

MECHANICS OF FLIGHT I
Project no. 2 – Aerodynamic Characteristics of the Wing

Geometry of the wing

1. Take from the airplane's drawing following basic dimensions of the airplane wing (fig. 2.1):

- wing span b ,
- root chord c_r ,
- tip chord c_t ,

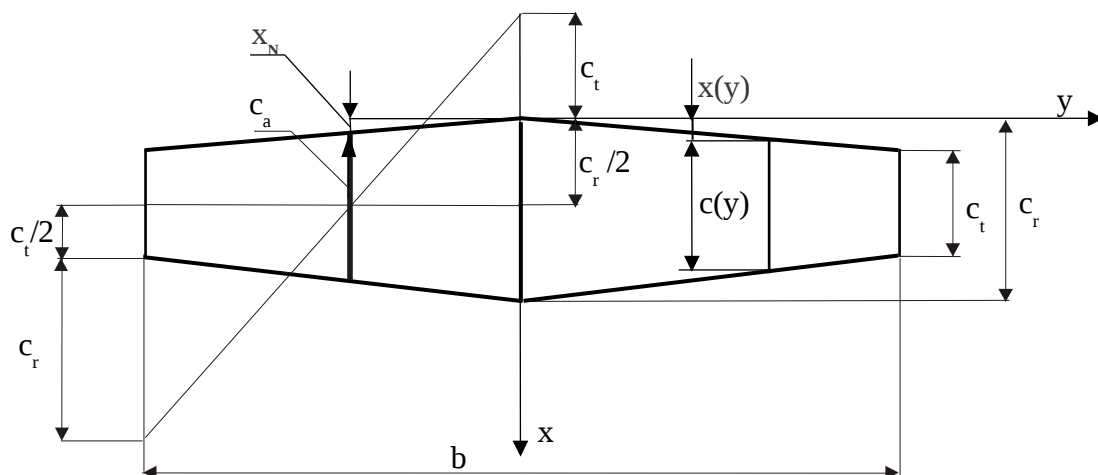


Fig. 2.1 Geometrical parameters of a wing

2. You need also the wing area S which usually is presented in the technical data set. If not, calculate it based on the wing dimensions.

3. Calculate (see fig. 2.1):

- wing aspect ratio $\Lambda = \frac{b^2}{S}$,
- wing taper ratio $\lambda = \frac{c_t}{c_r}$,
- mean aerodynamic chord (MAC) c_a , and coordinate of the tip of mean aerodynamic chord x_N :

$$c_a = \frac{\int_{-\frac{b}{2}}^{\frac{b}{2}} (c(y))^2 dy}{\int_{-\frac{b}{2}}^{\frac{b}{2}} c(y) dy},$$

$$x_N = \frac{\int_{-\frac{b}{2}}^{\frac{b}{2}} c(y) \cdot x(y) dy}{\int_{-\frac{b}{2}}^{\frac{b}{2}} c(y) dy}.$$

Note: for straight, moderate sweep tapered wings following formulas can be used:

$$c_a = \frac{2}{3} \cdot c_r \cdot \frac{1+\lambda+\lambda^2}{1+\lambda}, \quad x_n = \frac{1}{6} \cdot b \cdot \tan \nu_{x0} \cdot \frac{1+2\lambda}{1+\lambda}.$$

where ν_{x0} denotes the sweep angle of the leading edge of the wing.

Aerodynamic characteristics of the airfoil and the wing

1. Let assume that the wing of the airplane was designed using one of NACA wing sections published in NACA Report 824 (<http://ntrs.nasa.gov/>, also available on Mechanics Division home page <http://www.meil.pw.edu.pl/zm/ZM/Dydaktyka/Do-pobrania/Mechanika-Lotu-I/naca-824-3>).

2. From the technical description of the airplane take the minimum speed of the aircraft V_{S1} (stall speed). If this speed is not in the data set then calculate it using formula:

$$V_{S1} = \sqrt{\frac{2 \cdot m \cdot g}{\rho \cdot S \cdot C_{L_{max}}}}$$

where:

m – total take-off mass of the airplane,

g - normal Earth acceleration,

ρ - normal air density for altitude = 0 m STD,

S - wing area,

$C_{L_{max}}$ – maximum of lift coefficient, see airfoil data in NACA Report 824.

3. Using V_{S1} calculate Reynold number Re_1 for the wing

$$Re_1 = \frac{V_{S1} \cdot c_a}{\nu_0}$$

where $\nu_0 = 1.453 \cdot 10^{-5} \frac{m^2}{s}$ is the kinematic viscosity coefficient of the air for flight altitude 0 m STD.

