Prediction of transitional boundary layer flows with γ-Re_θ model

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Contents

1. Turbulent spots and intermittent signals
2. Narashimha low and a transport equation for intermittency
3. The γ-Re_θ model
   - Transition modeling in attached boundary layer
   - Transition modeling in separated boundary layer
   - Grid requirements and numerics
   - Results for wake-induced tranistion
4. Conclusions

Intermittency factor γ

The ***intermittency γ*** is the fraction of time that the flow is turbulent in a position in the boundary layer during the breakdown phase. It is zero in laminar flow and unity in fully turbulent flow.

$$\gamma(x,y) = \frac{1}{N}\sum_{1}^{N} I(x,y,t) \qquad I(x,y,t) = \begin{cases} 1: & \text{turbulent} \\ 0: & \text{non-turbulent} \end{cases}$$

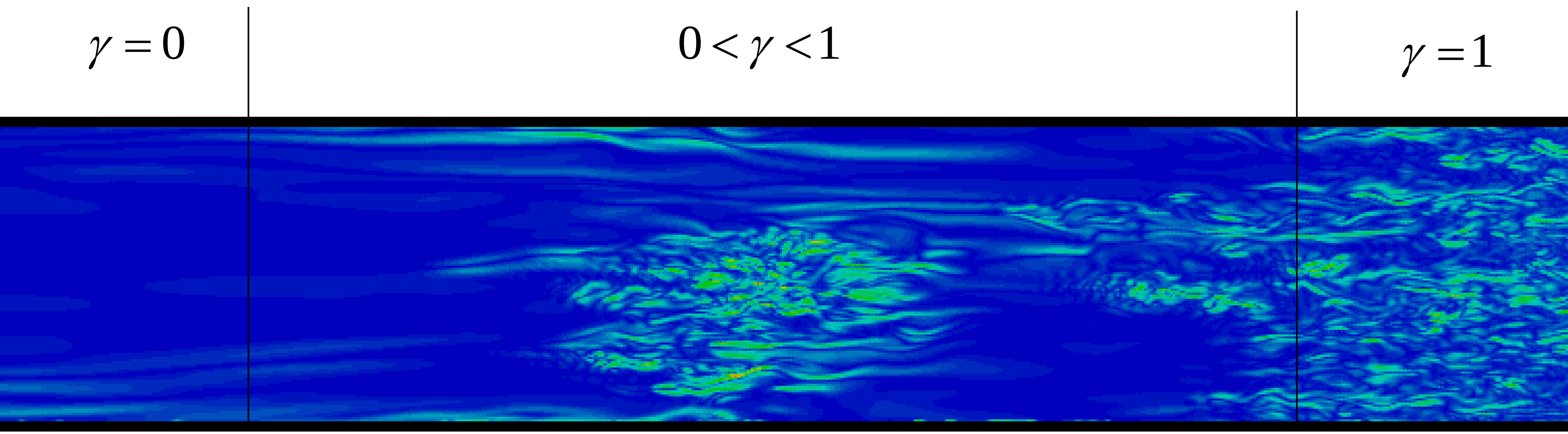


Fig. 1. DNS of transitional boundary layer flow. Visualisation of turbulent spot by means of vorticity magnitude.

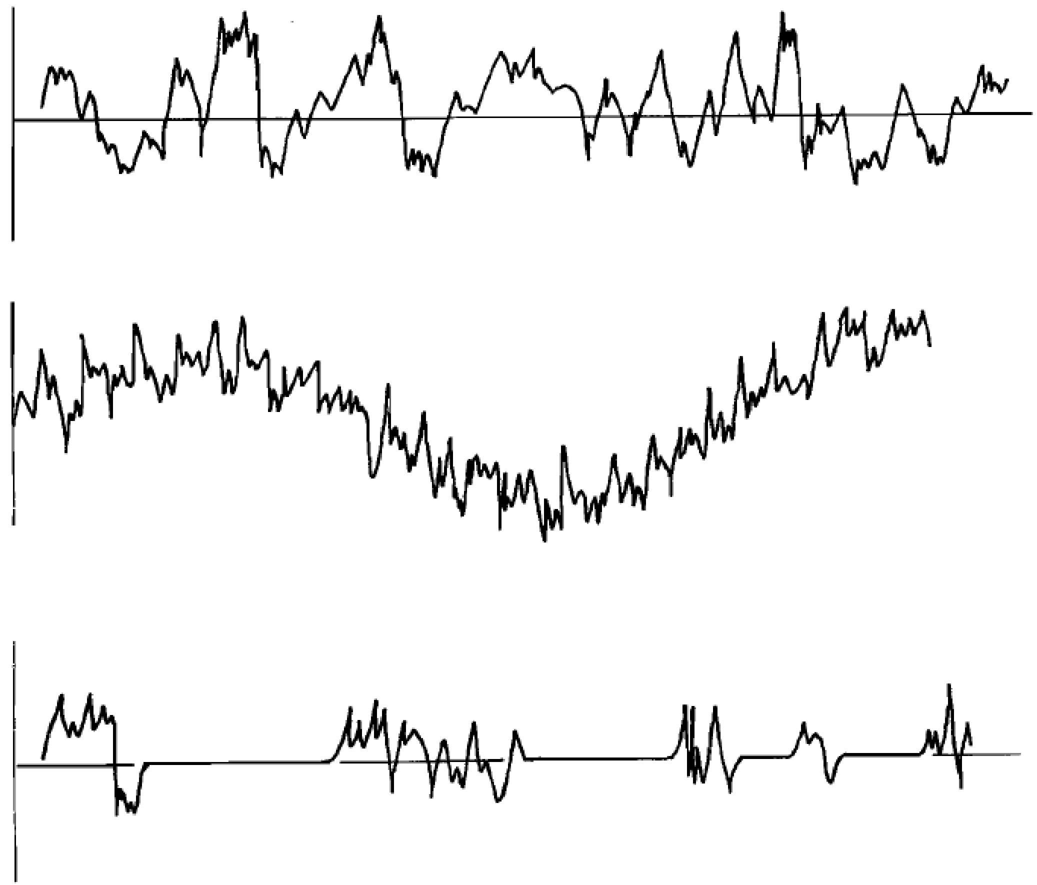


Fig. Time signals in turbulent flow

Narasimha low

$$\gamma(x) = 1. - \exp\left[-\frac{N\sigma}{U}(x - x_t)^2 \right] \qquad (1)$$

Dimensionless parameters:

- N is the spot production rate per unit distance in spanwise direction
- σ is the function which depends on the planform shape, the propagation velocities and the spreading angle

Other quantities:

- U is velocity at edge of boundary layer
- x is the streamwise distance
- x_t is the transition onset point

See Gostelow et al. and Solomon et al. for definitions of N and σ

Defining the spatial growth parameter, β_γ [1/m]

$$\beta_\gamma = \sqrt{N\sigma / U} \qquad \text{or} \qquad \beta_\gamma = \sqrt{\hat{N}\sigma}\,\frac{U}{\nu} \tag{2}$$

The Narashimha formula (1) reads

$$\gamma(x) = 1. - \exp\left[-\beta_\gamma^2 (x - x_t)^2\right] \tag{3}$$

or

$$1 - \gamma(x) = \exp\left[-\beta_\gamma^2 (x - x_t)^2\right] \tag{4}$$

From (3) follows:

$$\frac{d\gamma}{dx} = \exp\left[-\beta_\gamma^2 (x - x_t)^2\right] 2\beta_\gamma^2 (x - x_t). \tag{5}$$

With (4), this is:

$$\frac{d\gamma}{dx} = 2\beta_\gamma^2 (1 - \gamma)(x - x_t). \tag{6}$$

From (4), we also obtain:

$$\sqrt{-\ln(1-\gamma)} = \beta_\gamma (x - x_t). \tag{7}$$

Thus, (6) may also be written as

$$\boxed{\frac{d\gamma}{dx} = 2\beta_\gamma (1-\gamma)\sqrt{-\ln(1-\gamma)}.} \tag{8}$$

Typically, the linear law (7) is fitted to the experimental results. By this fitting, the onset position (x_t) and growth rate (β_γ) are determined.

Maximum production is inside the laminar boundary layer

Disadvantages:

- The transition onset x_t has to be known from experiments
- The evolution low (8) only applies for initial laminar flow (no possibility to enforce the laminar boundary layer starting from the fully turbulent flow).

More general a ***transport equation of intermittency*** can be formulated:

$$\vec{v}.\nabla\gamma = 2\sqrt{\hat{N}\sigma}\,\frac{U}{\nu}u(1-\gamma)\sqrt{-\ln(1-\gamma)} \tag{9}$$

$\vec{\mathbf{v}}$ is the local velocity vector,

U is the magnitude of the velocity at the edge of the boundary layer

u is the magnitude of the local velocity.

A further generalisation is

$$\frac{D(\rho\gamma)}{Dt} = 2\sqrt{\hat{N}\sigma}\,\rho(\frac{U}{\nu}u)(1-\gamma)\sqrt{-\ln(1-\gamma)}\,F_{onset} + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\gamma}\right)\frac{\partial\gamma}{\partial x_j}\right] \tag{10}$$

- F_{onset} function switches from zero to unity at transition onset x_t.
- The diffusion term is added to allow a profile across the boundary layer.

