Exercise 5b

Visualisation of the compressible flows.

1. Goal of the exercise

Students are getting familiar with some visualisation techniques of the compressible flows and interpretation basics of the obtained images.

2. Theoretical background

To visualise compressible flows ($M > 0.3$) one typically employs methods relying on the change of refractive index of light. In fact, this type of methods can be used to investigate flows with any velocity if the variation of fluid density is present (due to temperature or gas composition). The advantage of optical methods is that they are typically non-invansive, i.e. they do not require injecting any external elements (dyes, markers etc.).

Optical methods rely on the fact that variation of density ρ is related to a change of refractive index and is given by the Gladstone-Dale law:

$$\frac{n - 1}{\rho} = \text{const} \quad (1)$$

where: $n = c_0/c$ – refractive index

$\quad\quad c_0$ – light speed in vacuum

$\quad\quad c$ – light speed in a given medium characterized by density ρ

This dependency is valid for refractive index close to 1. This is true for gases and liquid vapours, e.g. when $p = 10^5 \, Pa$ and $T = 273K$ refractive index for air is $n = 1{,}000292$, oxygen $n = 1{,}000272$, methane $n = 1{,}000442$, water vapour $n = 1{,}000257$.

Optical methods of visualisation can be divided into three main groups:

a) streak method
b) shadowgraph method
c) interferometric method

The first two methods rely on the deflection of light ray passing through a variable density medium. Let's analyse this on the example (fig.1a).
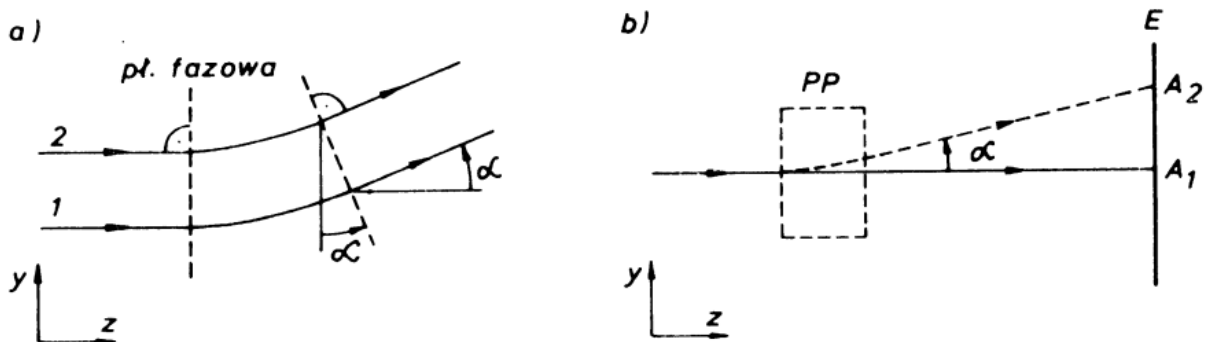

Figure 1 Ray passing through a variable density medium.

Accordingly to Eq.(1), any variation of density is accompanied by a change of refractive index. Therefore the speed of light c changes. Let's assume, that density change in a given field occurs only on y-direction (density increases). Let's consider two rays 1 and 2, both being in the yz-plane and parallel to z. Then, ray 2 will have lower speed. Optical paths of both rays will be different so distance covered by them will be different. Phase plane, at starting point of variable density region is parallel to y, will be deflected at the end by an angle α. Ray is deflected by the same angle (phase plane is perpendicular to the light propagating direction). Considering small deflections, i.e. small angles one obtains

$$\alpha = \int_L \frac{1}{n}\frac{\partial n}{\partial y}\,dz \quad (2)$$

where: L – width of variable density region.

To capture the difference between streak and shadowgraph method let's analyse fig.1b. We consider light ray passing through a test-section PP, in which density variation is present. If there is no density variation ray is present on the screen at point A_1. Density variation causes a ray deflection by an angle α (dashed line) and results in a point A_2. Shadowgraph method relies on the shift distance $\overline{A_1 A_2}$, whereas change of α angle is used in the streak method. One can prove, by considering eq.(1) and (2), that α is proportional to the first derivative of density in the perpendicular direction to the ray (here y), and line segment $\overline{A_1 A_2}$ is proportional to the second derivative in the same direction.

