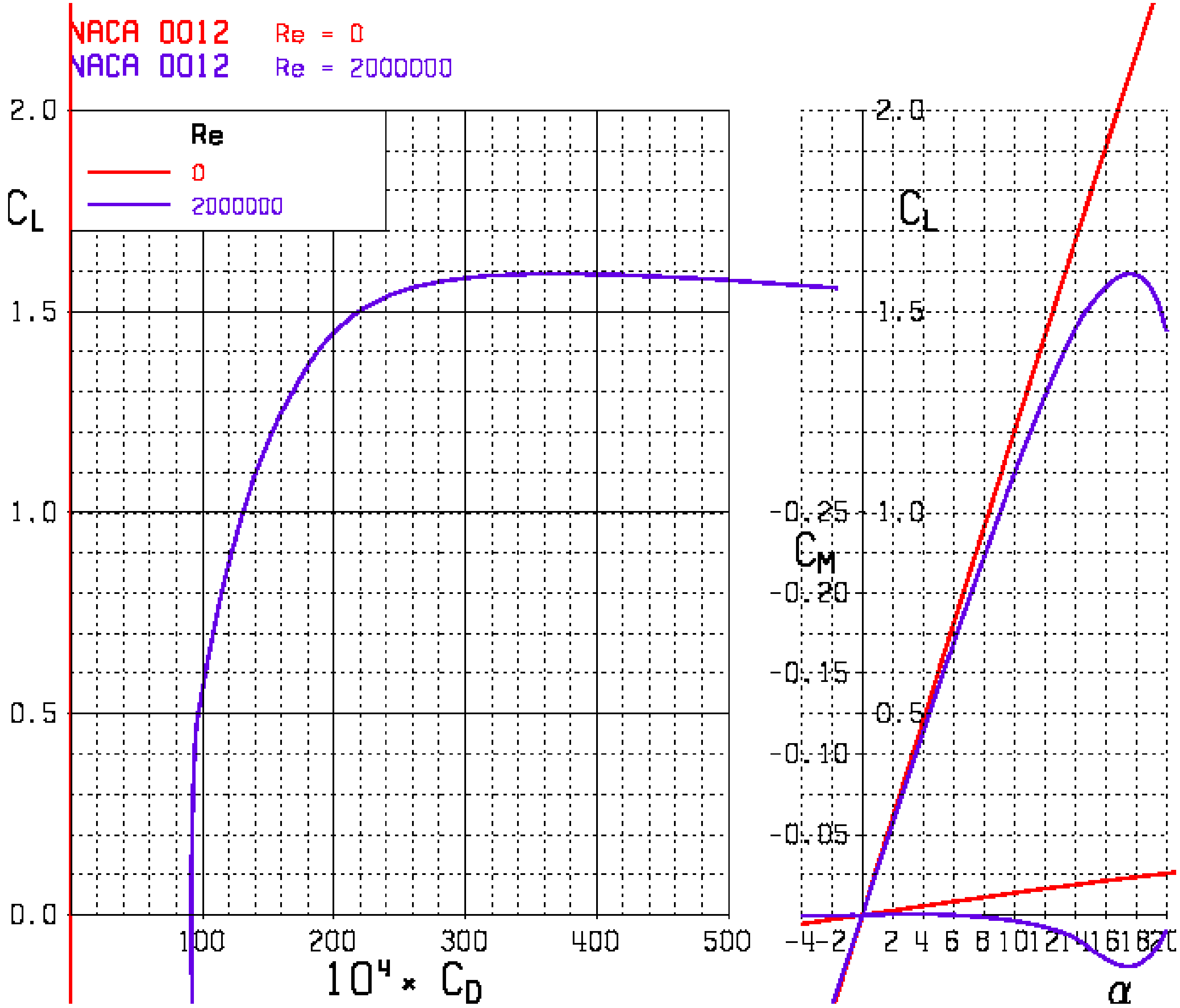
$$\frac{d}{dx}\left(u_e^2\theta\right) + \frac{du_e}{dx}u_e\delta^* - \frac{\tau_w}{\rho} = 0$$

$$u_e^2\frac{d\theta}{dx} + 2u_e\frac{du_e}{dx}\theta + u_e\frac{du_e}{dx}\delta^* - u_e^2\frac{\tau_w}{\rho_e u_e^2} = 0$$

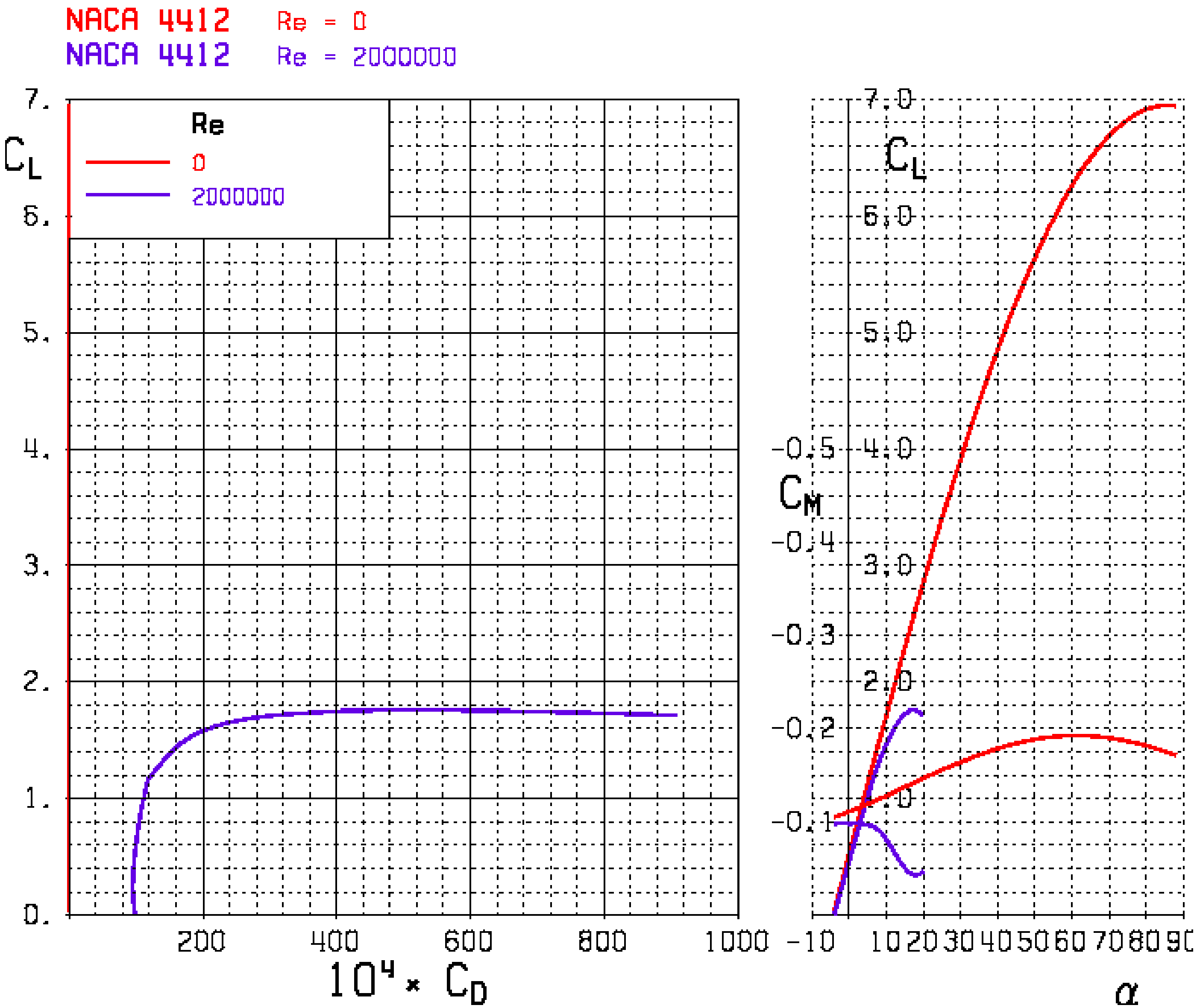
$$\frac{d\theta}{dx} + \frac{\theta}{u_e}\frac{du_e}{dx}\left(2 + \frac{\delta^*}{\theta}\right) - \frac{1}{2}C_f = 0$$

$$\frac{d\theta}{dx} + \frac{\theta}{u_e}\frac{du_e}{dx}(2 + H) - \frac{1}{2}C_f = 0$$

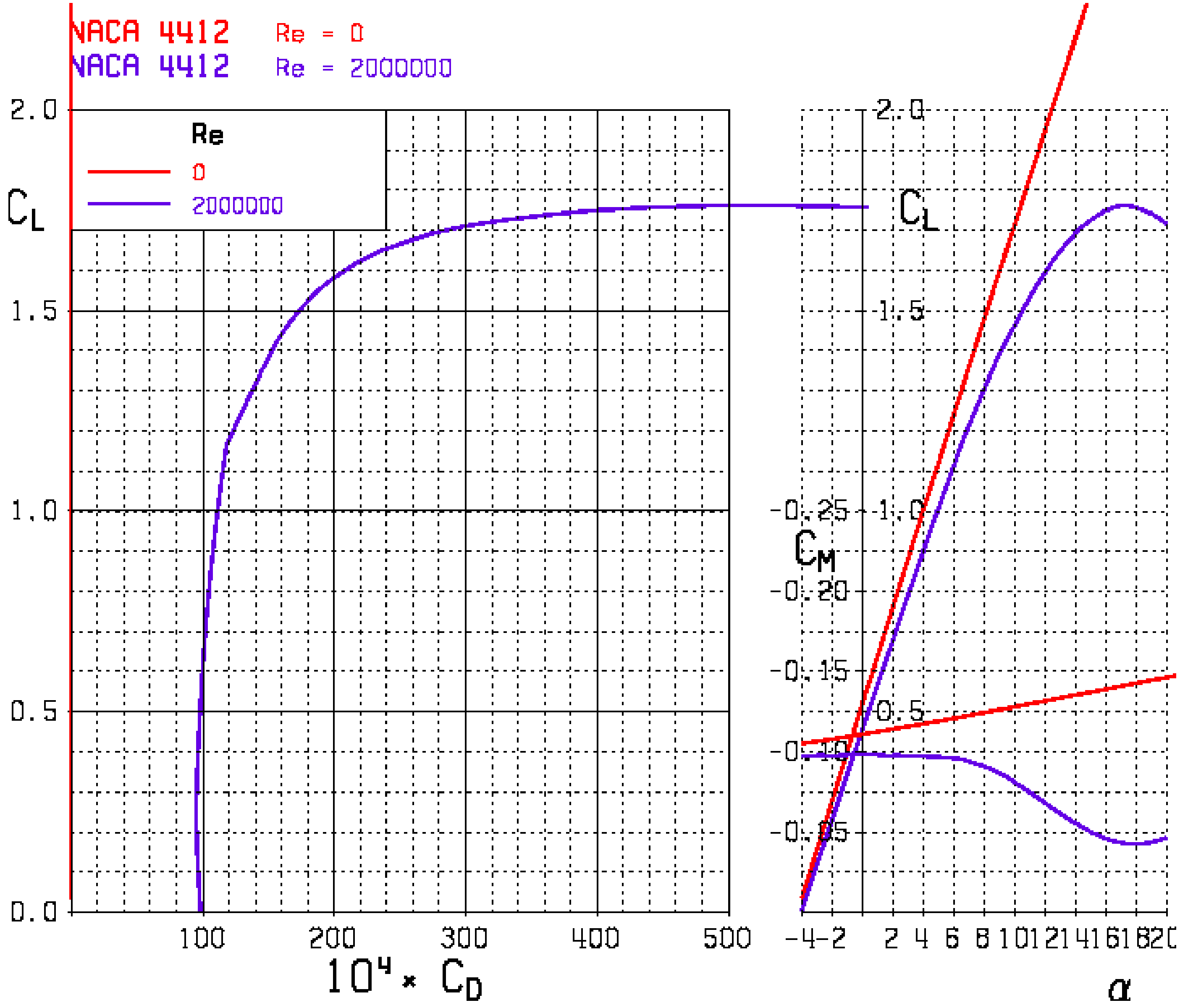

NACA 0012 Re = 0
NACA 0012 Re = 2000000
Re
0
2000000
$C_L$
$10^4 \times C_D$
$C_M$
$\alpha$

