

Fluid Mechanics - Laboratory instructions

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Pressure tubes velocimetry

key words: Dynamic, static and stagnation pressure, Bernoulli equation, water manometer, Pitot and Prandtl tubs.

goals: The objective of this exercise is to gain knowledge in velocimetry methods employing pressure tubes. Velocity and angular characteristics of pressure tube will be investigated.

1.1 Introduction

Reassure tubes are simple instruments used for local velocity measurements of the flow. By measuring static and stagnation pressure one might calculate flow velocity by employing the Bernoulli equation. For incompressible flow it takes the following form:

$$\frac{V_A{}^2}{2} + \frac{p}{\varrho} = \frac{p_0}{\varrho}$$

where: V - flow velocity, p, ρ pressure and density, p_0 - total or stagnation pressure. (pressure achieved when the flow comes to a stop.) Modification of the Bernoulli equation yields the expression for velocity:

$$V_A = \sqrt{\frac{2(p_0 - p)}{\varrho}}$$

The Difference $\Delta p = p_0 - p$ is often referred to as dynamic pressure.

For an incompressible flows (e.g. water measurements) density might be found in appropriate scientific tables. For compressible flows density should be calculated using the equation of state.

$$\varrho = \frac{p}{RT}$$

where: T stands for temperature in Kelvins and R is a gas constant.

In case of gas flow the governing equation must be modified as to consider the compressibility effect. The equation of the following form is used:

$$\frac{V_A{}^2}{2} + \frac{\kappa}{\kappa - 1}\frac{p}{\varrho} = \frac{\kappa}{\kappa - 1}\frac{p_0}{\varrho}$$

Where κ is a heat capacity ratio (adiabatic index or ratio of specific heats).



Figure 1.1: Sketch of a static Pitot tube.



Figure 1.2: Experimental set-up used for velocity characteristic measurements. 1 - reference an emometer, 2 - Prandtl tube.

Neglecting the compressibility effects might leads to measurement errors. For Mach numbers below 0.2 the error fall below 0.5%, while at M=0.4 it is around 2% and 12% for Mach number M=1.

1.2 Pressure tubes

Pressure tubes can be used to measure stagnation (total pressure), static pressure or both. Figure 1.1 shows a sketch of a tube capable of both stagnation and static pressure measurements (Static Pitot tube or Prandtl tube).

1.2.1 Velocity characteristic

To find the velocity characteristics of the pressure tube an incompressible flow will be considered. The tube will be placed measurement space of an open wind tunnel. The flow speed will be measured with the pressure tube and a reference anemometer. Figure 1.2 illustrates the experiment.

1. Note the atmospheric pressure p_a and ambient temperature t_a . Calculate density. (Assume $R = 287 \frac{m^2}{s^2 K}$).









Figure 1.3: Experimental set-up used to measure angle characteristic.

- 2. Calculate the Reynolds number for maximum and minimum velocity. ($Re = \frac{VD}{\nu}$ D tube diameter, ν kinematic viscosity)
- 3. Set the flow velocity at 20m/s, note the values of pressures and velocity.
- 4. Repeat previous point while decreasing the velocity of the flow.
- 5. For each measurement calculate dynamic pressure, velocity (from dynamic pressure) and error as compared to the reference anemometer.
- 6. Prepare an appropriate report.

1.2.2 Angular characteristic

Angular characteristic will be investigated at a constant flow velocity. Measurements will be performed at different angles. Figure 1.3 illustrates the experimental set up.

- 1. Set proper flow velocity.
- 2. Do pressure measurements for different angular alignments.
- 3. For each measurement calculate dynamic pressure, velocity (from dynamic pressure) and error as compared initial measurements.
- 4. Prepare an appropriate report.





Thermo-anemometry - Hot-Wire Technique

key words: Constant temperature thermo-anemometry, Kings curve,

convective heat transfer

<u>goals</u>: The objective of this exercise is to gain knowledge of velocimetry methods employing electro elements. Calibration of the device will be performed, velocity and angular characteristics of thermo-anemometer will be investigated.

