FLUID MECHANICS I - sample examination problems

Question/problems with number in the circle (like ①) belong to the basic cathegory, other problems belong to the extended part.

Fluid statics

① Write the differential equation of the statics of (Pascal) fluids. Check if an incompressible fluid can remain in rest in the following volumetric force field:

Check if it is potential, if it is potential, then it is in rest

$$\vec{f}(x,y,z) = \frac{x}{x^2+y^2}\vec{e}_x + \frac{y}{x^2+y^2}\vec{e}_y - g\vec{e}_z$$

② A Pascal fluid remains in rest in a potential force field. Show that surfaces of constant density and isopotential surfaces of the external force field are identical.

3. A Pascal fluid remains in rest in a potential force field. Show that surfaces $\rho = const$ and $p = const$ are the same.

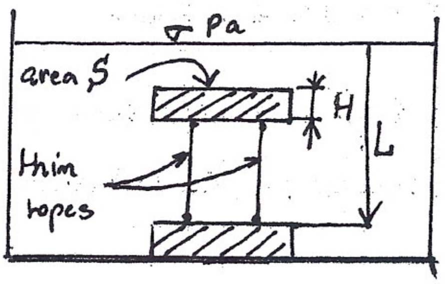
4. Derive the law of Archimedes.

5. Show that the displacement force is applied to the geometric center of the immersed body

6. Using differential equation of fluid statics, Clapeyron equation $p = \rho R T$ and the isentropic relation $p = const \cdot \rho^k$, $k = c_p/c_v$ show that, in an isentropic layer of Clapeyron gas under the action of a uniform gravity field, the temperature drops linearly with the altitude.

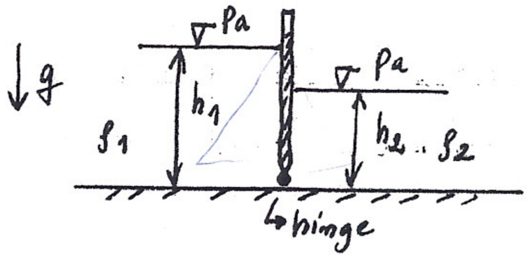
7. Isothermal layer of a Clapeyron gas in the uniform gravity field. Find $p = p(z)$ where z is a vertical coordinate (at the ground, $z=0$ and $p = p_0$). Given: k, R, g, temperature of the layer $T_0 = const$.

(8)



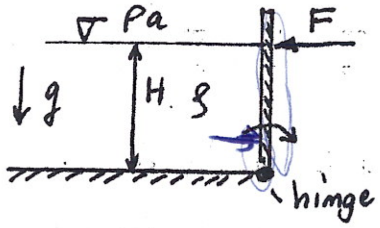
Is it possible to choose such density of the material so that the pair of blocks (linked by thin ropes) will rise to the free surface. Explain your arguments.

(9)



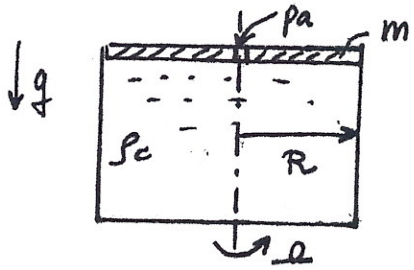
The flap remains in vertical position. The density ratio ρ_1/ρ_2 is given. Find the ratio h_1/h_2

(10)



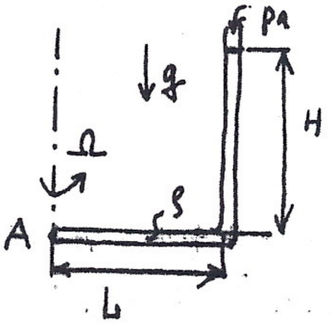
To keep the flap in vertical position the force F needs to be applied. Find the density ρ.

11.



Floating flap with the mass m remains balanced by the fluid which rotates with the angular velocity Ω. Find m knowing g, ρ_c, Ω, R.

12.



