Ex.7. Balance system measurement of aerodynamic forces. Determination of aerodynamic coefficients.

1. Aim of the exercise.

   The aim is to present how to measure aerodynamic forces and how to interpret the results.

2. Theoretical background.

   a. Aerodynamic force

      In the viewpoint of the interaction between a fluid and a body immersed in the fluid there is no difference if the body moves in a still fluid or non moving body is immersed in a uniform stream of fluid. This principle is the foundation for the idea of performing the aerodynamic experiments in special tunnels. Such tunnels can be equipped with stationary balance systems that allows to precisely measure the force generated by the fluid flow. The difference between flow conditions within a tunnel and natural conditions (finite size of the tunnel, turbulence intensity and pressure in the test-section) can be taken into account in the form of particular corrections. Transferring the results obtained in the tunnel to the objects at different scale can be done only if dynamic similarity is satisfied.

      Theory of similitude and dimensional analysis suggest that aerodynamic force, acting on a body immersed in a moving thermally non-conductive ideal gas, can be determined by knowing the following quantities:
      L – characteristic length of the body
      α, β, γ – angles determining the position the body with respect to the fluid stream direction
      V – undisturbed fluid flow
      T – temperature
      ρ – density
      μ – viscosity
      R – gas constant

      Aerodynamic force P_A is a combination of the mentioned quantities. Constructing nondimensional similarity criteria one obtains:

$$Re = \frac{\rho L V}{\mu} \quad \text{- Reynolds number} \qquad (1)$$

$$M = \frac{V}{\sqrt{kRT}} \quad \text{- Mach number} \qquad (2)$$

One can write

$$P_A = f\left(\frac{\rho V^2}{2}, \alpha, \beta, \gamma, S, Re, M\right)$$

where

$\frac{\rho V^2}{2}$ = q – dynamic pressure of undisturbed flow,

S – reference area

It needs to be pointed out that knowing the value of aerodynamic force P_A is not sufficient to describe its vector form. To describe this vector one may employ Cartesian coordinate system oriented in such a way that x-axis is co-linear with velocity vector. Sometimes, due to construction, one may use a system fixed with the investigated body (n, t, y) – fig.1. In the figure one may find the following symbols:

P_x – drag force (projection of aerodynamic force on V direction)

P_y – side force (projection of aerodynamic force on y-axis)

P_z – lift force (projection of aerodynamic force on axis perpendicular to V)

M_x – roll moment

M_y – pitching moment

M_z – yaw moment

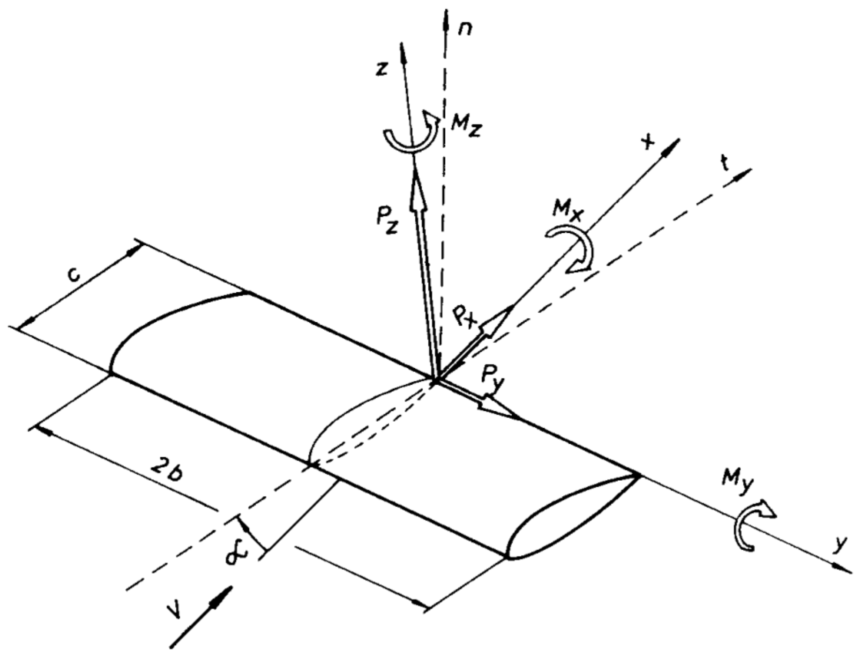

Figure 1. Components of force and moments

b. Aerodynamic coefficients

Aerodynamic force can be described by nondimensional coefficients of forces and moments. They are defined in the following way:

$$C_x = \frac{P_x}{\frac{\rho V^2}{2} S}, \quad C_y = \frac{P_y}{\frac{\rho V^2}{2} S}, \quad C_z = \frac{P_z}{\frac{\rho V^2}{2} S},$$

$$C_{mx} = \frac{M_x}{\frac{\rho V^2}{2} S c}, \quad C_{my} = \frac{M_y}{\frac{\rho V^2}{2} S c}, \quad C_{mz} = \frac{M_z}{\frac{\rho V^2}{2} S c}. \tag{4}$$