Interferometric methods employ optical pathlength differences between rays passing through distorted region and a so-called reference ray, leading to light interference. In this method, measured quantities are functions of refractive index so they also depend on density. All three visualisation method groups are complementary to each other and provide different information about the flow. Below, each method will be briefly presented.

Interferometric method

Research in aerodynamics is quite often performed by means of Mach-Zehnder interferometric system. Its main advantage is the separation of working beam from the reference beam that entirely passes through optically undistorted field. The scheme of the system is presented in the fig.2.
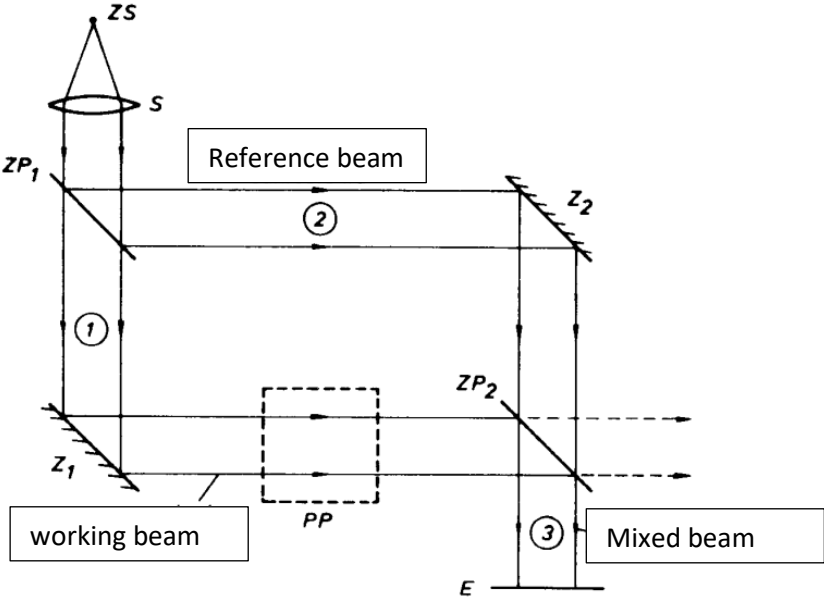

Figure 2 Scheme of Mach-Zehnder interferometer.

Parallel light beam can be obtained by setting a monochromatic source (ZS) in a focal point of a lens S. As a source one may use laser or diode (LED) with proper power and wavelength. Parallel beam reaches semi-transparent mirror ZP_1, which splits it into working beam (1) and reference beam (2). Semi-transparent mirror reflects half of the light intensity and passes through the other half. Mirror Z_1 directs the working beam through the test-section PP and onto semi-transparent mirror ZP_2. Reflected beam from ZP_1 (reference beam) is going to the mirror Z_2 and then to ZP_2. As a result one obtains two mixed light beams – one beam (3) travels to the screen and the second (dashed line) remains unused. To obtain the same optical pathlengths of the working and reference beams, the mirrors are located at the vertices of a rectangle and are set at an angle $45°$ to the initial parallel beam.

If all four mirror are perfectly parallel, and optical properties of the working beam and reference beam are the same, then both beams that create mixed beam will be parallel (as they are coming from the same source), they are coherent and can interfere with each other. Type of interference depends on the optical pathlengths $\overline{\Delta DL}$, which can be presented in non-dimensional form for the beams (1) and (3) as

$$\epsilon = \frac{\overline{\Delta DL}}{\lambda_0} = \frac{1}{\lambda_0} \int (n_r - n_{od}) \mathrm{d}z$$

where: λ_0 – light wavelength in vacuum
 n_r – refractive index of the working beam
 n_{od} – refractive index of the reference beam.

When there are no optical inhomogeneities in both beams and interferometer is set in such configuration that ϵ is 0 or integer number, then the screen E is uniformly illuminated. If at some place in the test-section density changes leading to a change of n_r, then $\overline{\Delta DL}$ becomes non-zero and causes change of illumination of part of the screen (when ϵ is ½ or integer number plus ½ - then there is no light). As a result one obtains a set of bright and dark regions (interferometric fringes), each representing a given value of ϵ. The difference between ϵ values of the adjoining fringes is equal to one.