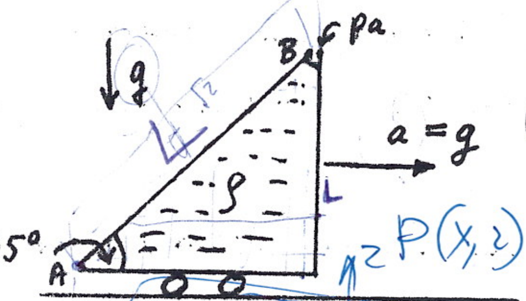
Find the angular velocity Ω such that:

a) pressure at A is equal p_a

b) pressure at A drops to zero.

$x = -s\frac{\sqrt{2}}{2}$ $z = -s\frac{\sqrt{2}}{2}$

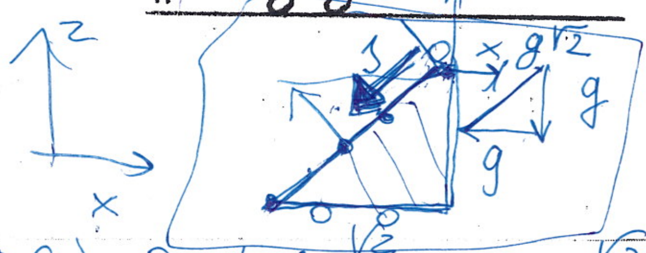
(13)

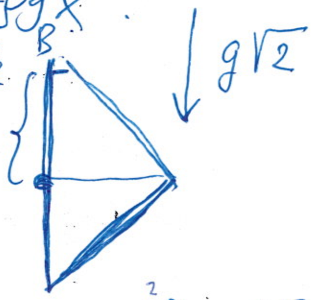


Find reaction on the wall AB (per 1m of span)

$p(x,z) = p_a - \rho g z - \rho g x$

$P(x,z) = -gz - gx + C$ $\frac{1}{2}L\sqrt{2}$





$F_{AB} = L\sqrt{2}\cdot D\cdot \rho g\sqrt{2}\cdot\frac{1}{2}L\sqrt{2}\ A = L^2 D\rho g\sqrt{2}$

$p(s) = p_a + \rho g\frac{\sqrt{2}}{2}s + \rho g\frac{\sqrt{2}}{2}s = p_a + \rho g\sqrt{2}\cdot s$

③

FLUID KINEMATICS, MASS CONSERVATION.

(14) given: • velocity field $\vec{V}(t,x,y) = [\sin t \cos x \sin y,\ -\sin t \sin x \cos y]$

• scalar field $f(t,x,y) = e^{-t}(x^2+y^2)$

Calculate substantial (material) derivative of f.

(15) $\vec{V}(t,x,y) = e^{-t}\sin\alpha x \cos\beta y\, \vec{e}_x + e^{-t}\cos\alpha x \sin\beta y\, \vec{e}_y$

Calculate y-component of the fluid acceleration

16. 2D motion of a fluid has the following description in Lagrangian form:

$$x(t,\xi,\eta) = \xi e^{\lambda t},\quad y(t,\xi,\eta) = \eta e^{-\lambda t},\quad \lambda \in R \text{ given}$$

Check if:

- this flow is stationary (steady)
- this flow conserves the volume (i.e., is incompressible)

?

(17) $$\vec{V} = \frac{y}{x^2+y^2}\vec{e}_x - \frac{x}{x^2+y^2}\vec{e}_y \qquad x^2+y^2 \geq R^2$$

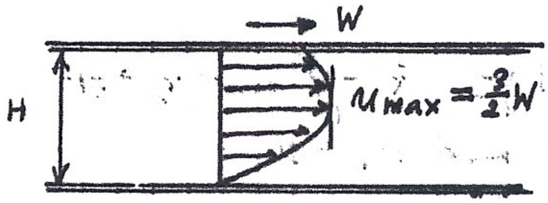
$\boxed{\nabla\cdot V = 0\ ?}$

Can an incompressible fluid move like this? Explain.

18. Define the stream function for 2D flow. Calculate streamfunction for the flow given in the problem 16.

?