Some simplifications: $\frac{D(\rho\gamma)}{Dt} = 2\sqrt{\hat{N}\sigma}\,\rho(\frac{U}{\nu}u)(1-\gamma)\sqrt{-\ln(1-\gamma)}\,F_{onset} + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\gamma}\right)\frac{\partial \gamma}{\partial x_j}\right]$

- Term $\sqrt{-\ln(1-\gamma)}$ might be replaced by $\sqrt{\gamma}$ (approximate proportionality in the range: $0<\gamma<0.35$)

- Term $\sqrt{-\ln(1-\gamma)}$ might also be replaced by γ (approximate proportionality in the range: $0.35<\gamma<0.95$)

- Factor $(\frac{uU}{\nu})$ has dimension [1/s]. This might be approximated by strain-rate S or rotation rate Ω magnitude. The latter formulation is Galilean invariant.

Thus, eq. (10) might be formulated by:

$$\frac{D(\rho\gamma)}{Dt} = F_{lenght}\rho S(1-\gamma)\sqrt{\gamma}F_{onset} + Diff(\gamma) \qquad (11)$$

F_{length} is a dimensionless function expressing the growth rate of the intermittency (similar as $\sqrt{\hat{N}\sigma}$ term).

The local correlation-based γ-Re_θ model

(Menter, Langtry et al., 2006, Langtry and Menter, 2009)

$$\frac{D(\rho\gamma)}{Dt} = P_\gamma - E_\gamma + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\gamma}\right)\frac{\partial \gamma}{\partial x_j}\right] \quad (12)$$

Production term

$$P_\gamma = c_{a1} F_{length} \rho S \left(1 - c_{e1}\gamma\right)\sqrt{\gamma F_{onset}} \quad (13)$$

Maximum production inside the B.L. Transition onset is activated with F_{onset} term.

Destruction term

$$E_\gamma = c_{a2} F_{turb} \rho \Omega \left(c_{e2}\gamma - 1\right)\gamma \quad (14)$$

Used to enforce the laminar solution prior to transition.

Coupling with the SST turbulence model

$$\frac{D(\rho\gamma)}{Dt} = P_\gamma - E_\gamma + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\gamma}\right)\frac{\partial\gamma}{\partial x_j}\right] \quad (15)$$

$$\frac{D\left(\rho\overline{Re}_{\theta t}\right)}{Dt} = P_{\theta t} + \frac{\partial}{\partial x_j}\left[\sigma_{\theta t}\left(\mu + \mu_t\right)\frac{\partial\left(\overline{Re}_{\theta t}\right)}{\partial x_j}\right]$$

$$\frac{Dk}{Dt} = P_k - D_k^* + \frac{\partial}{\partial x_j}\left[(\nu + \frac{\nu_t}{\sigma_\kappa})\frac{\partial k}{\partial x_j}\right] \quad (16)$$

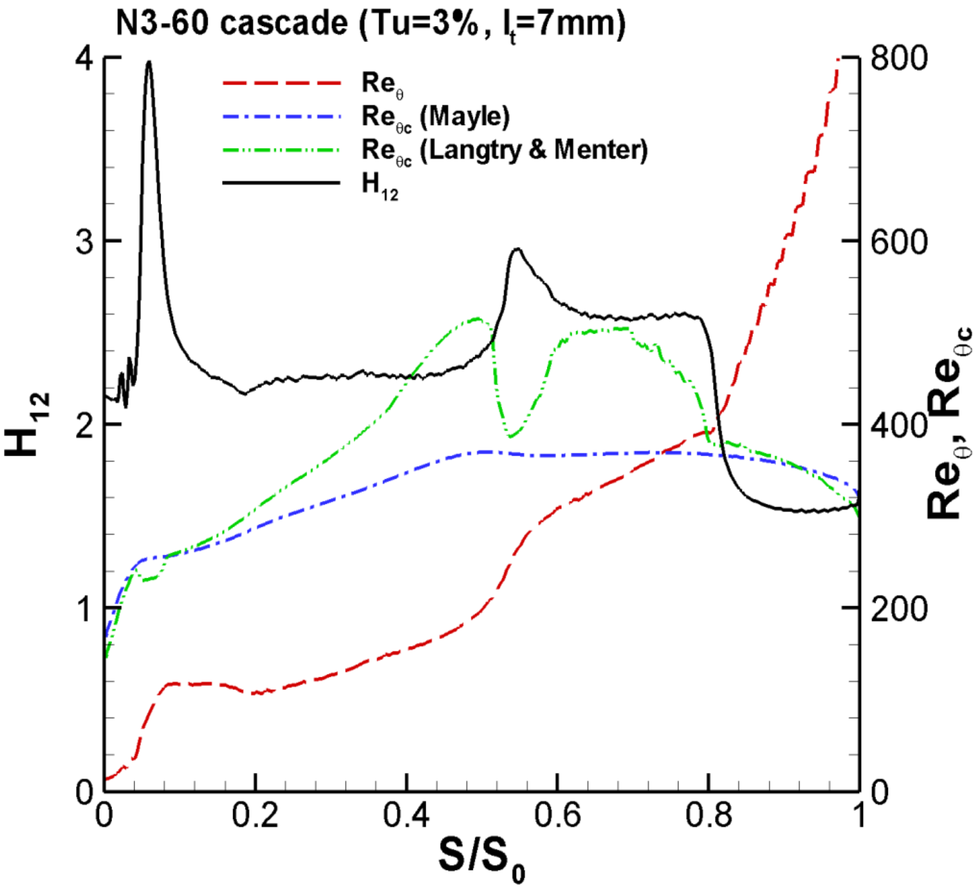
$$\frac{D\omega}{Dt} = \alpha P_\omega - \beta\omega^2 + \frac{\partial}{\partial x_j}\left[(\nu + \frac{\nu_t}{\sigma_\omega})\frac{\partial\omega}{\partial x_j}\right] + \frac{2\sigma_{\omega 2}}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial\omega}{\partial x_j} \quad (17)$$

$$P_k = \gamma_{eff}\,\nu_t S^2 \qquad \gamma_{eff} = \max\left(\gamma, \gamma_{sep}\right) \qquad \nu_t = \frac{a_1 k}{\max(a_1\omega, SF_2)} \qquad S^2 = 2S_{ij}S_{ij}$$

Transition in attached boundary layer

Example: Direct Correlation Based Models

Estimation of transition onset using experimental correlations for $Re_{\theta c}$. Re_{θ} calculated directly by integration of velocity profiles



Production term follows from Eq. (10)

$$P_{\gamma} = 2\sqrt{\hat{N}\sigma\rho}\,\frac{U}{\nu}\,u(1-\gamma)\sqrt{-\ln(1-\gamma)}F_{onset}$$

$$F_{onset} = 1 \quad \text{for} \quad Re_{\theta} > Re_{\theta c}$$

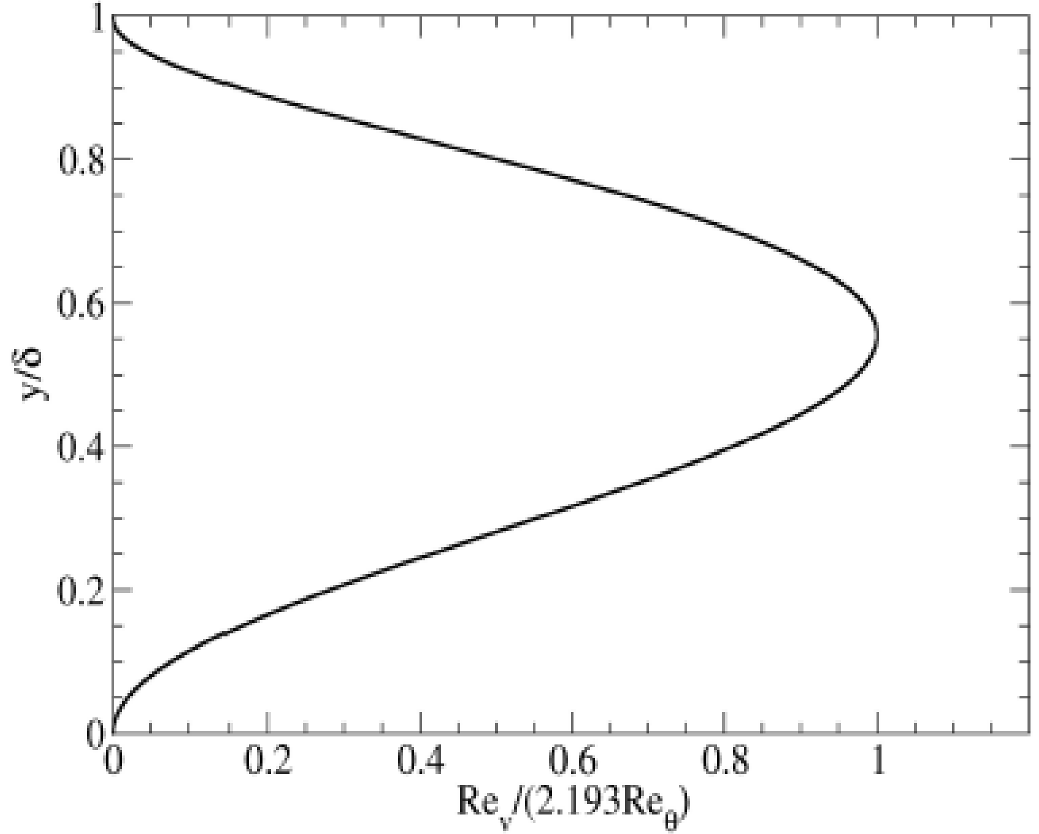
Estimation of Re_θ in γ-Re_θ model

$$Re_V = \frac{y^2}{\nu}\left|\frac{\partial U}{\partial y}\right| = \frac{y^2 S}{\nu}$$

