FLUID MECHANICS I – examination problems

Fluid Kinematics

1) Lagrangian versus Eulerian description of fluid motion.

2) Velocity, acceleration and condition of incompressibility in Lagrangian description

3) Velocity, acceleration and condition of incompressibility in Eulerian description

4) Definition of fluid element's trajectorie and streamlines of the velocity field. When these lines are exactly the same?

5) Calculate (Lagrangian) velocity and acceleration for the flow defined as:

$$(x_0, y_0) \xrightarrow{t} (x, y)$$

$$x(t) = -y_0 e^{-\lambda t}, \quad y(t) = x_0 e^{\lambda t} \quad (\lambda > 0).$$

Is this flow incompressible?

6) Calculate the (Eulerian) acceleration for the velocity field (in 2D)

$$\vec{v}(x,y) = \left[\frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2}\right]$$

Is this flow incompressible?

7) Let $\vec{v} = [v_1(x_1,x_2), v_2(x_1,x_2)] = [\cos\alpha x_1, \sin\alpha x_2]$

Calculate the substantial (material) derivative of

$$f = \sin t \cdot (x_1^2 + x_2^2).$$

8) Calculate x_1-component of the acceleration field for

$$\vec{v}(t, x_1, x_2) = \cos\omega t \cdot [\sin\alpha x_2, -\cos\alpha x_1].$$

9) Let $\vec{v} = [v_1(x_1,x_2), v_2(x_1,x_2)] = [\sin t \cdot \sin x_2, -\sin t \cos x_1]$
Calculate the vector $\vec{\omega} \times \vec{v}$ where $\vec{\omega} = \nabla \times \vec{v}$

10) Define the streamfunction ψ for 2D flow.
Calculate the velocity field knowing that.

$$\psi(x,y) = \sin \alpha x \cdot \cos \beta y.$$

Calculate the flow rate (in 2D sense) "crossing" the line segment joining the points (0,1) and (1,0).

11) Calculate the divergence of the velocity field

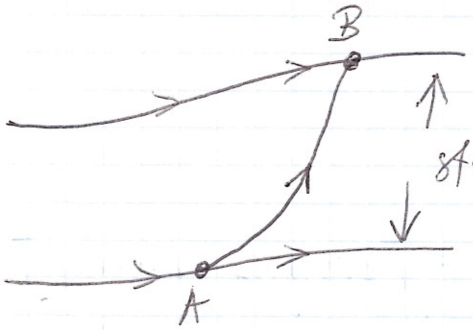
$$\vec{v}(x_1,x_2) = \left[\frac{-x_2}{x_1^2+x_2^2}, \frac{x_1}{x_1^2+x_2^2} \right]$$

12) Calculate the vorticity of the field in (11).

13). Check if the following vector field is an admissible velocity of an incompressible fluid

$$v_x(x,y) = -\beta \sin \alpha x \sin \beta y$$
$$v_x(x,y) = -\alpha \cos \alpha x \cos \beta y$$

14).

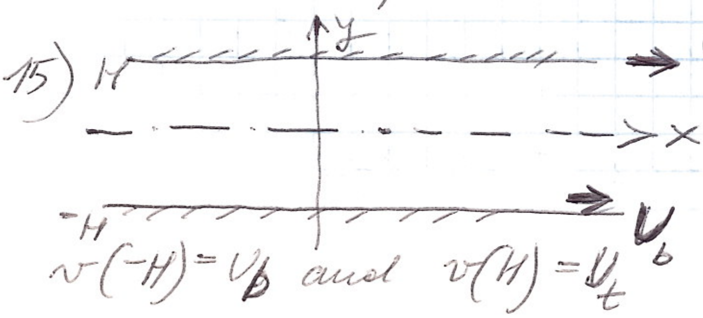


Show that the volumetric flow rate of flow through the line $\overset{\frown}{AB}$ is

$$Q_{AB} = \psi_B - \psi_A$$

where ψ is the streamfunction.