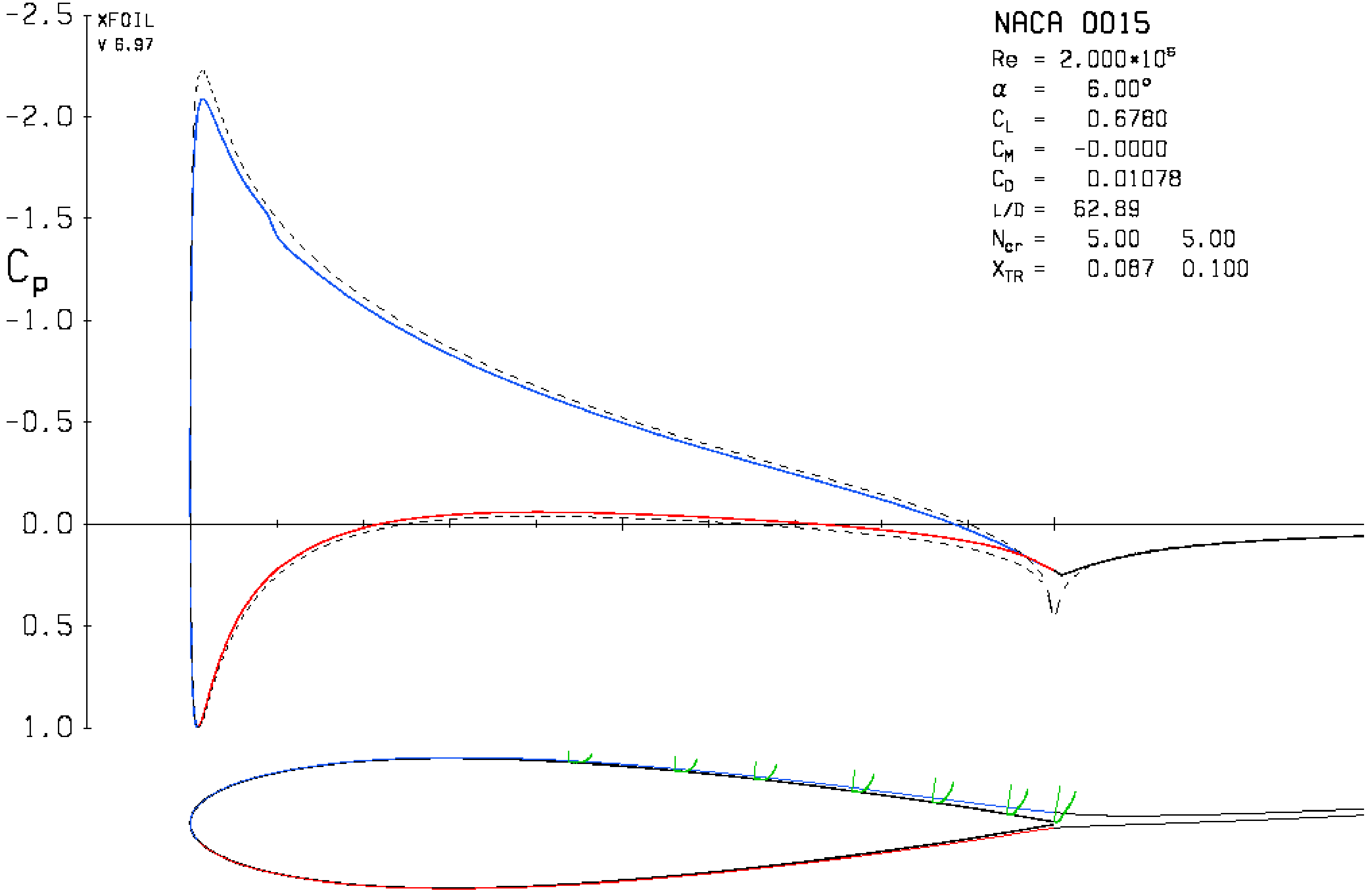


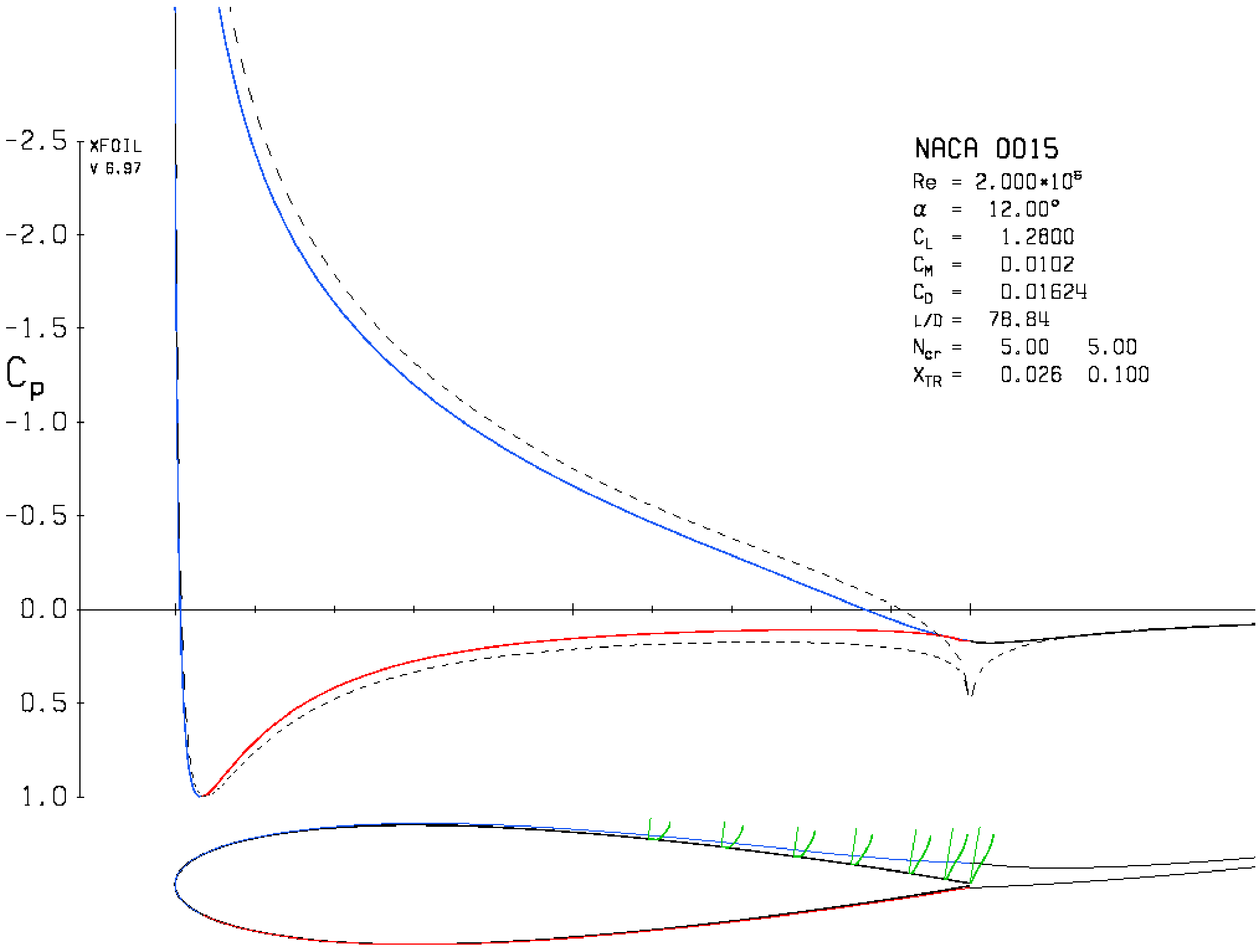
NACA 0012 Re = 0
NACA 0012 Re = 2000000
Re
0
2000000
$C_L$
2.0
1.5
1.0
0.5
0.0
100
200
300
400
500
$10^4 * C_D$
$C_M$
-0.25
-0.20
-0.15
-0.10
-0.05
-4 -2 2 4 6 8 10 12 14 16 18 20
α

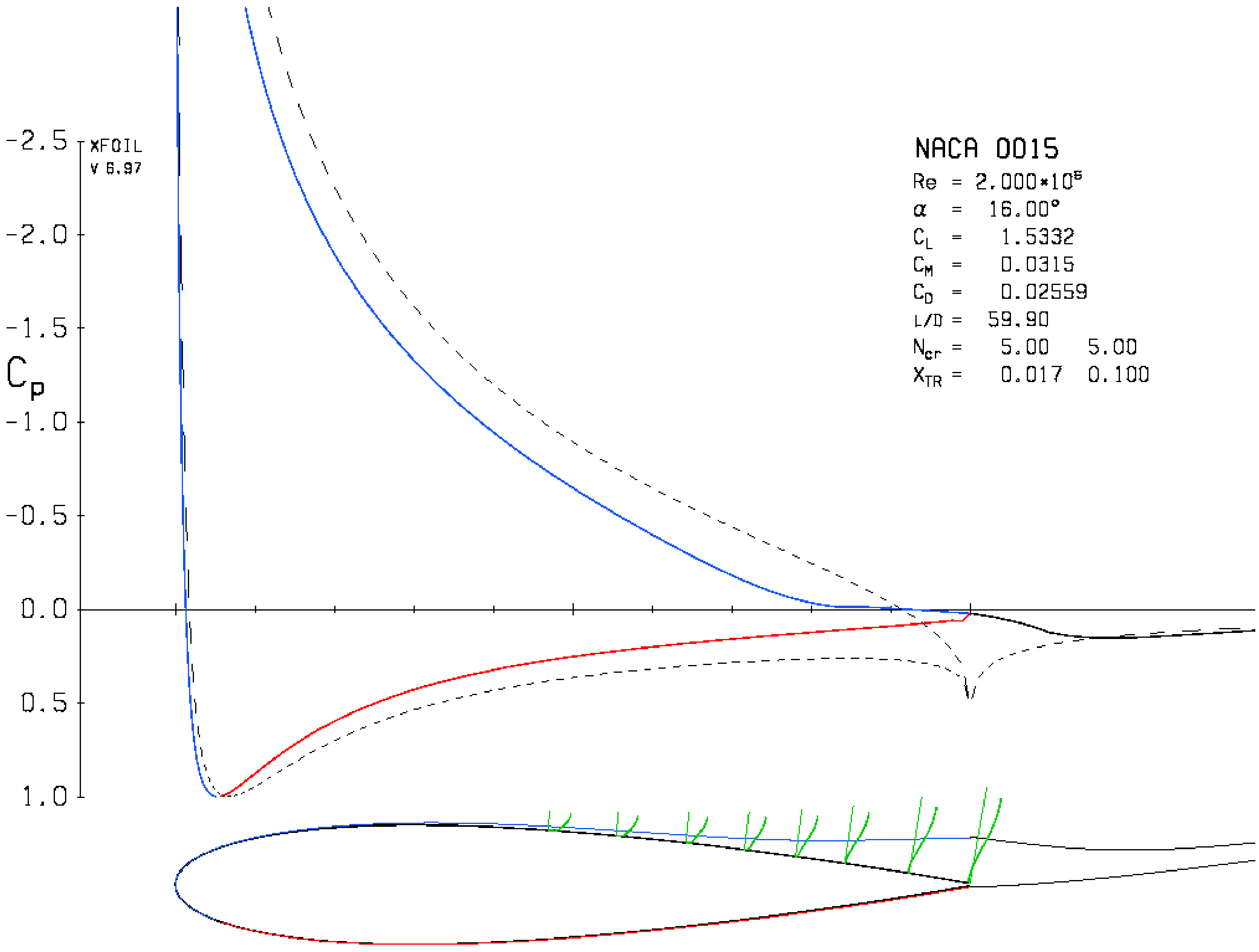


NACA 4412 Re = 0
NACA 4412 Re = 2000000
Re
0
2000000
CL
7.
6.
5.
4.
3.
2.
1.
0.
200
400
600
800
1000
10⁴ * CD
CL
CM
7.0
6.0
5.0
4.0
3.0
2.0
-0.5
-0.4
-0.3
-0.2
-0.1
-10
10
20
30
40
50
60
70
80
90
α

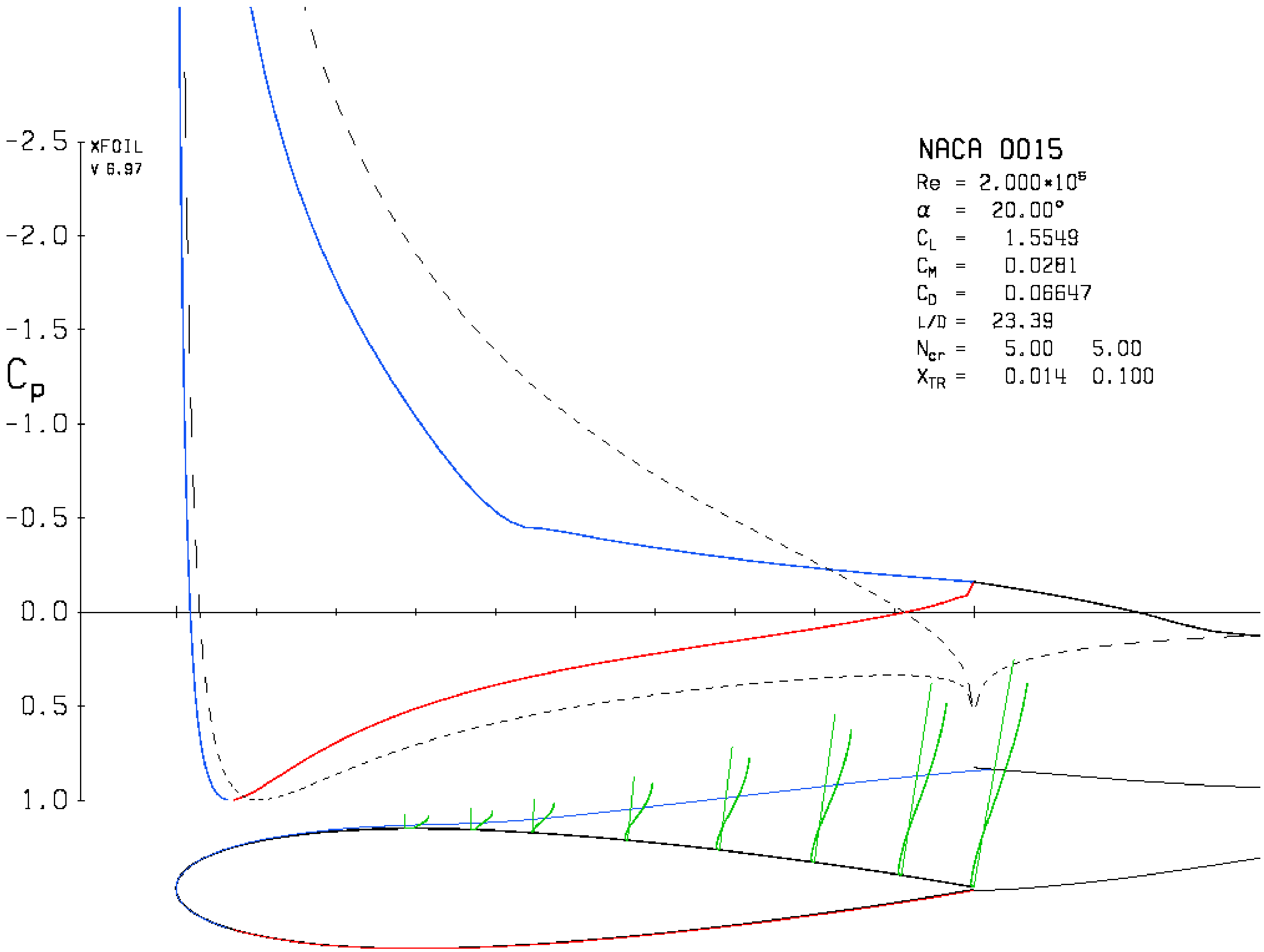

NACA 4412 Re = 0
NACA 4412 Re = 2000000
Re
0
2000000
CL
CL
CM
10⁴ * CD
α
0.0
0.5
1.0
1.5
2.0
100
200
300
400
500
-0.25
-0.20
-0.15
-0.10
-0.05
-4 -2 2 4 6 8 10 12 14 16 18 20


-2.5
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
Cp
XFOIL
V 6.97
NACA 0015
Re = 2.000*10^6
α = 6.00°
CL = 0.6780
CM = -0.0000
CD = 0.01078
L/D = 62.89
Ncr = 5.00 5.00
XTR = 0.087 0.100