When the test-section is 2D and density varies only in this plane, the optical pathlength difference is equal to

$$\epsilon = \frac{n_r - n_{od}}{\lambda_0} L \quad (4)$$

where L denotes test-section width.

Using Eq.(4) and (1) one can determine density difference corresponding to two adjoining fringes ($\epsilon = 1$)

$$\Delta\varrho = \frac{\lambda_0}{L \cdot \mathrm{const}} \quad (5)$$

where: $\mathrm{const} = \frac{n-1}{\rho}$ is Gladstone-Dale constant (its value depends on the wavelength and medium).

If pressure inside test-section is constant, then using $p = \rho R T$ and eq.(5) one can determine temperature difference between two adjoining fringes (k and $k + 1$).

$$T = T_{k+1} - T_k = \frac{\lambda}{\mathrm{const} \cdot \mathrm{L}} \frac{R}{p} T_k \cdot T_{k+1} \quad (6)$$

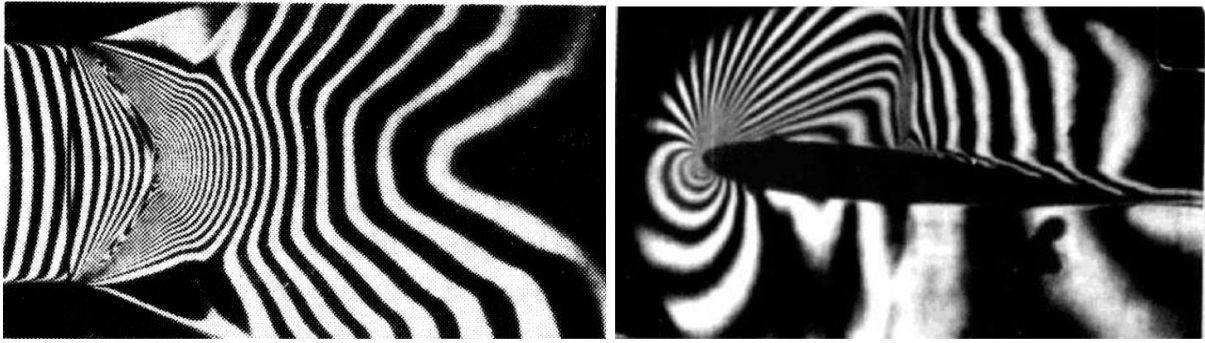Example of interferometric images are presented in the fig.3.

Figure 3 Interferometric fringes: (a) in de Lavale nozzle and (b) flow around transonic profile with a shockwave.

Setting interferometer in such a way that beam (1) and (2) are parallel after mixing provides infinite distance between fringes. This method can be used to visualise fields with very low density variation.

In flow visualisations, where large density variation is present (e.g. shockwave), it is better to set interferometer in such a way that beam (1) and (2) are not parallel after mixing. Then, one can observe a set of interferometric fringes. Optical inhomogeneity in a form of a wave causes distortion (deflection) of these fringes. This distortion is a measure of density variation in a wave. This requires employing light source with broader spectrum to distinguish colours of different fringes.

Mach-Zehnder interferometer present at the Department of Aerodynamics is presented in the fig.4 (part of the body is dismounted to present mirrors).
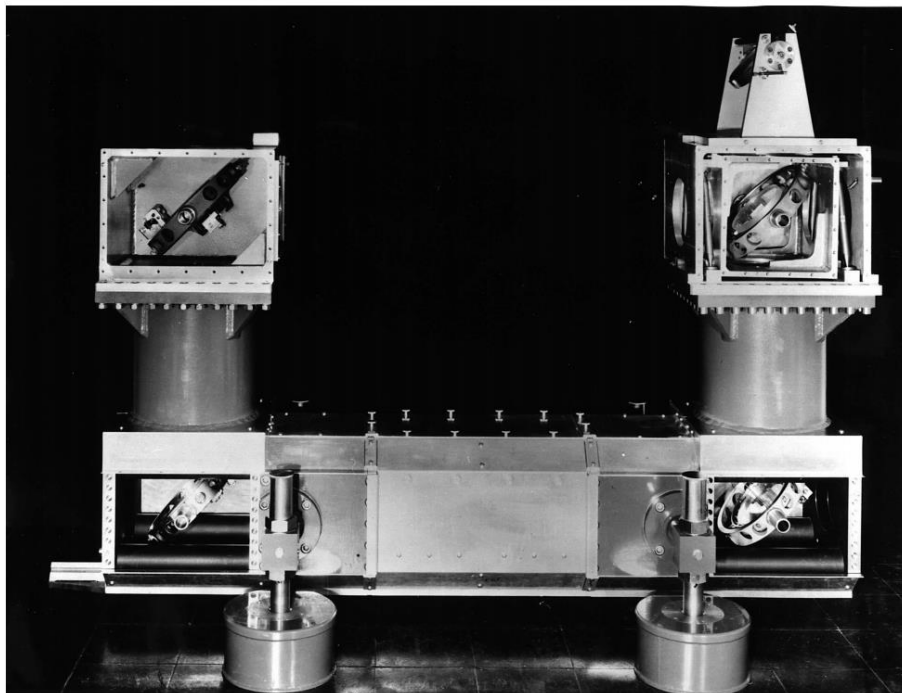