15)



$v(-H) = U_b$ and $v(H) = U_t$

Calculate volumetric flow rate in the channel (2D). $v_x(y) = A + By + y^2$

Fluid Statics

1) Write the integral and differential forms of the equation of fluid statics
2) The fluid remains in rest in the potential force field. What is the orientation of surfaces $\rho = const$ with respect to the force field vector $\vec{f}$?
3) The fluid remains in rest in the potential force field. What is a relation between surfaces $\rho = const$ and $p = const$?
4) Thermally isolated layer of Clapeyron gas remains in rest in a potential force field. Show that equipotential surfaces (which are perpendicular to $\vec{f}$) are izothermic ($T = const$)
5) Barotropic fluid remains in rest in a external force field. Show that the force field must be a potential one.
6) Can a barotropic fluid remain in rest in the following force field: (2D case)

$$\vec{f} = [f_x, f_y](x,y) = \left[\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}\right] \quad (x,y) \in R^2 - \{(0,0)\}$$

7) Calculate the pressure potential function $P = P(p)$ for the Clapeyron gas in izothermal conditions
8) As in 7, but in isentropic conditions...

9) Derive the formula for the pressure field of the incompressible motionless fluid in the following

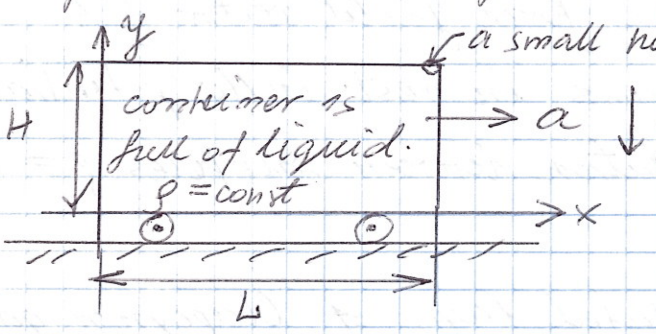
force field

$$\vec{f} = [f_r, f_z] = [\Omega^2 r, -g].$$

10) The same as in 9), but for

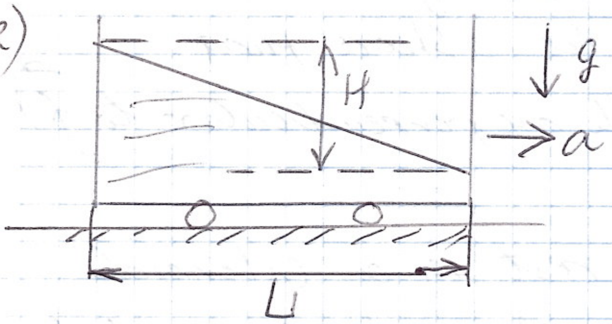
$$\vec{f} = [f_x, f_y] = [-a, -g]$$

11) Calculate the pressure at the point $(x,y) = (0,0)$ (see figure)



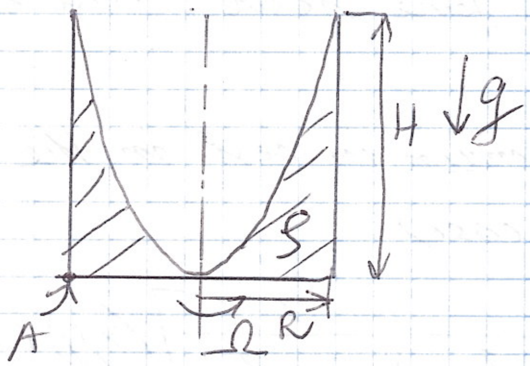
Given: H, L, ρ, a, g

12)



Find ΔH, given: L, g, a

13)

