Warsaw University of Technology

Faculty of Power and Aeronautical Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

Institute of Aeronautics and Applied Mechanics

Finite element method 2 (FEM 2)

Damping matrix

$$\underset{NDOF\times NDOF}{[M]}\cdot\underset{NDOF\times 1}{\{\ddot{q}\}}+\underset{NDOF\times NDOF}{[C]}\cdot\underset{NDOF\times 1}{\{\dot{q}\}}+\underset{NDOF\times NDOF}{[K]}\cdot\underset{NDOF\times 1}{\{q(t)\}}=\underset{NDOF\times 1}{\{F(t)\}}$$

MASS MATRIX

VECTOR OF NODAL ACCELERATIONS

DAMPING MATRIX

VECTOR OF NODAL VELOCITIES

STIFFNESS MATRIX

VECTOR OF NODAL DISPLACEMENTS

LOAD VECTOR

DAMPING MATRIX

$\boxed{\text{Rayleigh model}}$

$$[C] = \alpha \cdot [M] + \beta \cdot [K]$$

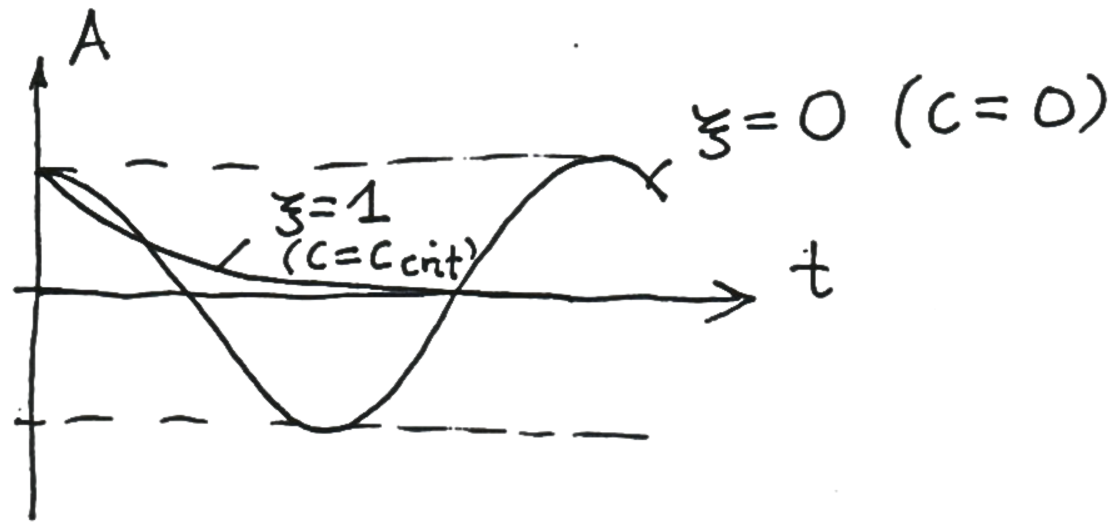
coefficient of mass damping (due to rigid body motion)

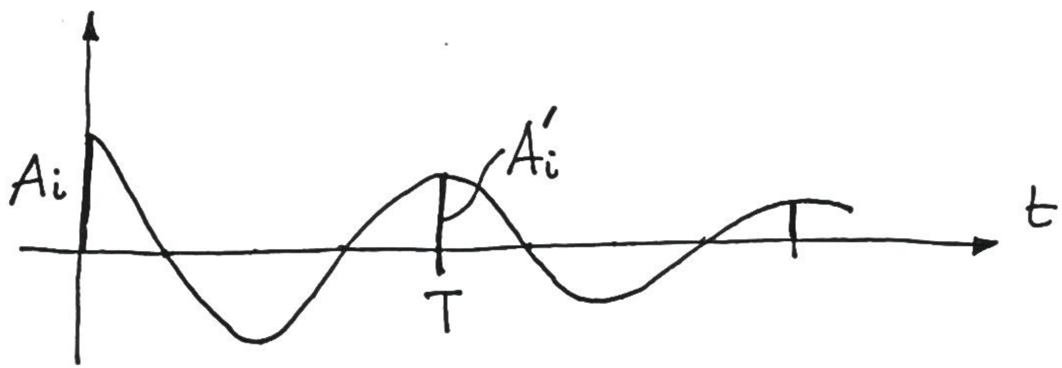
coefficient of structural damping

damping ratio:

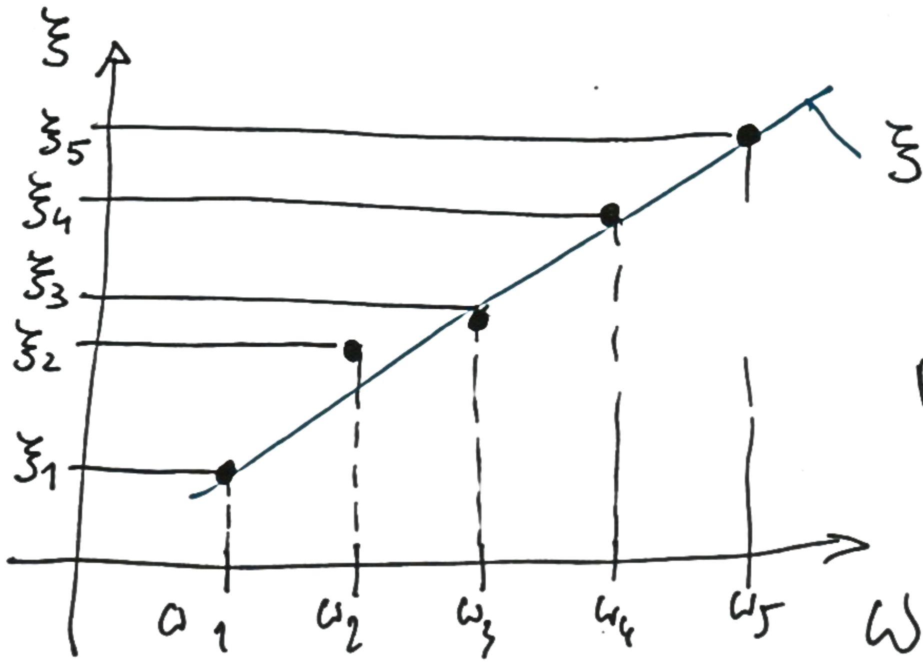
$$\xi = \frac{\alpha}{2\omega} + \beta \cdot \frac{\omega}{2}$$

$$\xi = \frac{c}{c_{crit}}$$





Damping ratio based on logarythmic decrement : $\xi_i = \dfrac{\ln \frac{A_i}{A_i'}}{2\pi}$



$$\xi = A \cdot \omega + B$$

FREQUENCY DEPENDENT DAMPING

FREQUENCY INDEPENDENT DAMPING

$$\begin{cases} \dfrac{1}{2\omega_1} \cdot \alpha + \dfrac{\omega_1}{2} \cdot \beta = A \cdot \omega_1 + B \\ \dfrac{1}{2\omega_5} \cdot \alpha + \dfrac{\omega_5}{2} \beta = A \cdot \omega_5 + B \end{cases}$$