If the considered body has a plane of symmetry then the coordinate system can be placed in such a way that $C_y = C_{mx} = C_{mz} = 0$ (plane of symmetry is then zx).
Such case is fundamental in the research of flight mechanics.
Aerodynamic coefficients, for particular Re and M, depend only on the angle of attack α (angle between the velocity vector and the chord). These dependencies are the so-called aerodynamic characteristics of a wing or, for a 2D flow – aerodynamic characteristics of an airfoil.

The moment coefficient C_m depends on the point (or axis), that was considered for the estimation of moment. Recalculation of the moment coefficient C_{m0} (defined with respect to y axis going through point 0 attached to the balance system) into C_{m01} (with respect to y_1 going through the point 0_1 attached to the investigated model) can be done in the following way (see fig.2):

$$M_1 = M_o - P_z x_1 - P_x z_1, \tag{5}$$

where:

$$z_1 = n_o \cos\gamma + t_o \sin\gamma,$$

$$x_1 = t_o \cos\gamma - n_o \sin\gamma.$$

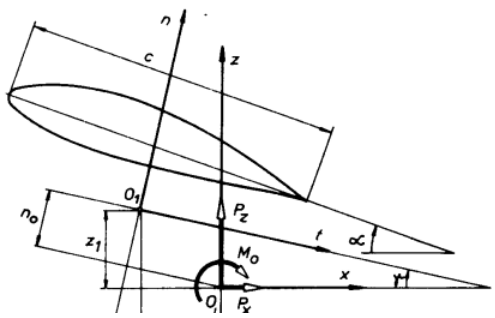

Figure 2 Coordinate system attached to the balance system

After some manipulations:

$$M_1 = M_o - t_o\,(P_z \cos\gamma + P_x \sin\gamma) - n_o\,(P_x \cos\gamma - P_z \sin\gamma).$$

Nondimensionalising this accordingly to the Eq.(4) one obtains

$$C_{m1} = C_{mo} - \frac{T_o}{c}\,(C_z \cos\gamma + C_x \sin\gamma) - \frac{n_o}{c}\,(C_x \cos\gamma - C_z \sin\gamma). \quad (6)$$

By means of Eq.(6) one may determine the location of characteristic points: (i) center of pressure and (ii) aerodynamic center.

c. Center of pressure

It is an intersection point of a chord and aerodynamic force, i.e. a point on a chord with respect to which aerodynamic moment is 0. For a wing model, it is convenient to select point "0" (balance axis) in such a way that it located on the chord and "t"-axis is co-linear with the chord. In such geometry $\gamma = \alpha$, $n_0 = 0$. Equation (6) for the center of pressure (M=0), has the following form:

$$0 = C_{mo} - \frac{T_p}{c}\,(C_z \cos\alpha + C_x \sin\alpha),$$

So

$$\frac{t_p}{c} = \frac{C_{mo}}{C_z \cos\alpha + C_x \sin\alpha}, \quad (7)$$

where t_p is the distance between the point "0" and the center of pressure.

Dependency of t_p/c as a function of α is one of the characteristics of a wing and it is the so-called movement of the center of pressure. For small angles of attack, i.e. where approximation $\cos\alpha \approx 1$ and $\sin\alpha \approx 0$, the position of center of pressure can be approximated as

$$\frac{t_p}{c} = \frac{C_{mo}}{C_z}. \quad (8)$$

Characteristics $C_z(\alpha)$ and $C_{m0}(\alpha)$ for the most common airfoils, in the range of small angles of attack, are linear.

Dependency $\frac{t_p}{c}(\alpha)$ will then have two asymptotes:

(i) when $C_z = 0$ and

(ii) $\frac{t_p}{c} = \frac{D C_{mo}}{d C_z}$.