Wilcox (1994):

$$Re_\theta \cong \frac{Re_{V,max}}{2.193}$$

- Re_V approximates Re_θ
- Maximum Re_V is observed inside the boundary layer



Estimation of Re_θ in γ-Re_θ model

$$Re_V = \frac{y^2}{\nu}\left|\frac{\partial U}{\partial y}\right| = \frac{y^2 S}{\nu}$$

Activate transition for

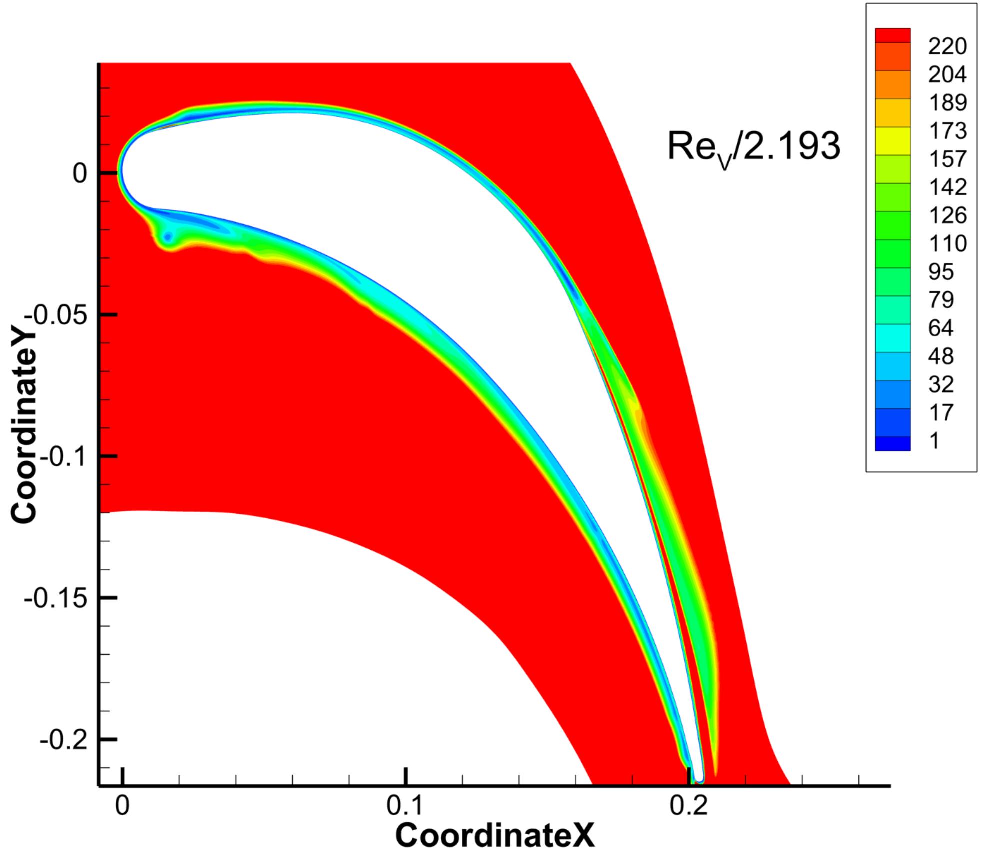
$$\frac{Re_{V,max}}{2.193 Re_{\theta c}} > 1$$

$Re_{\theta c}$ - critical value of the momentum thickness Reynolds number

In practice:

$$F_{onset} = \min\left[\frac{Re_V}{2.193 Re_{\theta c}}, 2\right]$$

$$P_\gamma = c_{a1} F_{length} \rho S \left(1 - c_{e1}\gamma\right)\sqrt{\gamma F_{onset}}$$



Estimation of critical value of $Re_{\theta c}$ in γ-Re_θ model

$Re_{\theta c}$ is obtained in three steps:

$$F_{onset} = \min[\frac{Re_V}{2.193 Re_{\theta c}}, 2]$$

First step: $Re_{\theta t}$ is calculated from experimental correlations outside of the boundary layer flow

$$Re_{\theta t} = [1173.51 - 589.428\,Tu + 0.2196\,Tu^{-2}]F(\lambda_\theta), \quad \text{for} \quad Tu \le 1.3$$

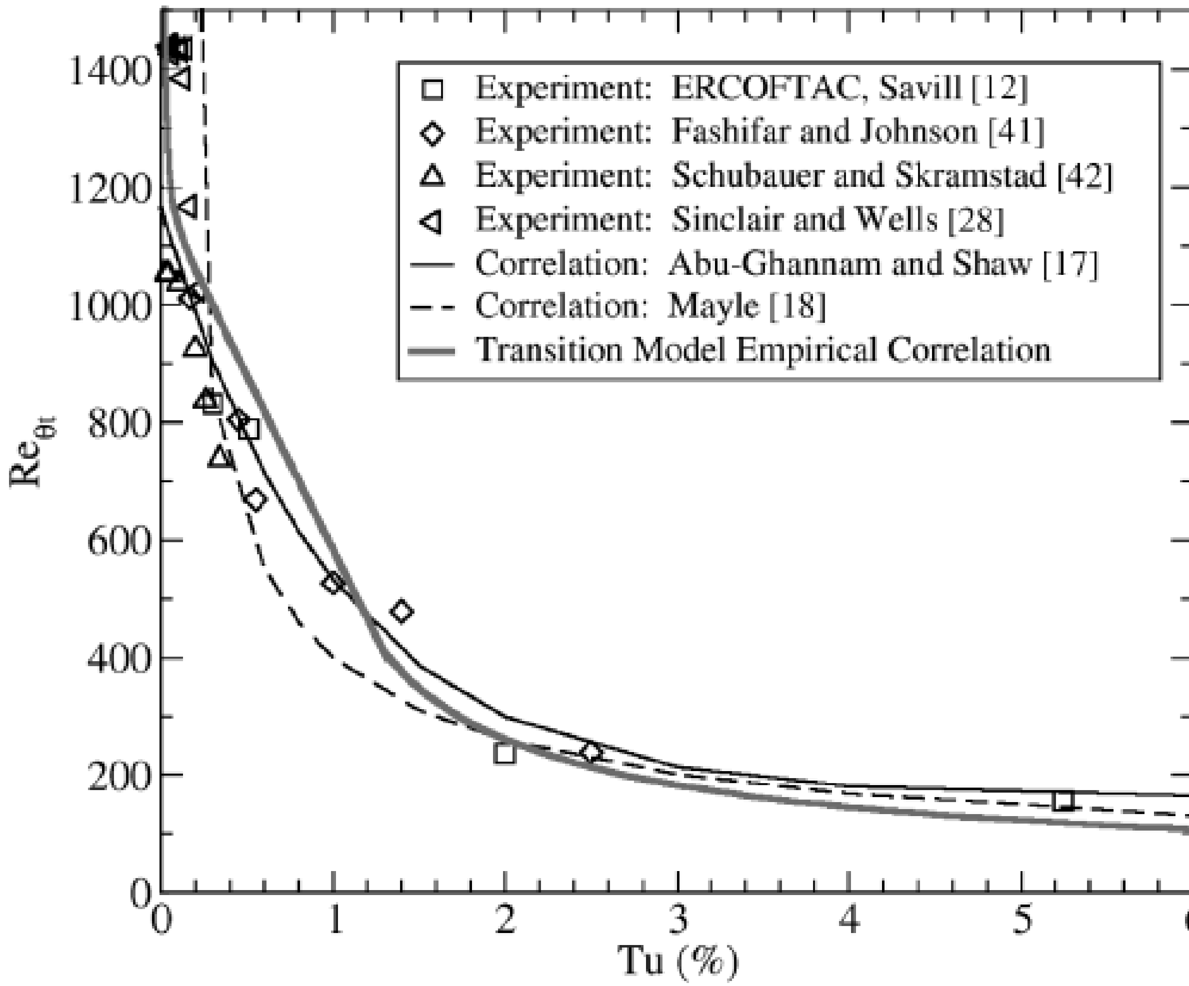
$$Re_{\theta t} = 331.50[Tu - 0.5658]^{-0.671} F(\lambda_\theta) \quad \text{for} \quad Tu > 1.3$$

$$F(\lambda_\theta) = 1 - [-12.986\lambda_\theta - 123.66\lambda_\theta^2 - 405.689\lambda_\theta^3]\exp[(\frac{Tu}{1.5})^{1.5}] \quad \text{for} \quad \lambda_\theta \le 0 \qquad (18)$$