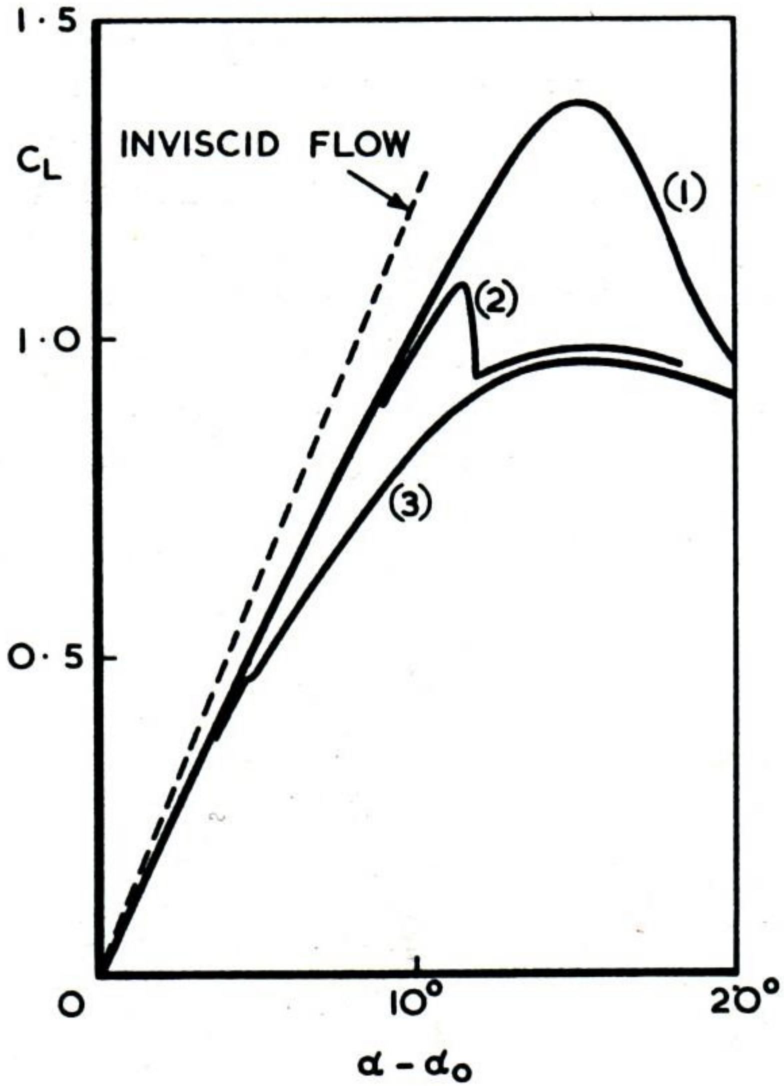

XFOIL
V 6.97
-2.5
-2.0
-1.5
Cp
-1.0
-0.5
0.0
0.5
1.0
NACA 0015
Re = 2.000*10^6
α = 12.00°
CL = 1.2800
CM = 0.0102
CD = 0.01624
L/D = 78.84
Ncr = 5.00 5.00
XTR = 0.026 0.100



XFOIL
V 6.97
-2.5
-2.0
-1.5
Cp
-1.0
-0.5
0.0
0.5
1.0
NACA 0015
Re = 2.000*10^6
α = 16.00°
CL = 1.5332
CM = 0.0315
CD = 0.02559
L/D = 59.90
Ncr = 5.00 5.00
XTR = 0.017 0.100

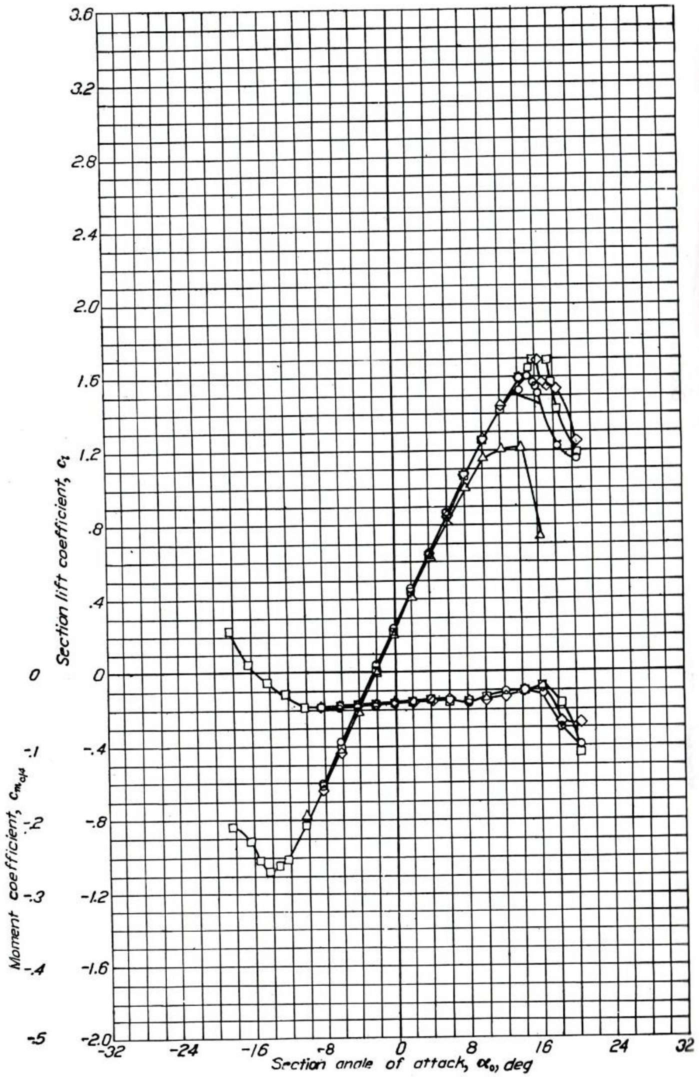


XFOIL
V 6.97
-2.5
-2.0
-1.5
Cp
-1.0
-0.5
0.0
0.5
1.0
NACA 0015
Re = 2.000*10^6
α = 20.00°
CL = 1.5549
CM = 0.0281
CD = 0.06647
L/D = 23.39
Ncr = 5.00 5.00
XTR = 0.014 0.100

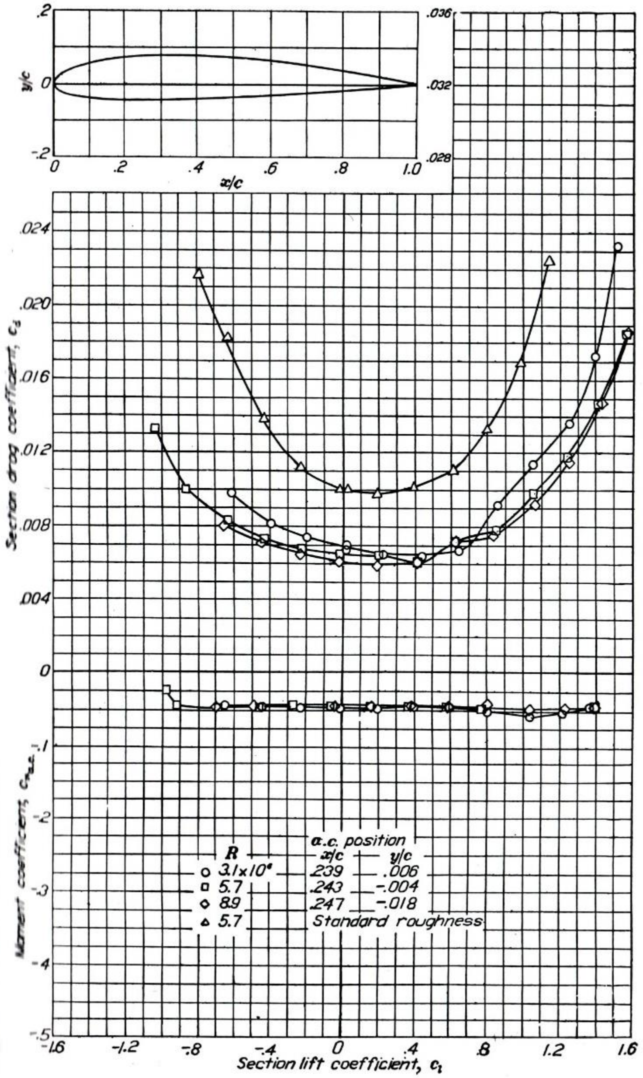



Aerodynamic Design of Aircraft – Pergamon Press, 1978

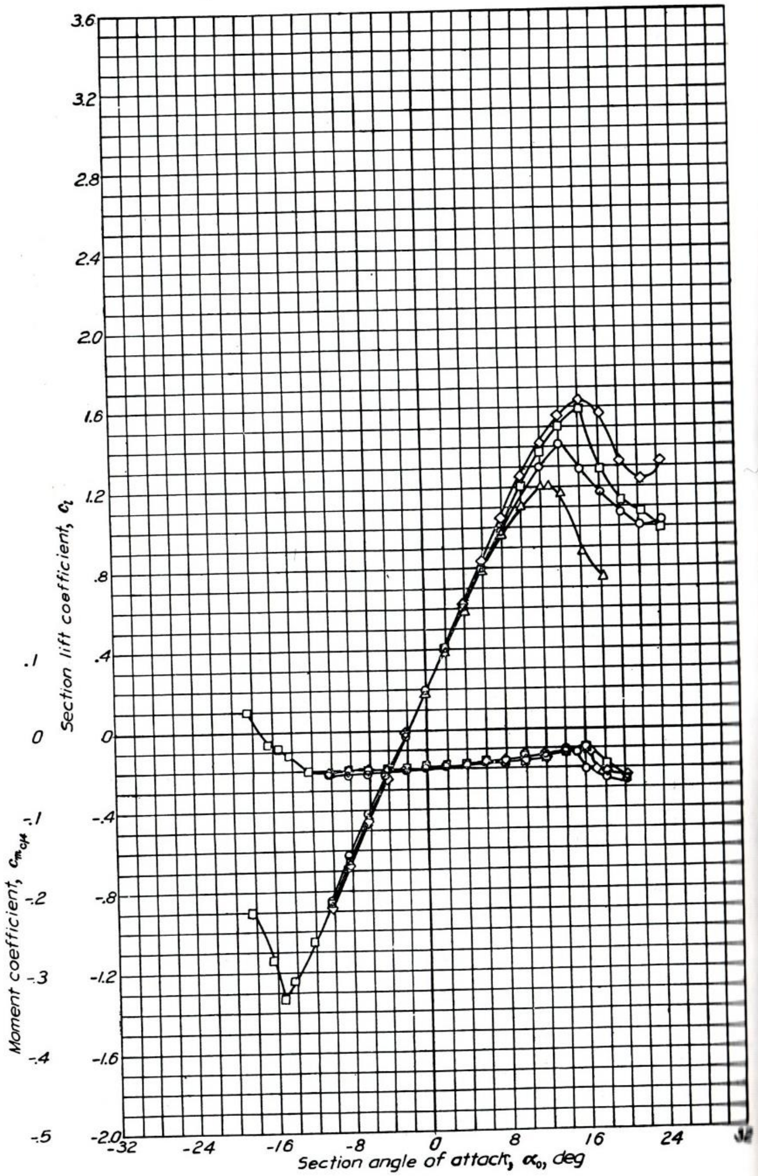
I.H.Abbott, A.E.von Doenhoff, Theory of Wing Sections, Dover Publications Inc., 1958 (presentation pp. 37-53)



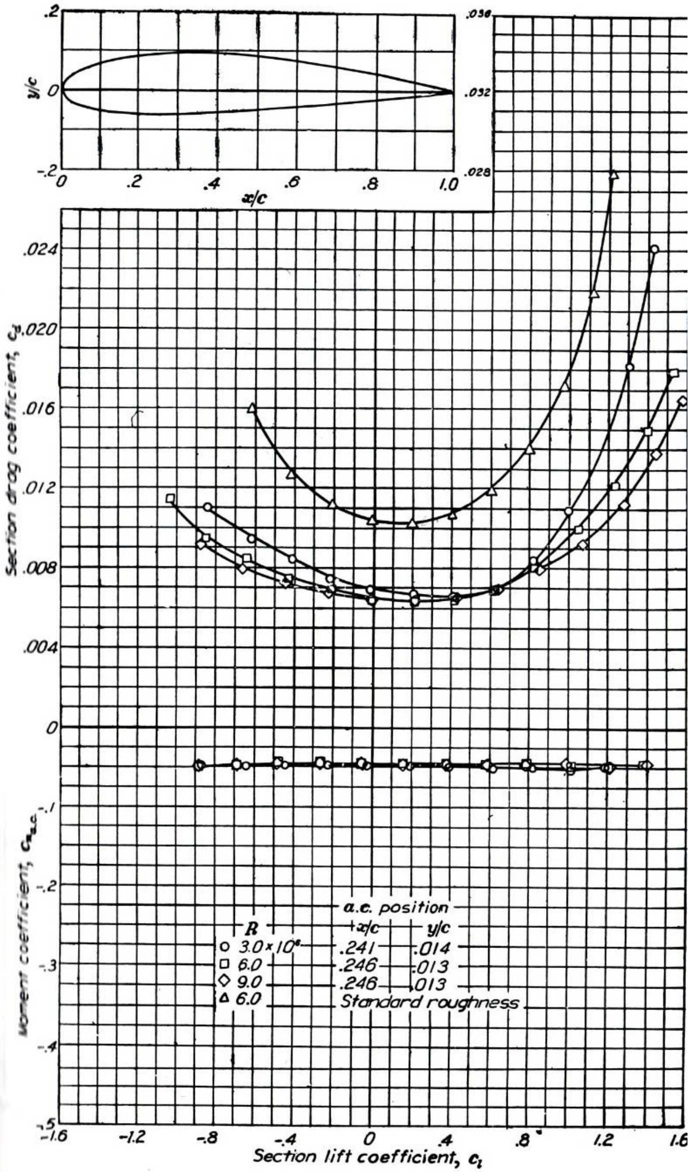

NACA 2412 Wing Section



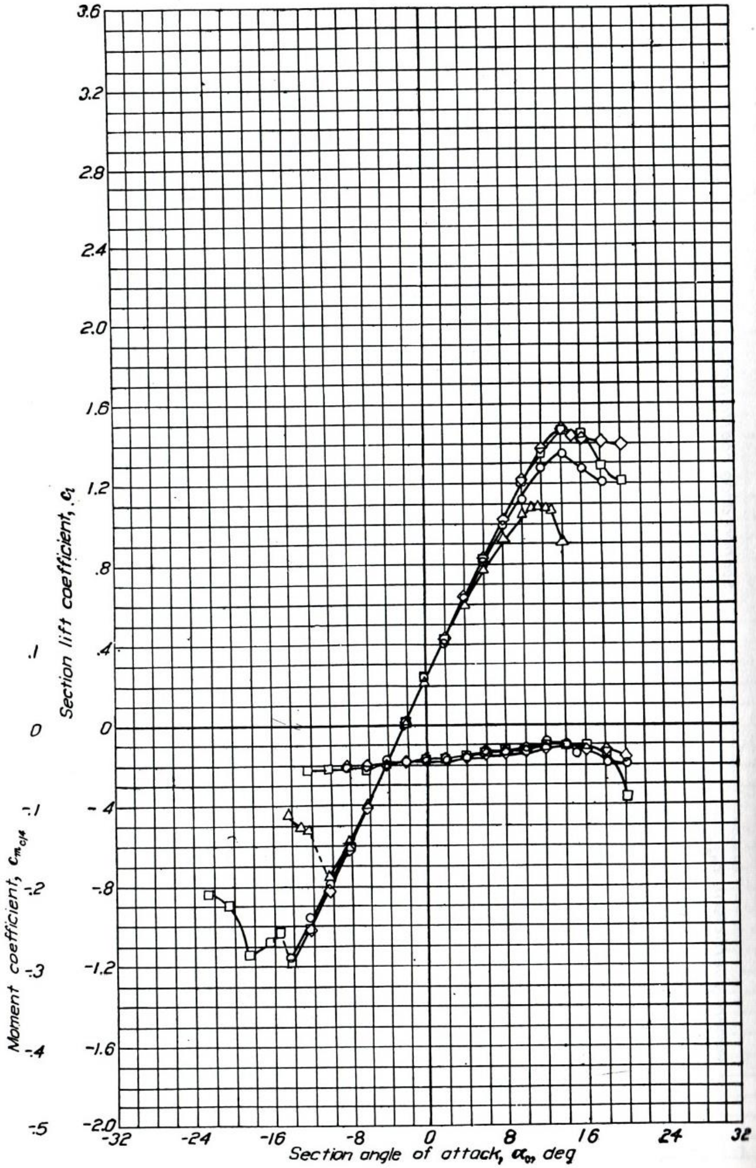

NACA 2412 Wing Section (*Continued*)



