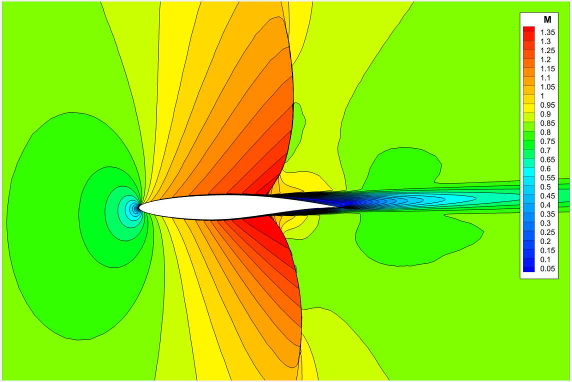
Aerodynamics I

Viscous effects in compressible flows.



flow past RAE-2822 airfoil ($M = 0.85$, $Re = 6.5 \times 10^6$, $\alpha = 2^\circ$)

Viscous effects in compressible flows

Equations of motion for compressible viscous fluid

Integral equations for a control volume Ω bounded by a boundary Γ (without body forces e.g., gravity):

Continuity equation

$$\frac{\partial}{\partial t}\int_\Omega \rho\, d\Omega + \oint_\Gamma \rho\, \mathbf{v}\cdot\mathbf{n}\, d\Gamma = 0 \qquad (1.1)$$

Momentum equation

$$\frac{\partial}{\partial t}\int_\Omega \rho\, \mathbf{v}\, d\Omega + \oint_\Gamma \rho\, \mathbf{v}\, (\mathbf{v}\cdot\mathbf{n})\, d\Gamma = -\oint_\Gamma p\, \mathbf{n}\, d\Gamma + \oint_\Gamma \mathcal{T}\cdot\mathbf{n}\, d\Gamma \qquad (1.2)$$

Total energy equation $E = e + \frac{v^2}{2}$

$$\frac{\partial}{\partial t}\int_\Omega \rho\, E\, d\Omega + \oint_\Gamma \rho\, E\, \mathbf{v}\cdot\mathbf{n}\, d\Gamma = \underbrace{\oint_\Gamma \kappa\, \nabla T\cdot\mathbf{n}\, d\Gamma}_{\text{thermal conductivity}}$$

$$\underbrace{-\oint_\Gamma p\, \mathbf{v}\cdot\mathbf{n}\, d\Gamma}_{\text{pressure forces work}} + \underbrace{\oint_\Gamma (\mathcal{T}\cdot\mathbf{v})\cdot\mathbf{n}\, d\Gamma}_{\text{friction forces work}} + \underbrace{\int_\Omega \rho\, \dot{q}\, d\Omega}_{\text{heat sources}} \qquad (1.3)$$

Equations of motion for compressible viscous fluid

Viscous stress tensor for Newtonian fluid:

$$\mathcal{T} = \mu \; (\nabla \mathbf{v} + (\nabla \mathbf{v})^{\intercal}) + \mathbb{1} \; \lambda \; \nabla \cdot \mathbf{v} \tag{1.4}$$

μ – dynamic viscosity coefficient
λ – volume viscosity coefficient; according to Stokes relation $\lambda = -2/3\,\mu$

Sutherland law:

$$\frac{\mu}{\mu_\infty} \approx \left(\frac{T}{T_\infty}\right)^{\frac{3}{2}} \frac{T_\infty + S}{T + S} \quad \text{where:} \quad S = 110.4^\circ K \tag{1.5}$$

thermal conductivity coefficient:

$$\kappa = \mu \frac{c_p}{Pr} \tag{1.6}$$

Pr – Prandtl number

Equations of compressible boundary layer

Continuity equation

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{1.7}$$

Momentum equation

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{1.8}$$

$$\frac{\partial p}{\partial y} = 0 \quad \longrightarrow \quad p = p_e(x) \tag{1.9}$$

Total energy equation $H = h + \frac{U^2}{2}$

$$\rho\left(u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\, u\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right) \tag{1.10}$$

Equations of compressible boundary layer - energy equation

Right hand side of (1.10) can be trasnformed:

$$\frac{\partial}{\partial y}\left(\mu\, u\,\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial y}\left(\kappa\,\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\,\frac{c_p}{Pr}\,\frac{\partial T}{\partial y}+\mu\, u\,\frac{\partial u}{\partial y}\right)$$
$$=\frac{\partial}{\partial y}\left(\frac{\mu}{Pr}\,\frac{\partial h}{\partial y}+\mu\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right)$$
$$=\frac{\partial}{\partial y}\left(\frac{\mu}{Pr}\,\frac{\partial H}{\partial y}+\mu\,\left(1-\frac{1}{Pr}\right)\,\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right) \qquad (1.11)$$

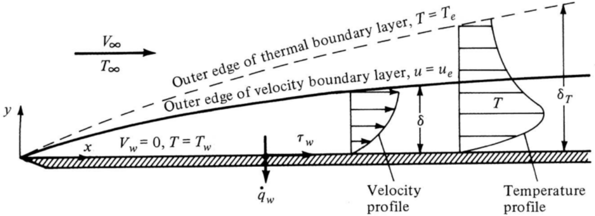
Then energy equation can be written as follows:

$$\rho\left(u\frac{\partial H}{\partial x}+v\frac{\partial H}{\partial y}\right)=\frac{\partial}{\partial y}\left(\frac{\mu}{Pr}\,\frac{\partial H}{\partial y}+\mu\,\left(1-\frac{1}{Pr}\right)\,\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right) \qquad (1.12)$$

If Prandtl number $Pr=1$:

$$\rho\left(u\frac{\partial H}{\partial x}+v\frac{\partial H}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\,\frac{\partial H}{\partial y}\right) \qquad (1.13)$$

Thermal boundary layer



$Pr < 1$ then $\delta_T > \delta$

$Pr > 1$ then $\delta_T < \delta$

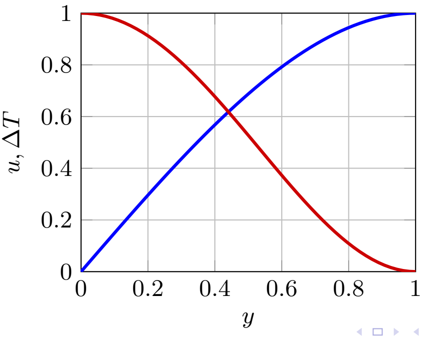
Compressible boundary layer - Busemann integral

Simplest possible solution to the energy equation is:

$$H = const \tag{1.14}$$

$$T_w = T_0 = T + \frac{u^2}{2\,c_p} \tag{1.15}$$

$$T = T_0 - \frac{u^2}{2\,c_p} \qquad \rightarrow \qquad \left(\frac{\partial T}{\partial y}\right)_w = 0 \tag{1.16}$$

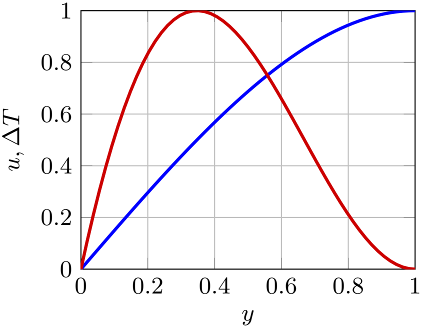


Compressible boundary layer - Crocco integral

It can be noted that if $\partial p/\partial x = 0$ then equations (1.8) and (1.13) are similar. If in addition the boundary conditions are also similar ($T_w = const$) then solutions to those equations are linearly dependent:

$$T_0 \equiv T + \frac{u^2}{2\,c_p} = A\,u + B \qquad \rightarrow \qquad T = A\,u + B - \frac{u^2}{2\,c_p} \tag{1.17}$$

Coefficients A and B must be chosen such that the boundary conditions are satisfied.