$\boxed{(19)}$



← Couette-Poiseuille flow (2D)
Find „volumetric" flow rate Q (in m^2/s)

(20) Show that in the Hagen-Poiseuille flow maximal velocity of fluid is twice bigger than the average velocity

21. Some 2D flow is potential (i.e., $\nabla\times\vec{V} = \vec{0}$) and we know the scalar field of the velocity magnitude. We also know that this flow is steady. Is such knowledge sufficient to determine the acceleration? Explain in details.

22 In some 2D flow the streamfunction is given by the formula

$$\psi(x,y) = \sin x \sin y$$

a) find „volumetric" flow rate crossing the straight line segment between the points $(0,0)$ and $(\frac{1}{4}\pi, \frac{1}{2}\pi)$

b) calculate vorticity at the point $(\frac{1}{2}\pi, \frac{1}{3}\pi)$

23. The streamfunction is $\psi(t,x,y) = \Psi(t) \sin x \sin y$. Show that the field of convective part of the acceleration is a potential field.

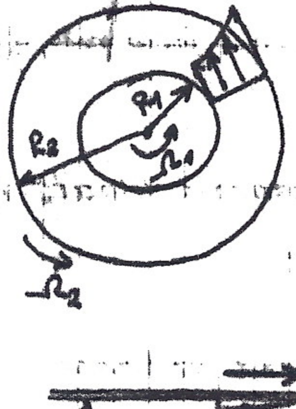
24. Write differential equation of mass conservation in a conservative and fully expanded forms.

25. In some stationary flow of a gas, the divergence $\nabla \cdot \vec{V} = 0$ at some point. Show that, at this point, the velocity is tangent to the surface of constant density

26. In the Taylor - Couette flow, the velocity has only circumpherential (asimuthal) component v_θ given as

$$v_\theta(r) = Ar + \frac{B}{r}$$

where A,B follow from angular velocities of inner and outer cylinders

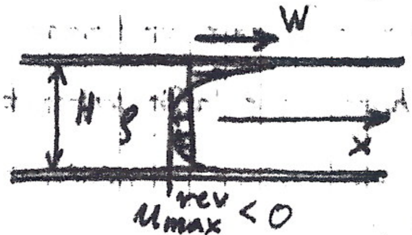


a) find A i B for $R_1 = R$ and $R_2 = 2R$, $\Omega_1 = \Omega$ and $\Omega_2 = 0$

b) find volumetric flow rate

c) calculate (in an intelligent way!) the field of acceleration

27.



Couette - Poiseuille flow. The volumetric flow rate is 0. Find $\frac{dp}{dx}$ (with sign!) and maximal velocity of reversed flow

Given: ξ, H, W

DEFORMATION AND INTERNAL FORCES. EQUATION OF MOTION

(28) Calculate deformation rate tensor $\mathbb{D}$ and rotation tensor $\mathbb{R}$ for the following motion

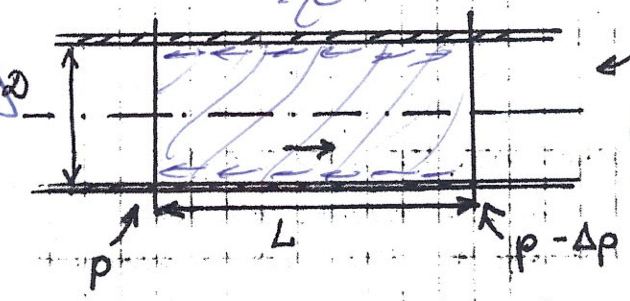
a) $\begin{cases} v_1(x,y) = \lambda x_1 \\ v_2(x,y) = -\lambda x_2 \end{cases}$ b) $\begin{cases} v_1 = \lambda x_2 \\ v_2 = -\lambda x_1 \end{cases}$

(29) Calculate wall shear stress for the Couette - Poiseuille flows from the problems 19 and 27. (both walls). Viscosity μ is known.

30. Calculate wall shear stress at both cylinders in the Taylor - Couette flow from the problem 26. Calculate the moment needed to maintain the stedy motion of the inner cylinder. Viscosity μ is given.