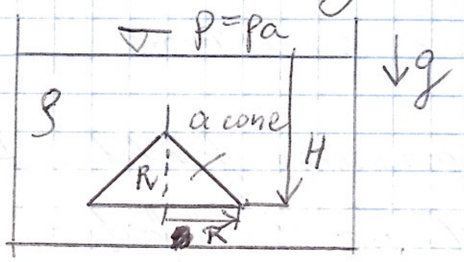


Find Ω and pressure p at the point A $(r = R)$. Given, H, R, ρ

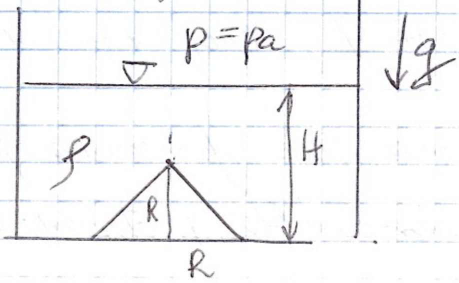
14) Derive the Archimedes low from general equation of the fluid statics.

15) Calculate the hydrostatic reaction force.

a)



b)



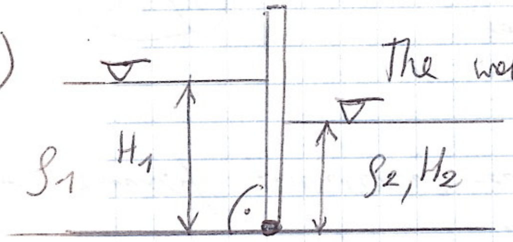
Fluid Dynamics - (Ideal fluid flow)

1) Write three equivalent, frame-invariant forms of the mass continuity differential equation
2) Express the relative rate of change of the fluid density ~~observed by~~ "measured" by an observer moving with the fluid. by an appropriate differential operator applied to the velocity field.
3) Write a fully expanded form of the mass conservation differential equation for stationary 3-dimensional motion.

4) Write the vectorial (frame-invariant) and index forms of the Euler Equation.

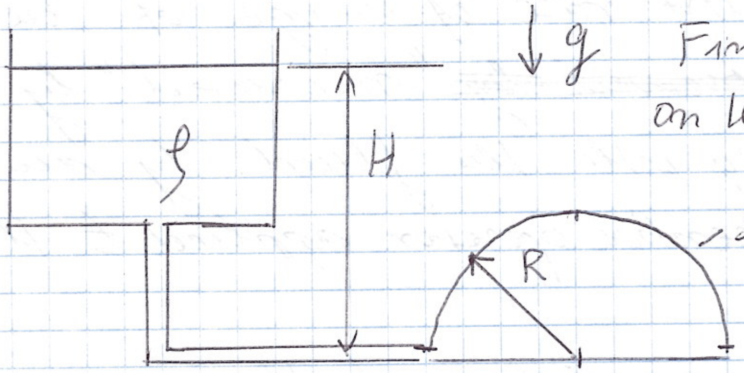
5) Write fully expanded form of all scalar Euler Equations for 2D stationary flow.
6). Enumerate all assumption necessary to derive the Bernoulli Equation. Write its general form and explain all symbols.

7) Write the Bernoulli equations for isentropic and isothermal Clapeyron gas.
8) Enumerate all assumption to derive the Cauchy-Lagrange first integral of the Euler Equations. Write ~~the~~ the general form of this integral and explain the meaning of used symbols.

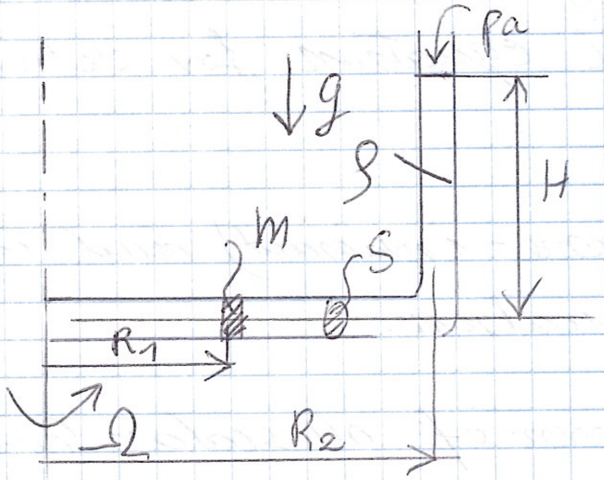
16) 