Compressible laminar boundary layer - similar solutions

The equations of the laminar compressible boundary layer can be transformed (like in case of incompressible equations) such that similar solution are obtained i.e. the flow parameters depend only on undimensional coordinate η.

The undimensional coordinates are defined:

$$\xi = \rho_e \, \mu_e \, u_e \, x \qquad \eta = \frac{u_e}{\sqrt{2\,\xi}} \int_0^y \rho \, dy \tag{1.18}$$

where index e denotes the parameters on the outer boundary of the domain.

Let us introduce functions:

$$f' = \frac{u}{u_e} \qquad g = \frac{H}{H_e} \tag{1.19}$$

and coefficient:

$$C = \frac{\rho\mu}{\rho_e \mu_e} \tag{1.20}$$

Compressible laminar boundary layer - similar solutions

Assuming the zero pressure gradient $\partial p_e/\partial\xi = 0$ Prandtl equations can be transformed to following form (the transformation is also possible for non-zero pressure gradient):

$$(C\,f'')' + f\,f'' = 0 \tag{1.21}$$

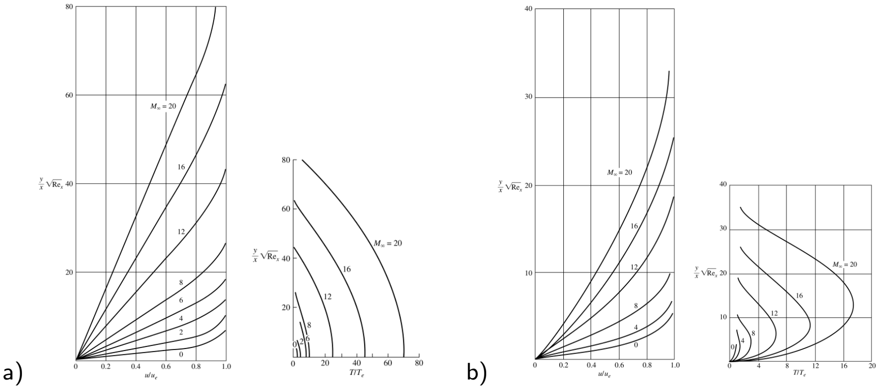
$$\left[\frac{C}{Pr}g'\right]' + f\,g' + \frac{u_e^2}{H_e}\left[\left(1-\frac{1}{Pr}\right)C\,f'\,f''\right]' = 0$$

with boundary conditions:

$$\text{dla } y = 0 \quad \begin{array}{l} f(0) = f'(0) = 0 \\ g(0) = g_w \leftarrow \text{ isothermal wall} \\ g'(0) = 0 \leftarrow \text{ adiabatic wall} \end{array} \tag{1.22}$$

$$\text{dla } y \to \infty \quad \begin{array}{l} f'(\infty) = 1 \\ g(\infty) = 1 \end{array} \tag{1.23}$$

Compressible laminar boundary layer



Example profiles of velocity and temperature for flat plate ($\partial p/\partial x = 0$, $Pr = 0.75$)

a) insulated wall b) "cold" wall

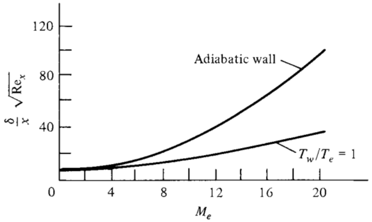
Compressible laminar boundary layer

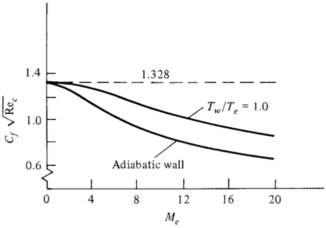
Flat plate for $\partial p/\partial x = 0$

$$C_f = \frac{1.328}{\sqrt{Re}} F\left(M_e, Pr, \frac{T_w}{T_e}\right) \tag{1.24}$$

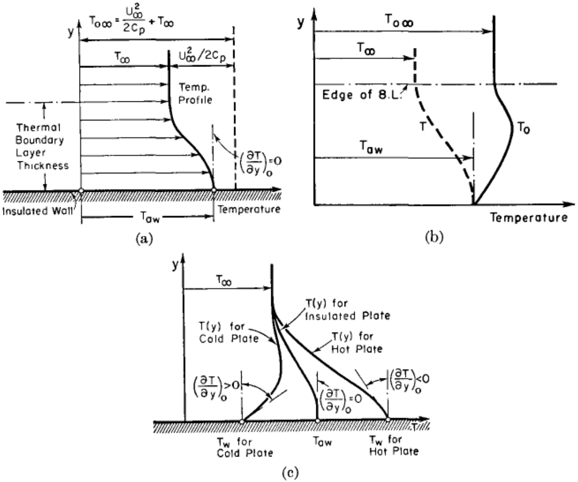
$$\delta = \frac{5\ x}{\sqrt{Re_x}} G\left(M_e, Pr, \frac{T_w}{T_e}\right) \tag{1.25}$$