Figure 4 Mach-Zehnder interferometer.

Mach-Zehnder interferometer is rather expensive device – it requires complicated rotation mechanism of mirrors and high rigidity of the entire construction. It also requires high optical quality of the mirrors. This is the reason for the development of different systems that also rely on the principle of interferometry, e.g. streak interferometer employing Wollastone prisms to obtain interferometric fringes.

Streak method

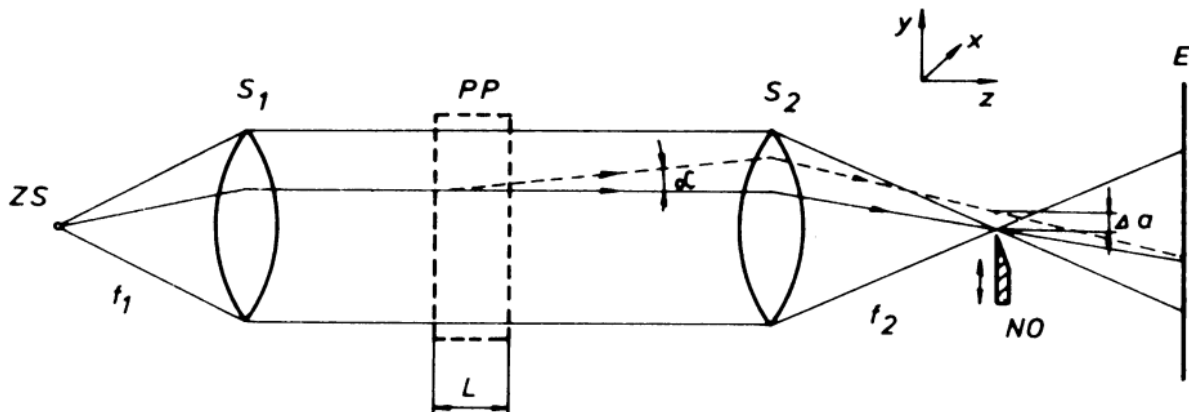Scheme of the optical setup is presented in the fig.5.

Figure 5 Scheme of the optical setup for the streak method.

Light source ZS (rectangular with the size $a_z b_b$) is set in the focal point of the lens S_1 and provide parallel beam passing through test-section PP. Light that is focused by the lens S_2 (with the focal length f_2) travels to the screen E that is set to provide sharp image of the test-section. At the focal point of the lens S_2 one sets a shutter, so-called optical knife NO, that can be precisely adjusted in the perpendicular plane to the optical axis of the system. Any ray going through a test-section characterized by uniform density is represented by solid line. Deflected ray by an angle α (when the density variation is present) is denoted by dashed line. The distance of the deflected ray from the optical axis at a focal point of the lens S_2 is equal to