$\rightarrow \lambda = \frac{64}{Re} = ?$

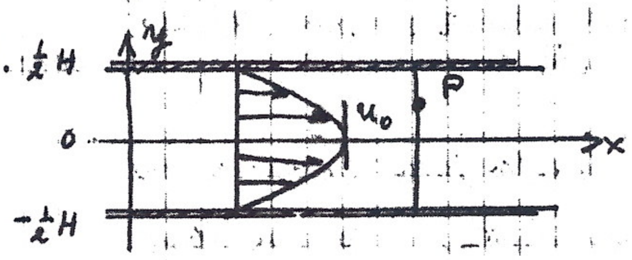
(31)



← stationary flow through the pipe segment. Find wall shear stress knowing Δp, $\mathbb{D}$ and L

λ μ

32.



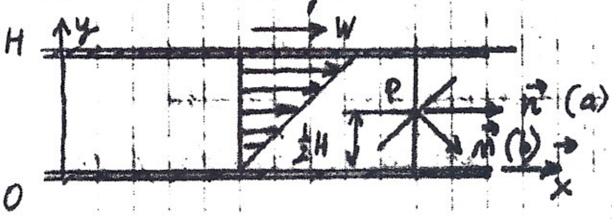
Poisseuille flow with the maximal velocity U_0

a) find the stress vector $\vec{\sigma}$ at the point P, at the surface oriented by the unit vector $\vec{n} = [1,0]$

b) as above but $\vec{n} = \left[\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right]$

Pressure at P is known and equal p_P. Viscosity μ is given.

33. As in 32 but for the Couette flow



$\vec{n} = \left[\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right]$, μ - given

$F_p = \frac{\pi D^2}{4} \Delta p$ $F_t = \pi D L \tau_w$

34 Write x-component of the Navier-Stokes equation for stationary 2D flow of Newtonian <u>liquid</u>. All differential operators must be written is a fully expanded form.

35 Write <u>complete</u> system of differential equations (scalar) which describes 2D, unsteady motion of incompressible newtonian fluid

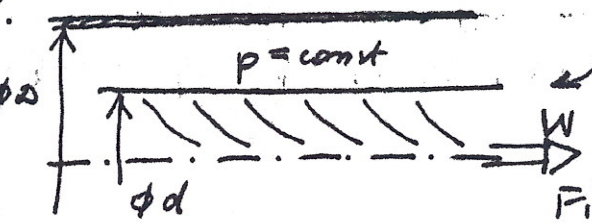
36 Write vector form of equation of motion of viscous gas. Assume that the second (bulk) viscosity ζ is zero

37. In some motion of newtonian liquid one can choose such reference frame that

$$\vec{v} = [V_1(t, x_1, x_2),\ V_2(t, x_1, x_2),\ V_3(t, x_1, x_2)]$$

Write full set of N-S equation taking into account that $\vec{v}$ does not depend on x_3. What can you say about the structure of the obtained system?

38.



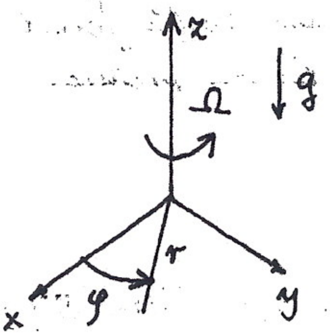
pipe with the solid core. The core is moving with the velocity W. Pressure is constant. Find the velocity profile $u = u(r)$. Calculate force per unary lenght (N/m) needed to drag the core.

39. The velocity profile of the Taylor-Couette flow (problem 26) is $u(r) = \Omega_1 R_1 \ln(r/R_2)/\ln(R_1/R_2)$. Assume that the gap between the cylinders is very narrow, i.e., $R_2 - R_1 \ll R_2$. Express $u(r)$ by the variable $\eta = R_2 - r$ where $\eta \in [0, \underbrace{R_2 - R_1}_{d}] \equiv [0, d]$. Next, use the Taylor expansion to get rid of the logarithm and derive the approximate formula for the T-C flow in a narrow gap. Using this formula calculate the moment of the "friction force" developed on the inner cylinder.