4. Take from the NACA Report 824 aerodynamic characteristics $C_{D\infty}$, $C_{L\infty}$ of chosen airfoil as a function of the airfoil angle of attack α_∞ . Use data for the Reynolds number closest to calculated Re_1 (ie. if you obtain $Re_1 = 5,28 \cdot 10^6$ then use values of $C_{D\infty}$ and $C_{L\infty}$ for $Re = 6 \cdot 10^6$). Take values of aerodynamic center position $\bar{x}_{a.c.}$ and $\bar{z}_{a.c.}$, the value of pitching moment coefficient (referenced to aerodynamic center) $C_{m_{a.c.}}$, and calculate very important aerodynamic coefficient - airfoil lift slope a_∞ - defined as

$$a_\infty = \frac{dC_{L\infty}}{d\alpha_\infty}$$

It can be easy done using standard linear approximation (or linear regression) function of a hand-held scientific calculator or a PC spreadsheet (ie. the function REGLINP in Open Office Calc). Please exclude from this calculation the non-linear part of the function $C_{L\infty}(\alpha_\infty)$ close to $C_{L_{min}}$ and $C_{L_{max}}$.

Remark

Due to convenience of future calculations the airfoil data should be collected as a function of lift coefficient $C_{L\infty}$ rather than angle of attack α_∞ , see table 2.1 below.

Table 2.1 - Aerodynamic Characteristics of the Wing

airfoil type:

wing area: m²

wing aspect ratio:.....

	Airfoil					Wing					
	$C_{L\infty}$	α_∞	$C_{D\infty 1}$	$\Delta C_{D\infty}$	$C'_{D\infty 2}$	α_i	α	C_{Di}	C'_{Dw}
1											
2											
...
n											

5. If the value of Re_1 is less than $10 \cdot 10^6$ (ie. $Re_1 = 5.28 \cdot 10^6$ is rather far from ten millions) then compute the correction of the airfoil drag coefficient $\Delta C_{D\infty}$ due to changes of Reynolds number for high speed regimes of flight using formulas:

$$C_{D\infty_{min2}} = C_{D\infty_{min1}} \cdot \left(\frac{Re_1}{10 \cdot 10^6} \right)^{0.11}$$

$$\Delta C_{D\infty}(C_{L\infty}) = (C_{D\infty_{min2}} - C_{D\infty_{min1}}) \cdot \left(1 - \left| \frac{C_{L\infty}}{C_{L\infty_{max}}} \right| \right)$$

and compute final value of the wing section drag coefficient:

$$C'_{D\infty_2} = C_{D\infty_1} + \Delta C_{D\infty}(C_{L\infty})$$

Assume that for this calculations as well as in future computations of wing and airplane characteristics, the independent variable is C_L , not the angle of attack α_∞ (see remark on previous page). Please note that for the negative region of C_L (ie. from 0.0 up to -1.1) use in above expression $C_{L_{min}}$ instead of $C_{L_{max}}$ (i.e. - 1.1). Please note that values of $\Delta C_{D\infty}$ are negative and are equal to zero for $C_{L_{min}}$ and $C_{L_{max}}$.

6. Compute Glauert's correctional coefficients τ , δ as follows:

$$\tau_1 = 0.023 \left(\frac{\Lambda}{a_\infty} \right)^3 - 0.103 \left(\frac{\Lambda}{a_\infty} \right)^2 + 0.25 \left(\frac{\Lambda}{a_\infty} \right)$$

$$\tau_2 = -0.18 \cdot \lambda^5 + 1.52 \cdot \lambda^4 - 3.51 \cdot \lambda^3 + 3.5 \cdot \lambda^2 - 1.33 \cdot \lambda + 0.17$$

$$\tau = \frac{\tau_1 \cdot \tau_2}{0.17}$$

$$\delta_1 = 0.0537 \frac{\Lambda}{a_\infty} - 0.005$$

$$\delta_2 = -0.43 \cdot \lambda^5 + 1.83 \cdot \lambda^4 - 3.06 \cdot \lambda^3 + 2.56 \cdot \lambda^2 - \lambda + 0.148$$

$$\delta_3 = (-2.2 \cdot 10^{-7} \cdot \Lambda^3 + 10^{-7} \cdot \Lambda^2 + 1.6 \cdot 10^{-5}) \cdot \nu_{25}^3 + 1$$

$$\delta = \frac{\delta_1 \cdot \delta_2 \cdot \delta_3}{0.048}$$

where ν_{25} is the wing quarter-chord line sweep angle, in degrees.

Please note that:

- in above expressions lift slope a_∞ units should be 1/radian, not 1/degree!
- for typical rectangular and straight tapered wings $0.0 \leq \tau \leq 0.25$ and $0.0 \leq \delta \leq 0.1$
- for elliptic wing (ie. Supermarine Spitfire) $\tau = \delta = 0.0$

7. Calculate induced angle of attack α_i and induced drag coefficient C_{Di} :

$$\alpha_i = \frac{C_{L\infty}}{\pi \cdot \Lambda} \cdot (1 + \tau), \quad C_{Di} = \frac{C_{L\infty}^2}{\pi \cdot \Lambda} \cdot (1 + \delta) \quad .$$