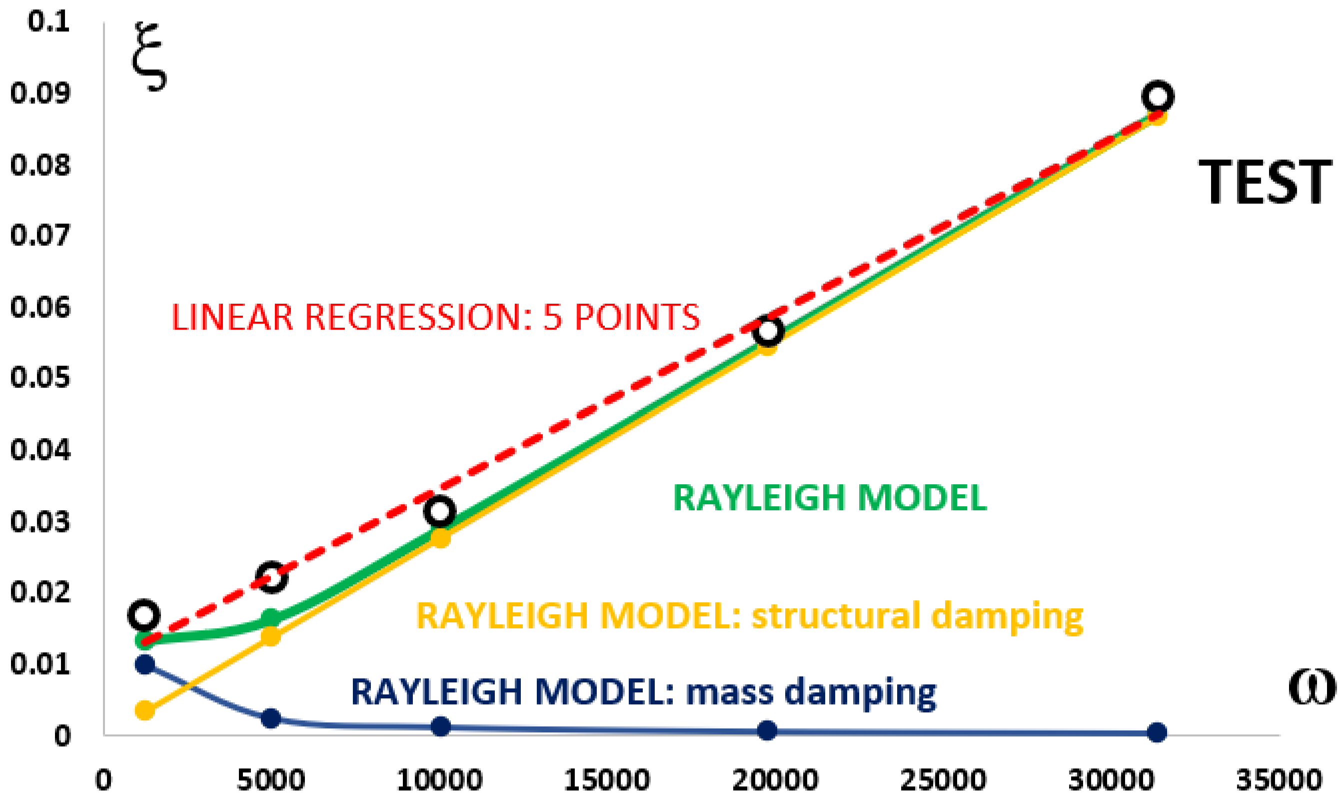
$$\alpha = \frac{2\omega_1\omega_5 B}{\omega_1+\omega_5}$$