BERNOULLI EQUATION

(40) Formulate all assumptions necessary to derive the B. Eq. Explain all concepts involved

(41) Write B. Eq. for the incompressible fluid ($\rho = \text{const}$) exposed to the force field $\vec{f} = a_1\vec{e}_1 + a_2\vec{e}_2 + a_3\vec{e}_3$, where $\vec{a}$ - constant vector

(42) 

Write B. Eq. for an incompressible fluid in noninertial reference frame rotating around z-axis (see Figure)

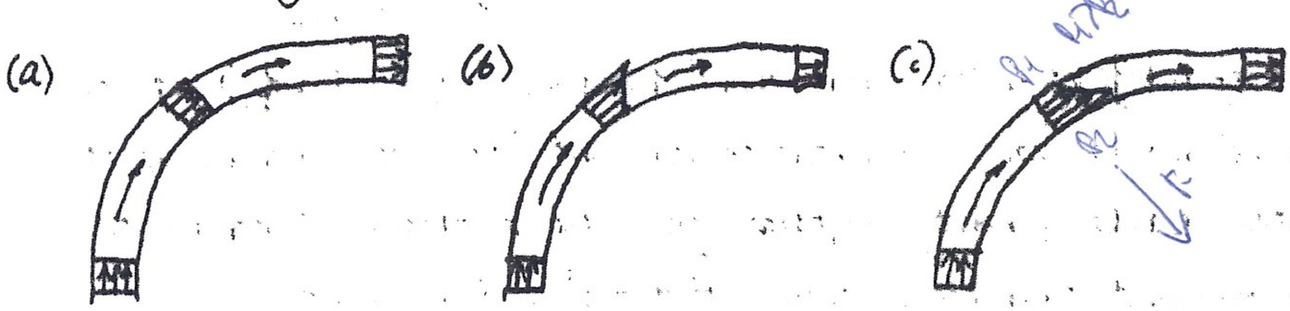
(43) Write (derive) the B. Eq. for the adiabatic stream of the Clapeyron gas. Neglect external force field term.

44. Write (derive) the B. Eq. for the isothermal stream of Clapeyron gas. Next, show that in such case the following relation

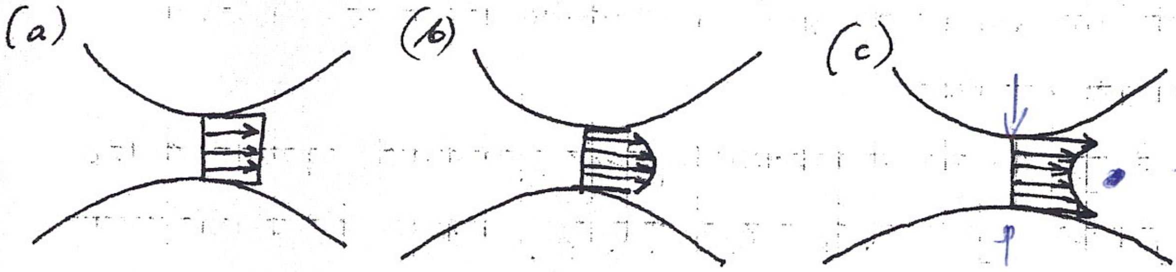
$$\ln \frac{p_0}{p} = kM^2$$

holds, where p_0 is the pressure at stagnation point ($V_0 = 0$) and p and M denote pressure and Mach number in any point. ($k = C_p/C_v$)

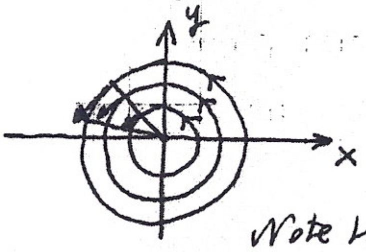
(45) Which of the velocity profile in the turning channel is qualitatively correct (for ideal fluid)? Explain your choice.