This can be proven by rearranging Eq.(8)

$$\frac{t_p}{c} = \frac{C_{m o \alpha_o} + \dfrac{\mathrm{d}\,C_{m o}}{\mathrm{d}\,\alpha}\,(\alpha - \alpha_o)}{\dfrac{\mathrm{d}\,C_z}{\mathrm{d}\,\alpha}\,(\alpha - \alpha_o)} = \frac{C_{m o \alpha_o}}{C_z} + \frac{\mathrm{d}\,C_{m o}}{\mathrm{d}\,C_z} \qquad (9)$$

where α_0 denotes angle, for which $C_z = 0$.

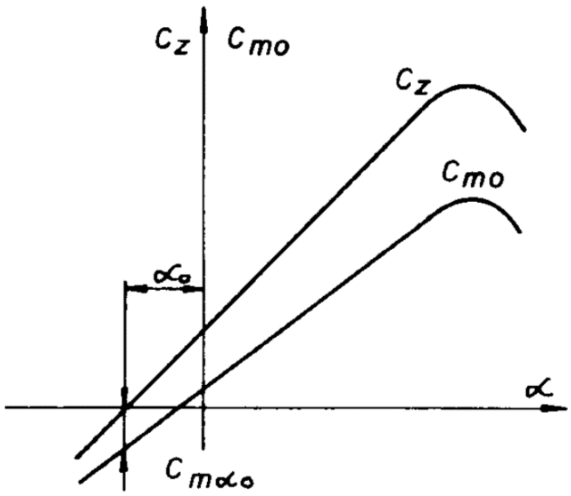

Figure 3 Characteristics Cz (alfa) and Cm0 (alfa)

As $C_{m o \alpha_o} / C_z$ is lower for higher angles of attack, and

$\dfrac{d\,C_{m o}}{\mathrm{d}\,C_z}$ in the linear range of $C_z(\alpha)$ and $C_{m0}(\alpha)$ has constant value, hence

$$\frac{t_p}{c} \dashrightarrow \frac{d\,C_{m o}}{\mathrm{d}\,C_z}.$$

For symmetric airfoil t_p/c has constant value and does not depend on the angle of attack (center of pressure does not change its location). There exist special non-symmetric airfoils (self-stabilizing profile) for which center of pressure changes its location in an opposite way to the classical.

In the fig.4 it is illustrated the variation of value, location and direction of aerodynamic force for different angles of attack, for 3 different airfoils: (a) symmetric , (b) nonsymmetric and (c) self-stabilizing profile,