The wall is in balance. Express

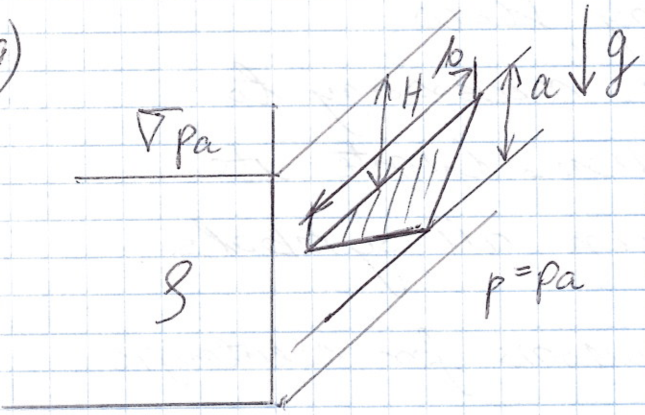
$$\frac{\rho_1}{\rho_2} = f\left(\frac{H_1}{H_2}\right)$$

17) 

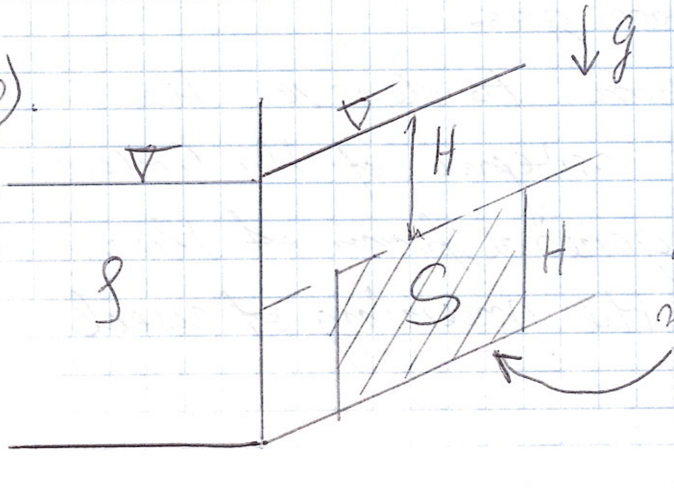
Find the vertical reaction on the semishevical cover

18) 

The mass m remains in balance. Find Ω.

19) 

Find reaction of the triangular part of the vertical sidewall of the container.

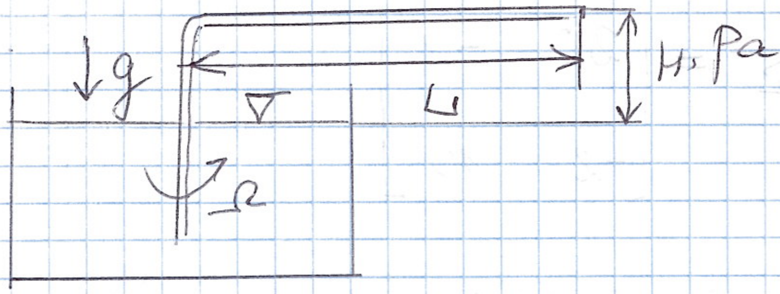
20). 

Calculate the depth of the point of application of the hydrostatic reaction on the rectangular part of the wall

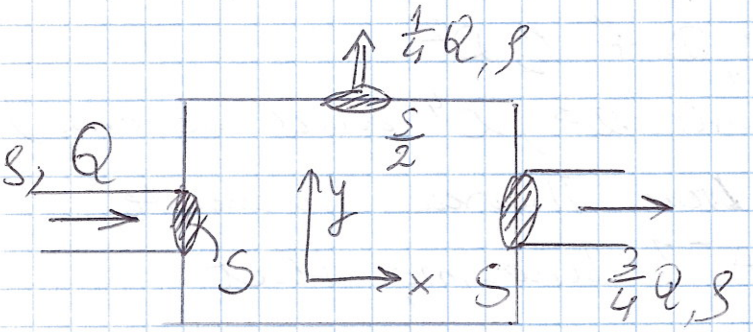
9). Write the Bernoulli equation for an incompressible fluid under the action of the following force field.