$$\Delta a = f_2 \alpha \quad (7)$$

To obtain required sensitivity of the system, image of the light source that is obtained at the focal point of the lens S_2 (with dimensions $a_0 b_0$) has to be partially hidden by the optical knife by a_k – see fig.6.
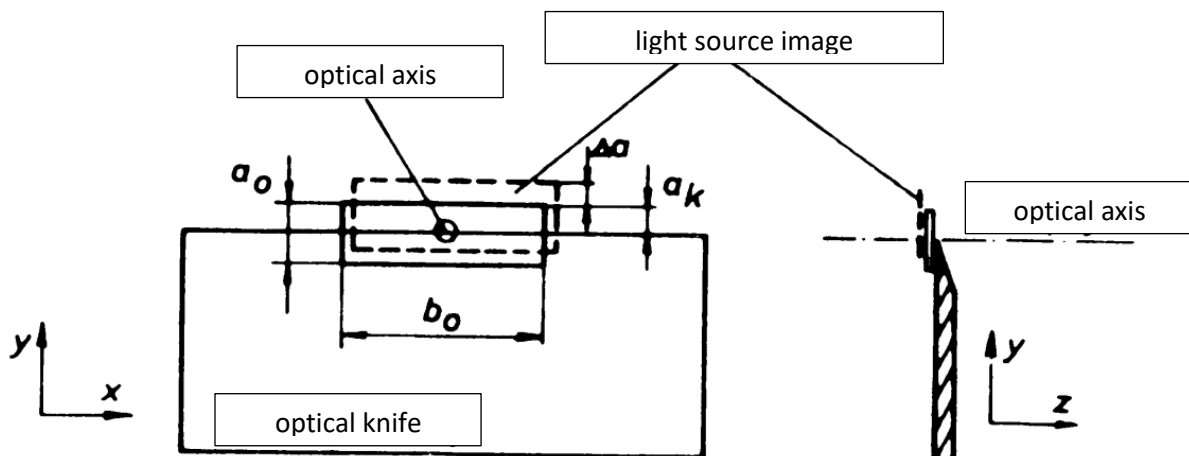

Figure 6 Working principle of optical knife.

As a result, rays deflected from the axis can increase illumination of the screen (if $\Delta\alpha$ is positive) or stop at the knife ($\Delta\alpha$ negative) locally decreasing the amount of light. Contrast at a given point on the screen can be expressed as

$$\frac{\Delta I}{I_k} = \frac{f_2}{a_k n_a} \int_L \frac{\partial n}{\partial y}\,\mathrm{d}z \quad (8)$$

where: I_k – light intensity on the screen in case of uniform density in the test-section PP and hiding part of the light source by a_k;

$\Delta I = I - I_k$ – light intensity difference at a given point with respect to I_k;

n_a – refractive index outside of PP.

Measuring light intensity at different positions of the screen (e.g. by means of photoelement) one can determine refractive index derivative. Next, by using eq.(1) calculate

$$\frac{\partial \rho}{\partial y} = \frac{\rho}{n-1}\frac{\partial n}{\partial y} = \frac{1}{\mathrm{const}}\frac{\partial n}{\partial y} \quad (9)$$

The method of streaks is mostly used for the purpose of qualitative analysis rather than quantitative. Obtained image of the flow field depends on the setting of light source and optical knife. When edge of the knife is parallel to x-axis (compare with fig.5), then the setup presents the variation of the gradient of refractive index in y-direction (light source needs to be parallel to x-axis). As rays are deflected towards increasing density, then, in situation where ρ increases in y-direction $\Delta\alpha$ will be positive. Brighter spots on the screen correspond to the increase of density, whereas darker spots – decrease of density. Switching the location of optical knife to another side of optical axis (inversely to fig.5) will inverse brighter and darker spots.