2.1 Introduction

Thermo-anemometry technique is used to measure **instantaneous velocity** of a turbulent flow. It employs the fact that electrical resistance varies with the change in temperature. The main element of a thermo-anemometer is thin wire (1-5 μ m) made of material with large resistance temperature dependence (e.g., Platinum, Tungsten).

In general a heated element (a thin electrically heated wire) surrounded by the fluid will experience heat transfer to the surrounding. Energy exchange mechanisms are the **free convection**, **forced convection** and **radiation**. In case of a stationary fluid free convection would play the main role. As the velocity of the flow increases forced convection would dominate. The influence of radiation is eliminated by relatively low working temperature (below 200 °C) of the instrument.

2.2 Constance Temperature Anemometer

In this type of an anemometer the wire is kept at almost constant temperature ($T_w = const$), independent of the flow velocity. It is achieved through feedback loop of the electrical elements. With proper calibration high sensitivity of the probe might be achieved. Figure 2.1 shows schematics of the experimental setup used to calibrate the thermal probe.

For constant temperature anemometer the voltage drop due to the wire resistance and the flow velocity are related through the so called **King's equation**. It has the following form:

$$E^{2} = E_{0}^{2} + BV^{n}orV = \sqrt[n]{\frac{E^{2} - E_{0}^{2}}{B}}$$

where:



Figure 2.1: Experimental set-up. 1 - probe, 1a - wire, 1b - supports, 2 - wind tunnel , 3 - bridge, 4 - RMS voltmeter, 5 - flow control, 6 - manometer.

- E voltage drop,
- \boldsymbol{V} the flow velocity,
- E_0 voltage drop for stationary flow (V~0)
- B, n calibration parameters, assumed to be constant.

2.2.1 Velocity characteristics and calibration

- 1. Note the ambient pressure and temperature.
- 2. Perform around 15 measurements of the voltage drop at different speeds.
- 3. Plot the Kings curve with the measured values.
- 4. Plot $log(E^2 E_0^2) = f(V)$ and determine n, and B parameters.
- 5. Using the King's equation calculate velocity for each measurement, plot the $V_{term} = f(V)$.
- 6. Estimate and plot the error according to the formula: $Er = \frac{V_{term} V}{V}$





2.2.2 Angular characteristics

- 1. Note the ambient pressure and temperature.
- 2. For a chosen velocity perform measurements of the voltage drop changing the probe alignment.
- 3. Using the King's equation calculate velocity for each measurement, plot the $V_{term} = f(\alpha)$.
- 4. Estimate and plot the error according to the formula: $Er = \frac{V_{term} V}{V}$





Venturi and orifice volumetric flow measurement

key words: Volumetric and mass flux, venturi and orifice meter, device calibration, . goals: The objective of this exercise is to gain knowledge in volumetric methods employing Venturi effect. Calibration of orifice and Venturi meter.

3.1 Introduction

As the fluid flows through a constriction it experiences a drop in fluids pressure. Although it is referred to as the Venturi¹ effect it was probably known in times of a Roman engineer Frontinus².

To satisfy the **continuity equation**, velocity of the flow through a constricting pipe will increase. Which in turn causes a drop in pressure (Bernoulli's principle). Figure 3.1 illustrates the phenomena named Venturi effect.



Figure 3.1: The Venturi effect. As the pipe contracts, the pressure of the fluid decreases. Source: www.wikipedia.org

Assuming a steady-state, inviscid, incompressible ($\rho = const$) and laminar flow, continuity and Bernoulli equation form the following set of equation:

 $^{^1{\}rm Giovanni}$ Battista Venturi (March 15, 1746 - April 24, 1822) was an Italian physicist.