$$\vec{f} = [f_r, f_z] = [\Omega^2 r, -g].$$

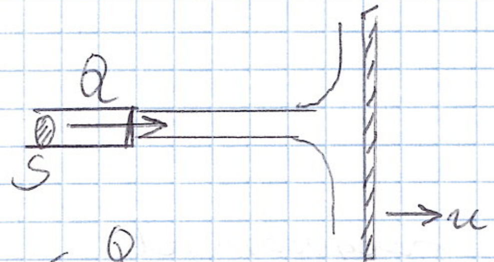
10) Derive the formula for the maximal velocity to be obtained by the incompressible fluid in the rotating pipe shown in the figure below. Assume that $p_{cavitation} = 0$.



11). Calculate the total reaction on the device hidden inside the control volume:



12)



$u < \frac{Q}{S}$

Find such u that the power $P = F \cdot u$ is maximal.

13) All I know about the Betz's limit...

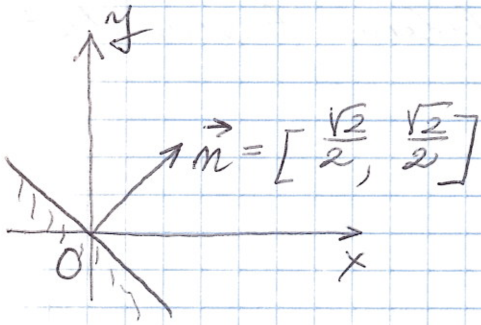
14) Write the formula for the stress at the material motionless surface using pressure and vorticity at this surface.

Fluid dynamics - viscous fluid

1) Let $\vec{v}(x,y) = [-\beta \sin\alpha x \sin\beta y, -\alpha \cos\alpha x \cos\beta y]$

Calculate $\nabla\vec{v}$, $\mathcal{D}$ and $\mathcal{R}$ tensors

2) Calculate normal and tangent stress at the point O (see figure).



The stress tensor at (0,0) is

$$\Sigma = \begin{bmatrix} 20 & -2 \\ -2 & 10 \end{bmatrix}$$

3) Explain why the most general constitutive relation for simple fluid has the form of the second order matrix polynomial:

$$\Sigma = c_0 I + c_1 \mathcal{D} + c_2 \mathcal{D}^2$$

(no need for terms with $\mathcal{D}^3$, $\mathcal{D}^4$ and so on...)

4) Using Cayley-Hamilton Theorem calculate the inverse matrix A^{-1} knowing that

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Hint: derive the characteristic polynomial

$$p_A(\lambda) = \det(A - \lambda I) = \lambda^2 + c_1\lambda + c_2$$

and use the

C-H Th.: $A^2 + c_1 A + c_2 I = 0$

5) Write the frame-invariant and index form of the constitutive relation for linear Newtonian fluids.

6). Write vector and index form of the Navier-Stokes Eq. for compressible and incompressible fluid.

7) Write fully expanded form of the complete set of differential equations describing general, nonstationary, 2D motion of incompressible fluid (Hint: there are 3 equations: 2 N-S and continuity equation).

8). All I know about Couette and (plane) Poiseuille flow ...

9) Write the reduced form of the Navier-Stokes and continuity equations, which describe the special case of flow, such that:

$$\vec{v} = [v_1(x_1, x_2), v_2(x_1, x_2), v_3(x_1, x_2)]$$

What can you say about the pressure field?

10). The flow has only one non-zero velocity component $\vec{v} = [0, 0, v_3(x_1, x_2)]$

What can you say about the pressure field?
Explain in details yours answer.