Example plots of C_f and δ for $Pr = 0.75$

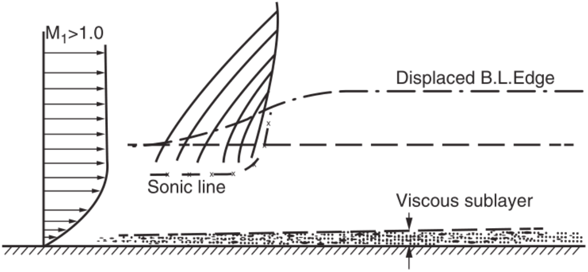




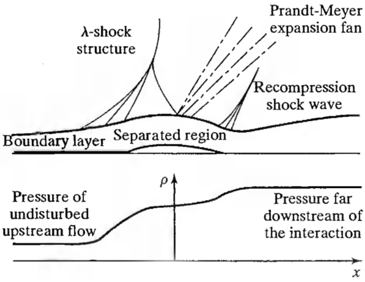
Temperature profile in boundary layer

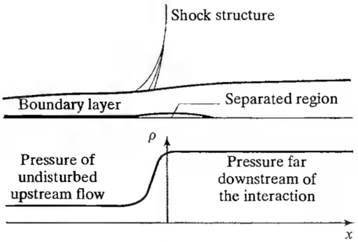


Boundary layer in supersonic flow

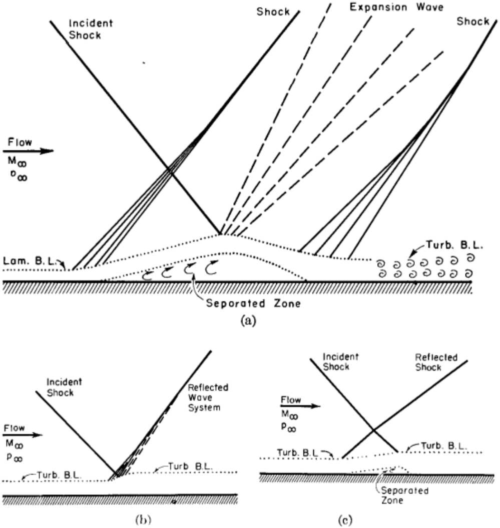


Shock wave boundary layer interaction



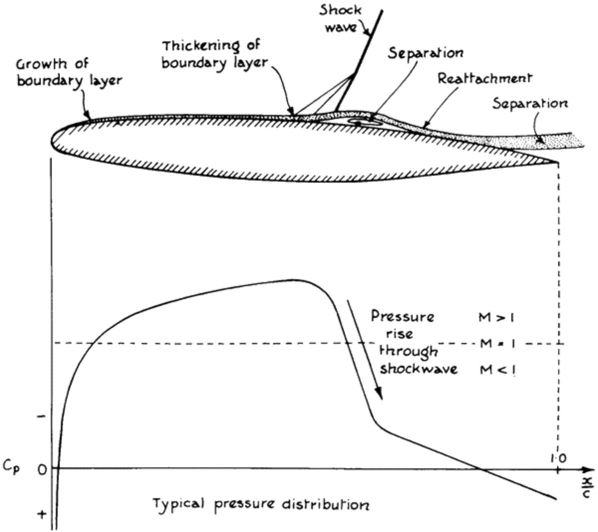


Shock wave boundary layer interaction



Viscous and compressible flow past an airfoil

Viscous and compressible flow past an airfoil



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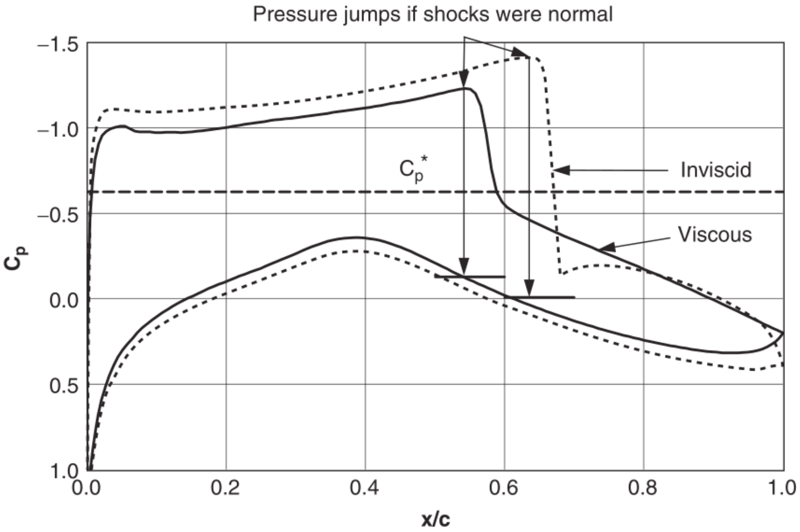
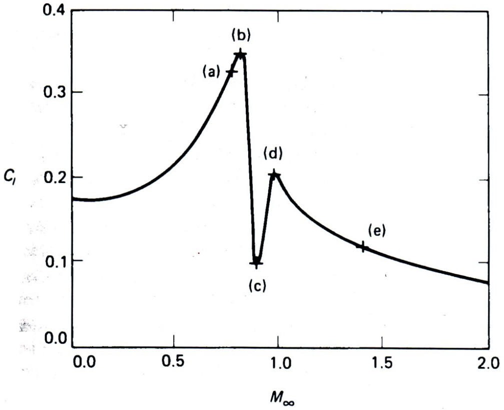


Viscous and compressible flow past an airfoil - lift coefficient



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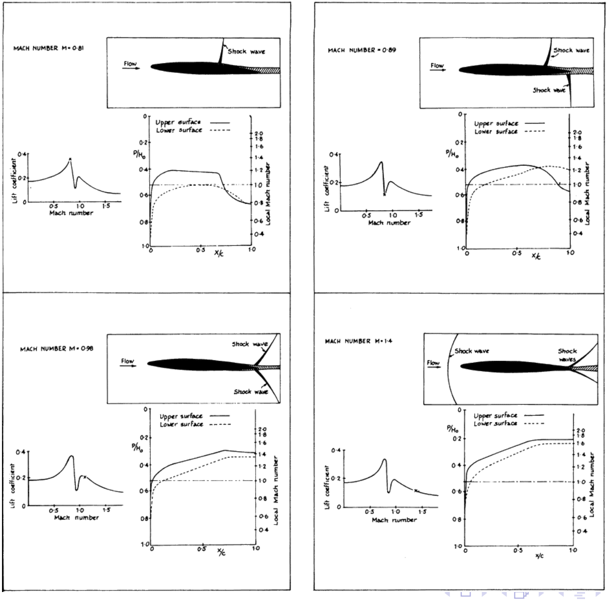
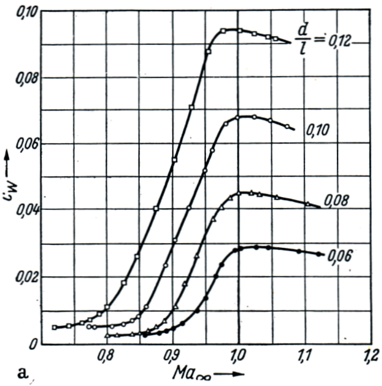
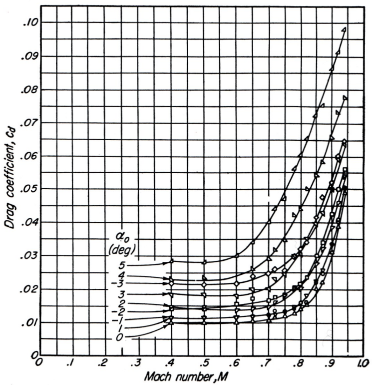


Viscous and compressible flow past an airfoil - drag coefficient

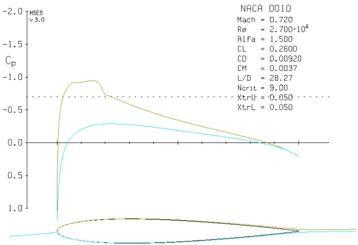


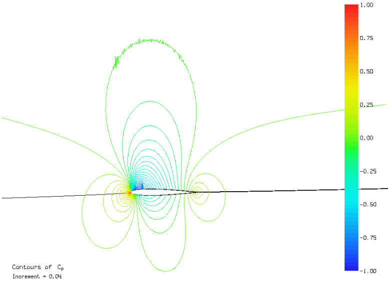
Relation of the wave drag coefficient to the Mach number

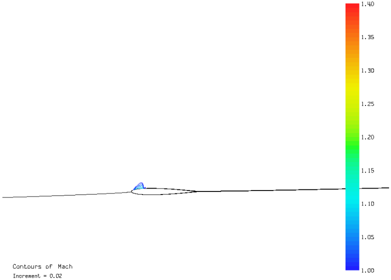


Relation of the drag coefficient to the Mach number (airfoil NACA-2306)

Transonic viscous flow past an airfoil







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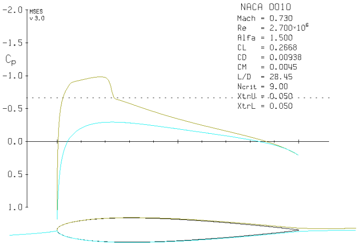

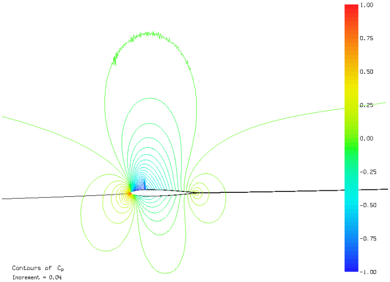

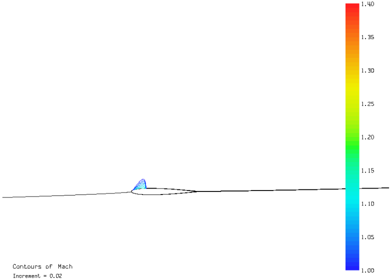

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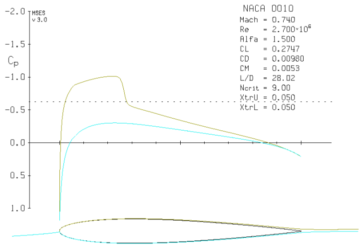