46 The same as in 45: (Bernoulli constant the same at each streamline)



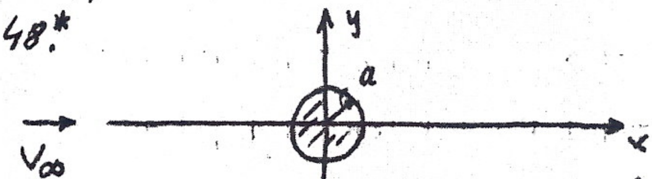
47. Consider 2D motion of an ideal incompressible fluid as in the picture.



$$\begin{cases} v_x(x,y) = -\Omega y \\ v_y(x,y) = \Omega x \end{cases}$$

Note that $\vec{v}(0,0) = \vec{0}$. From equations of motion (Euler Eq.) as well as elementary knowledge about circular motion we conclude that the pressure must rise with the distance from $(0,0)$ (i.e., $(0,0)$ is the point of the minimal pressure. On the other hand, from (naively used ?) B. Eq. we conclude completely opposite: $p(0,0)$ is the stagnation pressure hence the largest possible! Explain this (apparent) contradiction.

48.*



It can be shown that 2D stationary flow of ideal fluid, is such that a general formula for the velocity at the cylinder's contour is.

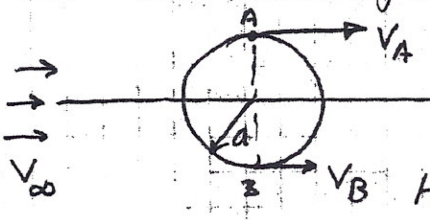
$$v_\theta(r=a,\theta) = -2V_\infty \sin\theta + \frac{\Gamma}{2\pi a}$$

↑ circumpherential component (obviously, $v_r\big|_{contour} \equiv 0$)

The quantity Γ is the circulation (the charge of vorticity) of the point vortex with the center located at $(0,0)$. The presence of this vortex breaks symmetry of flow wrt the x-axis and leads to appearance of transversal force $\vec{F} = F_y \vec{e}_y$. Show that

$$F_y = -\rho V_\infty \Gamma \quad \text{(so called Kutta-Youkovsky formula)}$$

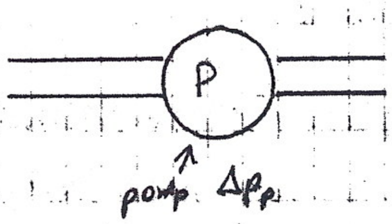
49. Consider the flow like in 48.



Knowing that $V_A = 3V_B$ find $F_y = -\rho V_\infty \Gamma$

Hint: the attached vortex with circulation Γ induces uniform circumpherential component at the contour equal $(V_\theta)_\Gamma = \frac{\Gamma}{2\pi a}$

(50)



The micropump is used to maintain the Hagen-Poiseuille flow (laminar) in the pipe of the diameter ϕD. The maximal velocity in the pipe is U_m and the power supplied to the flow is N_p. Find Δp_p.

FLOW SIMILTUDE. ELEMENTS OF HYDRAULICS

(51) Enumerate all conditions for full dynamic similtude of flows

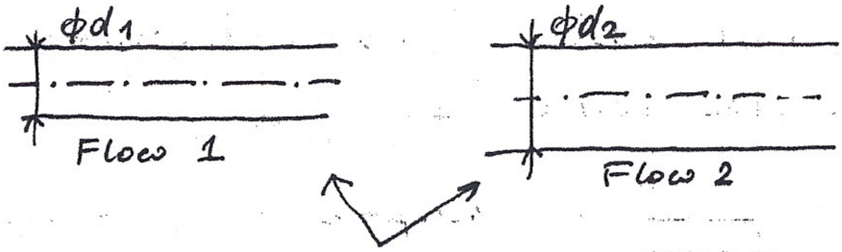
(52) Give a short example demontrating that fullfilment of full similtude of flows is - by the rule - impossible

(53) The ship's model is made in scale 1:16. The conditions of the model flow (or ship's motion) have been choosen, so that the wave resistence is properly estimated. Find the ratio Re_{real} / Re_{model} assuming that $\mu_{model} = \frac{1}{2} \mu_{real}$

(54) The model of an aircraft, scaled 1:10, has been placed inside the cryogenic wind tunnel with the flow temperature $T_\infty = 180K$. The aim of this research is to estimate the wave drag in the supersonic flight at low altitude: the tunnel conditions are supposed to correspond to a real flight at $M = 1.3$ in the atmosphere with $T_{real} = 270K$. What should the velocity in the tunnel be?