²Sextus Julius Frontinus (ca. 40-103 AD)

$$\begin{cases} Q_t = v_1 D^2 = v_2 d^2 \leftarrow \text{continuity equation} \\ \frac{\rho V_1^2}{2} + p_1 = \frac{\rho V_2^2}{2} + p_2 \leftarrow \text{Bernoulli equation,} \end{cases}$$
(3.1)

introducing an **area ratio** $m = (\frac{d}{D})^2$ and solving for the volumetric flow rate (in fact just a theoretical value) Q_t one would get:

$$Q_t = \frac{m}{\sqrt{1 - m^2}} \frac{\pi D^2}{4} \sqrt{\frac{2(p_1 - p_2)}{\rho}}$$
(3.2)

Equation 3.2 states that for a given geometry the flow rate might be calculated simply by measuring the pressure difference $p_1 - p_2$. While deriving equation 3.2 some idealistic assumptions were made. Due to that equation 3.2 will **overestimate** the actual flow rate if used for a real fluid. Therefore to calculate the actual flow rate a modification is required. The following equation is used:

$$Q = C_d Q_t \tag{3.3}$$

where C_d is so called coefficient of discharge. In general it is a function of device type and geometry, but also depends on the Reynolds number of the flow. Figure 3.2 shows a values of $\frac{C_d}{\sqrt{1-m^2}}$ plotted as a function of Re for different area ratio of a Venturi meter³.

3.2 Venturi effect based meters

The flow meters employing the Venturi effect do come in different shapes and sizes. From crude orifices (just a plate with a drilled hole) having high hydraulic loses, lower accuracy and high discharge coefficient dependence on area ratio and the type of flow. To more sophisticated Venturi meters and nozzles with strictly defined geometry. Figure 3.2 shows some examples of orifices (a, b), a Venturi meter (d) and a nozzle (c, e).

3.3 Orifice and Venturi meter calibration

- 1. Note the dimension of device to be calibrated (orifice and/or Venturi meter).
- 2. Start the flow through the installation, manipulate the flow rate with a valve.
- 3. Using a stopwatch measure a time t necessary to fill the reference tank of volume $V.^4$
- 4. Measure the pressure drop at the device being calibrated.
- 5. Repeat the measurements at different flow rates. Ask the tutor for the number of test you should perform.
- 6. Calculate the theoretical volumetric flow rate using eq.3.2.





³Value of $\frac{C_d}{\sqrt{1-m^2}}$ instead of C_d is used for better plot resolution. ⁴Now how do you coloridate the volumetric flow rate?

⁴Now how do you calculate the volumetric flow rate?



Figure 3.2: Discharge coefficient dependency on geometry and Reynolds number for a Venturi meter.

- 7. Calculate the actual flow rate using data from experiment and the discharge coefficient.
- 8. Calculate the Reynolds number. $Re = \frac{VD}{v}$
- 9. Repeat the calibration for each of the measurements and plot C_d as a function of Re.







Figure 3.3: Different types of constriction flow meters. a,b - orifices, c - nozzle, d - Venturi constriction meter, e - Venturi nozzle.





Hydraulic pressure loss coefficient

key words: Darcy friction factor, pressure loss, turbulent and laminar friction coefficient, Blasius' equation, Nikuradse or Moody diagram.

<u>goals</u>: The objective of this exercise is to measure pressure loss coefficient for an incompressible, turbulent and laminar flow through a pipeline.

4.1 Introduction

Any real life engineering phenomena is accompanied by energy dissipation. It is no different with a real fluid in motion. In case of a liquid flowing through a straight pipe energy losses are an effect of shear stresses (τ) acting against the flow. Those energy losses manifest itself by drop in **hydraulic head**¹ at consecutive cross-sections of the flow, and are referred to as **head losses**². This is shown in Fig.4.1.

Pressure losses Δp_{loss} resulting from liquid's flow through a pipe of diameter D and length L are proportional to liquids dynamic pressure. And are defined by the so called **Darcy-Weisbach equation**

²Also referred to as resistance head or friction head



Figure 4.1: Drop in liquids pressure at distance L, as it flows through a pipe of diameter D.

 $^{^{1}}$ Pressure expressed as **hight** of a static column of fluid. Expressed in meters, millimeters ...

$$\Delta p_{loss} = \lambda \frac{L}{D} \frac{\rho V^2}{2} \tag{4.1}$$

The proportionality coefficient λ is the so called **Darcy friction factor**.