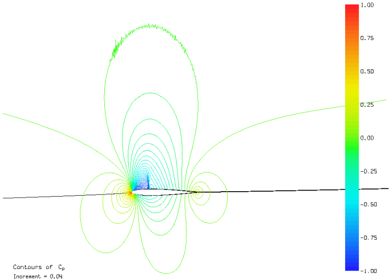

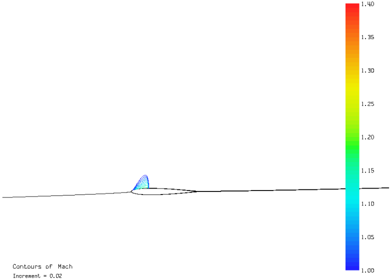

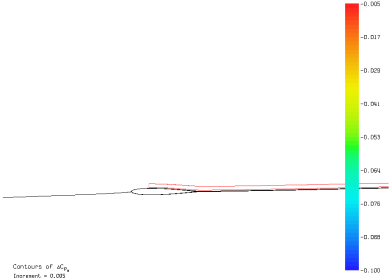

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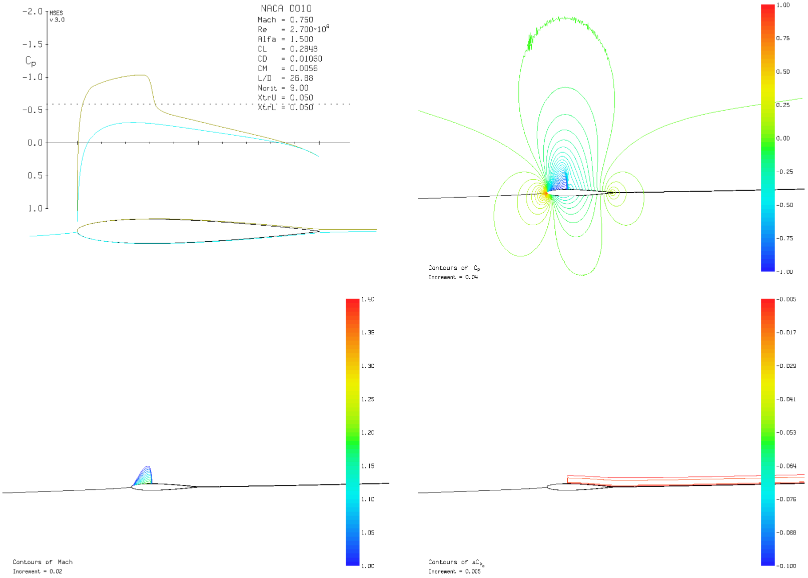

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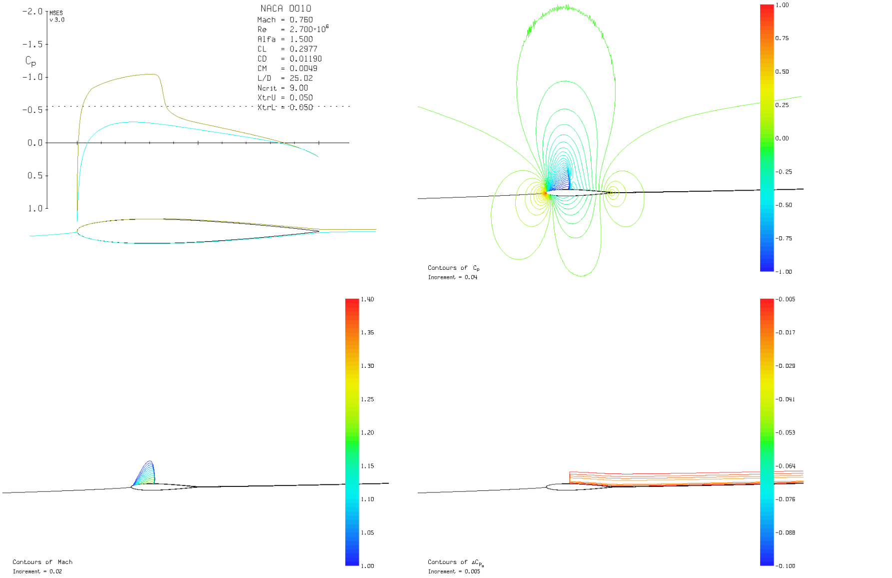

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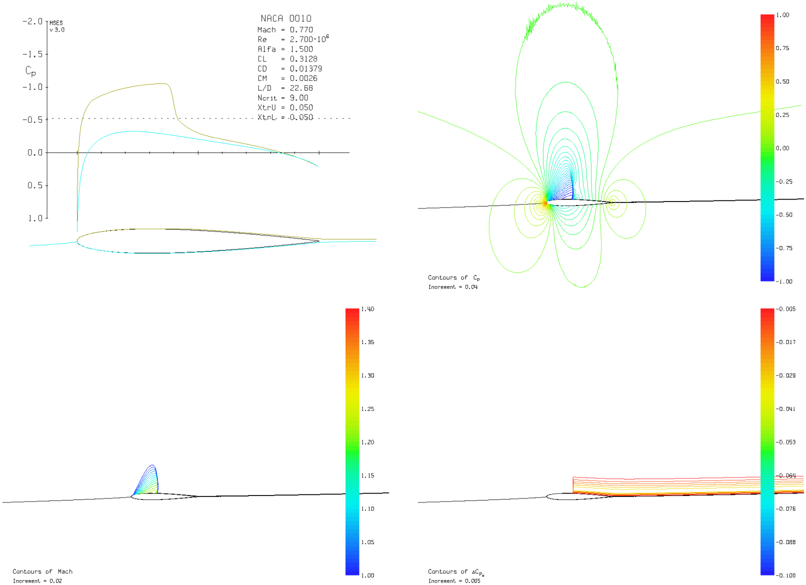

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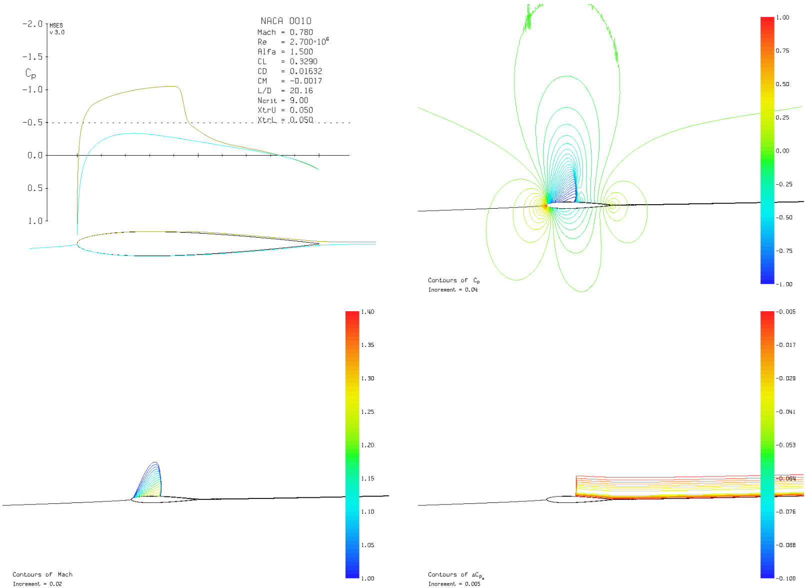