$$\beta = 2\left(A + \frac{B}{\omega_1+\omega_5}\right)$$

$$[C] = \alpha\cdot[M] + \beta\cdot[K]$$

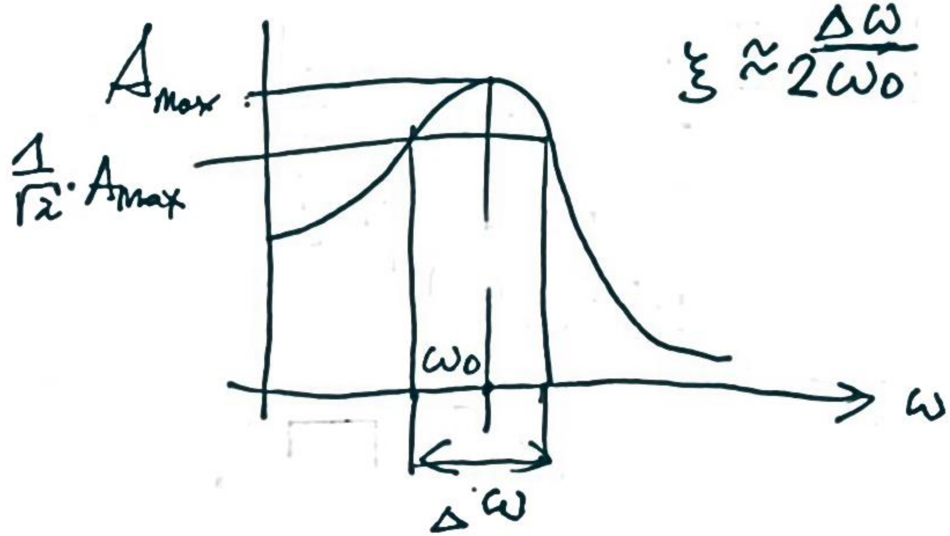
				LINEAR REGRESSION		A=	2.451E-06	α= 24.75304351 [s]
						B=	1.024E-02	β= 5.52849E-06 [1/s]

TEST no.	f [Hz]	ω [1/s]	A1 [mm]	A1' [mm]	ξ	Rayleigh	mass_part	structural_part
1	200	1256.637	1	0.9	0.016769	0.013323	0.009848923	0.00347365
2	800	5026.548	1	0.87	0.022164	0.016357	0.002462231	0.013894601
3	1600	10053.1	1	0.82	0.031584	0.02902	0.001231115	0.027789201
4	3150	19792.03	1	0.7	0.056767	0.055335	0.000625328	0.05470999
5	5000	31415.93	1	0.57	0.089464	0.087235	0.000393957	0.086841254



Half power bandwidth method

$0 < \xi < 0.2$



$\xi \approx \frac{\Delta\omega}{2\omega_0}$

1 DOF

Estimating Damping Values Using the Half Power Method

Tech Brief 150401 D

Integrated Systems Research, Inc.

April, 2015

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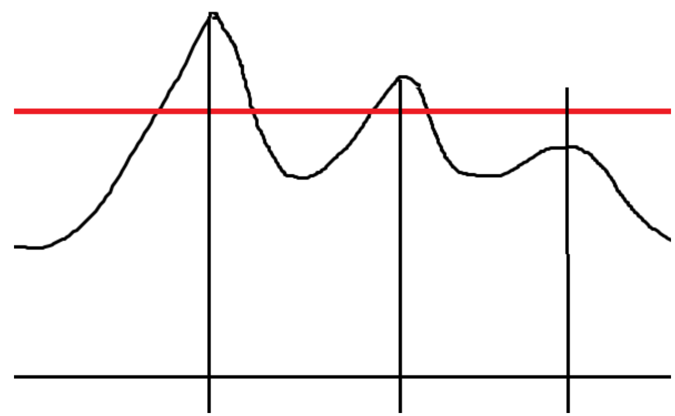
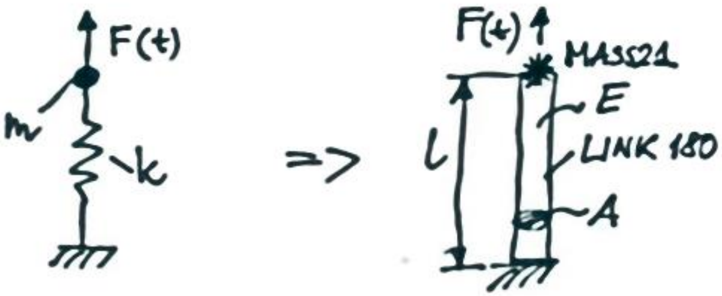
3 DOF



1 DOF mass and spring system in ANSYS

MASS MULTIPLIER	ALPHAD
STIFFNESS MULTIPLIER	BETAD
FREQUENCY INDEPENDENT	DMPR

$$m\ddot{x} + c\dot{x} + kx = F_0 \cdot \sin\omega t$$



$$k = \frac{EA}{L}$$

DAMPING RATIO :

$$\xi = \frac{C}{C_{crit}} = \frac{\beta \cdot k + \alpha \cdot m}{2\sqrt{k \cdot m}}$$