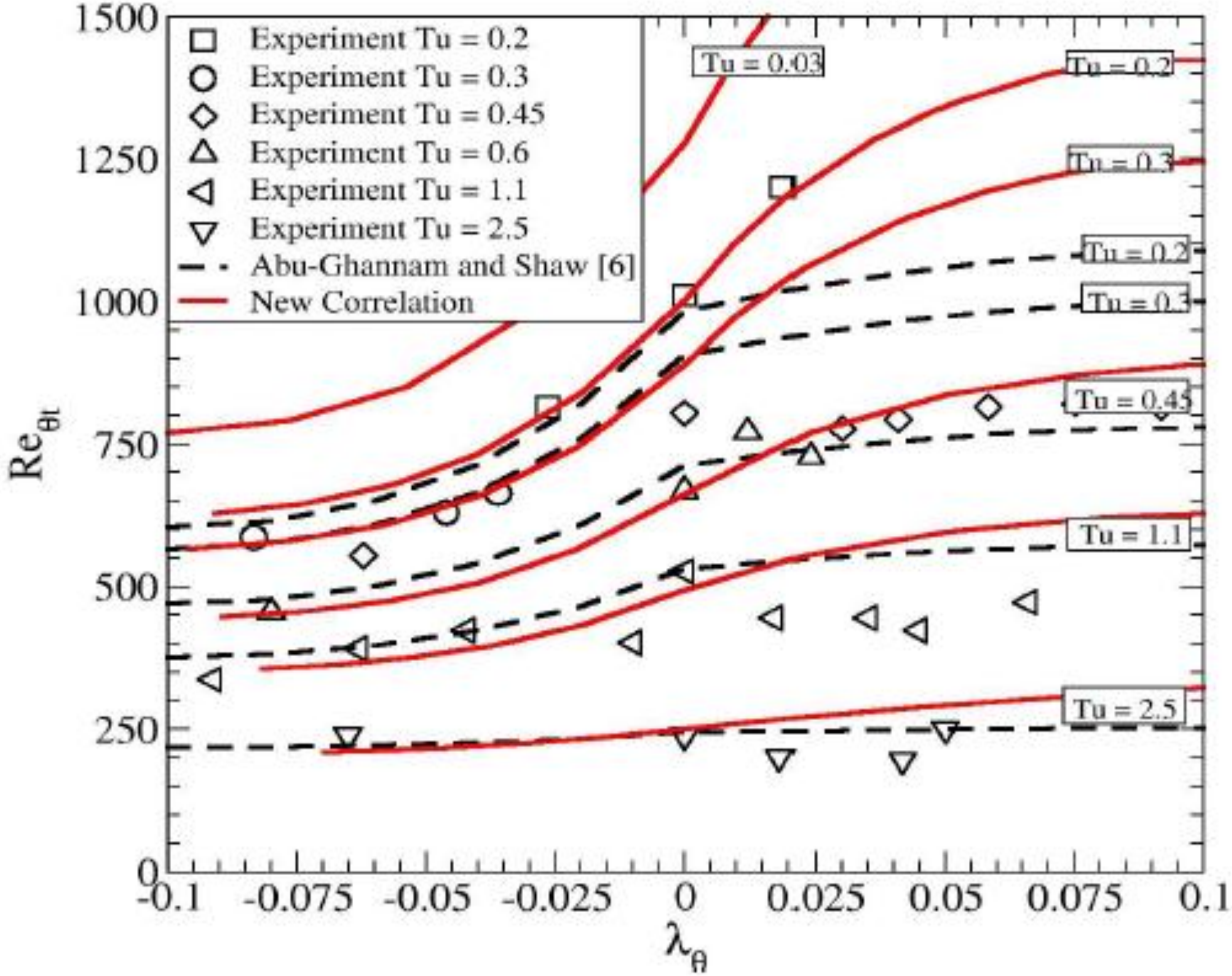
$$F(\lambda_\theta) = 1 + 0.275[1 - \exp(-35.0\lambda_\theta)]\exp(-\frac{Tu}{0.5}) \quad \text{for} \quad \lambda_\theta > 0$$

$$\lambda_\theta = (\theta^2 / \nu)(dU / ds) \qquad Tu = \frac{\sqrt{2k/3}}{U}$$

Estimation of critical value of $Re_{\theta c}$ in γ-Re_θ model (dependence on turbulence level)



Estimation of critical value of $Re_{\theta c}$ in γ-Re_θ model (dependence on pressure gradient)



Estimation of critical value of $Re_{\theta c}$ in γ-Re_θ model

<u>Second step</u>: Transfer of freestream values of $Re_{\theta t}$ towards the wall

$$\frac{D\left(\rho\overline{Re}_{\theta t}\right)}{Dt} = P_{\theta t} + \frac{\partial}{\partial x_j}\left[\sigma_{\theta t}\left(\mu+\mu_t\right)\frac{\partial\left(\overline{Re}_{\theta t}\right)}{\partial x_j}\right] \qquad (19)$$

- This operation can also be done without solving the partial differential equation (19).

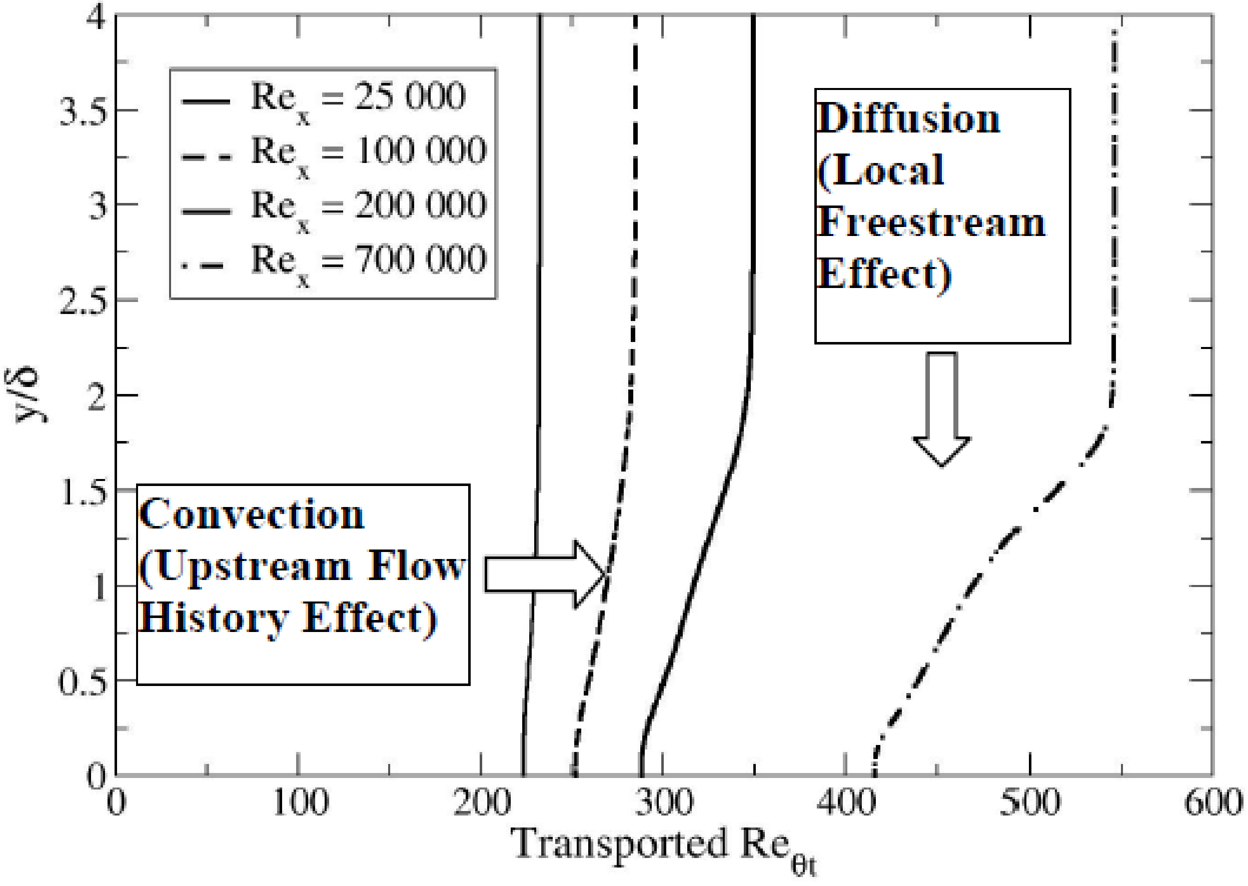
- The source term $P_{\theta t}$ enforces the free-stream values of $Re_{\theta t}$ to be equal to $\overline{Re}_{\theta t}$ and is set to zero in boundary layers,

- Diffusion term. The transport equation processes the value of $Re_{\theta t}$ such that $\overline{Re}_{\theta t}$ inside a boundary layer is influenced by the values at the edge of the boundary layer

- Difficulty with imposing the inlet conditions for $\overline{Re}_{\theta t}$. Now, the inlet conditions are reproduced from correlations.