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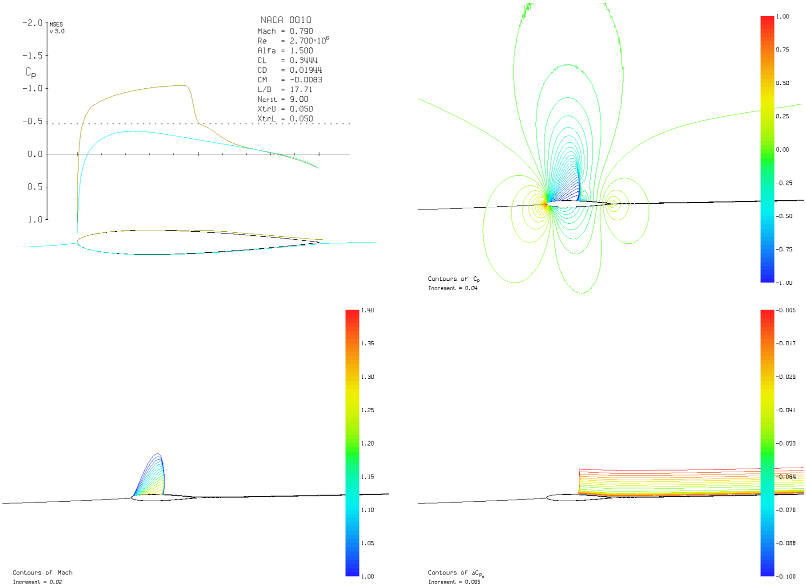

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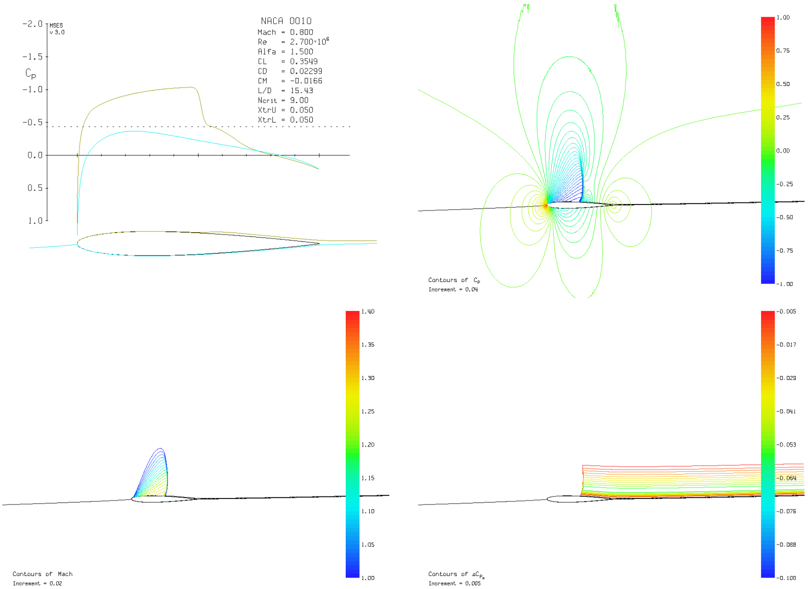

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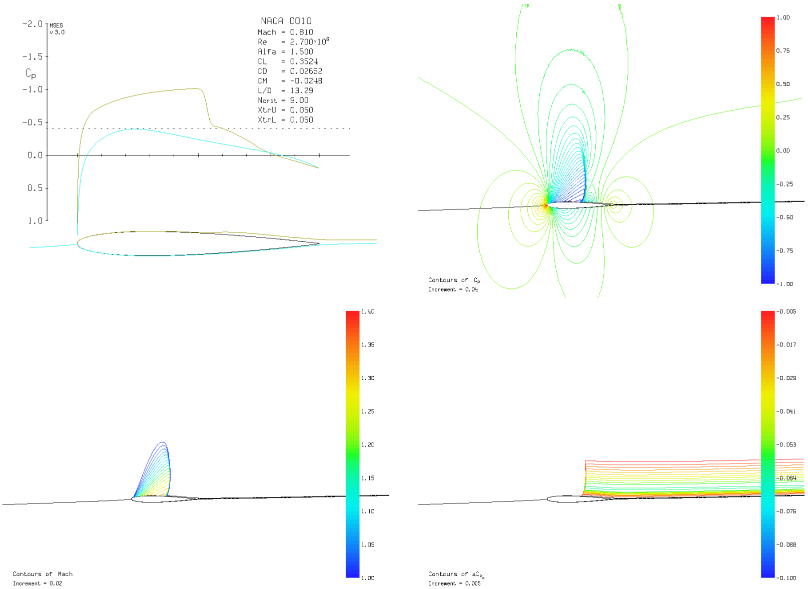

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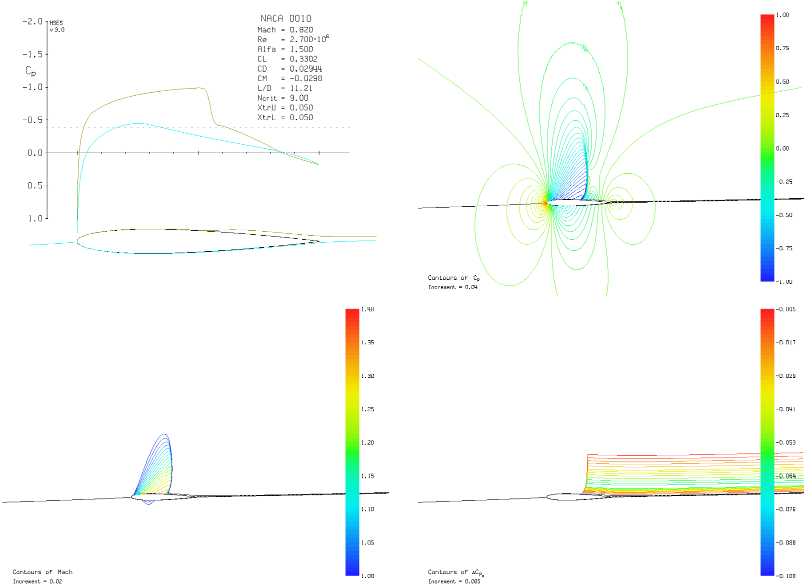

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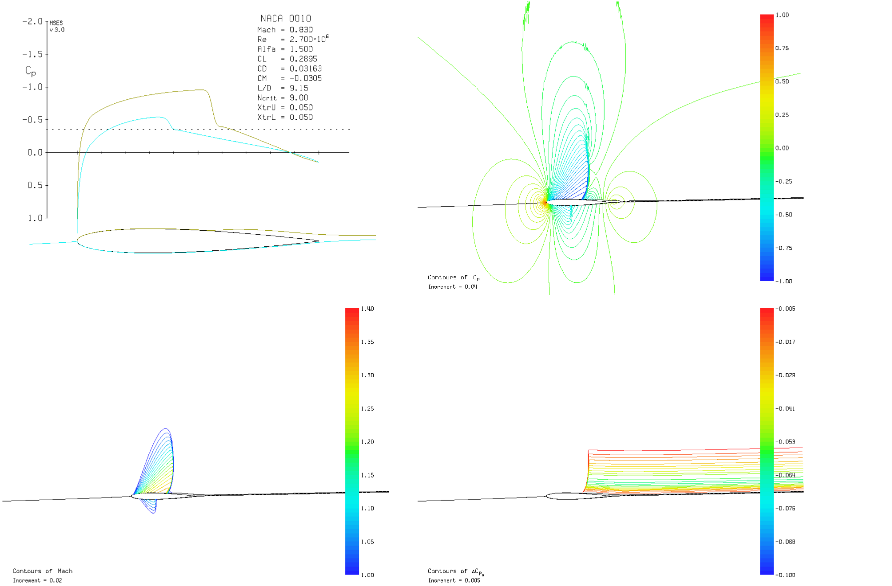