Visualisation of density gradient changes in x-direction requires switching position of the optical knife to a position in which its edge would be parallel to y-axis. Similarly one needs to alter light source position. Example of images obtained by means of streaks method is presented in the fig.7.
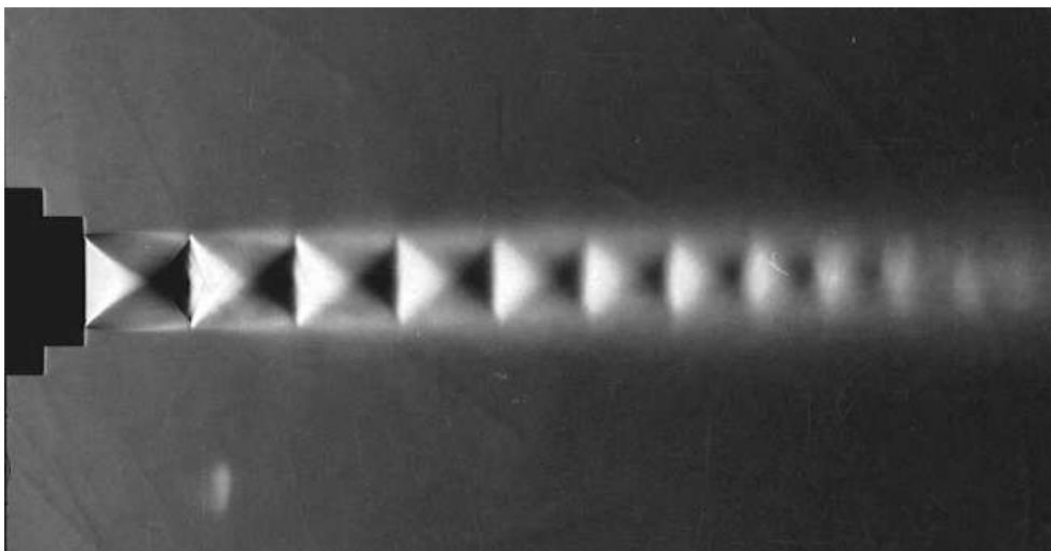

Figure 7 Streak method visualisation of a jet flowing out of the converging nozzle (constant illumination).

Images obtained with constant illumination show only stationary phenomena. To visualise unsteady flows one needs to employ triggered flash light. To capture instantaneous state of the high-speed flow of gases the light needs to be able to flash for about 1microsecond. In such case the image can also visualise acoustic waves. Example of such visualisation is presented in the fig.8.
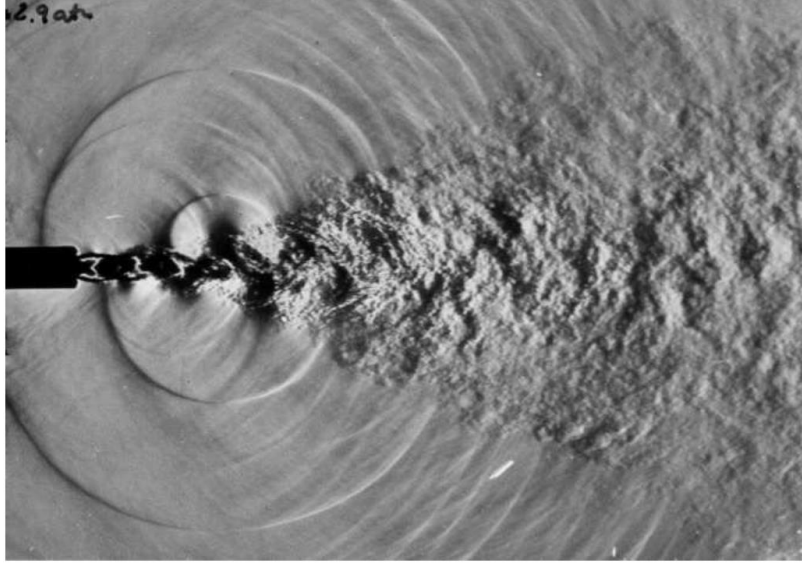
Figure 8 Streak method visualisation - triggered flash light. Stream of air flowing out of the flat nozzle (visible boundaries of the stream and acoustic waves generated by the stream).

Shadowgraph method.

In contrast to streak method, where contrast depends on the angle of ray deflection α, in the shadowgraph method one employs linear deflection of the rays. The simplest scheme of the optical setup used in this method is presented in the fig.9.
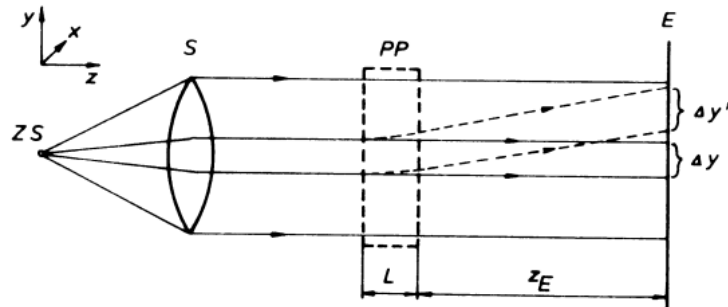

Figure 9 Shadowgraph method - scheme.

