# Longitudinal equilibrium of the aircraft in the horizontal steady flight

Deflection angle of the horizontal control surface

Compute the value of $\delta_H$ as a function of lift coefficient $C_L$ and flight speed V using following formulas:

$$\delta_H=\frac{C_{mw+b}}{\kappa_H a_2}-\frac{a_1}{a_2}\left[\frac{C_L}{a}\left(1-\frac{d\epsilon}{d\alpha}\right)+\alpha_{sett\,H}\right],$$

$$\kappa_H=\frac{S_H\cdot x_H}{S\cdot c_a},\quad a_1=\frac{a_{1\infty}}{1+\frac{a_{1\infty}}{\pi\cdot\Lambda_H}},\quad a_2=0.6\cdot a_1,\quad \frac{d\epsilon}{d\alpha}=\frac{2\cdot a}{\pi\cdot\Lambda}\ ,\tag{1}$$

where (see also Project No. 1(8)):

- $S_H$ - the horizontal tailplane area (stabilizer and elevator),
- $x_H$ - the horizontal tailplane arm (distance between the mass center C and ¼ $c_{aH}$ ),
- S - the wing area,
- $c_a$ - the wing mean aerodynamic chord,
- a - the wing lift slope,
- $a_{1\infty}$ - the horizontal tailplane airfoil lift slope (assume NACA 0009 or NACA 0012 airfoil type, see NACA Technical Report No. 824),
- $\Lambda_H$ - the horizontal tailplane aspect ratio.

Assuming that for the airplane's cruise speed (see airplane data and Project No. 5 for the value of cruise speed) the elevator deflection angle $\delta_H=0$ , the horizontal tailplane setting angle $\alpha_{sett\,H}$ may be calculated using formula:

$$\alpha_{sett\,H}=\frac{C_{mw+b}}{\kappa_H a_1}-\frac{C_L}{a}\left(1-\frac{d\epsilon}{d\alpha}\right)\ ,\tag{2}$$

for the lift coefficient $C_L$ corresponding to the cruise speed Of course, the speed of aircraft and lift coefficient must satisfy the equation:

$$\frac{1}{2}\cdot\rho\cdot S\cdot V^2\cdot C_L=m\cdot g,\tag{3}$$

Figure 1 and 2 show an example result of calculation for a low-wing light airplane $(\bar{z}_C=+0.2)$ for three different positions of the center of mass (12%, 25% and 37% of MAC).

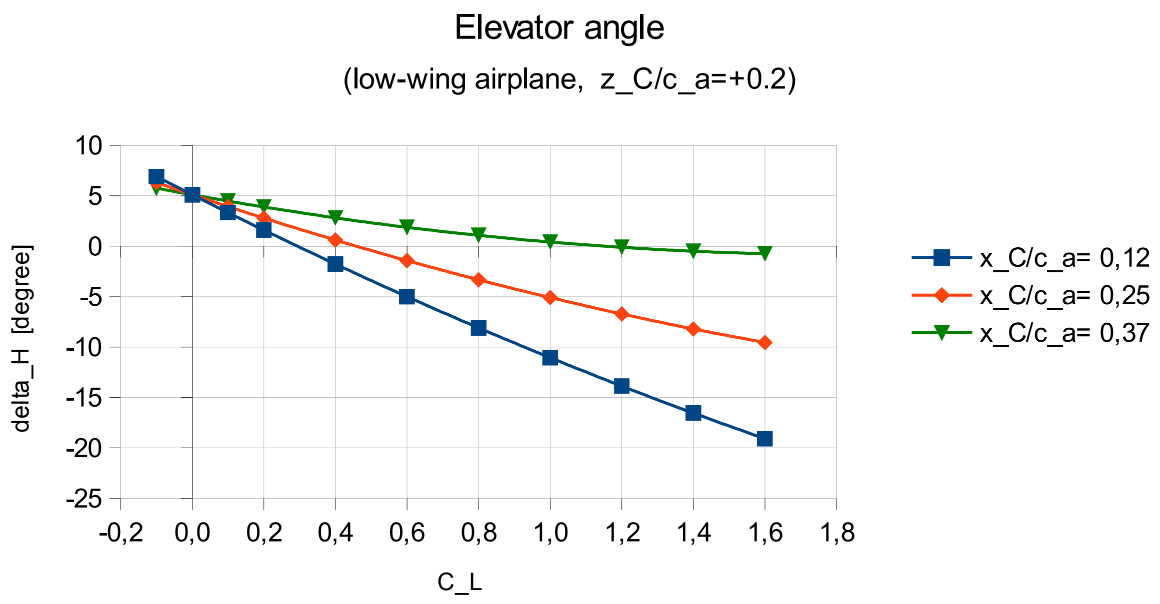
Elevator angle
(low-wing airplane, z_C/c_a=+0.2)
10
5
0
-5
-10
-15
-20
-25
delta_H [degree]
-0,2
0,0
0,2
0,4
0,6
0,8
1,0
1,2
1,4
1,6
1,8
C_L
x_C/c_a= 0,12
x_C/c_a= 0,25
x_C/c_a= 0,37