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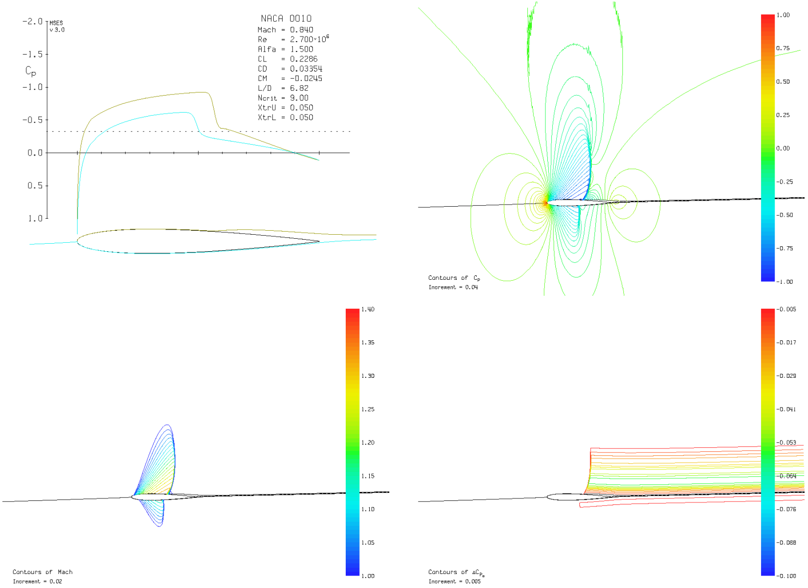

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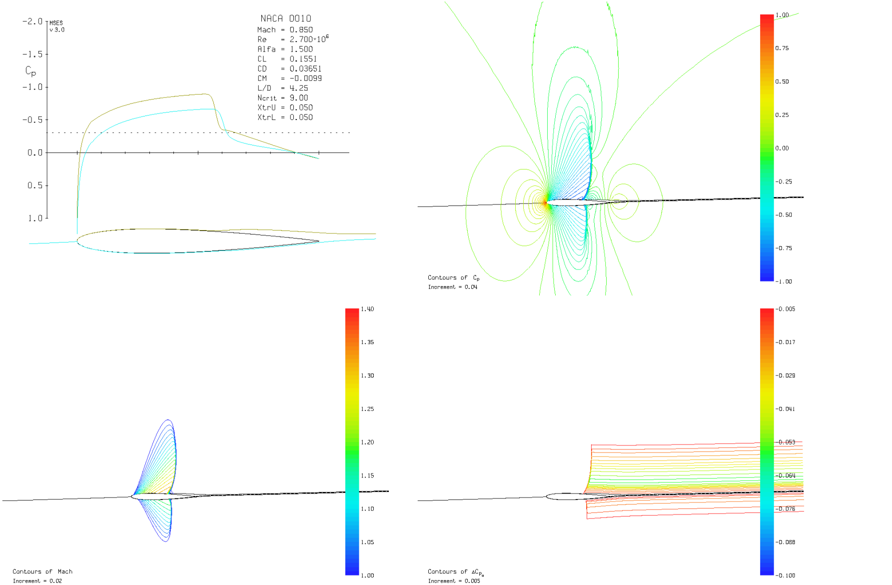

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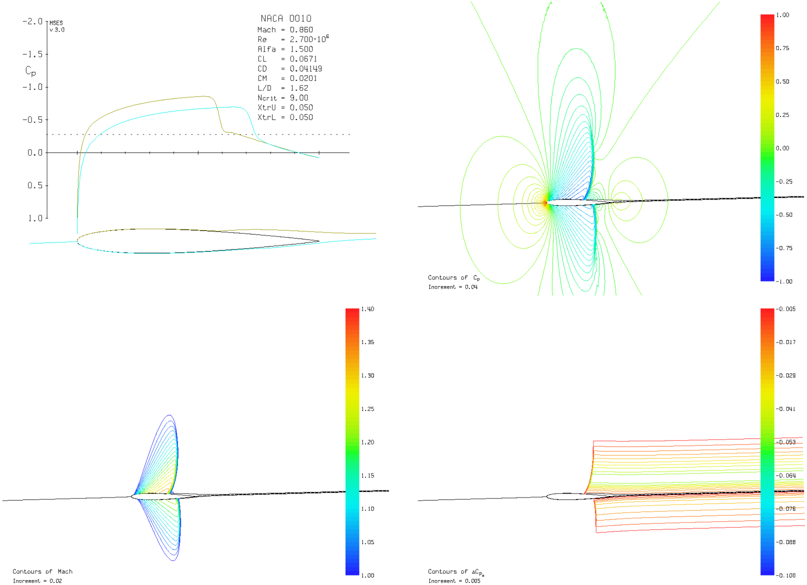

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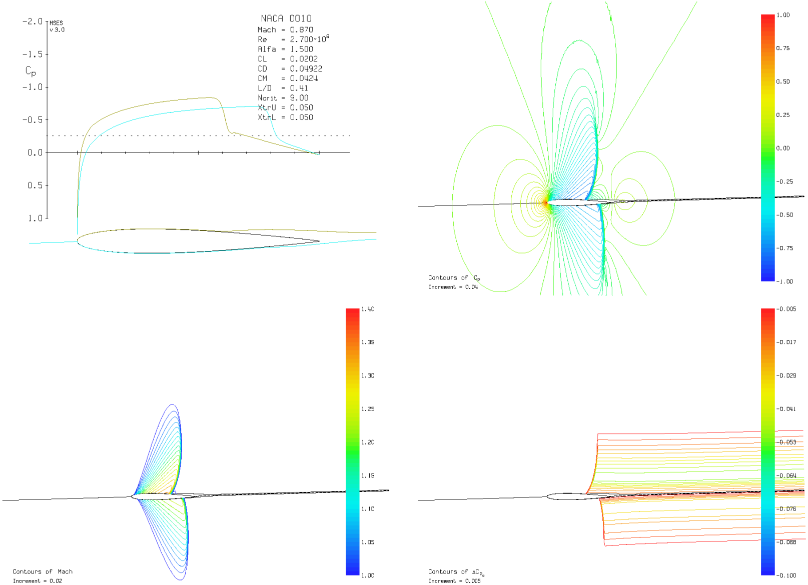

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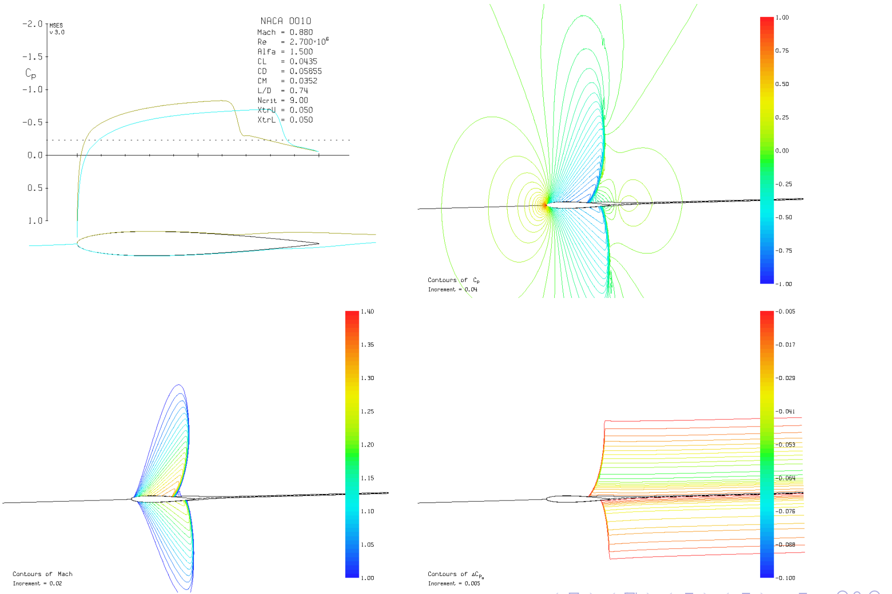