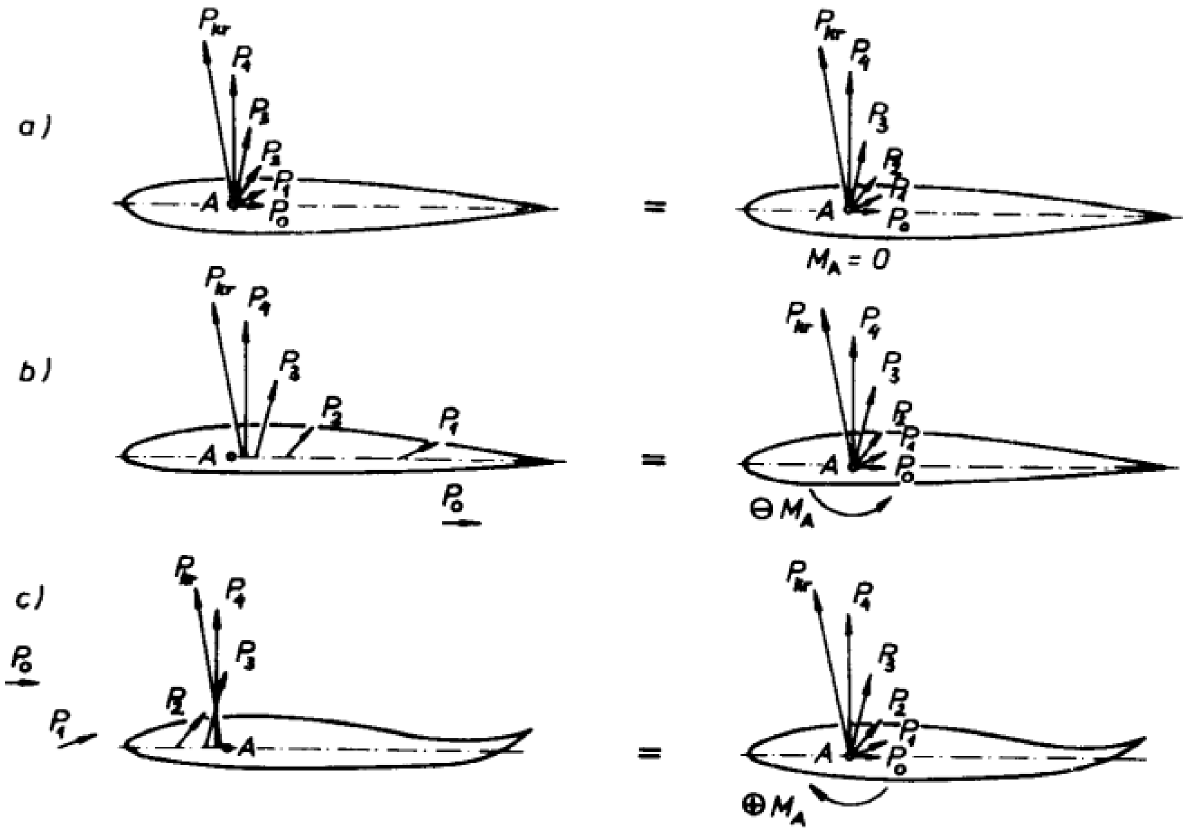

Figure 4 Movement of the center of pressure: (a) symmetric, (b) nonsymmetric and (c) self-stabilizing profiles.

d. Aerodynamic center

Aerodynamic center is a point A on the model, for which the aerodynamic moment is independent of the angle of attack (in the linear range of $C_z(\alpha)$ and $C_m(\alpha)$). Knowing its location is of great interest in the aircraft stability research as this is the point of neutral equilibrium. For the coordinate system "t,n" oriented in such a way that, $\gamma = \alpha$, by using eq.(6) one can write

$$\frac{d\,C_{m1}}{d\,\alpha} = \frac{d\,C_{mo}}{d\alpha} - \frac{t_A}{c}\left(\frac{dC_z}{d\alpha}\cos\alpha - C_z\sin\alpha + \frac{dC_x}{d\alpha}\sin\alpha + C_z\cos\alpha\right) +$$

$$+\frac{n_A}{c}\left(\frac{dC_x}{d\alpha}\cos\alpha - C_x\sin\alpha - \frac{dC_z}{d\alpha}\sin\alpha - C_z\cos\alpha\right) = 0, \quad (10)$$

When $\frac{d\,C_{m1}}{d\,\alpha} = 0$, the coordinates of the aerodynamic center are t_A and n_A.

Knowing the characteristics $C_z(\alpha)$ and $C_x(\alpha)$ one can determine all components of eq.(10) for two different angles of attack α and solve a set of equations with respect to t_A/c and n_A/c. The set of equations will be simpler if one of the angles will be the angle of attack for which $C_x = C_{x\,min}$ (then $dC_x/d\alpha$=0).

Approximate position of aerodynamic center can be estimated if one assumes that n_A=0, and $\cos\alpha \approx 1$ and $\sin\alpha \approx 0$. From Eq.(10) we obtain in such case

$$\frac{t_a}{c} = \frac{\frac{dC_{m0}}{d\alpha}}{\frac{dC_z}{d\alpha}} = \frac{dC_{m0}}{dC_z}. \qquad (11)$$

For majority of airfoils aerodynamic center is located at ¼ of chord from the leading edge. By analyzing full form of Eq.(10) one can notice that even for symmetric airfoil $n_A \neq 0$ due to nonlinear dependency of $C_x(\alpha)$.

3. Experimental setup.

Experiments are performed in the tunnel no 1 of Department of Aerodynamics. The diameter of the test-section is 1.16m. Tunnel is equipped with the balance system designed by prof. Witoszyński (balance type – JAW). Axonometric scheme of the balance system is depicted in the fig. 5.