Estimation of critical value of $Re_{\theta c}$ in γ-Re_{θ} model

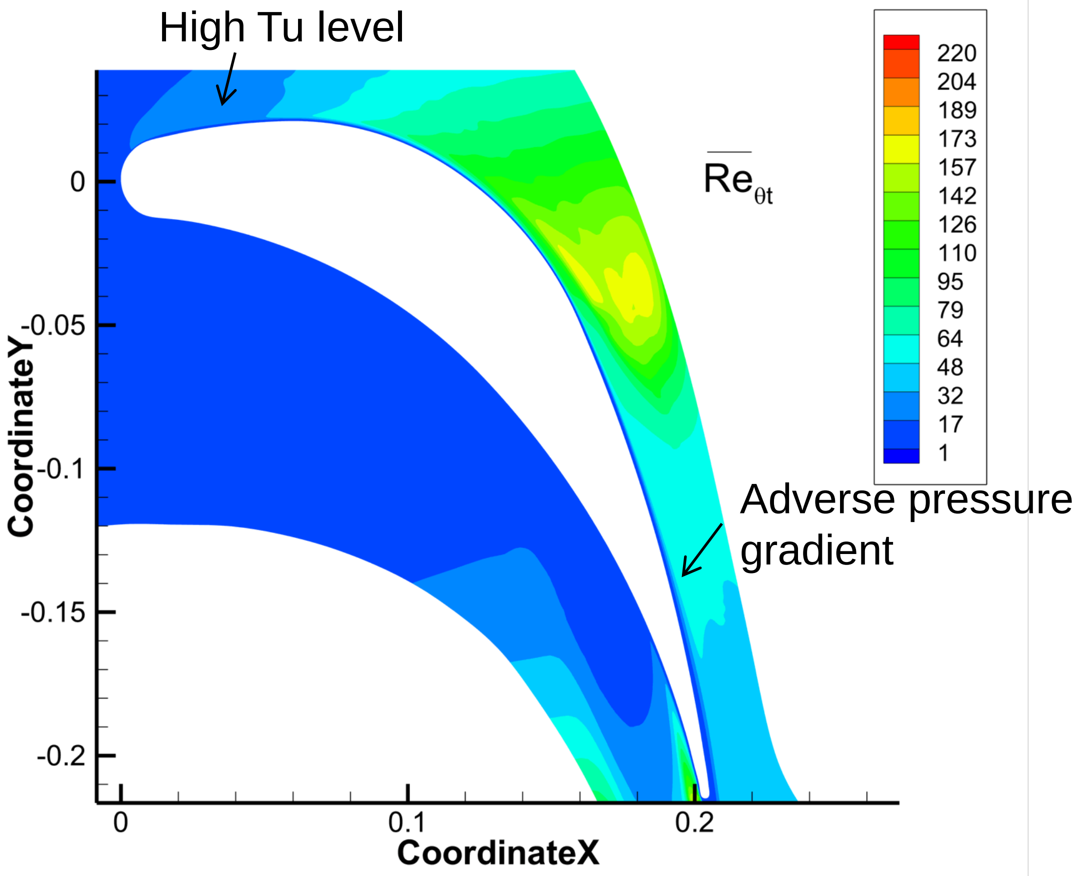
Second step: Transfer of freestream values of $Re_{\theta t}$ towards the wall



Estimation of critical value of $Re_{\theta c}$ in γ-Re_θ model

$\overline{Re}_{\theta t}$ very gradual change in the whole computational domain

Low $\overline{Re}_{\theta t}$ - means the flow is susceptible to transition



Estimation of critical value of $Re_{\theta c}$ in γ-Re_θ model

Third step: Activation of turbulence production in pseudo-laminar boundary layer

$$Re_{\theta c} = f(\overline{Re}_{\theta t})$$

The flow disturbances (secondary instabilities), precursor to flow transition, are initiated upstream of transition onset point

–this causes increase of modelled shear stress in pseudo-laminar boundary layer

Correlations are proposed based on numerical experiment (see Langtry and Menter, 2009):

Estimation of critical value of $Re_{\theta c}$ in γ-Re_θ model

Third step: Activation of turbulence production in pseudo-laminar boundary layer

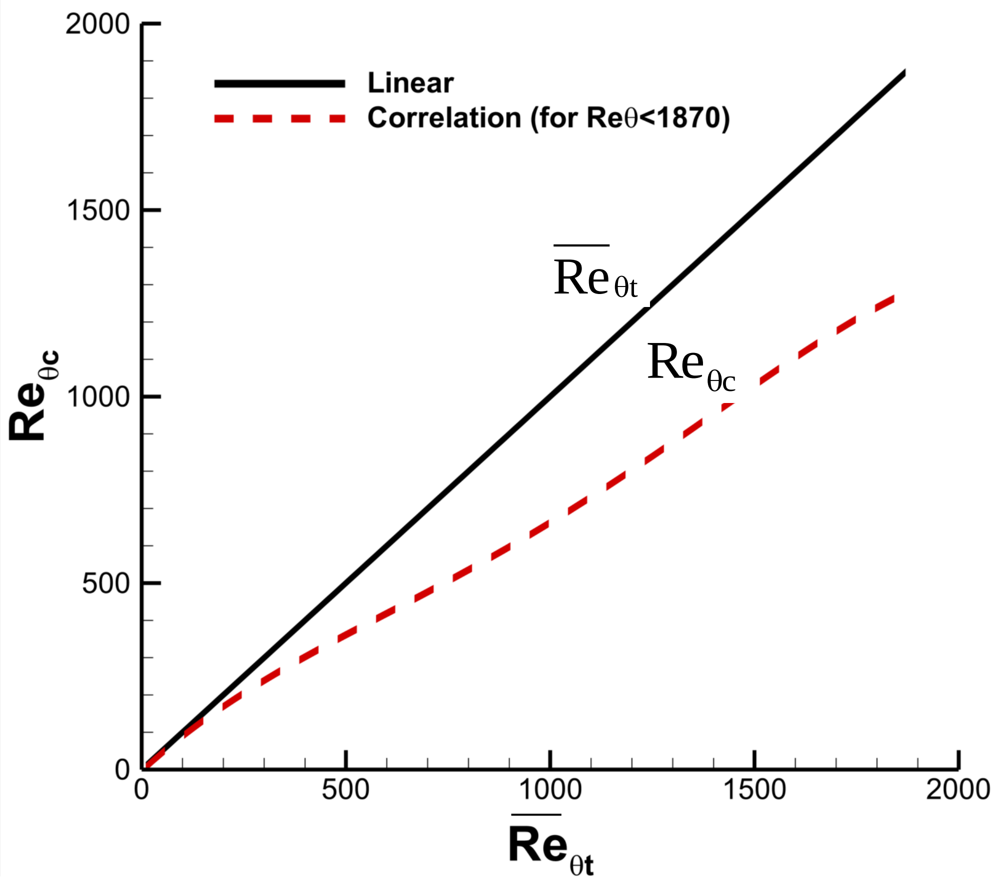
Correlation $Re_{\theta c} = f(\overline{Re}_{\theta t})$ gives a reduction of critical value of momentum thickness Reynolds number $Re_{\theta c}$ with respect to $\overline{Re}_{\theta t}$ (by 30%)

According to experimental correlations:

$$F_{onset} = \min[\frac{Re_V}{2.193\overline{Re}_{\theta t}}, 2]$$

Corrected formula

$$F_{onset} = \min[\frac{Re_V}{2.193 Re_{\theta c}}, 2]$$



Coupling of γ-equation with correlations for $Re_{\theta c}$

$$\frac{D(\rho\gamma)}{Dt} = P_\gamma - E_\gamma + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\gamma}\right)\frac{\partial\gamma}{\partial x_j}\right]$$

$$P_\gamma = c_{a1}F_{length}\rho S\left(1 - c_{e1}\gamma\right)\sqrt{\gamma F_{onset}}$$

$$\frac{D\left(\rho\overline{Re}_{\theta t}\right)}{Dt} = P_{\theta t} + \frac{\partial}{\partial x_j}\left[\sigma_{\theta t}\left(\mu + \mu_t\right)\frac{\partial\left(\overline{Re}_{\theta t}\right)}{\partial x_j}\right]$$

