MECHANICS OF FLIGHT II
Project no. 8 – Pitching Moment of the Airplane without Horizontal Tailplane

As usually in the analysis of flight properties of an aircraft, the pitching moment of the airplane at an arbitrary point A located in the plane of symmetry of the aircraft will be presented in non-dimensional form using aerodynamic coefficient:

$$C_{m_A}=\frac{M_A}{\frac{1}{2}\rho\cdot A\cdot V^2\cdot c_a} \tag{8.1}$$

In the analysis of longitudinal equilibrium, static stability and control of the aircraft the moment and the moment coefficient will be calculated always at the center of mass C.

The moment coefficient C_{m_C} can be split into two basic parts:

$$C_{m_C}=C_{m_C\ w+b} + C_{m_C\ tail} \tag{8.2}$$

where:

$C_{m_C\ w+b}$ - the moment coefficient of all parts of the airplane excluding horizontal tailplane,
$C_{m_C\ tail}$ - the moment coefficient of the horizontal tailplane.

Moment coefficient of the airplane without the tailplane

For large number of the classic subsonic airplanes the pitching moment coefficient $C_{m_C\ w+b}$ of the in-flight aerodynamic configuration depends only on wing, fuselage (body) and wing nacelles (if any).

A. Wing

For regular speed of flight, far from the stall speed, the wing pitching moment coefficient can be computed using formula:

$$C_{m_C\ w}=C_{m\ a.c.}+\bar{x}_s\cdot C_L-\bar{z}_s\cdot(C_D-\alpha_a\cdot C_L), \tag{8.3}$$

or (better!):

$$C_{m_C\ w}=C_{m\ a.c.}+\bar{x}_s\cdot C_L-\bar{z}_s\cdot(C_D-\frac{C_L^2}{a}), \tag{8.4}$$

where:

$$\bar{x}_s=\bar{x}_C-\bar{x}_{a.c.}\ ,\quad \bar{z}_s=\bar{z}_C-\bar{z}_{a.c.}\ ,\quad \bar{x}_C=\frac{x_C}{c_a}\ ,\quad \bar{z}_s=\frac{z_C}{c_a}\ . \tag{8.5}$$

x_C and z_C are center of mass C coordinates, respectively along and perpendicular to the mean aerodynamic chord of the wing . The z_C coordinate is positive for low-wing airplanes (mass center is located above the wing). C_D and C_L are drag and lift coefficients of the wing (not the airplane!). The angle

$$\alpha_a=\alpha-\alpha_0=\frac{C_L}{a} \tag{8.6}$$

is called as *the aerodynamic angle of attack* of the wing.

$C_{m\ a.c.}$ denotes the moment coefficient of the wing section over it's aerodynamic center, $\bar{x}_{a.c.}$ and $\bar{z}_{a.c.}$ are (non dimensional!) coordinates of the wing section aerodynamic center (see airfoil data, ie. NACA Report 824), .

B. Fuselage (the body)

Large number of tests done in aerodynamic tunnels show that for small angles of attack the moment coefficient of airplane's body can be calculated using following linear formula:

$$C_{m_C\ b}=(C_{m\ 0})_b+C_L\cdot(-\Delta\bar{h}_n)\ . \tag{8.7}$$

Coefficients $(C_{m\ 0})_b$ and $(-\Delta\,\bar{h}_n)$ can be estimated using method described in the Appendix B of "Dynamics of Flight" by B. Etkin and L. D. Reid, 3-rd edition, pages 335 until 337.

C. Engine nacelles

The nacelle of the engine mounted on the wing produce similar influence on the pitching moment as the fuselage body. Please use this same formula and this same graphs for estimating the value of $C_{m_C\ n}$.

The total value of pitching moment coefficient is equal to sum all main components described above:

$$C_{m_C\ w+b}\ =\ C_{m_C\ w}+C_{m_C\ b}+n_n\cdot C_{m_C\ n} \tag{8.8}$$

where n_n is the total number of nacelles on the wing.

<u>Calculation of the moment coefficient of the airplane without the tailplane</u>

The pitching moment coefficient shall be calculated using above formulas as a function of the lift coefficient C_L for three different positions of center of mass C along the MAC: forward, mid and backward (rear) position and one z_C.. Ask the teacher for advise of particular values of x_C.

<u>Note:</u> vertical position of the center of mass z_C need to be estimated based on side and front/rear drawings of the aircraft. It can be assumed that the center of mass of the airplane is placed on propeller rotation axis or turbojet engine axis.

An example results of calculation of the pitching moment coefficient for three different positions of the center of mass $\left(\ \bar{x}_C=0.12,\quad 0.25,\quad 0.37\ \right)$ and for $\bar{z}_C=0.0$ is shown on Fig.8. 1 .