*Fig. 1*

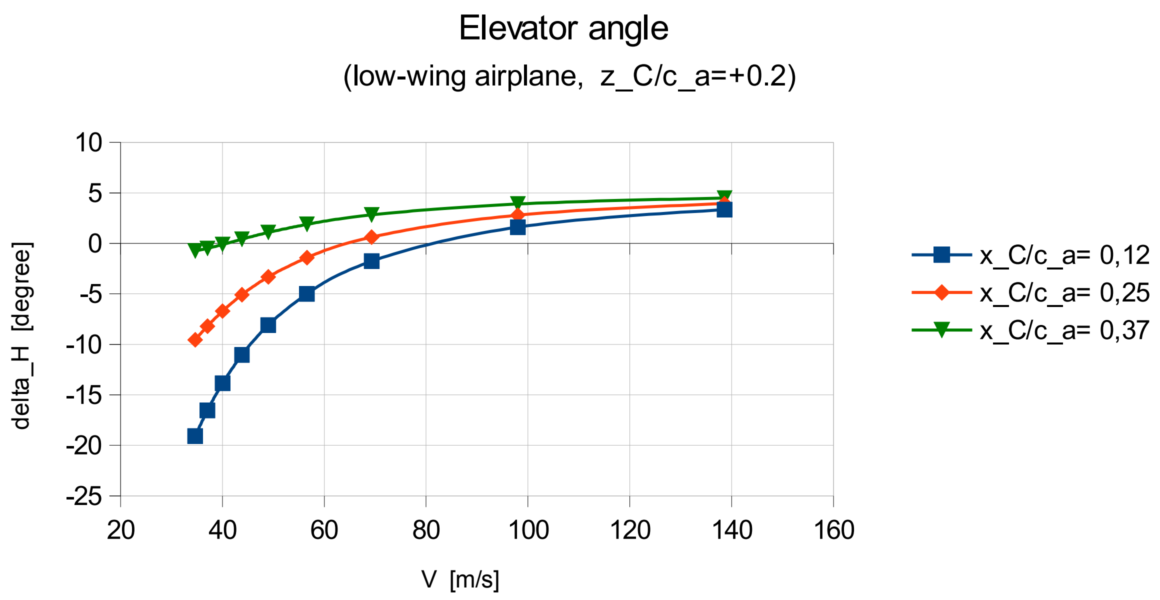
Elevator angle
(low-wing airplane, z_C/c_a=+0.2)
10
5
0
-5
-10
-15
-20
-25
delta_H [degree]
20
40
60
80
100
120
140
160
V [m/s]
x_C/c_a= 0,12
x_C/c_a= 0,25
x_C/c_a= 0,37

*Fig. 2*

<u>Control stick force $P_{sH}$</u>

The control force acting on the control stick as the function of flight speed can be calculated using formulas:

$$P_{s\ H}=\frac{1}{2}\cdot\frac{1}{l_{s\ H}}\cdot\rho\cdot S_e\cdot c_e\cdot V^2\cdot C_{m\ h\ H},\quad C_{m\ h\ H}=b_1\cdot\alpha_H+b_2\cdot\delta_H+b_3\cdot\delta_{H\ t}\ ,\qquad (4)$$

where:

- $S_e$ - the elevator (horizontal control surface) area, fig. 4 ,
- $c_e$ - the mean chord of the elevator, fig. 3,
- $l_{s\ H}$ - length of the control stick.