$$Re_{\theta c} = f\left(\overline{Re}_{\theta t}\right)$$

$$F_{onset} = \min\left[\frac{Re_V}{2.193 Re_{\theta c}}, 2\right]$$

Transition in separated boundary layer

Incresed production in separated boundary layer flows

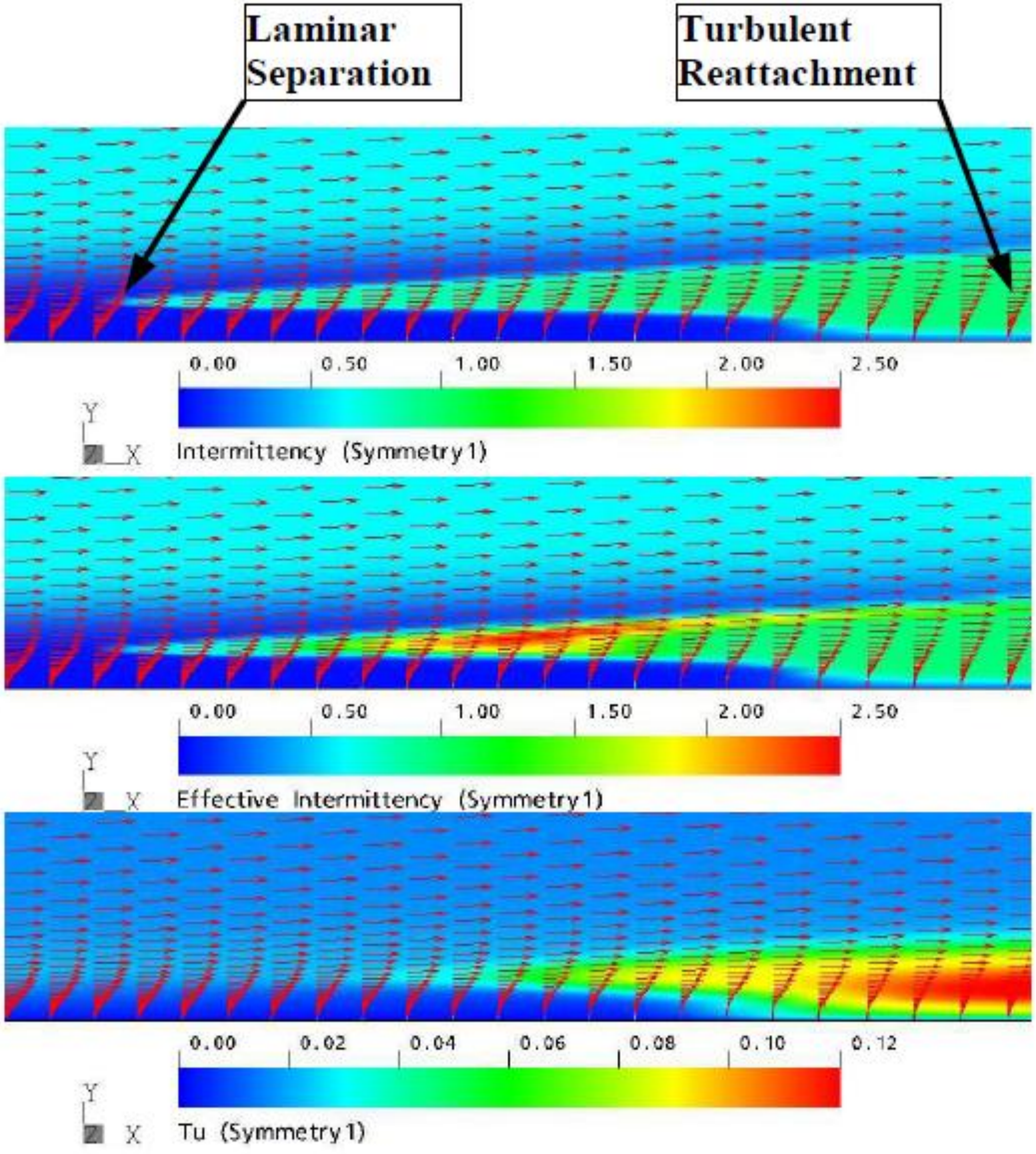
$$\frac{Dk}{Dt} = P_k - D_k^* + \frac{\partial}{\partial x_j}\left[(\nu + \frac{\nu_t}{\sigma_\kappa}) \frac{\partial k}{\partial x_j} \right]$$

$$P_k = \gamma_{eff}\, \nu_t S^2 \qquad \gamma_{eff} = \max\left(\gamma, \gamma_{sep}\right)$$

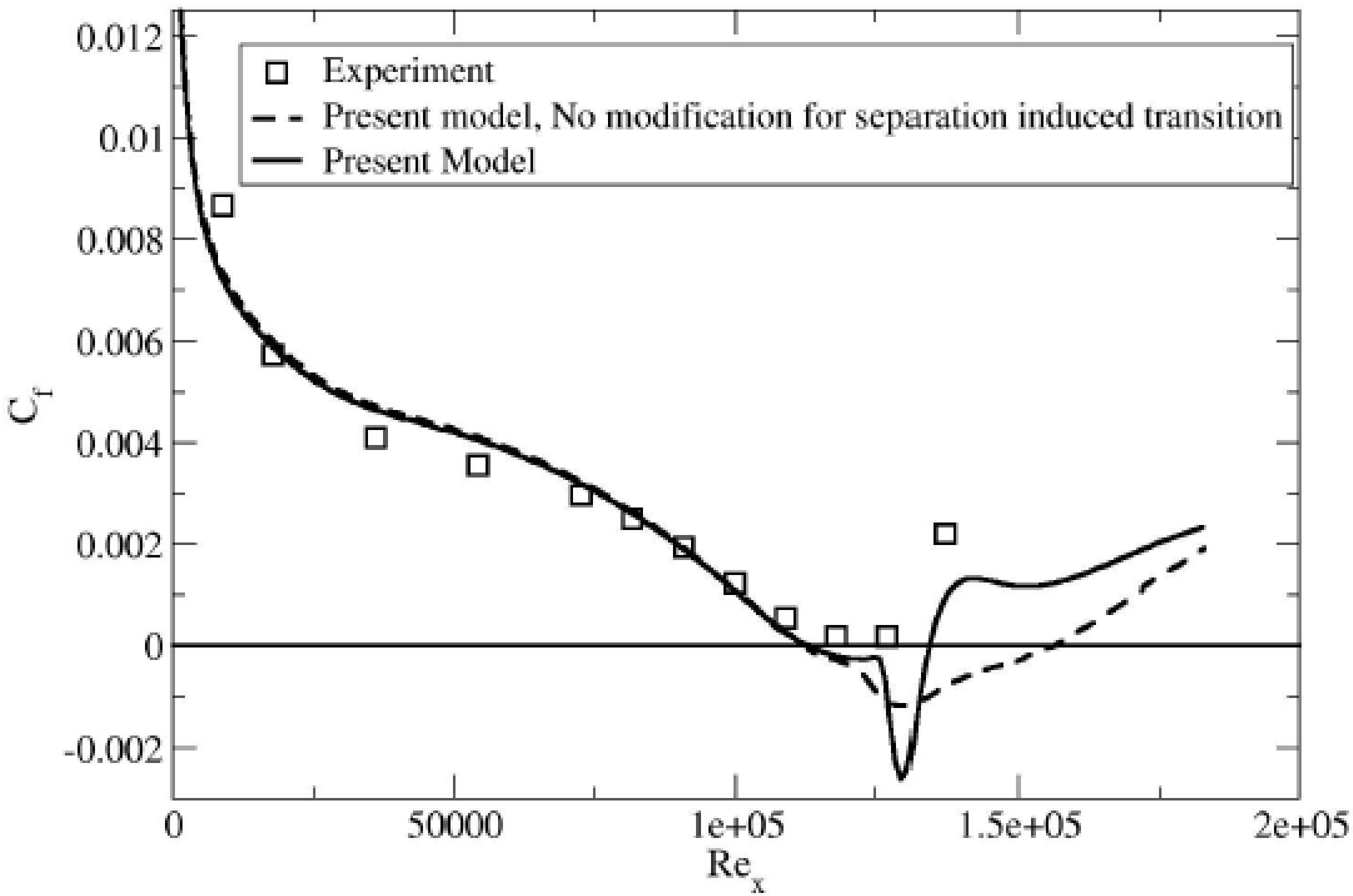
- In separted laminar boundary layer flow, an activation of tranistion model, typically results in delayed reattachement (provided it occurs)
- This is cured by alowing the production term to be locally twice as high as in reference fully turbulent flow

$$\gamma_{sep} = \min\left(\max\left(\frac{Re_V}{3.235 Re_{\theta c}} - 1, 0 \right) Funct_1, 2 \right) Funct_2$$

