Computer Science 2

Exercise 6: Cubic spline interpolation.

1. Copy the C function ***tridiag*** (the solver of a general 3-diagonal linear system) to the hard disk of your PC.

2. Read the instructions included in the comment block of this function – it explains assumed notation and the structure of the 3-diagonal system.

3. Create a sample data file on your local HD containing a list of (x,y) of the interpolating nodes. Define 8-10 arbitrary located nodes. The first line of the data file should contain the number of nodes.

4. Write a complete program which calculates the cubic spline interpolant for the set nodes you have defined in the data file. This task will contain two basic steps:

First, you have to evaluate 2nd derivative of the spline function at the nodes. Assume natural end-point conditions at both ends of the interpolation interval. The equations to be solved can be written as follows

$$\begin{cases} m_0 = 0 \quad , \quad k = 0 \\ h_{k-1} m_{k-1} + 2(h_{k-1} + h_k) m_k + h_k m_{k+1} = 6 \cdot \left(\dfrac{y_{k+1} - y_k}{h_k} - \dfrac{y_k - y_{k-1}}{h_{k-1}} \right) \quad , \quad k = 1,..,M-2 \\ m_{M-1} = 0 \quad , \quad k = M-1 \end{cases}$$

where $h_k = x_{k+1} - x_k$, k = 0,1,..,M-2. You have to establish correspondence between the coefficients of the above system and the "standard" system assumed by the ***tridiag*** function.

After ***tridiag*** returns the vector **m** of the nodal values of the 2nd derivative you have to tabulate the spline function in a reasonably dense mesh of the points in the interpolation interval. Assume that the tabulation points are defined by dividing each interval $\lfloor x_k, x_{k+1} \rfloor$ into 10 equal pieces. The value of the spline function C at $x \in \lfloor x_k, x_{k+1})$ can be computed using the following formula

$$C(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{y_k}{h_k} - \frac{m_k h_k}{6} \right)(x_{k+1} - x) + \\ + \left(\frac{y_{k+1}}{h_k} - \frac{m_{k+1} h_k}{6} \right)(x - x_k)$$

5. Make a Grapher or Excel plot of the spline function. The plot should also contain the given nodes so you can check whether the spline function really interpolates data you have created.

Extension: After the main task is completed, try to modify your program to implement the end-point conditions for the 1st derivative (see your lecture notes). You can introduce such condition only at $x = x_{M-1}$. Try to use 2-3 different values of the 1st derivative at this nodes and make nice Grapher (or Excel) plots.