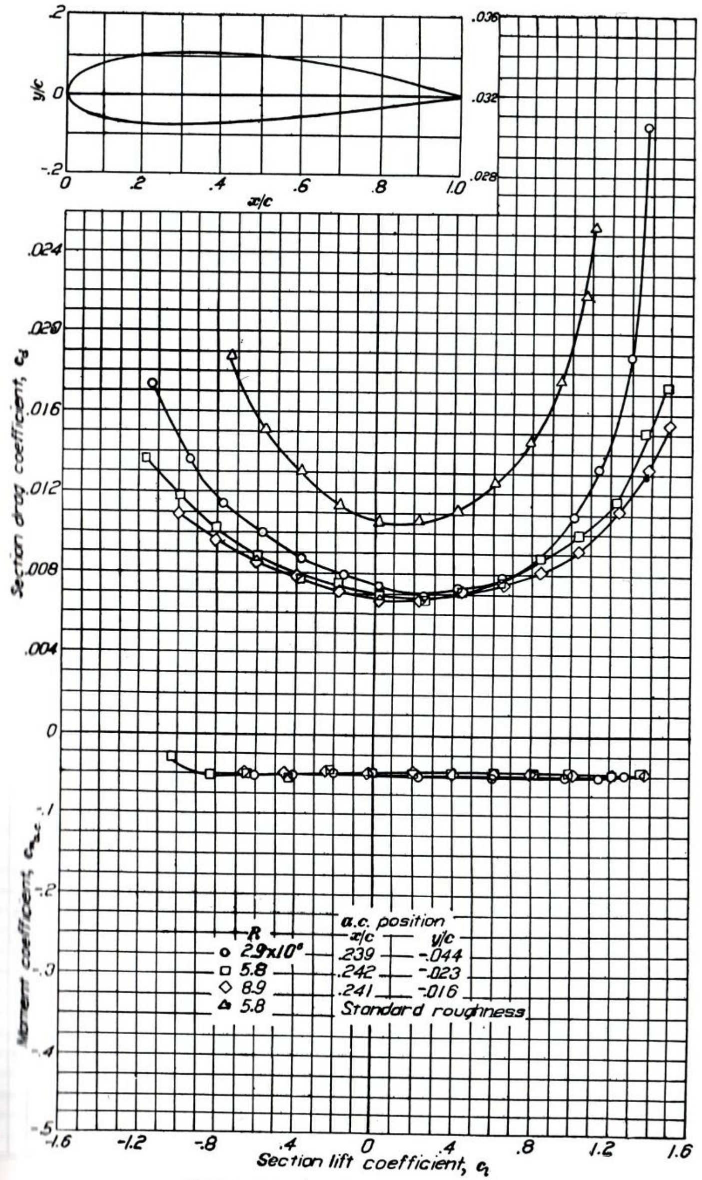

NACA 2415 Wing Section



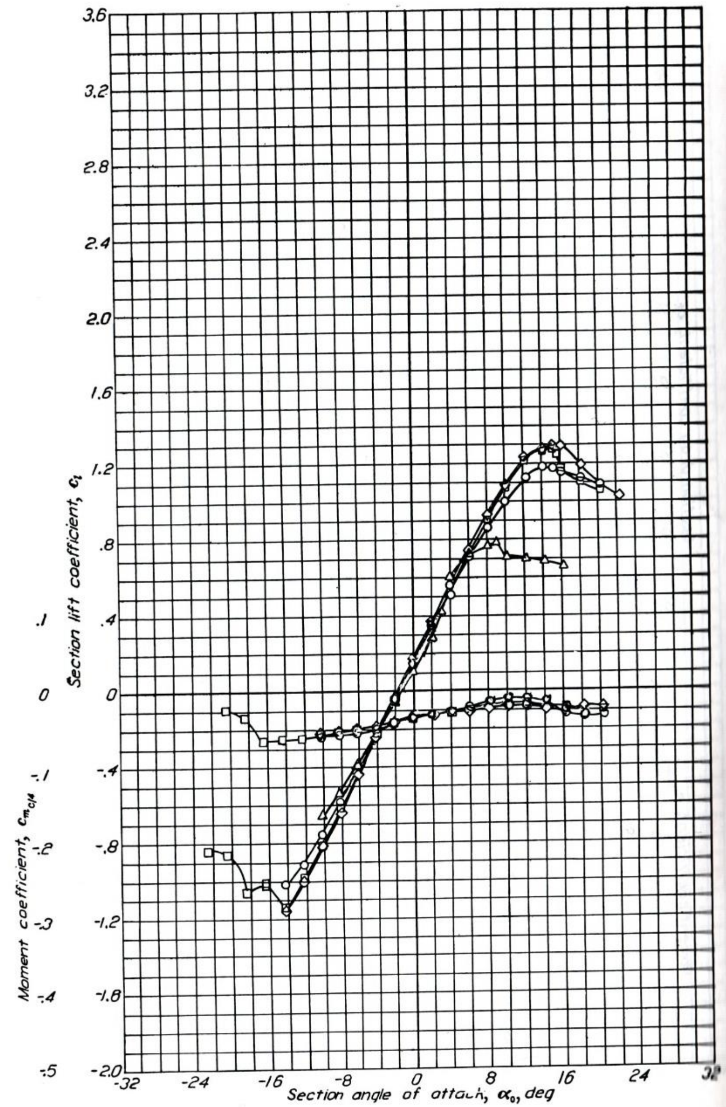

NACA 2415 Wing Section (*Continued*)




NACA 2418 Wing Section

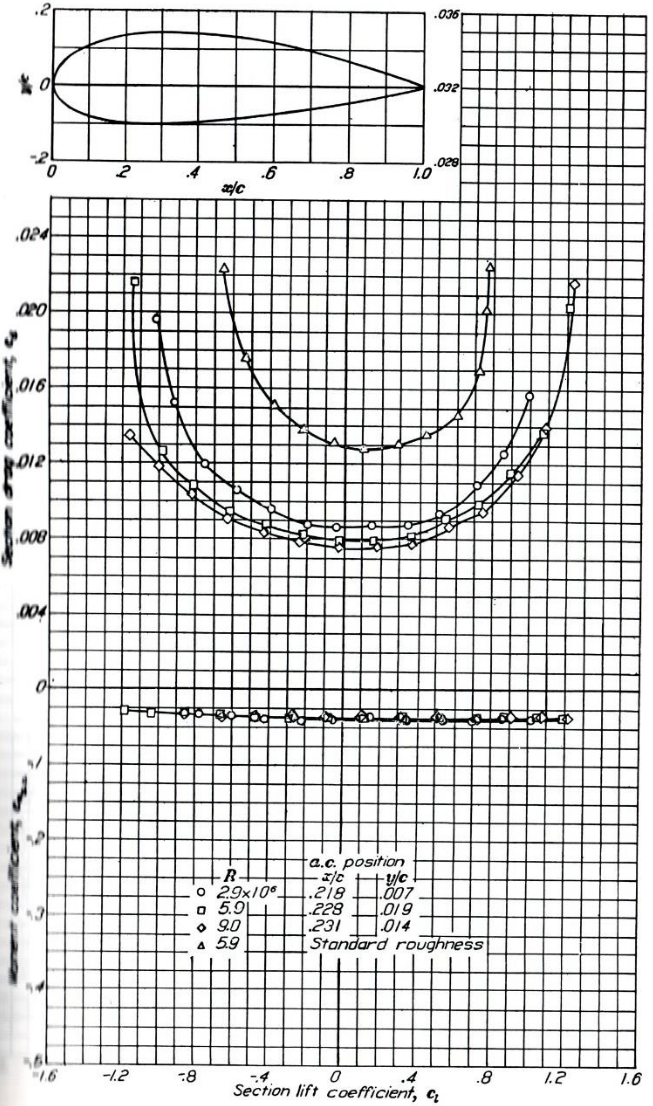



NACA 2418 Wing Section (*Continued*)

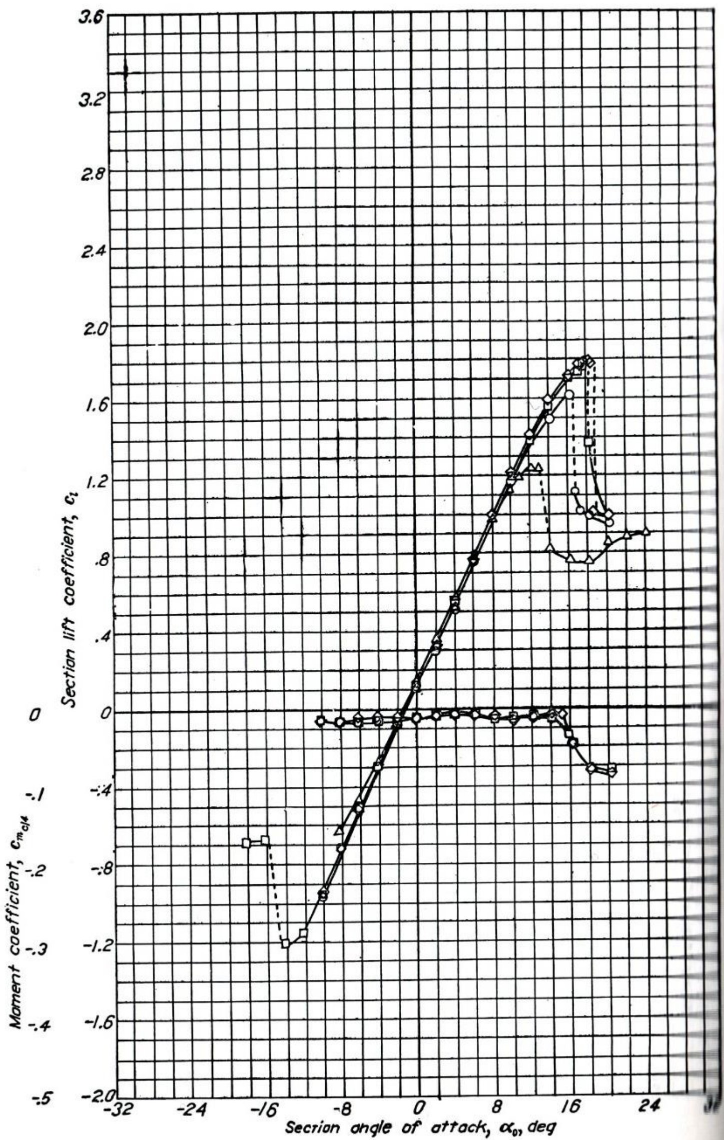



NACA 2424 Wing Section



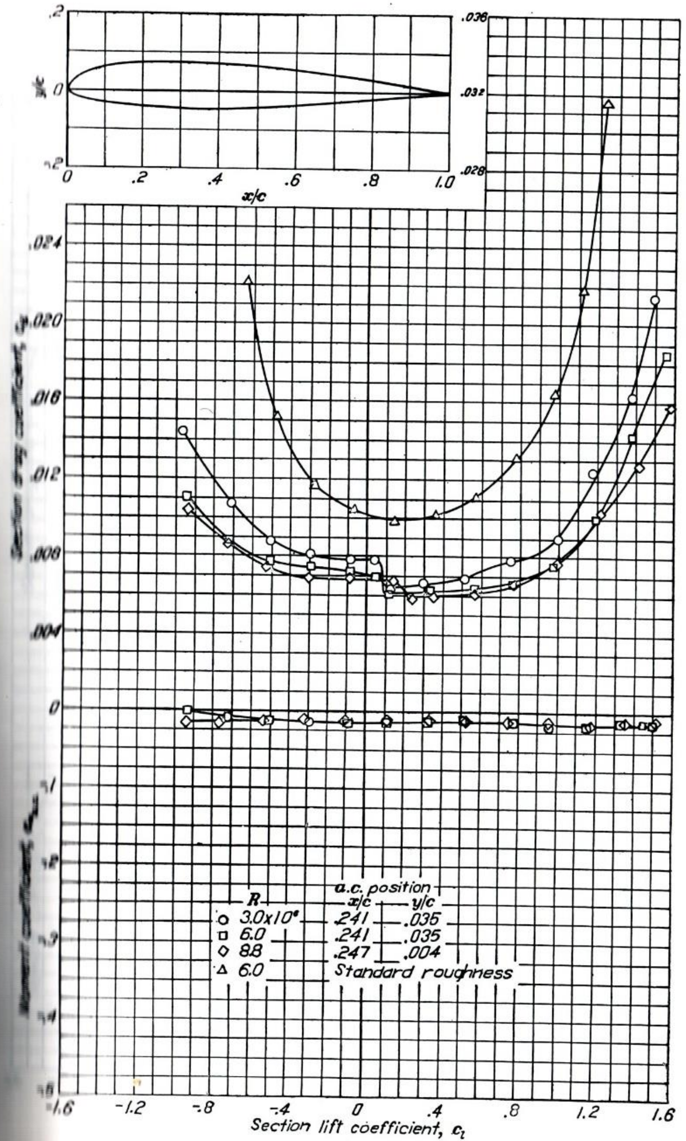



NACA 2424 Wing Section (*Continued*)

