Equation of continuity: $\dfrac{\partial \rho}{\partial t} + \dfrac{1}{r^2}\dfrac{\partial}{\partial r}(\rho r^2 u_r) + \dfrac{1}{r \sin \theta}\dfrac{\partial}{\partial \theta}(\rho u_\theta \sin \theta) + \dfrac{1}{r \sin \theta}\dfrac{\partial}{\partial \varphi}(\rho u_\varphi) = 0.$

Navier-Stokes equations with constant ρ, constant ν, and no body force:

$$\dfrac{\partial u_r}{\partial t} + (\mathbf{u} \cdot \nabla)u_r - \dfrac{u_\theta^2 + u_\varphi^2}{r}$$

$$= -\dfrac{1}{\rho}\dfrac{\partial p}{\partial r} + \nu\left[\nabla^2 u_r - \dfrac{2u_r}{r^2} - \dfrac{2}{r^2 \sin \theta}\dfrac{\partial(u_\theta \sin \theta)}{\partial \theta} - \dfrac{2}{r^2 \sin \theta}\dfrac{\partial u_\varphi}{\partial \varphi}\right],$$

$$\dfrac{\partial u_\theta}{\partial t} + (\mathbf{u} \cdot \nabla)u_\theta + \dfrac{u_r u_\theta}{r} - \dfrac{u_\varphi^2 \cot \theta}{r}$$

$$= -\dfrac{1}{\rho r}\dfrac{\partial p}{\partial \theta} + \nu\left[\nabla^2 u_\theta + \dfrac{2}{r^2}\dfrac{\partial u_r}{\partial \theta} - \dfrac{u_\theta}{r^2 \sin^2 \theta} - \dfrac{2 \cos \theta}{r^2 \sin^2 \theta}\dfrac{\partial u_\varphi}{\partial \varphi}\right],$$

$$\dfrac{\partial u_\varphi}{\partial t} + (\mathbf{u} \cdot \nabla)u_\varphi + \dfrac{u_\varphi u_r}{r} + \dfrac{u_\theta u_\varphi \cot \theta}{r}$$

$$= -\dfrac{1}{\rho r \sin \theta}\dfrac{\partial p}{\partial \varphi} + \nu\left[\nabla^2 u_\varphi + \dfrac{2}{r^2 \sin \theta}\dfrac{\partial u_r}{\partial \varphi} + \dfrac{2 \cos \theta}{r^2 \sin^2 \theta}\dfrac{\partial u_\theta}{\partial \varphi} - \dfrac{u_\varphi}{r^2 \sin^2 \theta}\right],$$

where

$$\mathbf{u} \cdot \nabla = u_r\dfrac{\partial}{\partial r} + \dfrac{u_\theta}{r}\dfrac{\partial}{\partial \theta} + \dfrac{u_\varphi}{r \sin \theta}\dfrac{\partial}{\partial \varphi},$$

$$\nabla^2 = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial}{\partial r}\right) + \dfrac{1}{r^2 \sin \theta}\dfrac{\partial}{\partial \theta}\left(\sin \theta\dfrac{\partial}{\partial \theta}\right) + \dfrac{1}{r^2 \sin^2 \theta}\dfrac{\partial^2}{\partial \varphi^2}.$$