~~11) Show the pressure of~~

10/14

11) Derive the nondimensional form of the incompressible Navier-Stokes equations. Assume that the scale of the pressure is defined as $P = \rho V^2$ where V is the scale of the velocity. The scale of time is T and the lenght scale is L.

12) Enumerate conditions of dynamic flow similtude.

13) The model of the boat is 5 times smaller than the real object. The experiment is being carried out in the towing basin filled with the sea water. We need to estimate the drag of the real boat in two different flow conditions:

a) small velocities (basically no wave generation)

b) large velocity (immense wave generation)

Formulate the approximate similtude criterion which should be satisfied in each of these cases. What should be the velocity of the model if the velocity of the real boat is:

a) 1 m/s b) 10 m/s

14) Using definitions of appropriate similtude numbers show that simultaneous similtude with respect to viscous and gravitational effects in sailing boat hydrodynamics cannot be achieved (in practice).

15) Using the dimensional arguments show that the pressure drop along the pipe can be expressed as

$$\Delta p = C\left(Re, \frac{l}{d}\right) \rho V^2$$

where $Re = \dfrac{\rho V d}{\mu}$. In the above: ρ - density, V - averaged velocity, d - diameter, l - lenght of the pipe, μ - dynamic viscosity.

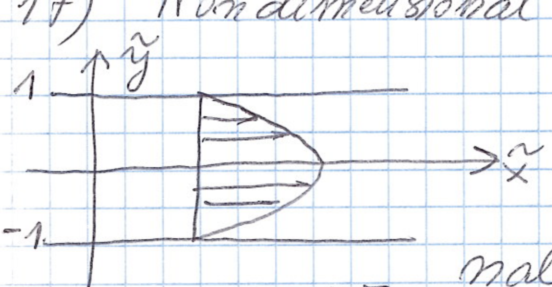
16) Using dimensional arguments derive the formula for the hydrodynamic drag of the sailing boat. the parameters involved are: F_D - drag force, L - lenght of the boat's waterline, B - width of the waterline, Q - volume of water displaced by the boat, ρ - water density, V - speed of the boat, g - gravity acceleration. the formula should have the form:

$$F_D = C(\;?\;)\, \rho V^2 L \cdot B.$$

nondimensional coefficient

What are the nondimensional quantities involved?

17) Nondimensional (plane) Poiseulle flow is.



$$\tilde{u}(\tilde{y}) = 1 - \tilde{y}^2 \qquad \tilde{y} \in [-1, 1]$$

and the corresponding nondimensional pressure gradient $\dfrac{d\tilde{p}}{d\tilde{x}} = -\dfrac{2}{Re}$.

where $Re = \dfrac{V_{max} \cdot D}{\nu}$ (V_{max} - maximal velocity and D - half-height of the channel). Calculate Δp along the channel's segment having 2 m, $D = 5$ cm, $\rho = 1000\ kg/m^3$, $\nu = 10^{-6}\ m^2/s$. The scale for pressure is $P = \rho V_{max}^2$

~~18) The Write the formula for the.~~

12/14

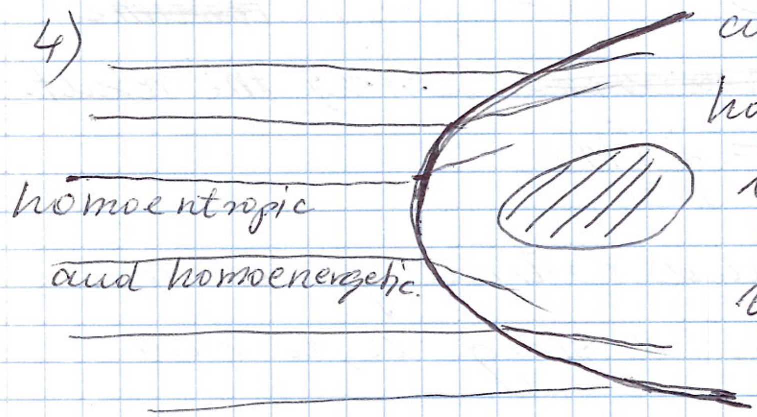
Energy dissipation ...