8. Now you can calculate aerodynamic characteristics for the wing $C_L = f_1(\alpha_w)$, $C_D = f_2(C_L)$:

- the wing angle of attack as a sum of α_∞ and α_i :

$$\alpha_w = \alpha_\infty + \alpha_i \quad ,$$

- total drag coefficient as the sum:

$$C_{D_w} = C_{D_{\infty 2}} + \Delta C_{D_{tech}} + C_{Di} \quad .$$

Second component of the sum above, $\Delta C_{D_{tech}}$, is called as drag increment due to manufacturing (technological) effects on a real wing of an airplane. It can be estimated as:

- $0.15 \cdot (C_{D_{\infty 2}})_{min}$ for all-metal or fiberglass composite wings
- $0.50 \cdot (C_{D_{\infty 2}})_{min}$ for wings of old-style airplanes made of wood and fabric (World War I and 1920-1930).

9. Results of the calculations have to be shown on graphs consists of C_L and C_D both for airfoil (with Re effect) and for the wing.

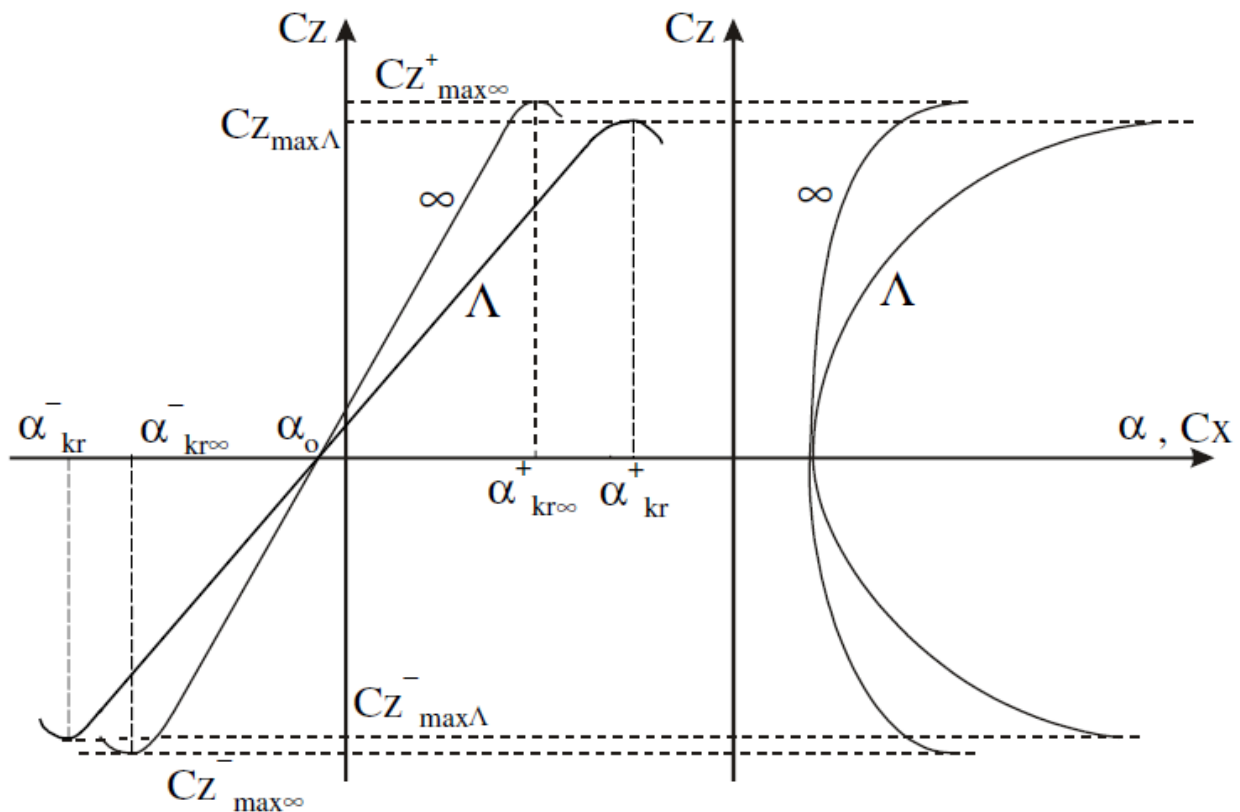


Fig. 2.2. Lift and drag characteristics of the wing

Notes for the figure 2.2:

- $C_x == C_D$ – drag coefficient, $C_z == C_L$ – lift coefficient;
- the symbol ∞ denotes data valid for wing section (airfoil),
- the subscript **kr** by angle of attack symbol is equivalent to **cr** (critical);
- lower values of $C_{L_{min}}$ and $C_{L_{max}}$ for the wing you can see on the graph were obtained from more sophisticated method taking into account effects of variable distribution of angle of attack and lift force along wing span; simple method described in this paper does not "see" all these effects and because of this for our characteristics always $C_{L_{min\infty}} = C_{L_{min\text{wing}}}$ as well as $C_{L_{max\infty}} = C_{L_{max\text{wing}}}$.

[end-of-text]