LOSS FACTOR :

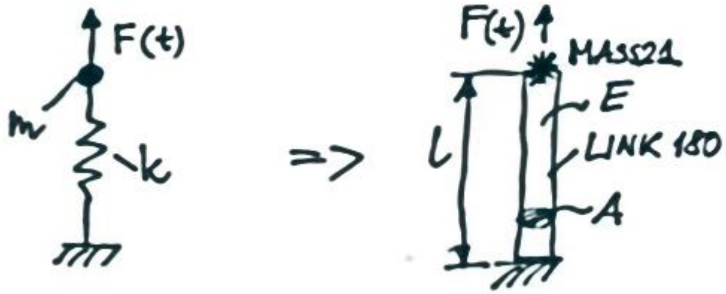
$$tg\,\varphi = 2\xi = \frac{\beta \cdot k + \alpha \cdot m}{\sqrt{k \cdot m}}$$

$$C = BETAD \cdot k + \frac{2K}{\omega} DMPR + ALPHAD \cdot m$$

$$= \underbrace{\left(BETAD + \frac{2}{\omega} \cdot DMPR\right)}_{\beta} \cdot k + \underbrace{ALPHAD}_{\alpha} \cdot m$$

1 DOF mass and spring system in ANSYS

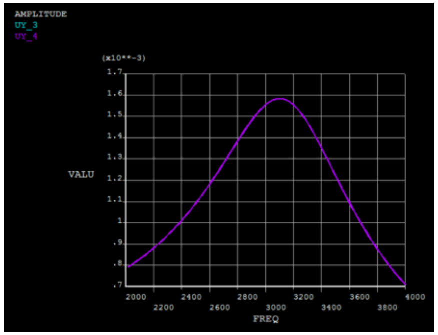
$$m\ddot{x} + c\dot{x} + kx = F_0 \cdot \sin\omega t$$

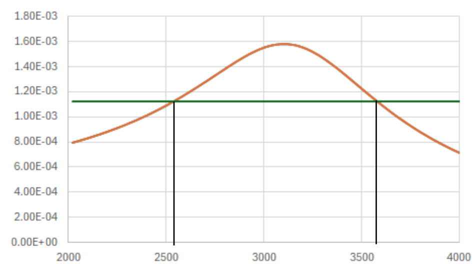


$$k = \frac{EA}{L}$$

alpha	0	
beta	1.60E-05	
E	1000	N/mm2
A	100	mm2
l	50	mm
k	2000	N/mm
m	5.00E-06	Ns2/mm
omega0	20000	rad/sek
f	3183.1	Hz
c	0.032	
c_crit	0.2	
ξ	0.16	

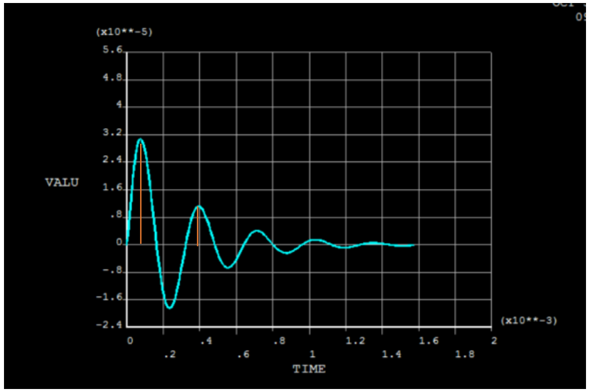
Harmonic analysis





Half power bandwidth method		
f1	2532.31	Hz
f2	3579.65	Hz
fmax	3183.098862	Hz
ξ	0.165	
BETD	1.65E-05	

Transient analysis



logarythmic decrement	
A1	1.54E-05
A2	5.58E-06
ln	1.018425032
ξ	0.162087378
BETD	1.62E-05