Find the ratio between the Reynolds numbers Re_{real}/Re_{model} using the formula $\mu(T) = C\frac{T\sqrt{T}}{T_{ref}+T}$, $T_{ref} = 110K$, C – see internet or the lecture. (the Sutherland formula), $a = \sqrt{kRT}$, $k = 1.4$, $R = 287 \frac{J}{kgK}$.

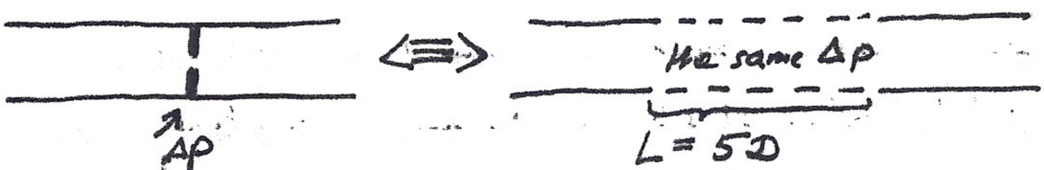
(55)



both are laminar pipe flows (Hagen-Poiseuille flows)

These flows are dynamically similar. We also know that $\mu_1 = \mu_2$ and $\rho_1 = \rho_2$. Find the ratios of: volumetric flow rates Q_1/Q_2 pressure gradients $(dp/dx)_1 / (dp/dx)_2$ and wall shear stresses τ_1/τ_2.

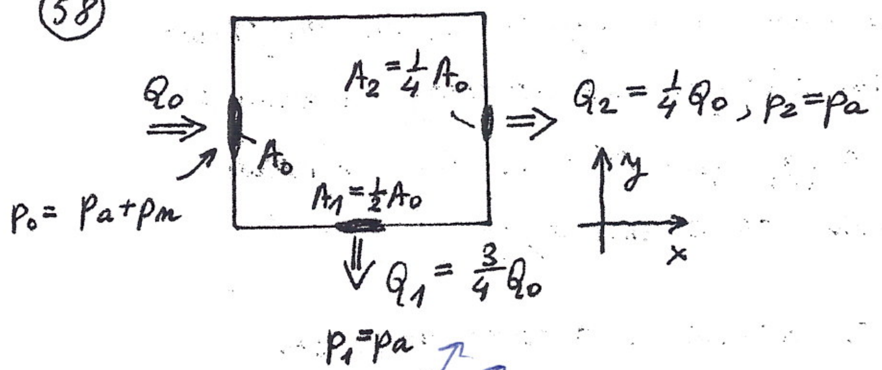
(56) In a certain pipe the local pressure drop on the orifice inside the pipe is equivalent to the distributed pressure lose at the distance $L = 5D$ at the Reynolds number Re_{ref}. (turbulent flow). Find the coefficient of the local lose ζ for this orifice.



(57) How does λ depend on the Reynolds number when the inner surface of the pipe has substantial roughness and the Reynolds number is high? How, in such case, does pressure lose change with the volumetric flow rate?

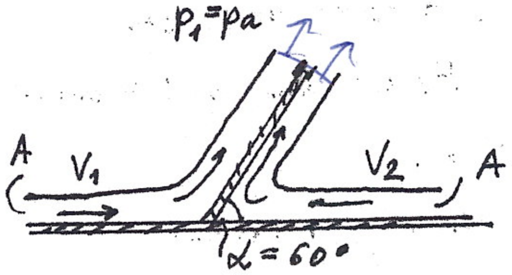
DYNAMICAL REACTIONS

58



Find both components of the reaction (R_x, R_y)

59



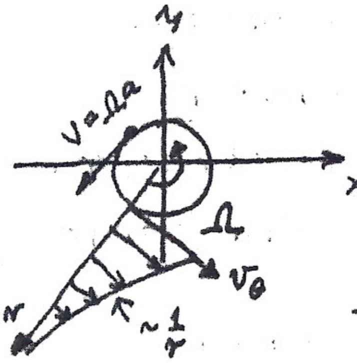
The horizontal reaction is $R_x = 0$
Find V_2/V_1.
(and other similar problems ...)