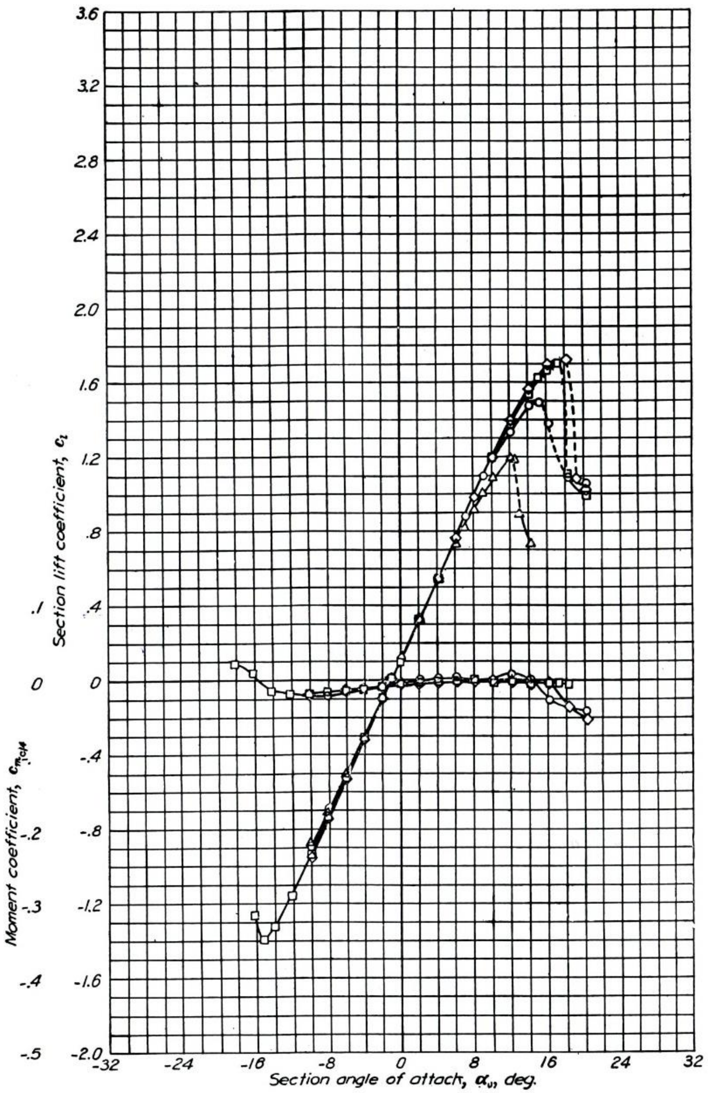



NACA 23012 Wing Section



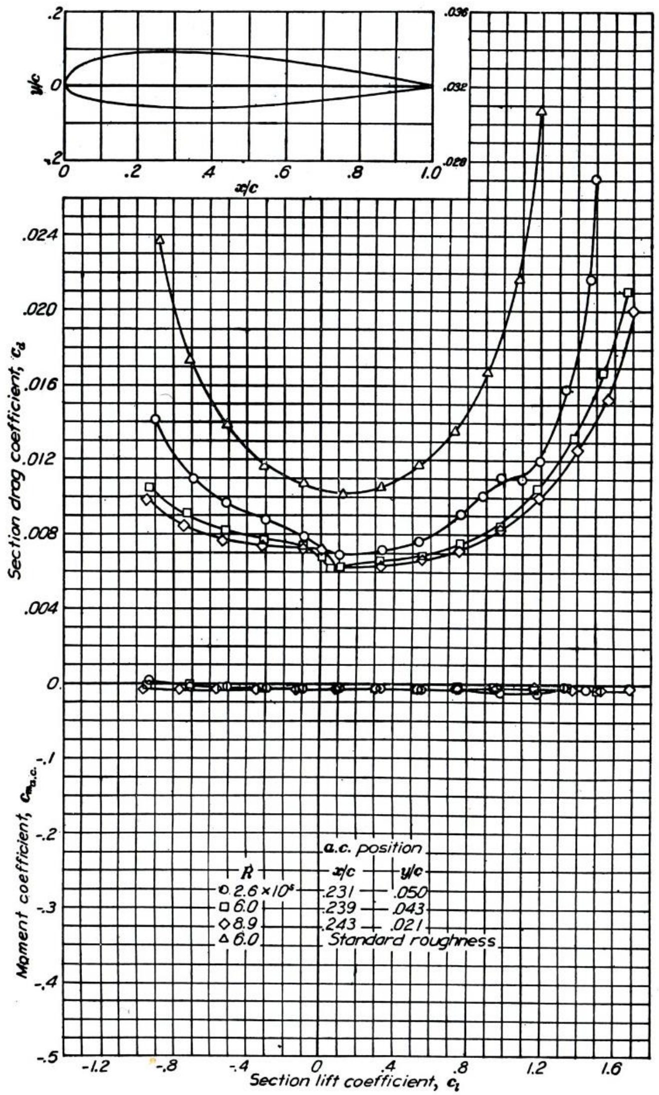



NACA 23012 Wing Section (*Continued*)

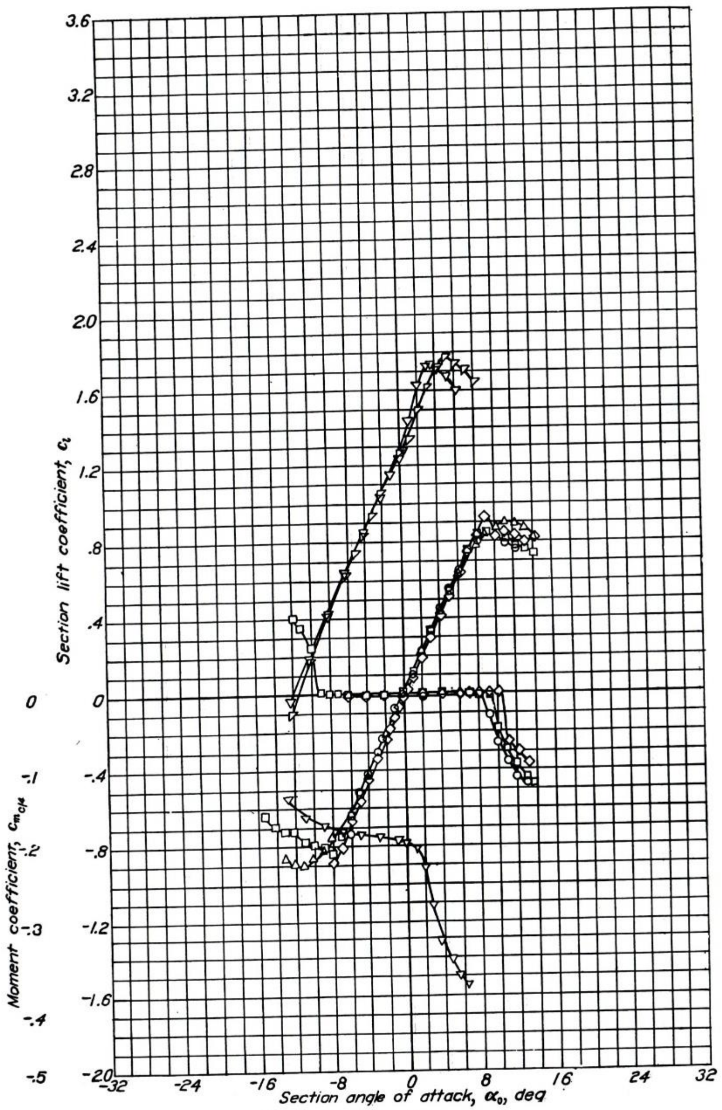



NACA 23015 Wing Section



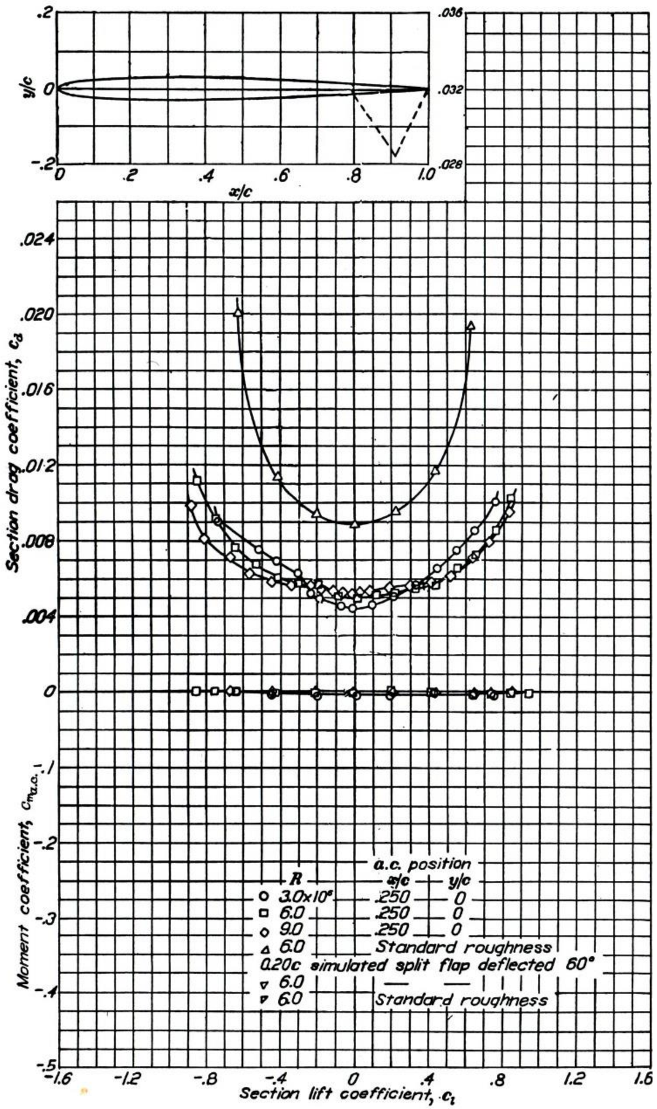



NACA 23015 Wing Section (*Continued*)




NACA 0006 Wing Section






NACA 0006 Wing Section (*Continued*)