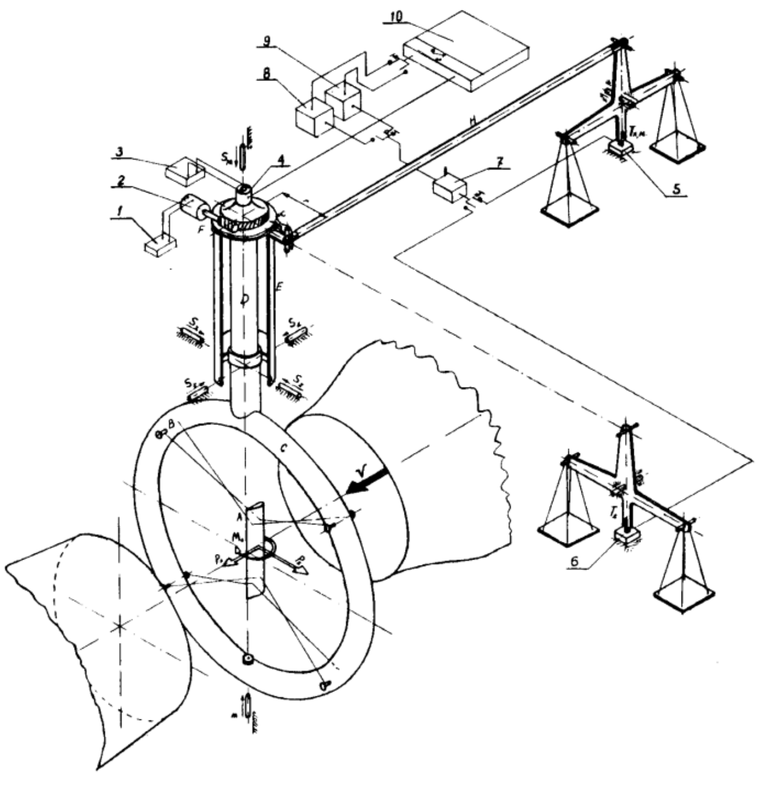

Figure 5 Scheme of the aerodynamic balance system

The investigated model is attached to the frame, covering the entire test-section, by means of metal wires. Tightly adjusted wires holding the model are transferring all the forces acting on the model to the frame. Frame is mounted in such a way that its rotation is allowed in the range of $\pm 30°$.
Force acting on a model is transferred by a lever system to a calibrated tensometer. To measure lift force, one enters S_Z pivots (with the axis colinear to the stream direction) and joins by means of rod H the pipe E, being the extension of the frame C, with the balance being in the plane perpendicular to the tunnel axis. To measure drag force, one enters horizontal pivots S_X (with axis perpendicular to the tunnel axis). During the measurement of moment one uses vertical pivots S_M and the same tensometer as for the drag force measurement. Moment measurement is possible because the rod H was connected to the frame eccentrically with respect to the pivots S_M. This way we can measure a force required to keep the balance at a given position depending on the moment acting on the model.
To sum up, the presented balance system allows to measure 3 components of aerodynamic force. However, these components cannot be measured simultaneously.
The measured values of aerodynamic forces are gross quantities. They need to be corrected by subtracting the tare of the balance and the force or moment acting on the system of wires holding the model. Correction due to wires are typically given in a form of table or a plot for each position and standard dynamic pressure q. For different velocities correction due to wires is recalculated accordingly to dynamic pressure. Balance tare is non zero due to the fact that respective balance axes are not colinear with balance center of gravity with the model. Also, tare depends on the angle of attack. This quantity is measured when there is no flow in the tunnel.

4. Performing experiments.
   a. Calculating aerodynamic coefficients
      Net forces are calculated accordingly:

$$Z = Z_B - Z_T - Z_D$$
$$X = X_B - X_T - X_D$$
$$M = M_B - M_T - M_D$$

Where Z_B, X_B, M_B are the gross quantities dependent on α,
Z_T, X_T, M_T – balance tares
Z_D, X_D, M_D – corrections due to wires

Aerodynamic moment can be determined as:

$$M_0 = M \cdot r$$

where r=0,196m.

The aerodynamic coefficients are defined as

$$C_z = \frac{P_z}{\frac{\rho V^2}{2} S}, \quad C_x = \frac{P_x}{\frac{\rho V^2}{2} S}, \quad C_{mo} = \frac{M_o}{\frac{\rho V^2}{2} Sc}.$$

Movement of the center of pressure will be calculated by means of Eq.(7) or (8).

Position of aerodynamic center will be determined on basis of Eq.(11).

REPORT SHOULD CONTAIN:

1. Protocol with the measurements.
2. Plots:

$$C_z(\alpha),\ C_x(\alpha),\ C_{mo}(\alpha),\ C_{mo_{0.25c}}(\alpha)$$

3. Drag polar

$$C_z\ (C_x)$$

   with marked values of angle of attack.
4. Lift-to-drag

$$\frac{C_z}{C_x}\ (\alpha)$$

5. Movement of center of pressure $\bar{e} = t_p/c$
   with both asymptotes marked.