Point light source ZS placed in the focal point of the lens S_1 provides parallel light beam, which travels through the test-section PP and illuminates the screen E. The screen is placed at a distance z_E from the test-section. Let's consider a situation, in which inside the test-section there is density increase in y-direction (corresponding to the increase of refractive index). Two, initially parallel light beams separated by a distance Δy are deflected inside the test-section with different angles $-\alpha$ and $\alpha + \mathrm{d}\alpha$, respectively. Distance between them on the screen will be given as

$$\Delta y' = \Delta y + z_E \mathrm{d}\alpha > \Delta y \quad (10)$$

Which is equivalent to the illumination on the screen. Contrast between two adjoining regions on the screen depends on the second derivative of the refractive index and is equal to

$$\frac{\Delta I}{I_p} = \frac{I - I_p}{I_p} = \frac{\Delta y'}{y} - 1 - z_E \int_L \frac{\partial \alpha}{\partial y} \mathrm{d}z = -\frac{z_E}{n_a} \int_L \frac{\partial^2 n}{\partial y^2} \mathrm{d}z \quad (11)$$

Where: I – light intensity on the screen,

I_p – light intensity

z_E – distance between the test-section and the screen,

L – width of the test-section.

Equation (11) implies that image contrast can increase with the distance z_E, but it is accompanied by decrease of the image sharpness. Shadowgraph method is especially useful for the visualisation of flows, where the second derivative of density is high, e.g. shockwaves. It is not useful for the flows where this derivative is low.

Scheme presenting changes of density for a shockwave is depicted in the fig.10. Additionally, in the figure deflection direction of rays passing a shockwave is presented.
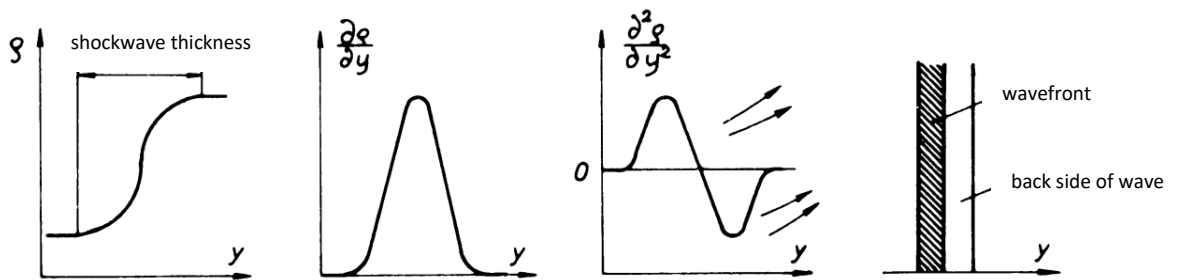

Figure 10 Density variation in a shockwave.

For the wavefront one obtains $\frac{\partial^2 \rho}{\partial y^2} > 0$, hence $\frac{\partial^2 n}{\partial y^2} > 0$ (see eq.(1)). Initially parallel, rays passing through wavefront becomes divergent. At the back side of the wave $\frac{\partial^2 \rho}{\partial y^2} < 0$ (so $\frac{\partial^2 n}{\partial y^2} < 0$) and rays are converging – light intensity increases on the screen. Shadowgraph image of a shockwave can be characterized by the presence of two lines (brighter and darker). The darker correspond to the wavefront.

In comparison to the streak method (showing refractive index variation only in perpendicular direction to an optical knife), in shadowgraph method one may capture variation in both y and x directions.
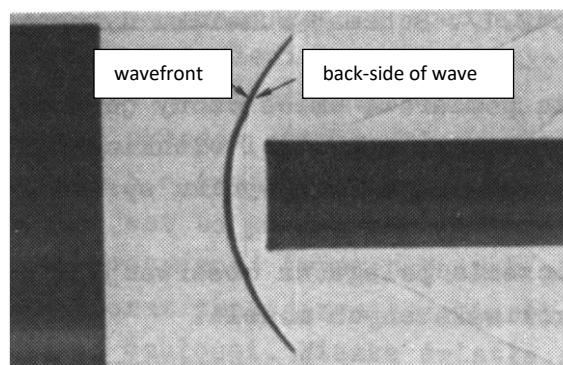
Example of shadowgraph method is presented in the fig.11.

Figure 11 Shadowgraph visualisation - shockwave.