1) First integral of energy equation: assumptions and different forms.

2) Concept of homoenergetic and homoentropic flows. Relations between such flows and barotropic flows.

3) Crocco Equations and conclusions.

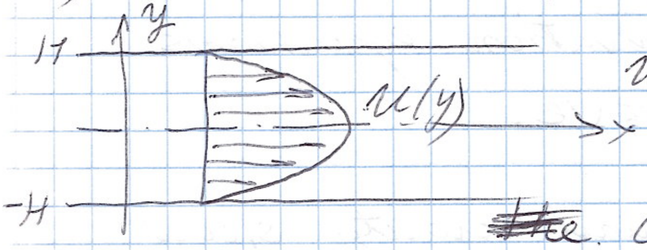
4)



Use Crocco Eq. to show that $\vec{\omega} \neq 0$!

5). Write the formula for the Rayleigh dissipation function. Explain the physical interpretation

6) Consider the plane Poiseuille flow:



$$u(y) = u_0\left(1 - \left(\frac{y}{H}\right)^2\right)$$

Viscosity is μ. ~~Show that~~ ~~the~~ Calculate the energy dissipated per 1 sec in the volume of fluid inside the channe's segment of the lenght L. Compare it to the power developed by pressure forces.

Elements of gas dynamics

1) Define the speed of sound a. Write the formulae for a in the Clapeyron gas using: a) p and ρ b) using T.

2) Using the energy equation (energy integral) express the ratio T/T_0 as the function of the Mach number.

3) In the isentropic motion of the Clapeyron gas one has $p \sim \rho^k$ $k = C_p/C_v$. ~~Assume that the flow is also adiabatic~~. Using the result of (2) derive $P/P_0 = f(M)$.

4) Define stagnation and critical parameters. Write energy equation using a, u and a_*.

5) Calculate special ratios: $\frac{T_*}{T_0}$, $\frac{P_*}{P_0}$ and $\frac{\rho_*}{\rho_0}$.

6) Write algebraic conservation equations for the normal shock wave.

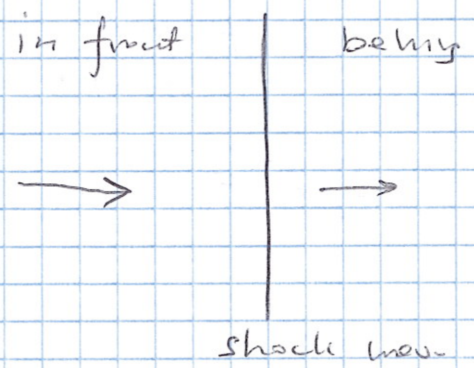
7) Using definition of the sound velocity show that the principle of linear momentum conservation at the normal shock wave can be written as

$$p(\varkappa M^2 + 1) = \text{const.}$$

8) Write Poisson and Hugoniot adiabats on the same plot. ~~What is~~ $p_2/p_1 = f\left(\frac{\rho_2}{\rho_1}\right)$
What can you say about the relation between these two lines at the point (1,1)?

9) Put the right relation operator into the following table. (normal shock wave)

in front	>, <, =	behind
M_1		M_2
p_1		p_2
p_{01}		p_{02}
T_1		T_2
T_{01}		T_{02}
s_1		s_2
ρ_1		ρ_2
u_1		u_2
a_1		a_2
T_{*1}		T_{*2}
a_{01}		a_{02}.



10). Explain why the stagnation pressure behind the shock wave is smaller than in front of it.

11) The Prandtl's relation on the normal shock wave.

$$\begin{bmatrix} 10\sqrt{3} & 2\sqrt{3} \\ -2\sqrt{3} & 4\sqrt{3} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{10}{2} \cdot 3 - \sqrt{3} \end{bmatrix}$$

$$C_p T + \frac{V^2}{2} = C_p T_0$$

$$1 + \frac{V^2}{2 C_p T} = \frac{T_0}{T}$$

$$C_p T =$$

~