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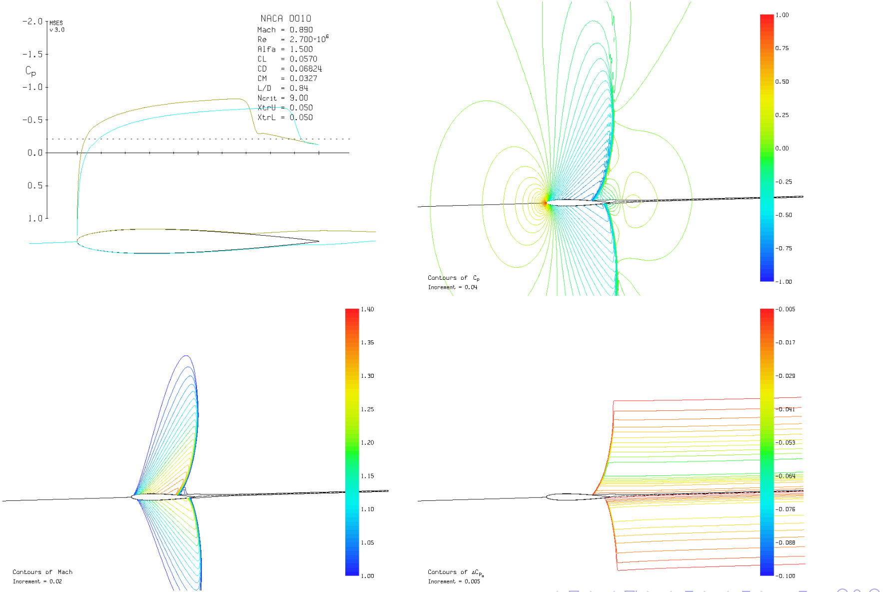

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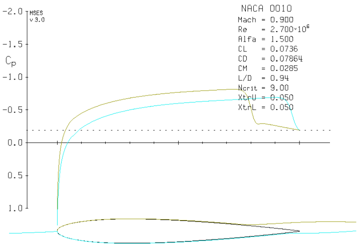

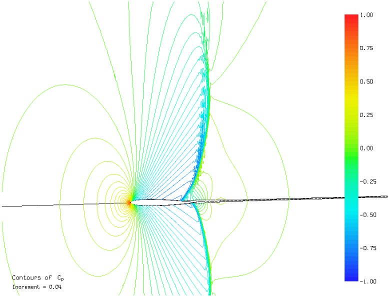

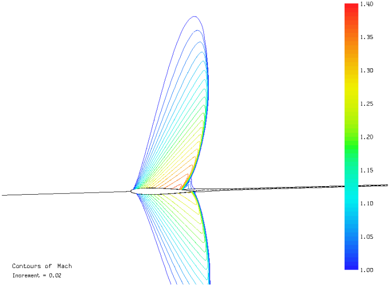

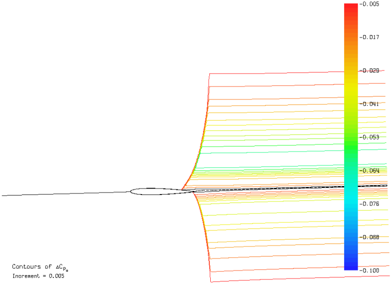

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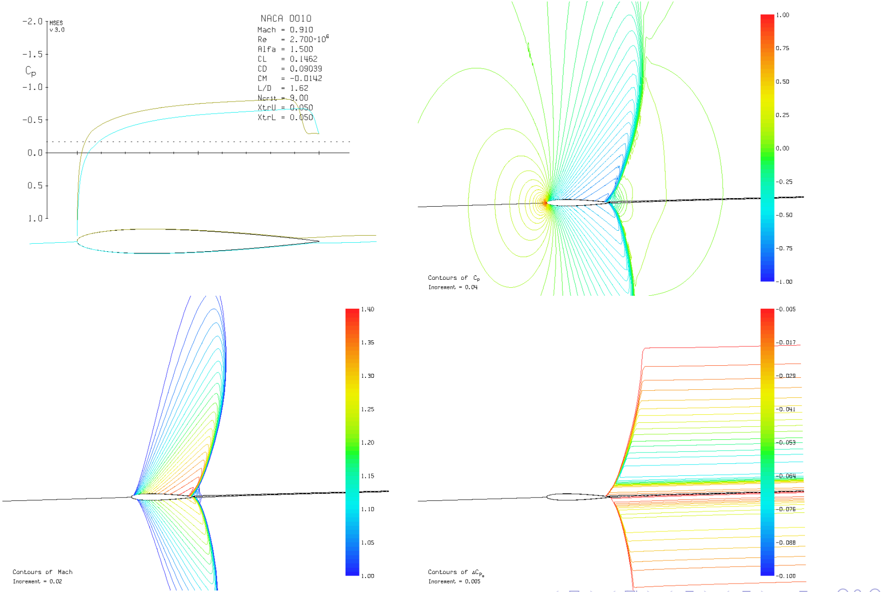

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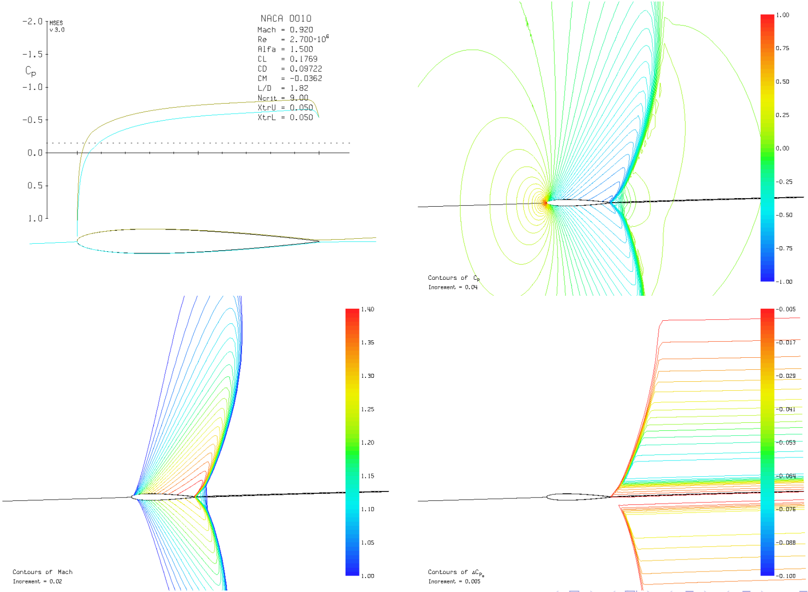

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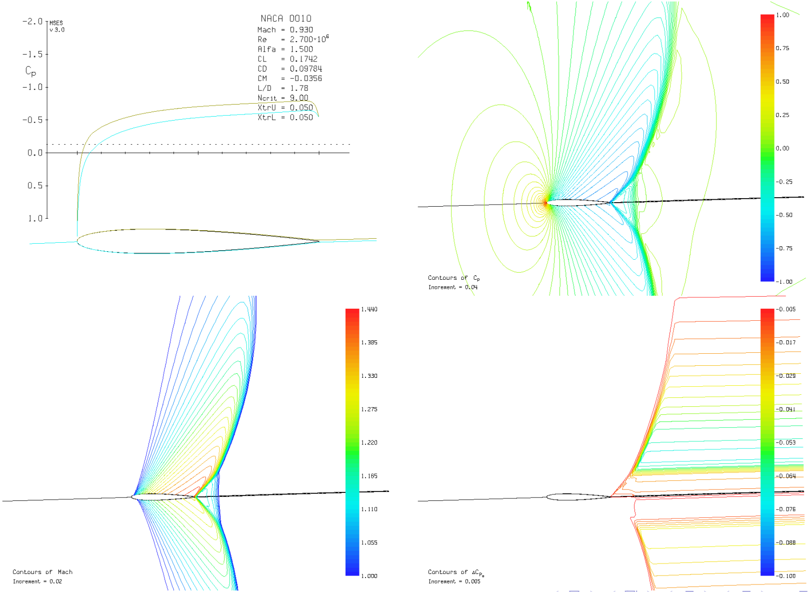