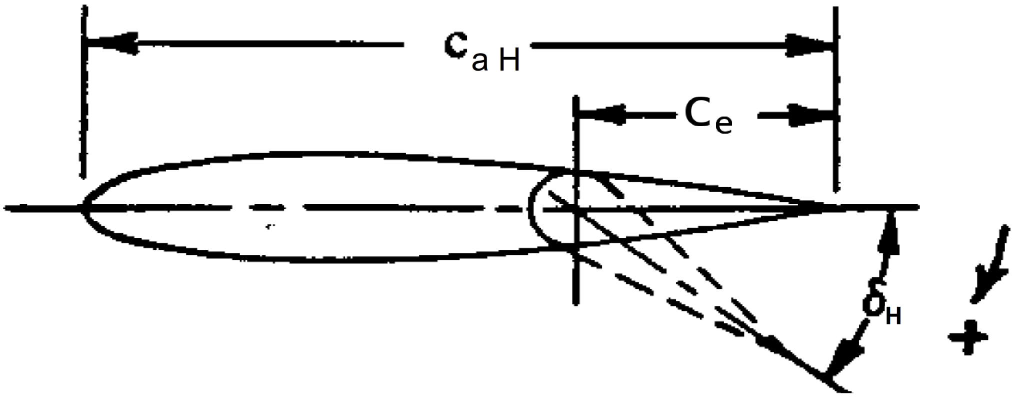


*Fig. 3*

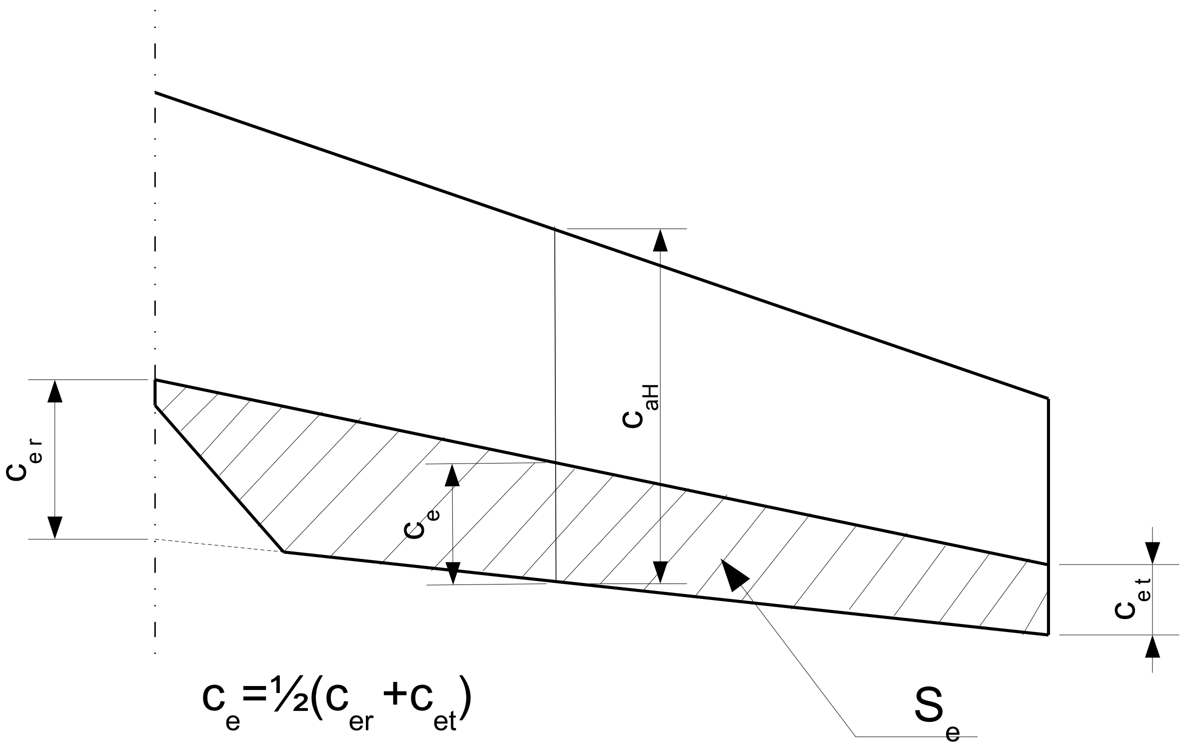


*Fig. 4*

The horizontal tailplane angle of attack

$$\alpha_H = \frac{C_L}{a} \cdot \left(1 - \frac{d\epsilon}{d\alpha}\right) + \alpha_{sett\ H} \ , \qquad (5)$$

the elevator angle $\delta_H$ is given by formula (1).

The trim tab deflection angle $\delta_{H\ t}$ is to be calculated with additional assumption: $P_{sH} = 0$ for a chosen airspeed (called as *trim speed,* $V_{trim}$) and middle position of center of mass $\left(\bar{x}_C = 0.25\right)$:

$$\delta_{H\ t} = -\frac{b_1}{b_3} \cdot \alpha_H \ - \ \frac{b_2}{b_3} \cdot \delta_H \ . \qquad (6)$$

Assume:

$l_{s\ H} = 0.35\ \ m\ \ ....\ \ 0.7\ \ m,$ depend on mass of the aircraft
(small aircraft - small length of the control stick)

$b_1 = \ -0.2\ \dfrac{1}{radian}\ ,$ $\quad b_2 = \ -0.3\ \dfrac{1}{radian}.$ $\quad b_3 = \ -0.1\ \dfrac{1}{radian}.$

Figure 5 shows the results of calculations for the light aircraft ($V_{trim} \approx 70$ m/s).

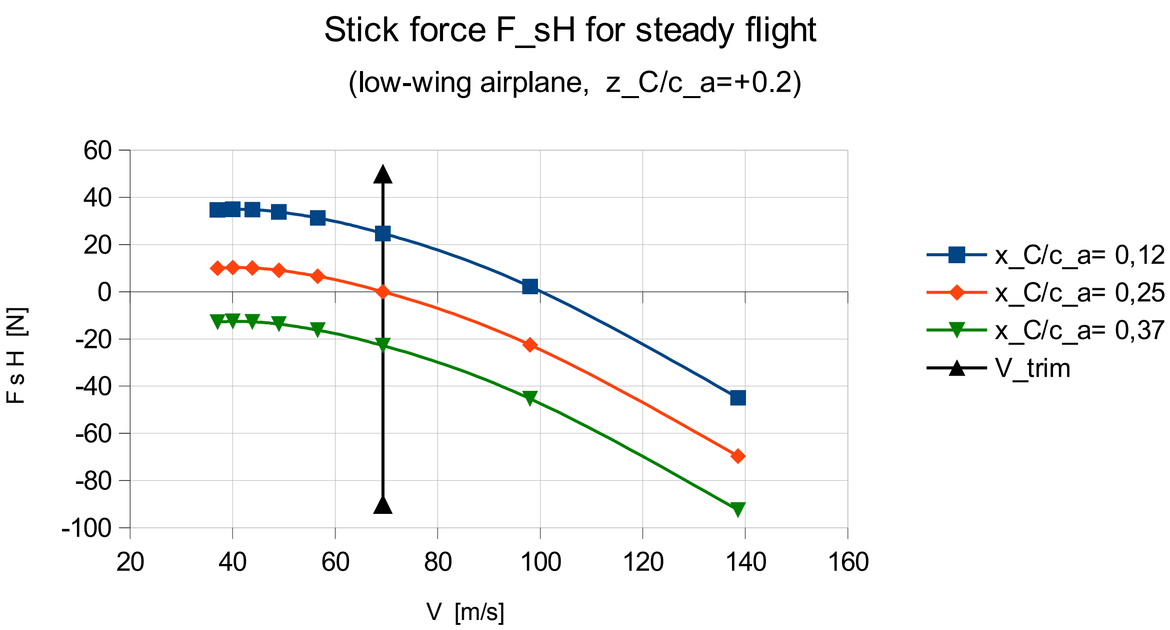


*Fig. 5*

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