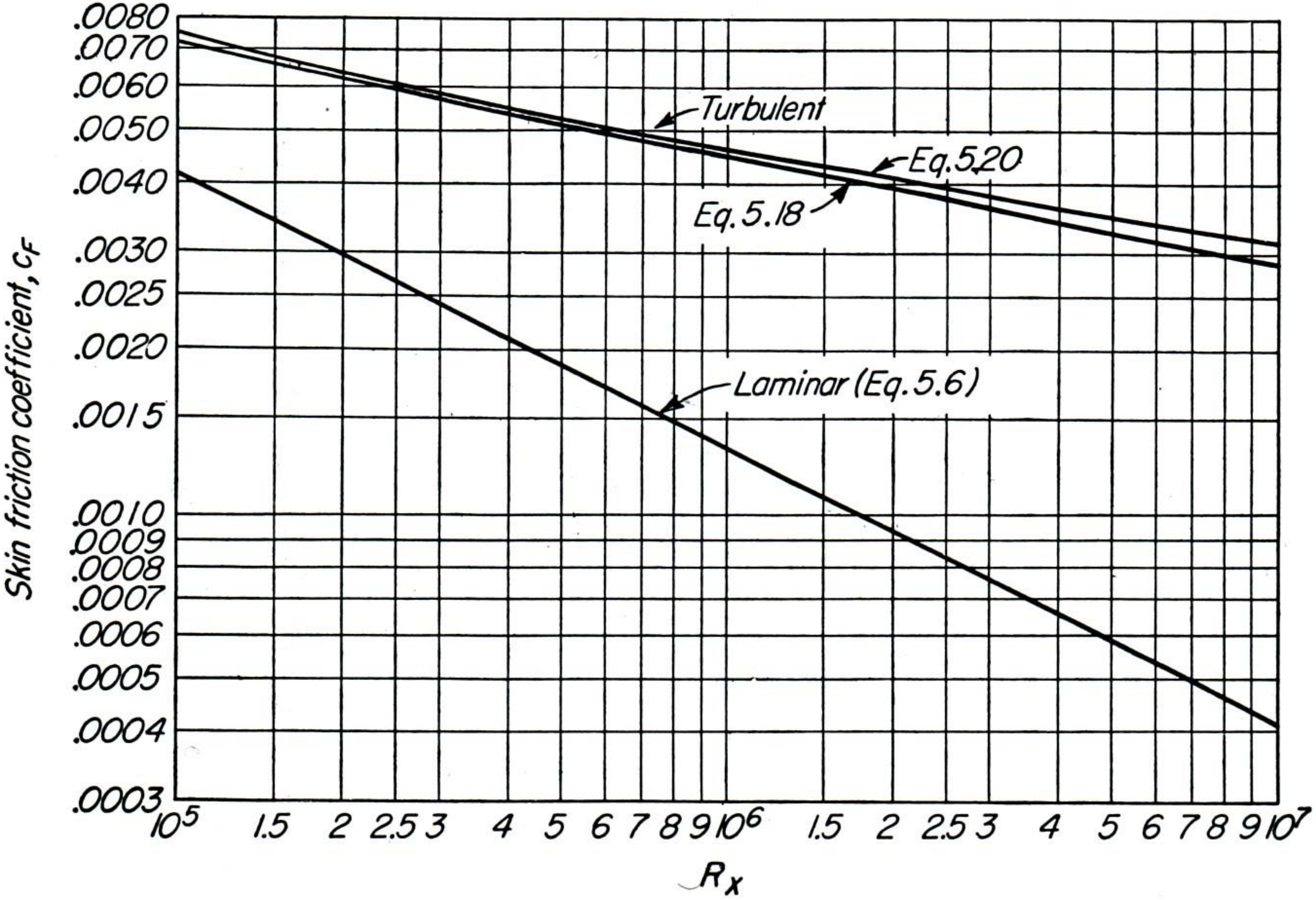



**Fig. 46.** Laminar and turbulent skin-friction coefficients for one side of a flat plate.

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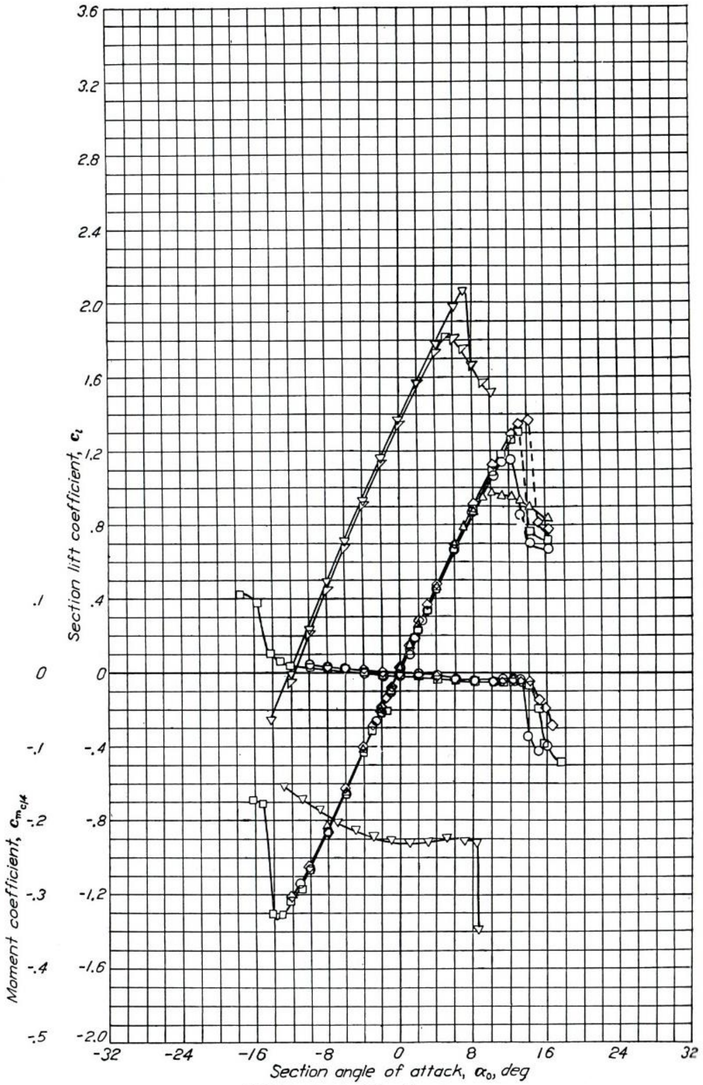
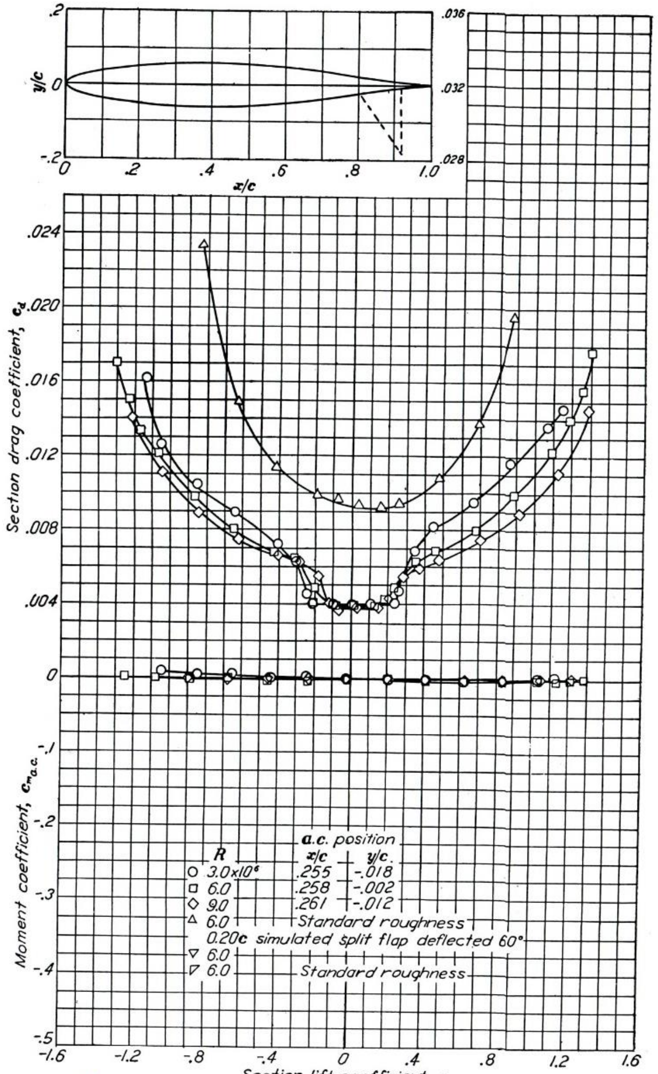
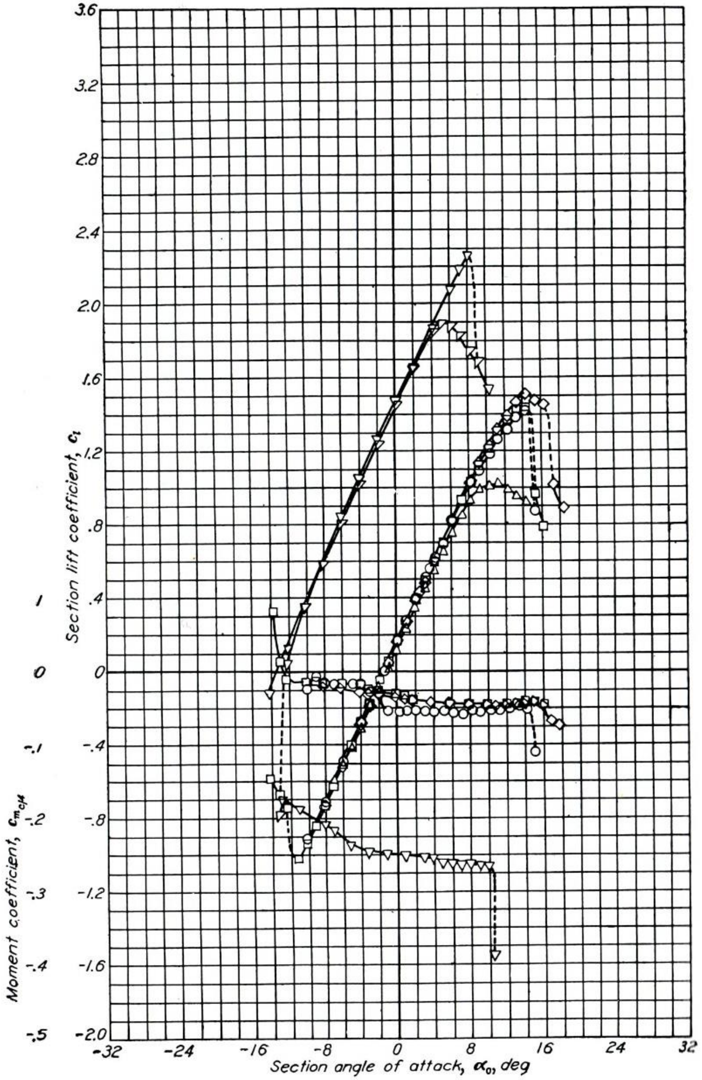



NACA $65_1$-012 Wing Section

APPENDIX IV 6

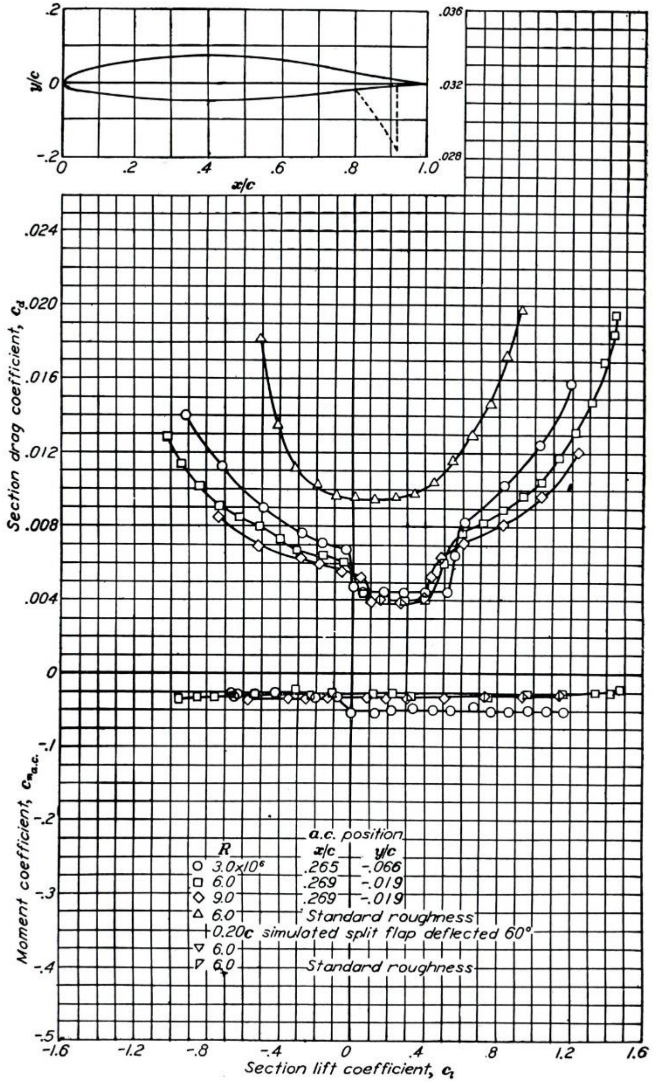



NACA $65_1$-012 Wing Section (*Continued*)

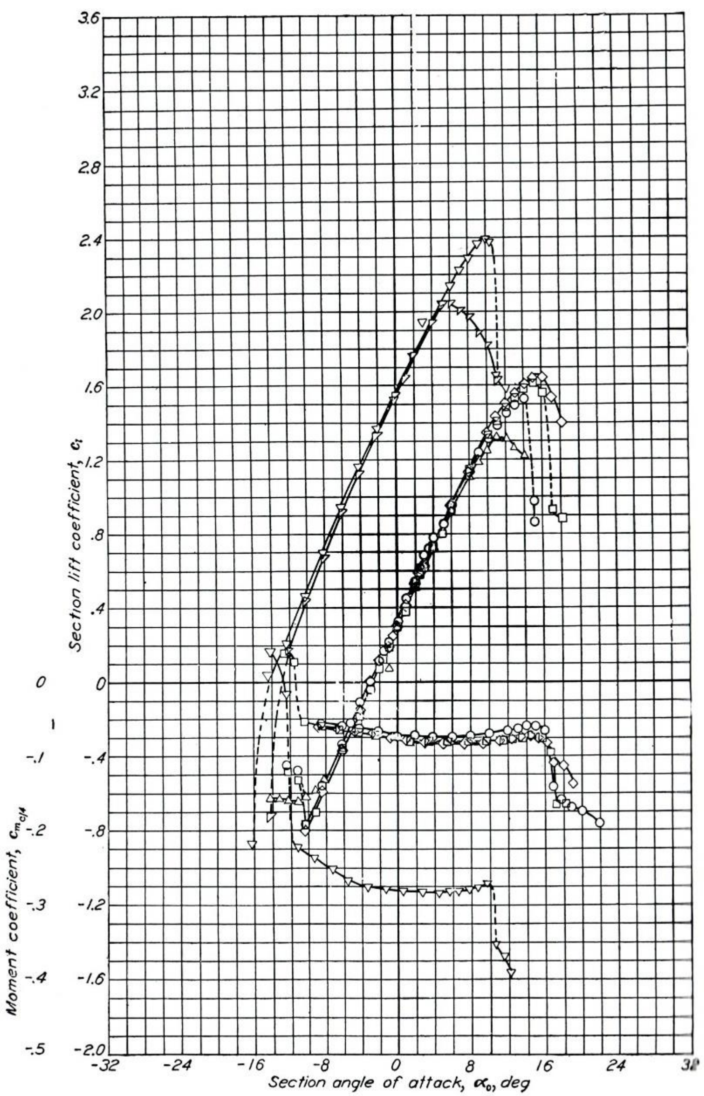



NACA $65_1$-212 Wing Section
$a = 0.6$



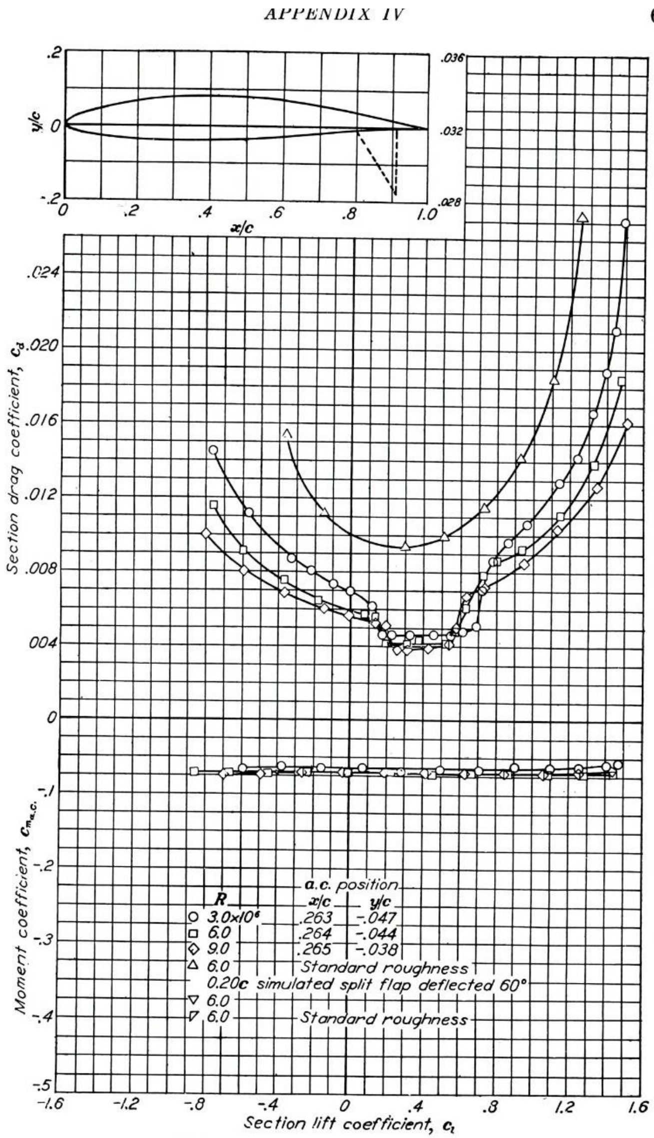



NACA $65_1$-212 Wing Section
$a = 0.6$ (*Continued*)