Friction coefficient λ varies strongly with the Reynolds number³. For laminar flows (Re < 2300) it might be calculated analytically from HagenPoiseuille equation to be:

$$\lambda = \frac{64}{Re} \tag{4.2}$$

However for turbulent flows empirical formulas are used. A good approximation is given by the Blasius⁴ formula (for flows where $Re < 8 * 10^4$):

$$\lambda = \frac{0.316}{\sqrt[4]{Re}} \tag{4.3}$$

Equation 4.1 and 4.2 are valid only in case of **smooth** and **circular** pipes. In case of rough surfaces values of the friction factor are found by experiment. Such experiment were performed by Johann Nikuradse⁵. Figure 4.1 shows the so called Nikuradse diagram⁶ relating friction factor λ , Reynolds number and relative roughness of the pipe.

4.2 Measurements and calculations

Using the experimental setups for laminar and turbulent flows investigate pressure losses. Perform appropriate measurements and calculations.

4.2.1 Pressure loss coefficient for turbulent flow

- 1. Get acquainted with the experimental setup, check how manometers are connected and what is being measured.
- 2. Copy all the necessary dimensions of the investigated installation to your report.
- 3. Start the flow through the installation, manipulate the flow rate with a valve.
- 4. Using a stopwatch measure a time t necessary to fill the reference tank of volume V.
- 5. Note the pressure losses.
- 6. Repeat the measurements at different flow rates. Ask the tutor for the number of test you should perform.
- 7. Stop the flow through the installation when finished.

⁶In English nomenclature Moody chart.





 $^{{}^{3}}Re = \frac{V*D}{\nu}$, Pipe diameter D is used as characteristic length.

⁴Paul Richard Heinrich Blasius (1883-1970) was a German fluid dynamics engineer.

⁵Johann Nikuradse (November 20, 1894 July 18, 1979) Georgia-born German engineer and physicist.Lived mostly in Gttingen.



Figure 4.2: Nikuradse or Moody diagram. Relates friction factor, Reynolds number and relative roughness for flows through circular pipes. Source: www.wikipedia.org





- 8. Note water temperature.
- 9. For each of the measurements calculate: pressure loss, flow velocity, Reynolds number and friction factor λ .
- 10. Plot the results on a $\lambda(Re)$ plot, and compare with Blasius formula 4.3.
- 11. Comment on the results.

4.2.2 Pressure loss coefficient for laminar flow

- 1. Get acquainted with the experimental setup, check how manometers are connected and what is being measured.
- 2. Copy all the necessary dimensions to your report.
- 3. Start the flow, manipulate the flow rate by changing hight H of the water source.
- 4. Using a stop watch measure a time t necessary to fill graduated cylinder used for reference.
- 5. Measure the pressure drop.
- 6. Repeat the measurements at different flow rates. Ask the tutor for the number of test you should perform.
- 7. Note water temperature.
- 8. For each of the measurements calculate: pressure loss, flow velocity, Reynolds number and friction factor λ .
- 9. Plot the results on a $\lambda(Re)$ plot, and compare with formula 4.2.
- 10. Comment on the results.





Drag measurement

key words: Air resistance, pressure and friction drag, bluff and stream line body, surface roughness.

goals: Estimate air resistance of a cylinder. Pressure on the cylinder surface and momentum loss in the cylinders wake will be measured.

5.1 Introduction

As an object moves through a fluid it experiences forces from the interaction with the fluid. Forces in the direction of the object's motion are referred to as **drag**. Drag forces act against the object motion, and are in general dependent on the velocity. Depending on the flows conditions different phenomena could be responsible for drag creation. In this exercise we will focus on **pressure** and **friction** drag.

Unsymmetrical pressure distribution on the surface of a body will manifest itself as a force acting on that body. Part of that pressure force¹ acting against the objects movement could be considered as pressure drag P_{xp} . By friction drag we will refer to the force resulting from shear stresses on the body's surface. Lets designate it by P_{xf} . The sum of the two yields the **profile** drag P_x .

