Training examples CFD

1. On one-dimensional equidistant mesh with step *h* given are:

$$f(h), f(-h), f'(h), f'(-h)$$

a) provide a most accurate approximation for $f''(0)$. The formula is given by

$$f''(0) = \alpha f(h) + \beta f(-h) + \gamma f'(h) + \delta f'(-h)$$

b) Show a leading error term. Provide order of approximation.

2. Given is the boundary value problem

$$\begin{cases} \dfrac{d^2 y}{dx^2} + e^{-x}\dfrac{dy}{dx} = \cos(x) \\ y(x=-3) = 1 \\ y(x=3) = 1 \end{cases}$$

a) Provide a discretisation using the finite difference formulas.
b) Give a step size *h* for which the matrix of coefficients is weakly diagonally dominant.

3. Given is the following system of equations. Nonzero coefficients are indicated by a,b,c and e:

$$\begin{bmatrix} a_1 & b_1 & e_3 & e_4 & \cdots & \cdots & e_n \\ c_2 & a_2 & b_2 & & & & \\ & \ddots & \ddots & \ddots & & & \\ & & c_j & a_j & b_j & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & c_{n-1} & a_{n-1} & b_{n-1} \\ & & & & & c_n & a_n \end{bmatrix} \bullet \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_j \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_j \\ \vdots \\ f_{n-1} \\ f_n \end{bmatrix}$$

a) Provide an exact (non-iterative) algorithm for solution of this problem.

4. Given is the following equation:

$$\frac{\partial^2 u}{\partial x^2} + 5\frac{\partial^2 u}{\partial y^2} - e^{-y}\frac{\partial u}{\partial x} = \cos(x+2y)$$

The size of the computational domain is Ω= <0;3> x <-3,3>. At boundaries ∂Ω : u=0.

a. Provide discretisation using the finite difference method for $h=2h_x=h_y$.
b. Provide minimum number of grid points for which the matrix of coefficients is weakly diagonally dominant.
c. Provide the Gauss-Seidel (Jacobi) algorithm (written in C language) for solution of this problem.

5. Show that in the log layer the following law of the wall is valid:

$$U^+ = f(y^+)$$

Take into account that the friction velocity u_τ can be expressed as follows:

$$u_\tau = l_{mix}\frac{dU}{dy}$$

where $l_{mix}=\kappa y$ is the mixing length and κ is the von Karman constant. Notice that nondimensional distance to the wall is $30 < y^+ < 300$.

6. Simplify the x-momentum (velocity component parallel to wall) and the turbulent kinetic energy equations for the boundary layer flow. Provide justification for these simplifications. Show that close to wall ($30 < y^+ < 300$) the turbulent kinetic energy k can be obtained from the following relation:

$$k = \frac{u_\tau^2}{C_\mu^{1/2}}$$

7. a)Discretise the following one-dimensional convection-diffusion and continuity equations using the finite volume method. Use the 'upwind' scheme for discretisation of the convective terms (the flow is from left to right).

$$\frac{\partial}{\partial x}(\rho U \psi) = \frac{\partial}{\partial x}\left(\Gamma \frac{\partial \psi}{\partial x}\right)$$

$$\frac{\partial}{\partial x}(\rho U) = 0$$

b)Provide the Gauss-Seidel (Jacobi) algorithm (written in C language) for solution of this problem.