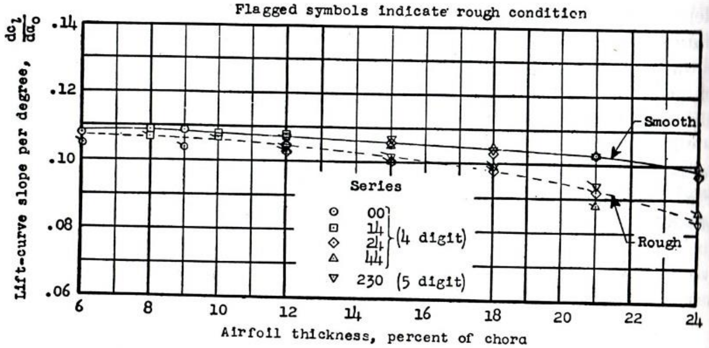



NACA $65_1$-412 Wing Section

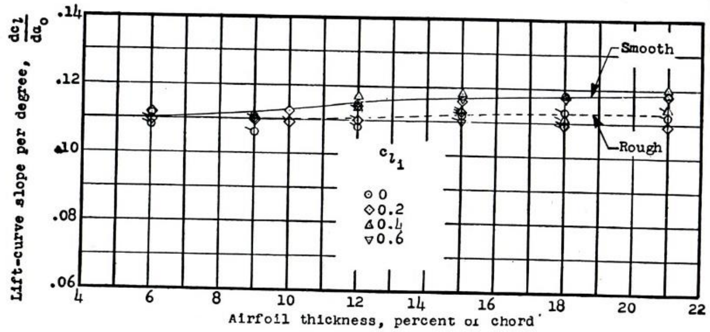




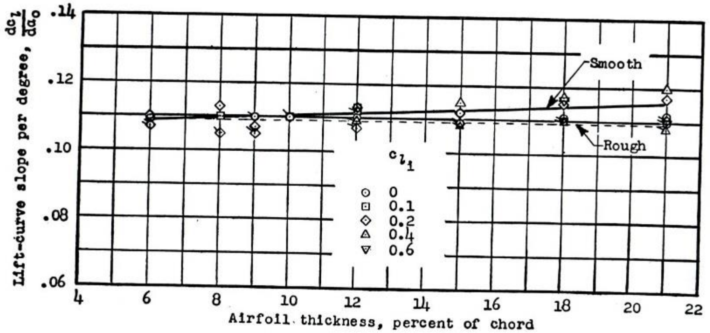

NACA $65_1$-412 Wing Section (*Continued*)




(a) NACA four- and five-digit series.

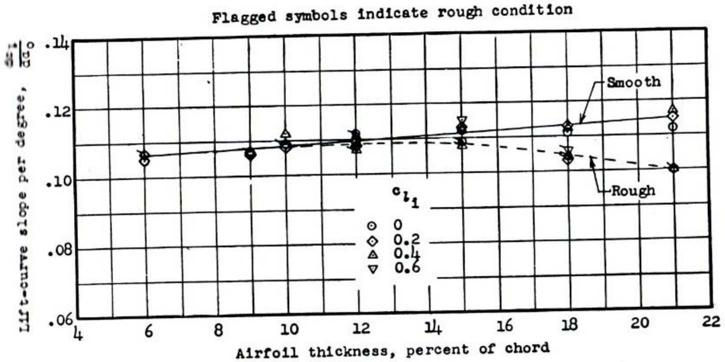



(b) NACA 63- series.

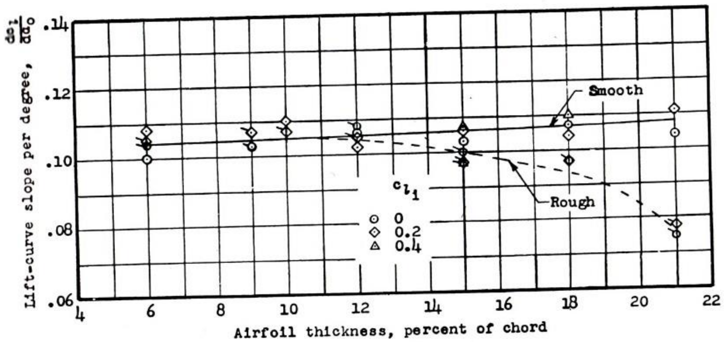



(c) NACA 64- series.



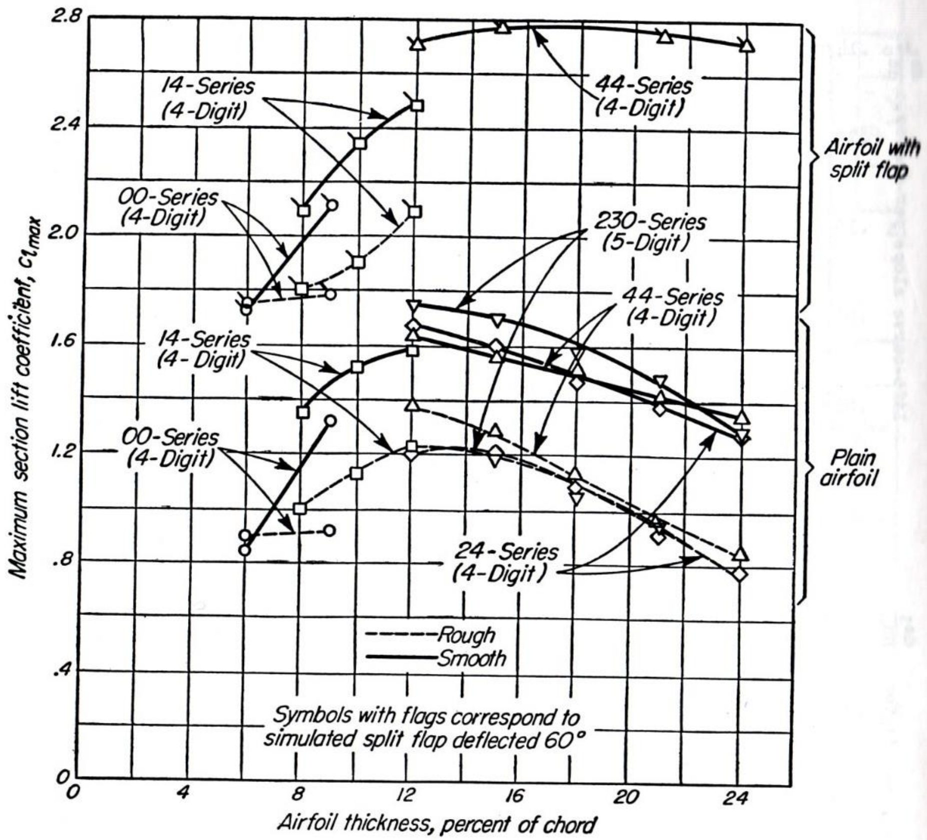



(d) NACA 65-series.




(e) NACA 66- series.

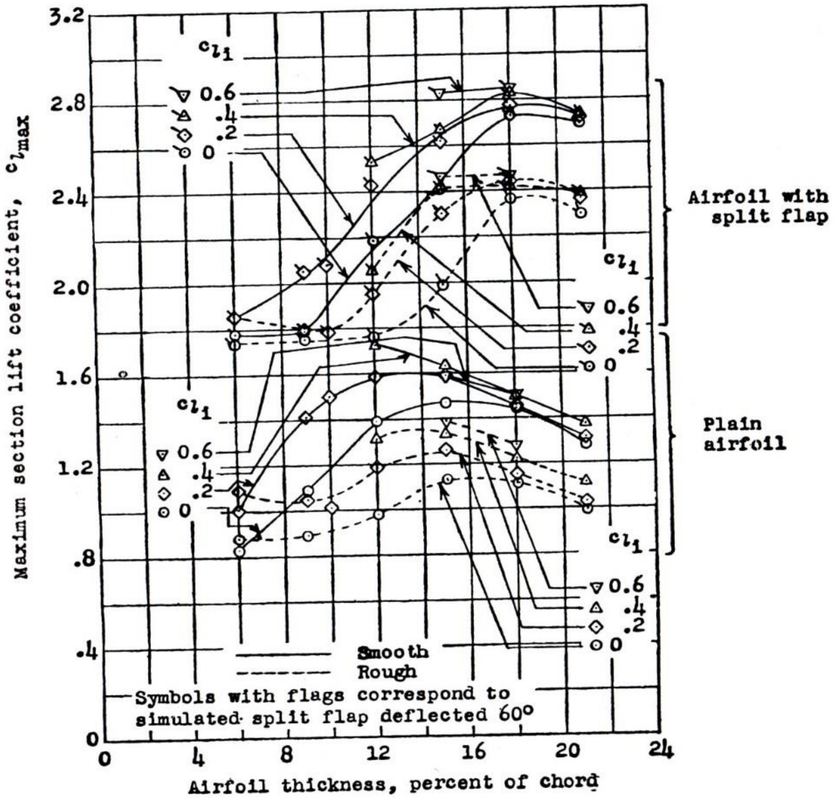
FIG. 57. Variation of lift-curve slope with airfoil thickness ratio and camber for a number of NACA airfoil sections in both the smooth and rough conditions. $R$, $6 \times 10^6$.




(a) NACA four- and five-digit series.

FIG. 58. Variation of maximum section lift coefficient with airfoil thickness ratio and camber for several NACA airfoil sections with and without simulated split flaps and standard roughness. $R$, $6 \times 10^6$.

practically nonconst...




(b) NACA 63-series.

FIG. 58. (*Continued*)

sections that would be considered for practical application. This jog n

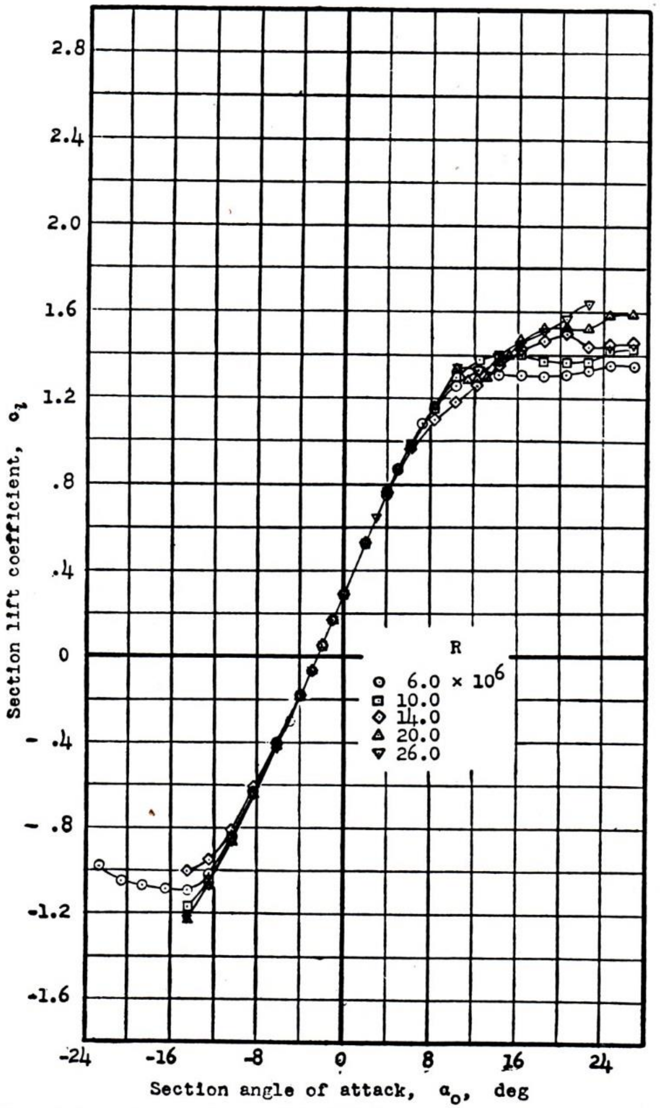



Fig. 61. Lift and drag characteristics of the NACA 63(420)-422 airfoil at high Reynolds number; TDT tests 228 and 255.

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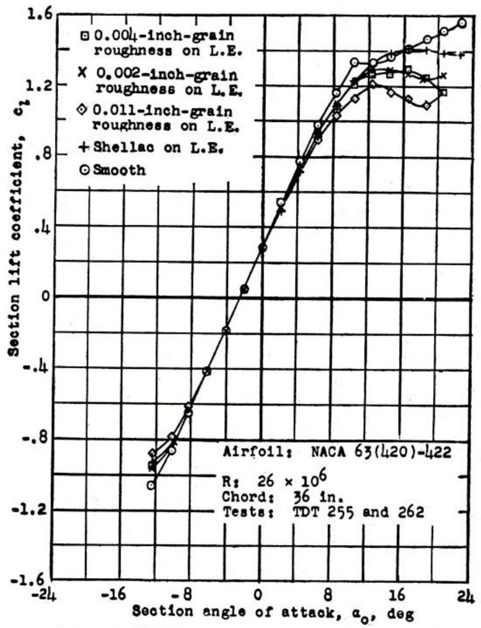



Fig. 62. Lift characteristics of a NACA 63(420)-422 airfoil with various degrees of roughness at the leading edge.

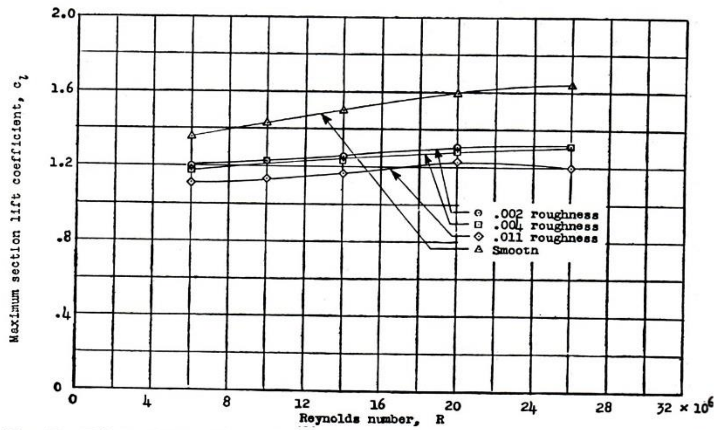



Fig. 63. Effects of Reynolds number on maximum section lift coefficient $c_l$ of the NACA 63(420)-422 airfoil with roughened and smooth leading edge.