Which of the two drag components is dominative for the profile drag depends on the body's shape and position against the flow. A good example is a flow past a flat plate. If it is aligned with the flow the drag force is an effect of friction (tangential stresses) only. While it being perpendicular to the flow will result with pressure drag being the important one.

In general it is customary to refer to bodies as **bluff** if pressure drag is higher than friction drag $(P_{xp} \gg P_{xf})$ and **streamlined** if the opposite is true $(P_{xp} \ll P_{xf})$. See Fig.5.1 for reference.

To investigate aerodynamic forces the following methods could be used:

- 1. Employment of momentum conservation principle.
- 2. Pressure distribution and tangential stresses measurement.
- 3. Direct force measurement (e.g. weighing method which is covered in exercise 7).

¹The other part could be responsible for lift force.



Figure 5.1: Flat plate and an airfoil as an example of stream line bodies (left) with low pressure drag. Bluff shapes (right) have relatively high pressure drag. Source: www.wikipedia.org



Figure 5.2: Illustration to the drag measurment using the momentum method.

5.2 Momentum method

The rate of change of momentum must be equal to the sum of forces acting on that system. Or so the Newton's second law states. Therefore by measuring momentum change one might calculate the the forces exerted on the flow (or on the object being flown by).

Consider an example in Fig.5.2. Force F is a force with which a cylindrical object acts against the flow. As the fluid moves around the investigated body some of its momentum is lost. This is reflected by change in the velocity profile in the wake of an object. By writing the momentum equation for the ABCD contour one might calculate the drag forces². The resulting formula would be (assuming $p_1 = p_2$):

$$-F = P_x = \int \rho V_2 (V_1 - V_2) dy$$
 (5.1)

Lets introduce the drag coefficient C_d . It relates drag force P_x , square of the velocity, density and the cross-sectional area A of a body. It is defined as:

$$C_d = \frac{2Px}{\rho A V_\infty^2} \tag{5.2}$$

 $^{^2 \}mathrm{You}$ should be able to do that after Fluid Mechanics I.





Assuming a unitary length of the cylinder (A = d * 1) and modifying the equation so dynamic pressure $(q = \frac{\rho V^2}{2})$ is used one would get the following equation for the drag coefficient.

$$C_d = \frac{2}{d} \int \frac{\sqrt{q_2}}{\sqrt{q_\infty}} (1 - \frac{\sqrt{q_2}}{\sqrt{q_\infty}}) dy \tag{5.3}$$

As one might notice it is enough to measure the dynamic pressure in the wake to calculate drag coefficient.

5.3 Pressure method

To calculate the pressure drag it is necessary to know the pressure distribution on the surface of the body being investigated. In general this can only be achieved through experiment. Once this is know pressure drag can be found. In case of a cylindrical shape of radius s the following formula is valid:

$$P_{xp} = \int_0^{2\pi} p(r) r \cos(\Theta) d\Theta$$
(5.4)

Further using dynamic pressure q, for a cylinder of length L (area A = Ld) it is possible to define the pressure drag coefficient C_{dp} .

$$C_{dp} = \frac{P_{xp}}{qdL} \tag{5.5}$$

5.4 Measurements and calculations

5.4.1 Momentum method

- 1. Note ambient pressure, temperature and undisturbed flow conditions.
- 2. Measure pressure distribution in the wake of the investigated object.
- 3. Do plots of pressure distribution in the wake.
- 4. Apply numerical integration procedure of your choice to calculate drag coefficient in accordance with formula 5.3.
- 5. Calculate method calibration coefficient $K = \frac{C_{model}}{C_{momentum}}$ using model values of drag coefficient.
- 6. Repeat pressure measurements for different surface roughness.
- 7. Perform the experiment at different flow regime.





5.4.2 Pressure distribution method

- 1. Measure pressure distribution on the surface of the investigated cylinder.
- 2. Plot pressure distribution as a function of Θ
- 3. Calculate pressure drag coefficient.
- 4. Compare the result with the drag coefficient measured with the momentum method.