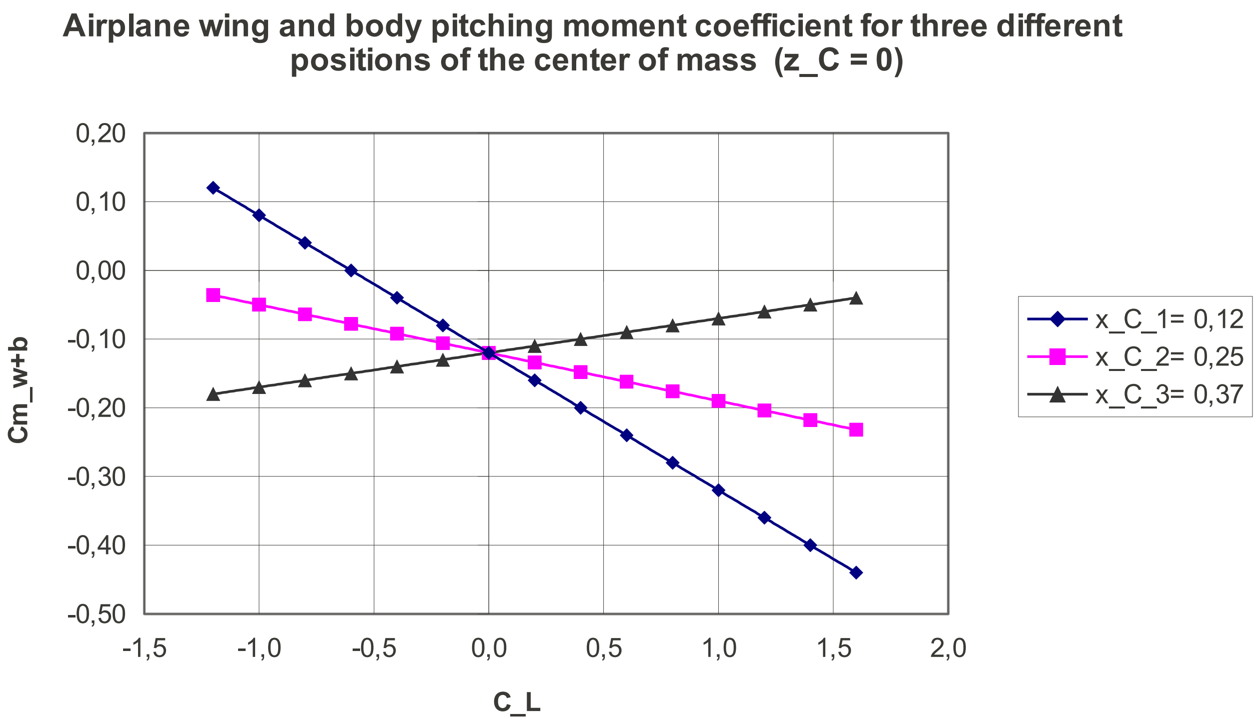

Fig.8. 1

The effect of vertical shift of the center of mass (low-wing airplane, 10% of MAC above the MAC line) for similar airplane as for previous case is presented on the figure Fig.8. 2 .

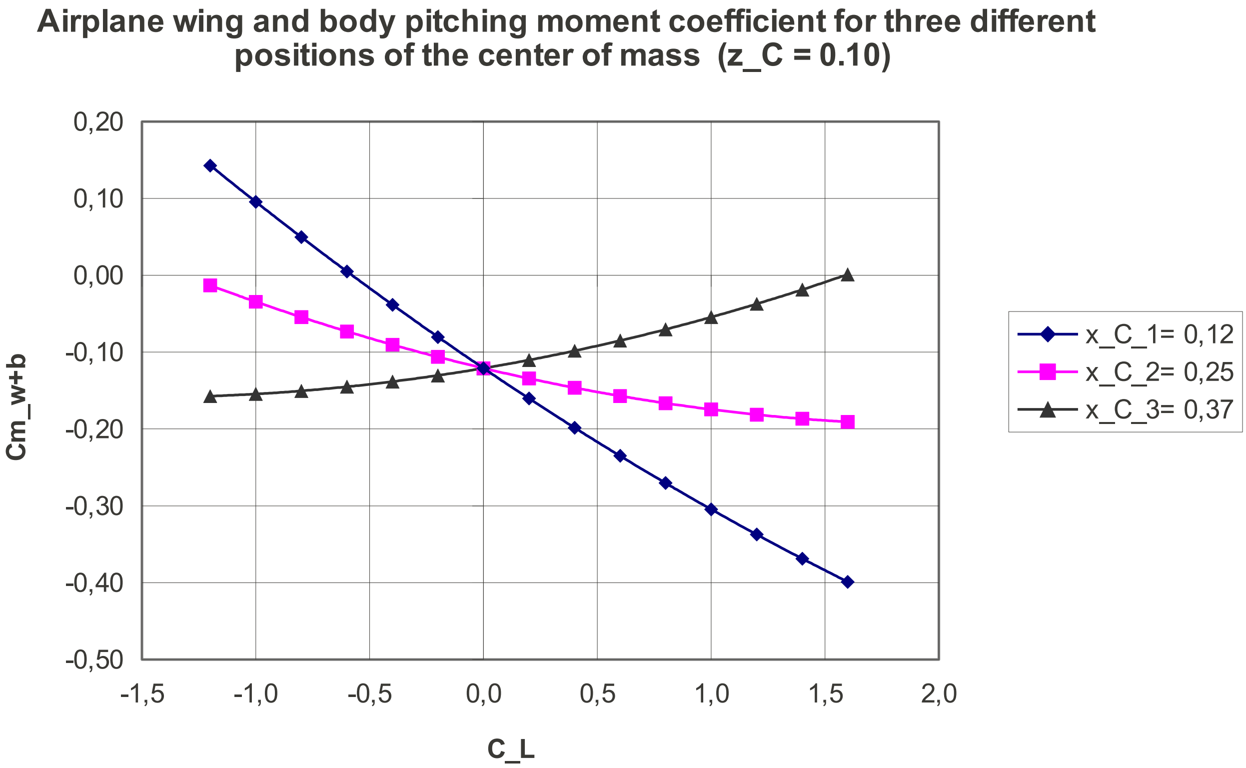

Fig.8. 2

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