Separation induced transition in γ-Re_θ model



Separation induced transition in γ-Re_θ model



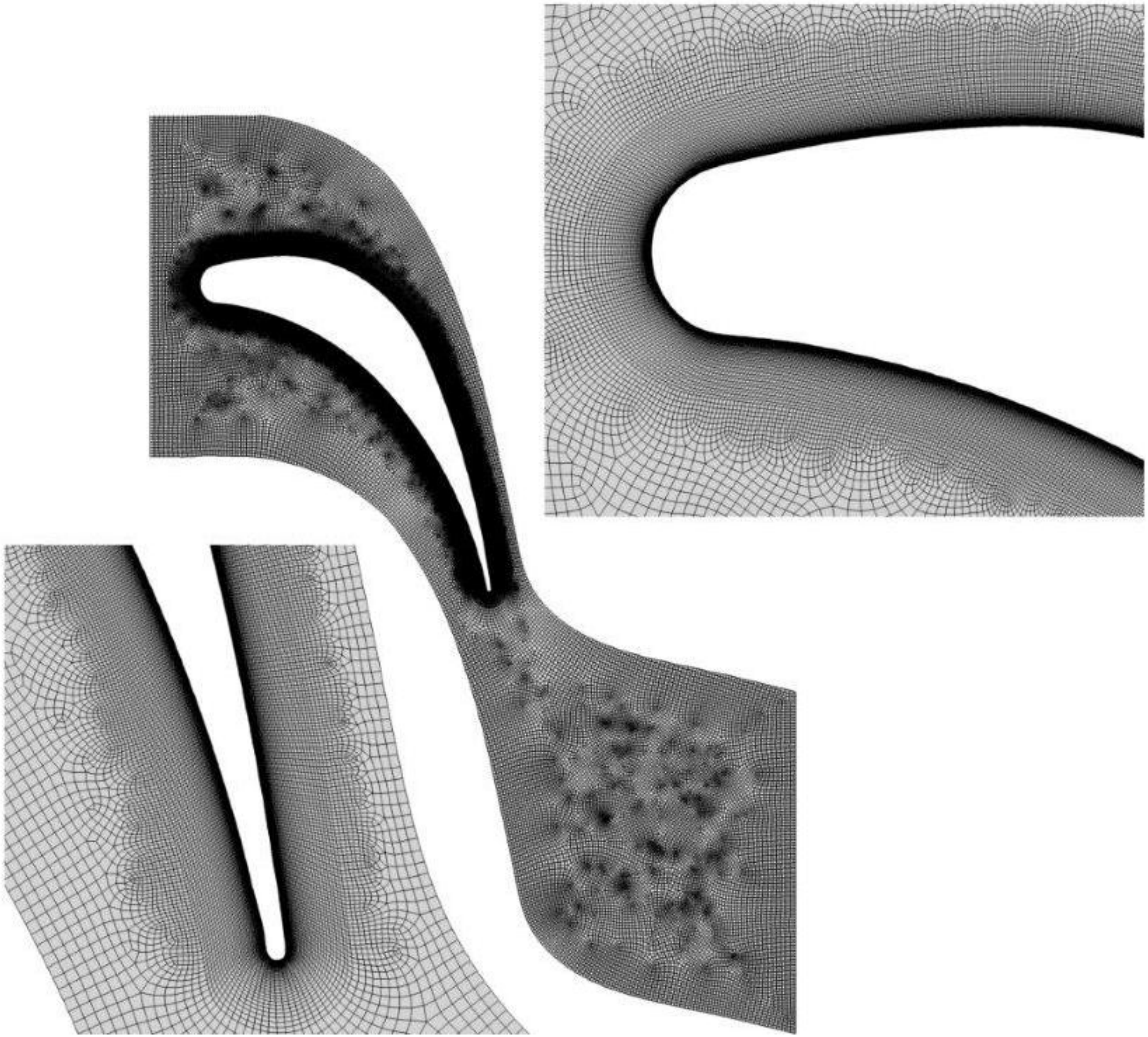
Grids and numerics

Grid requirements

- Necessity to resolve the viscous sublayer: $y^+=1$
- About 20-40 points inside the boundary layer
- Gradual variation of points in wall-normal direction: Ratio 1.05-1.2
- Grid-sensitivity study required

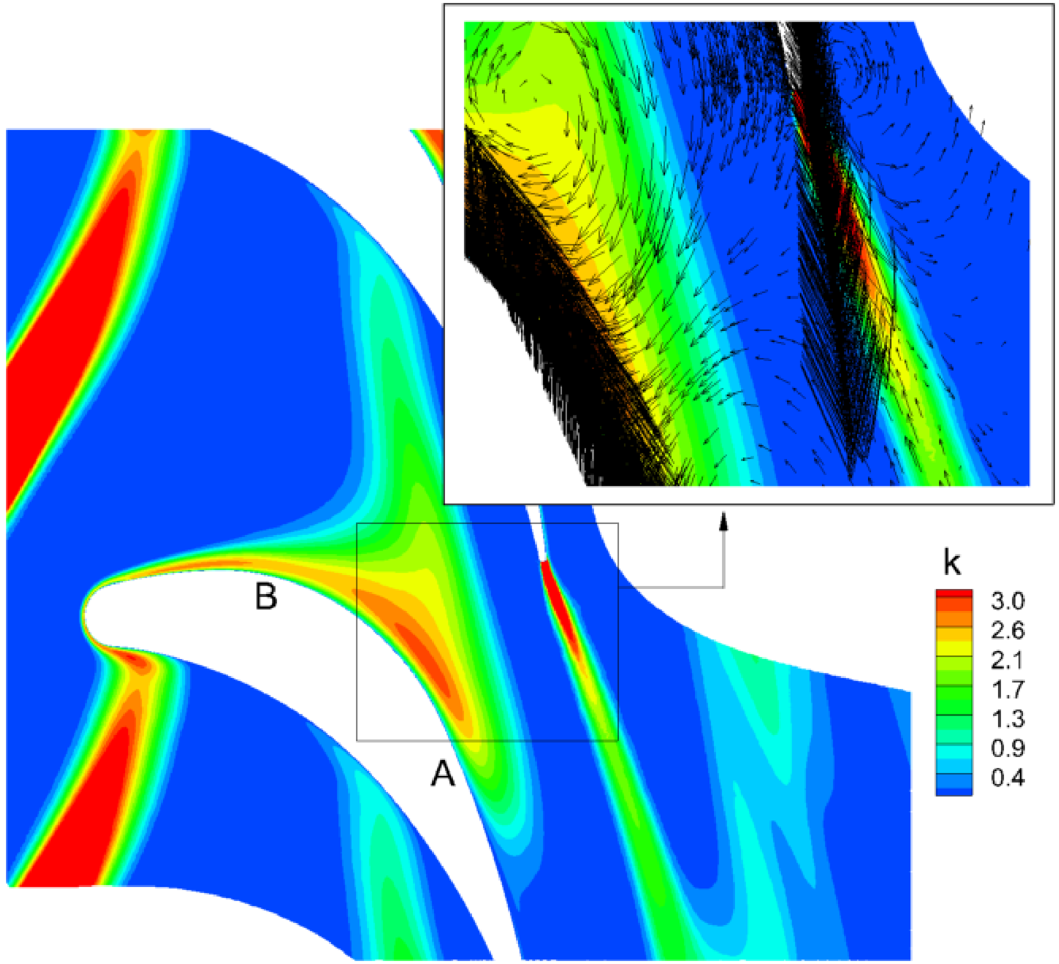
Numerical algorithms

- Transition models are developed and tuned for second-order schemes
- Use the second-order upwind scheme for discretization of convective terms in momentum equations
- Use also the second-order upwind scheme for discretization of convective terms in transport equation
- At first step the first-order upwind schemes are also acceptable in some cases for disctetization of transport equations