Aerodynamic force measurement

key words: Aerodynamic coefficients, scale measurement, airfoil goals: Aerodynamic forces on an airfoil will be measured using a scale method

6.1 Introduction

Consider an airfoil model, just like the one shown in Fig.6.1. Let 2b be the span (Rozpito pata) of the airfoil and c its chord (Ciciwa). As the model is investigated in the wind tunnel it experiences aerodynamic force and moment from the flow.

Considering components of those in the (x,z,y) reference frame (see Fig.6.1) one might distinguish:

- P_x the drag force or simply drag (Sia oporu), force component parallel to the flow velocity vector **V**.
- P_y the lateral force (Sia boczna), force component projected on to the y axis.
- P_z the lift force (Sia nona), force component perpendicular to the flow velocity vector **V** and in the direction of the z axis.
- M_x the rolling moment (Moment przechylajcy).
- M_y the pitching moment (Moment pochylajcy).
- M_z the yawing moment (Moment odchylajcy).

6.1.1 Aerodynamic coefficients

In engineering and scientific practice one uses dimensionless coefficient to describe aerodynamic forces and moments. They are defined as follows:

$$C_i = \frac{P_i}{\frac{\rho V^2}{2}S} \tag{6.1}$$

$$C_{mi} = \frac{M_i}{\frac{\rho V^2}{2}Sc} \tag{6.2}$$



Figure 6.1: An example of an airfoil with aerodynamic forces and moments components. (x,z,y) is a reference frame related with the wind tunnel (free stream velocity **V** is parallel to the X axis). (n,t,y) is a reference frame related to the model under investigation (t axis is parallel to the airfoil's chord, and is at angle α to the flow velocity vector **V**).





For fixed flow conditions aerodynamic coefficients become dependent on the angle of attack only (angle between the chord and the velocity vector). It is customary to plot aerodynamic coefficients as functions of the angle. Those are referred to as aerodynamic characteristics.

Often the investigated model has a plane of symmetry. If so it is possible to choose the reference frame in such a way that the plane (z,x) is a symmetry plane, and as a consequence $C_y = C_{mx} = C_{mz} = 0$.

Values of moment coefficient C_m will be dependent on the reference used. The experiment is setup so the reference axis y_0 passes through the trailing edge of the model. Moment coefficient in respect to this point is denoted as C_{m0} .

Assume axis y_1 (parallel to y_0) intersects the models chord c at a distance t_0 from the axis y_0 . To recalculate it to value of C_{m1} (moment coefficient in respect to the axis y_1) one might use formula 6.3.

$$C_{m1} = C_{m0} - \frac{t_0}{c} (C_z \cos(\alpha) + C_x \sin(\alpha))$$
(6.3)

6.1.2 Center of pressure

Point where the aerodynamic force vector intersects with the chord is called the center of pressure. Moment calculated in reference to this point vanishes. Knowing that and rearranging formula 6.3 accordingly one will get the formula to calculate the position of the center of pressure.

$$\frac{t_0}{c} = \frac{C_{m0}}{C_z cos(\alpha) + C_x sin(\alpha)} \tag{6.4}$$

6.2 Experimental setup

Figure 6.2 shows the schematic drawing of the experimental setup that is to be used in this experiment. Model is fixed to a stiff frame and further, through shafts to the tensometric scale.

It is possible to manipulate the degrees of freedom of the device by locking and unlocking the screws S. Therefore one might select the force components to be recorded.







Figure 6.2: Schematic view of the aerodynamic scale. Forces from the model are transfered through the frame to the tensometers installed at 5 and 6.





6.3 Measurements

You will perform all the necessary measurements under the supervision of the technician. All the necessary data will be recorded for further manipulation. The calculation to be performed are listed below.

- 1. Using equations 6.1 and 6.2 plot $C_z(\alpha), C_x(\alpha), C_{m0}(\alpha), C_{m25}(\alpha), \ldots$
- 2. Plot the dependency $\frac{C_z}{C_x}$, on the plot mark the angle. Plot the dependency $\frac{C_z}{C_x}(\alpha)$.
- 3. Plot the position of the center of pressure $e = \frac{t_p}{c}$.