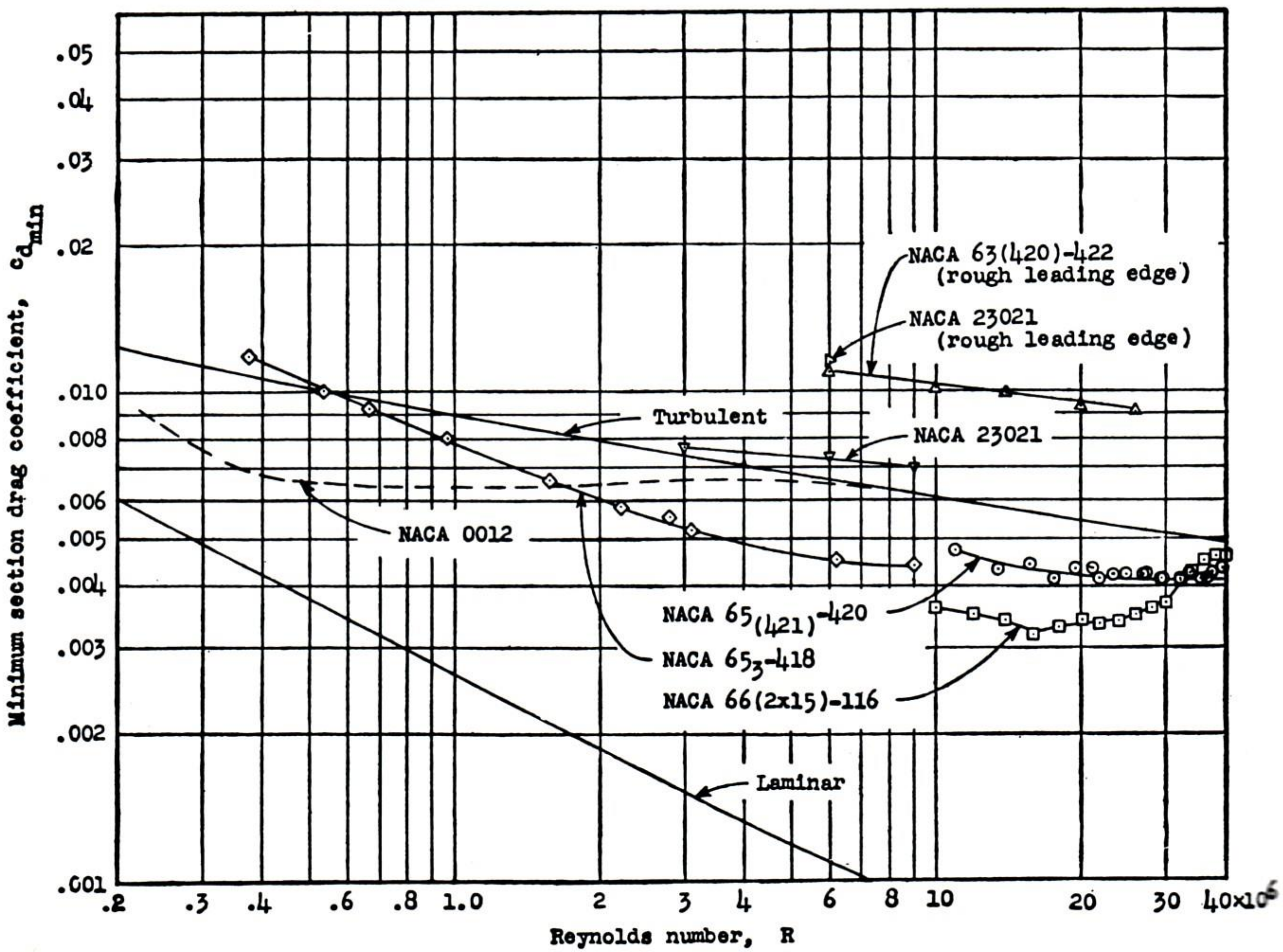



FIG. 66. Variation of minimum drag coefficient with Reynolds number for several airfoils, together with laminar and turbulent skin-friction coefficients for a flat plate.

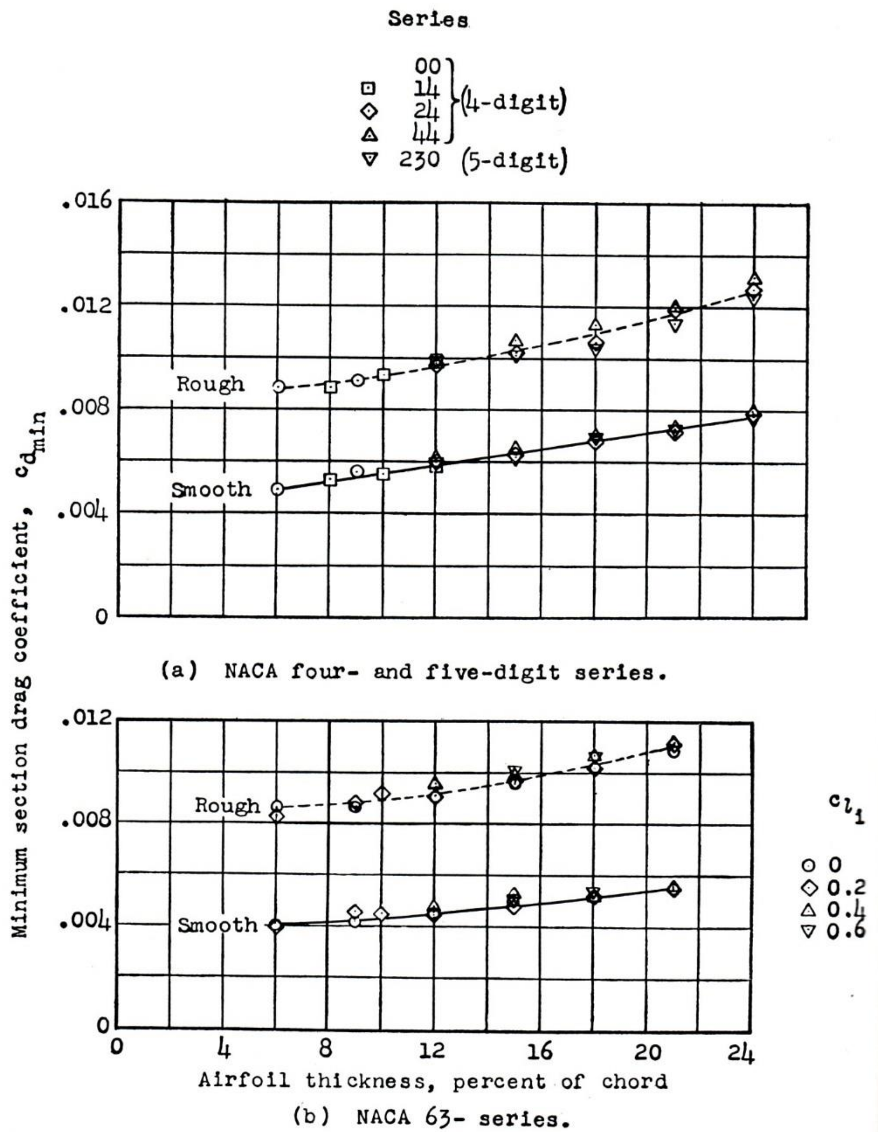
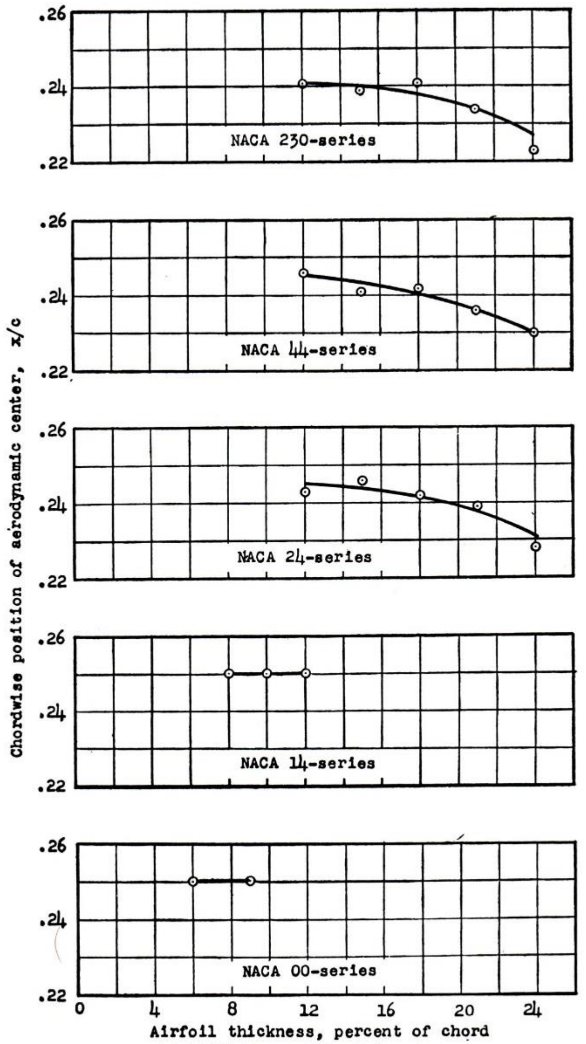


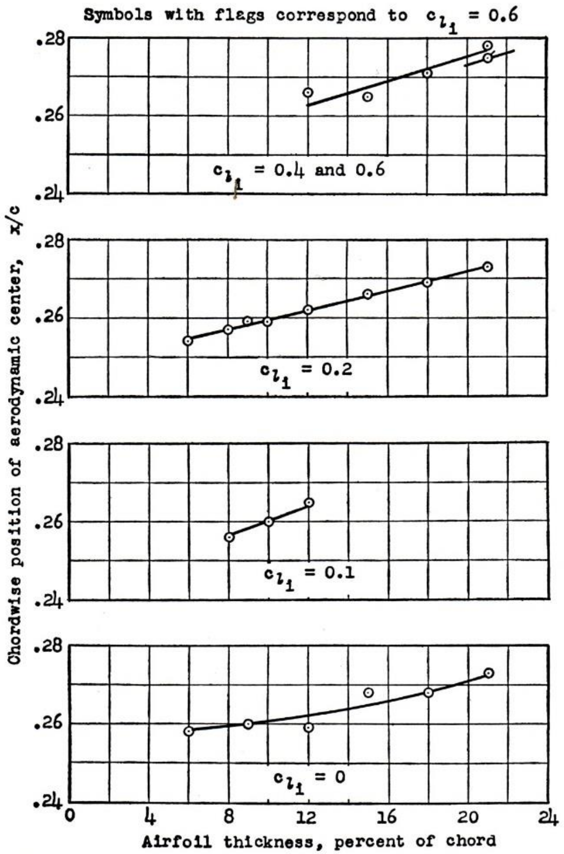

FIG. 68. Variation of section minimum drag coefficient with airfoil thickness ratio for several NACA airfoil sections of different cambers in both smooth and rough conditions. $R$, $6 \times 10^6$.




(a) NACA four- and five-digit series.

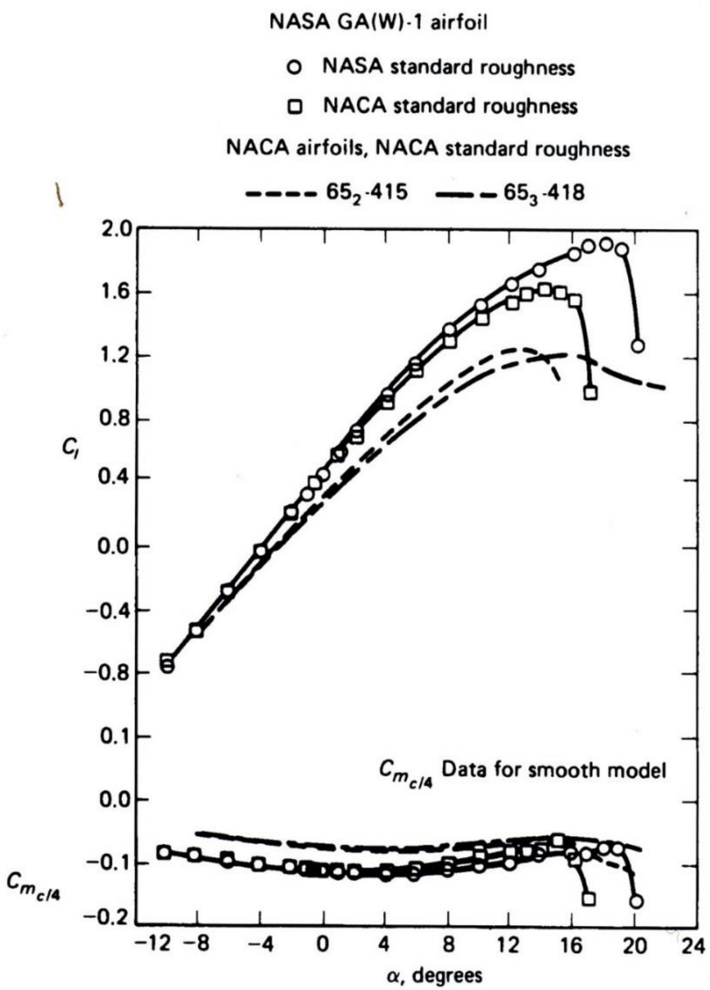
Fig. 94. Variation of section chordwise position of the aerodynamic center with airfoil thickness ratio for several NACA airfoil sections of different cambers. $R$, $6 \times 10^6$.

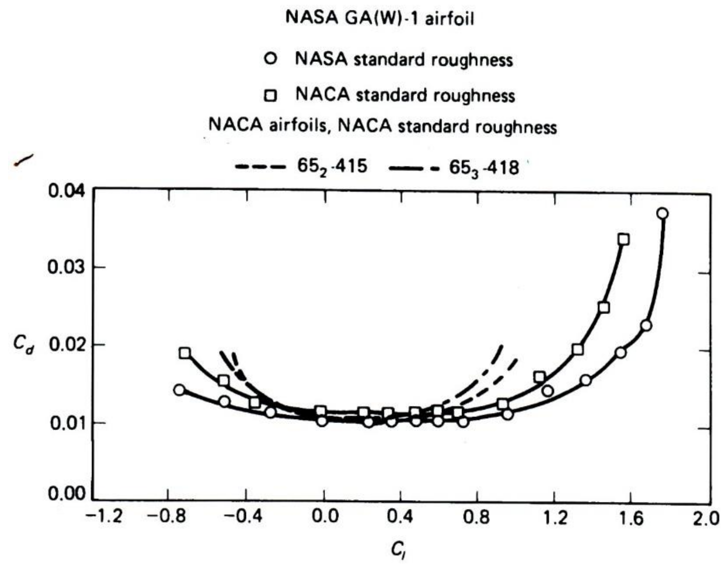





(c) NACA 64-series.

Fig. 94. (*Continued*)




R.J.McGhee, W.D.Beasley,- Low Speed Aerodynamic Characteristics of a 17-Percent-Thick Section Designed for General Aviation Application (NASA TN D-7428, 1973) [from J.J.Bertin, M.L.Smith – Aerodynamics for Engineers, Printice-Hall International, Inc., 1998]