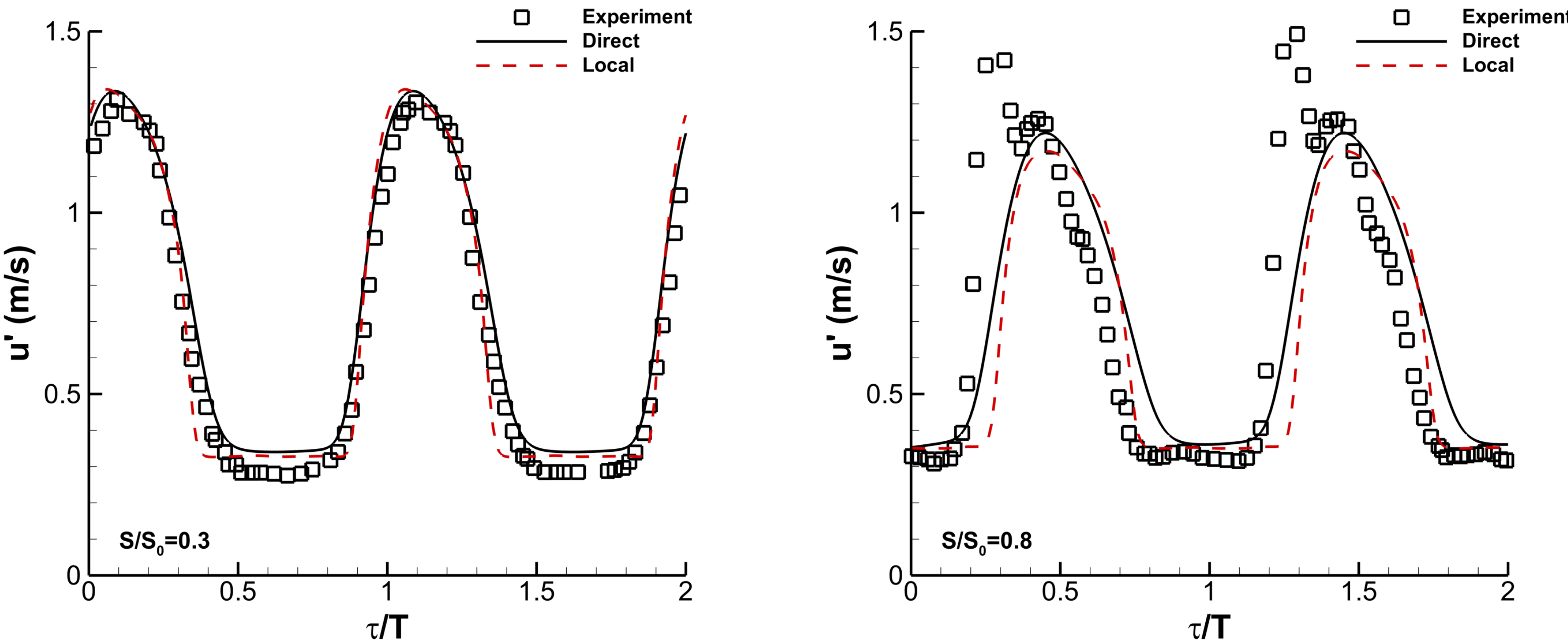


Results for wake-induced transition

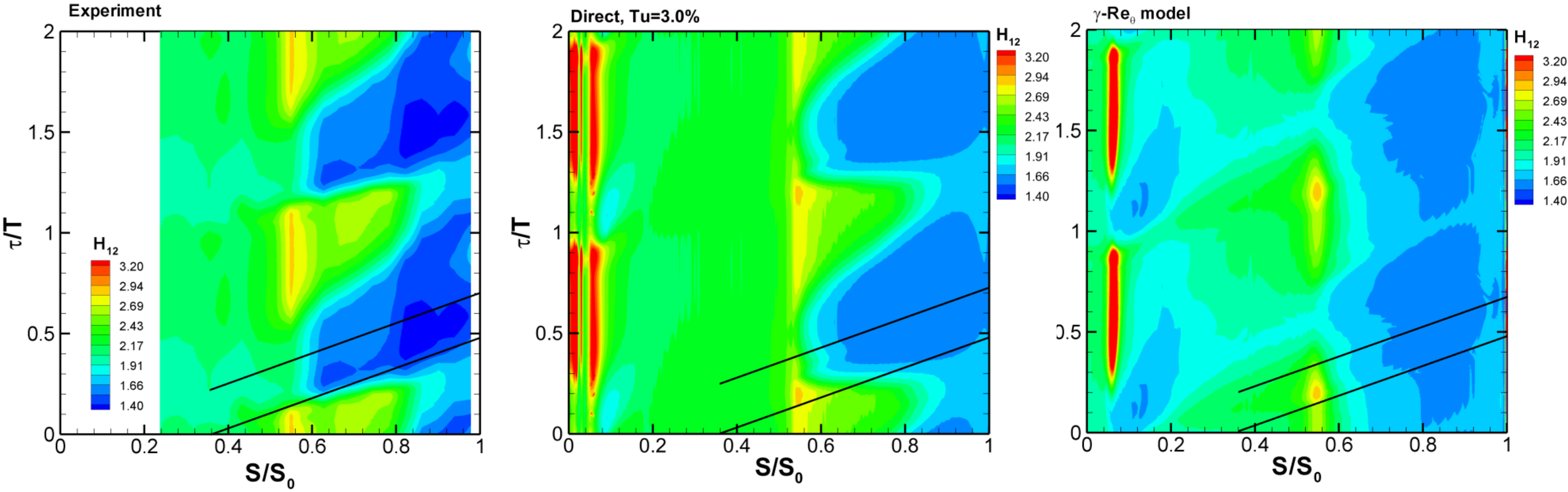
Jet effect



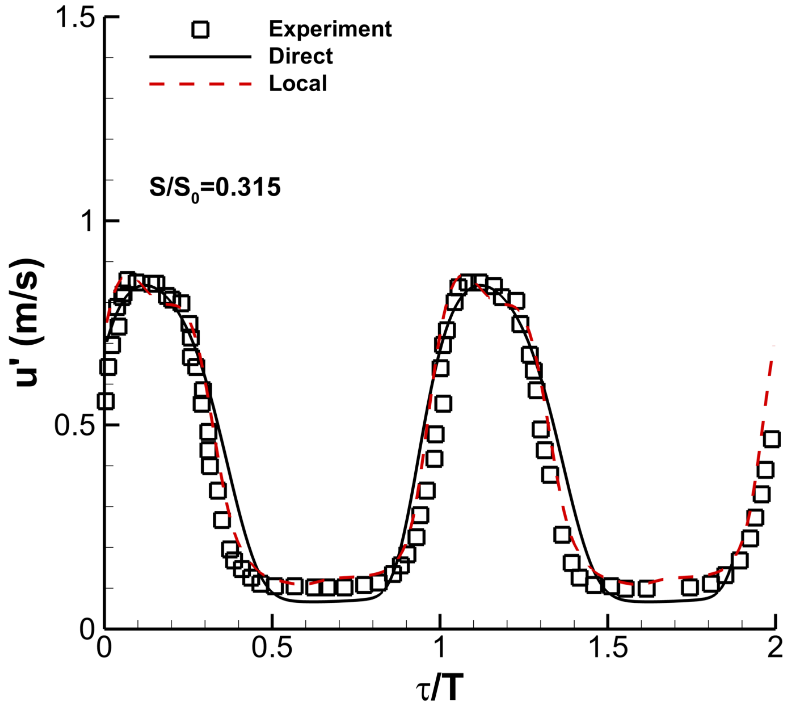
Direct Correlation vs. γ-Re_θ model (high turbulence level, Tu=3.0%)

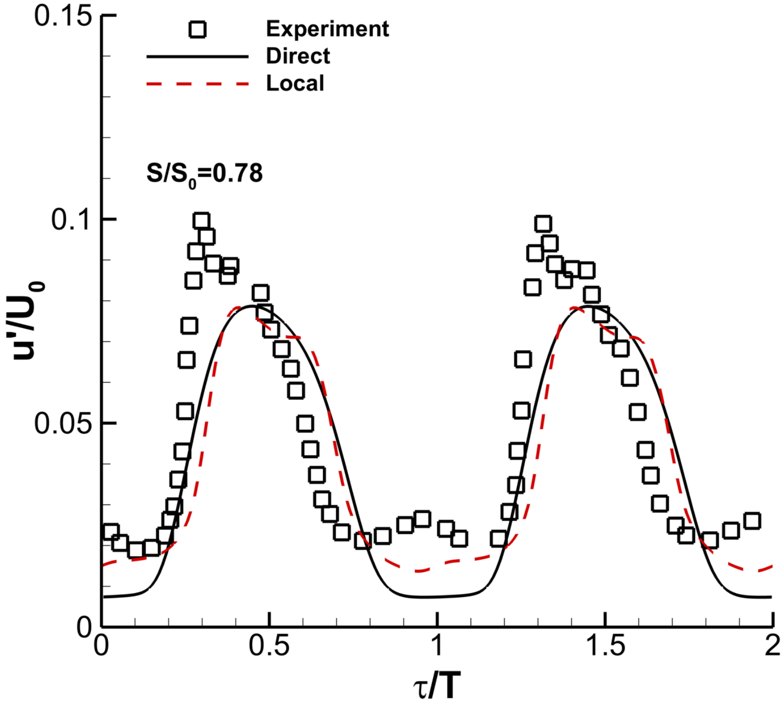


Direct Correlation vs. γ-Re_θ model (high turbulence level, Tu=3.0%)

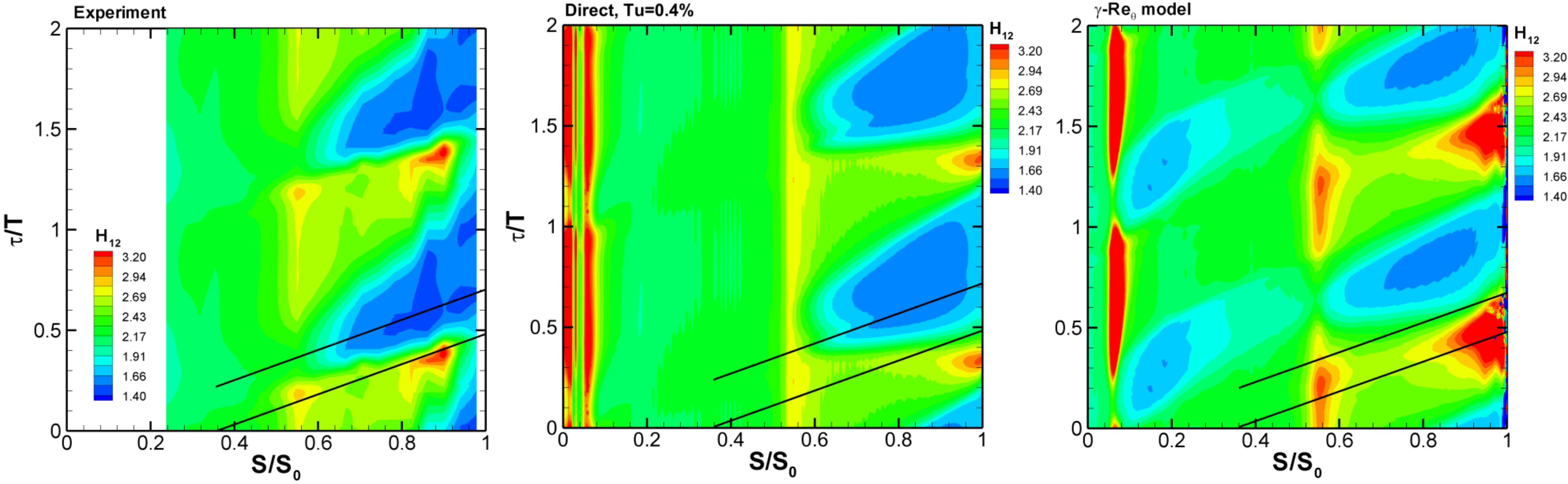


Direct Correlation vs. γ-Re_θ model (low turbulence level, Tu=0.4%)





Direct Correlation vs. γ-Re_θ model (low turbulence level, Tu=0.4%)



Final remarks:

- The transition models are very sensitive to inlet turbulence level and inlet turbulent length scale
  - The boundary conditions for turbulence quantities have to be correctly specified
  - It is recommended (if possible) to check the evolution of freestream turbulence with reference experiments/DNS/LES
- In some cases the predictions of transition models are strongly limited by shortcomings of underlying turbulence model (insufficient turbulence production in the freestream – moving wakes)
- High quality grids are required ($y^+=1$)
- Transition models are developed to improve the global flow characteristics – less accurate in capturing the local flow details (kinematic effect of impacting wake)

Final remarks:

- More standard direct methods are still reliable techniques, but require local operations