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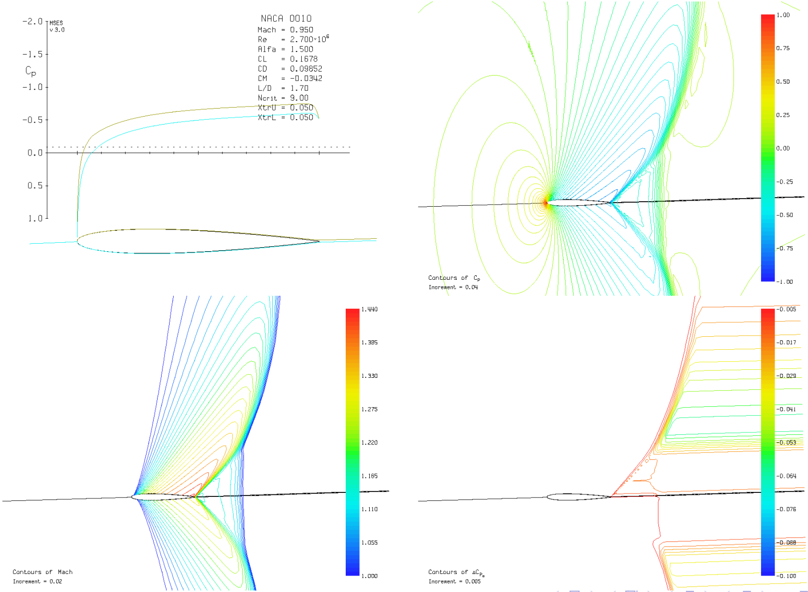

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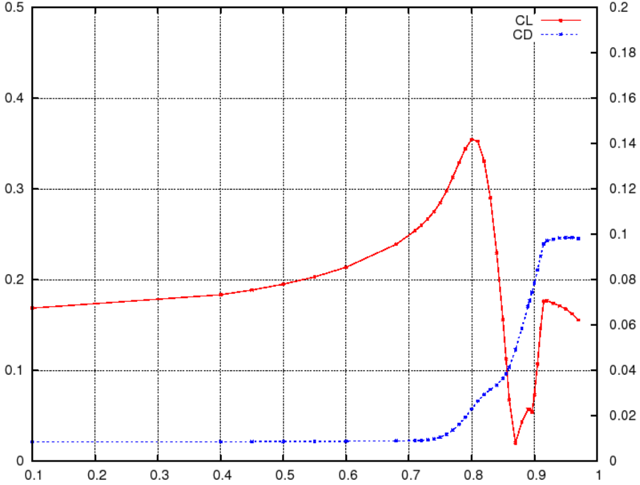
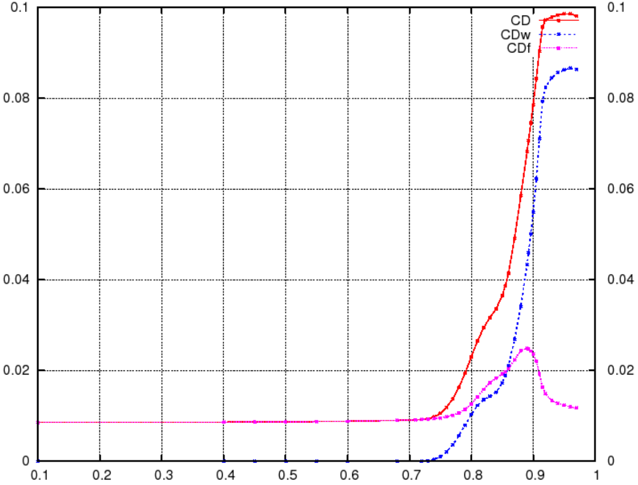
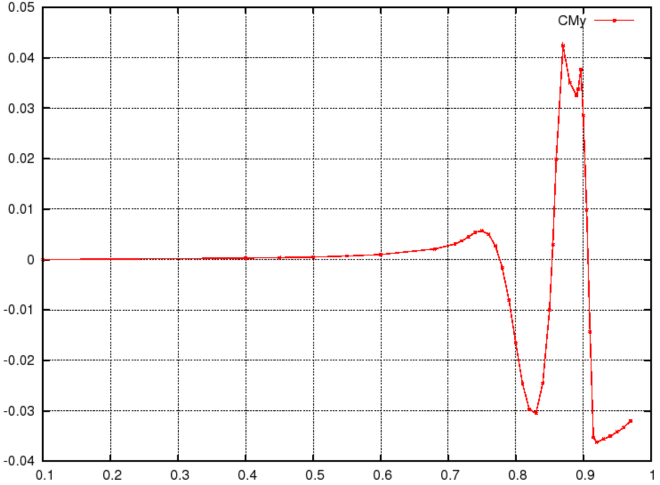


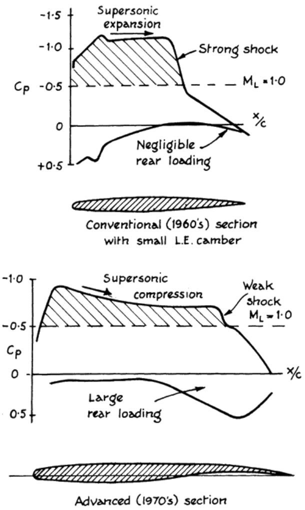
Transonic viscous flow past an airfoil

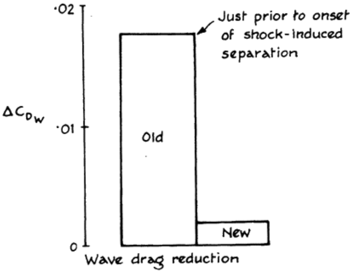


Transonic viscous flow past an airfoil

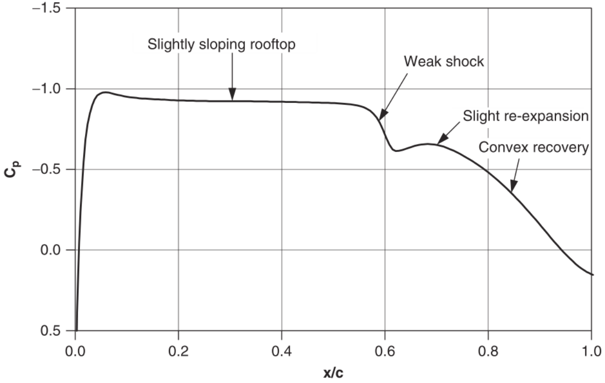


Supercritical airfoils



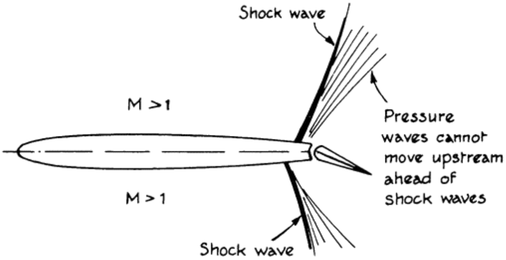


Supercritical airfoils



Example of the C_p distribution for supercritical airfoil

Flap efficiency in transonic flow



Flap efficiency in transonic flow