60. In the lecture about dynamical reactions we have derived the formula for the wall stress

$$\vec{\sigma} = -p\vec{n} - \mu\,\vec{n} \times \vec{\omega} \quad (\vec{\omega} = \nabla \times \vec{V} \text{ at the wall})$$

It follows that tangent stress at the wall vanishes, wherever $\vec{\omega}\big|_{wall} = \vec{0}$. Consider now the viscous flow outside the rotating 2D cylinder. It can be shown that such flow may have only circumpherential component V_θ which is

$$V_\theta \sim \frac{1}{r}.$$



But for such flow $\vec{\omega} \equiv \vec{0}$ everywhere! On the other hand, wall shear stress at the cylinder contour is evidently nonzero – – there exists the velocity gradient $\neq 0$ and the fluid is viscous. Thus, we arrived at contradiction! Find an error in this argument.

ENERGY, DISSIPATION, ELEMENS OF GAS DYNAMICS

61. Enumerate all assumptions necessary to derive the first integral of the differential equation of energy conservation

62. Write the (algebraic) energy equation is this or other form (e.g. conteing only u, a and a_*).

63. The plane flies with the velocity $V = 1800$ km/h in the stratosphere (temp. $-53°C$). Find maximal theoretical temperature on the plane's surface.

64. The Clapeyron gas flows out of the big container through the thermally isolated pipe. Temperature of gas is $T = 300K$. Find the Mach number in such point where $u = 200$ m/s ($k = 1.4$, $R = 287$ J/kg K)

65. In a certain stationary and adiabatic gas flow $u_* = 300$ m/s. Find maximal theoretical gas velocity in such flow

66. Show by direct calculations that power developed by pressure forces in the 2D Poiseuille flow is equal to the power dissipated inside the viscous liquid.

67. Show by direct calculation that power developed by the force driving the upper moving wall in the Couette flow is equal to the power dissipated inside the viscous liquid.

68. Write algebraic forms of: mass, linear momentum and energy conservation principles for steady 1D motion of the Clapeyron gas.

69. 1D, steady flow of the Clapeyron gas. Show that

$$\frac{pM}{\sqrt{T}} \quad \text{and} \quad p(1 + kM^2) \quad \text{are constants of this motion.}$$

70. The Clapeyron gas outflows from the big container with $T_{container} = 300\,K$. In the duct we observe the stationary normal shock wave. The velocity behind the shock wave is $u_2 = 200$ m/s. What is the velocity of gas in front of the shock wave?

71. $u_1 = 1000$ m/s $\longrightarrow$ | $u_2 = 100$ m/s $\longrightarrow$ NSW

Is such situation possible? Explain.

No shock wave that can increase density 6 times.

72. Make careful, qualitative drawing of the Hugoniot adiabat. Explain why expansion shock waves do not exist.

73. Show that in any adiabatic, stationary gas flow the temperatures at two points T_1 and T_2 are related with the Mach numbers M_1 i M_2 as follows

$$\frac{T_2}{T_1} = \frac{1 + \frac{k-1}{2} M_1^2}{1 + \frac{k-1}{2} M_2^2}$$

74. Insert an appropriate relation operator (<, > or =)

in front of the NSW	>, <, or =	behind the NSW
M_1		M_2
u_1		u_2
T_1		T_2
a_1		a_2
ρ_1		ρ_2
p_1		p_2
T_{01}		T_{02}
a_{*1}		a_{*2}
s_1		s_2
p_{01}		p_{02}
ρ_{01}		ρ_{02}

(usually the list has 5 selected parameters)