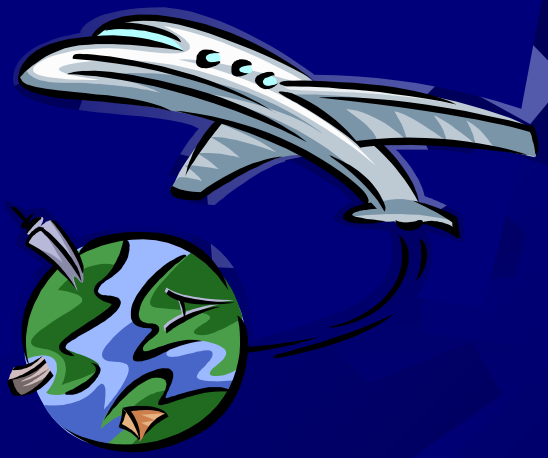


# RISK and RELIABILITY in AVIATION

Marek Matyjewski

# Books



2009

**Oficyna Wydawnicza Politechniki Warszawskiej**

2016

# Contents of the course:

- Introduction
- Risk definitions and measures
- Reliability
- Human Reliability
- Risk analysis

# Safety engineering

# M-T-E system

**M**an

**T**echnology

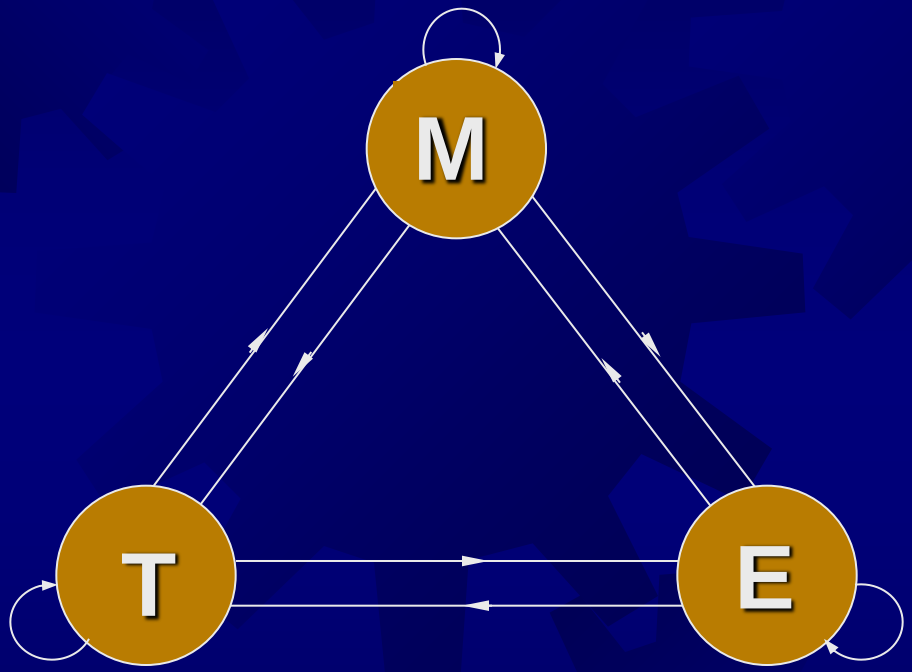
**E**nvironment

**System** – an integrated composite of people, products, and processes that provide a capability to satisfy a stated need or objective.

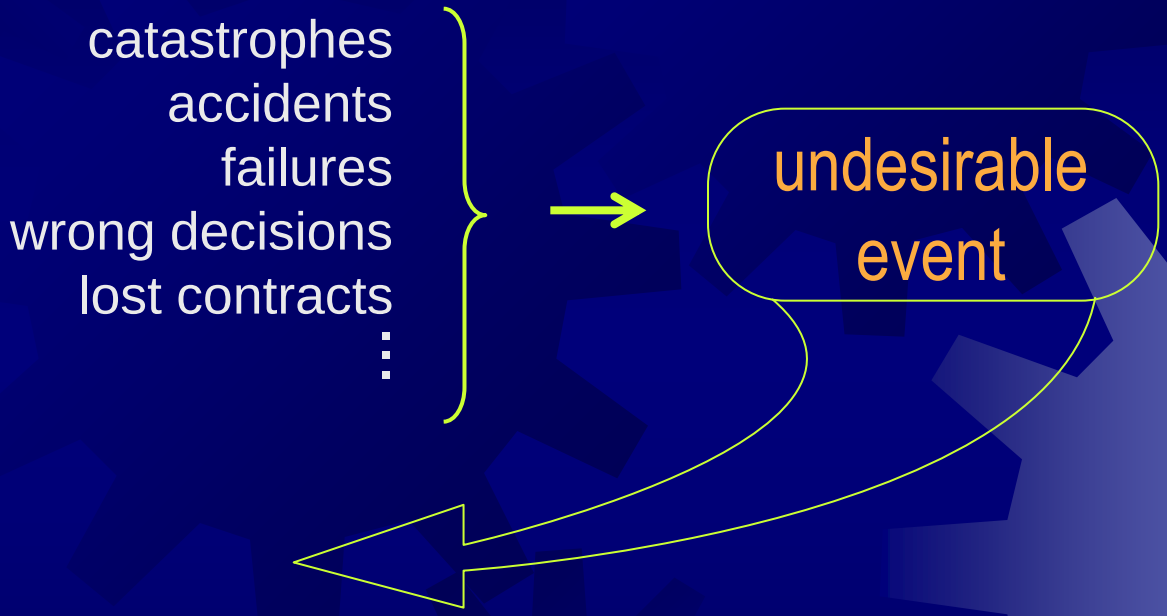
(MIL-STD882D)

# Loss in M-T-E system

*C-T-O*




# Categories of loss (consequences)




**Loss:**

**human (health & life**)
material (possessions, buildings)
national heritage
environment
opinion and trust
financial

# Loss

- Individual
  (probability of loss or injury)
- Collective
  (number of casualties)

**Example:**

**Individual loss** – probability of death in traffic accident in Poland (2014) $8 \cdot 10^{-5}$

**Collective loss** – 3202 fatalities

# FAR

## **F**atal **A**ccident **R**ate

Measure of the risk level

**FAR** is statistically expected number of accidental deaths per 100 million ($10^8$) exposed hours

1400 persons lifetime work

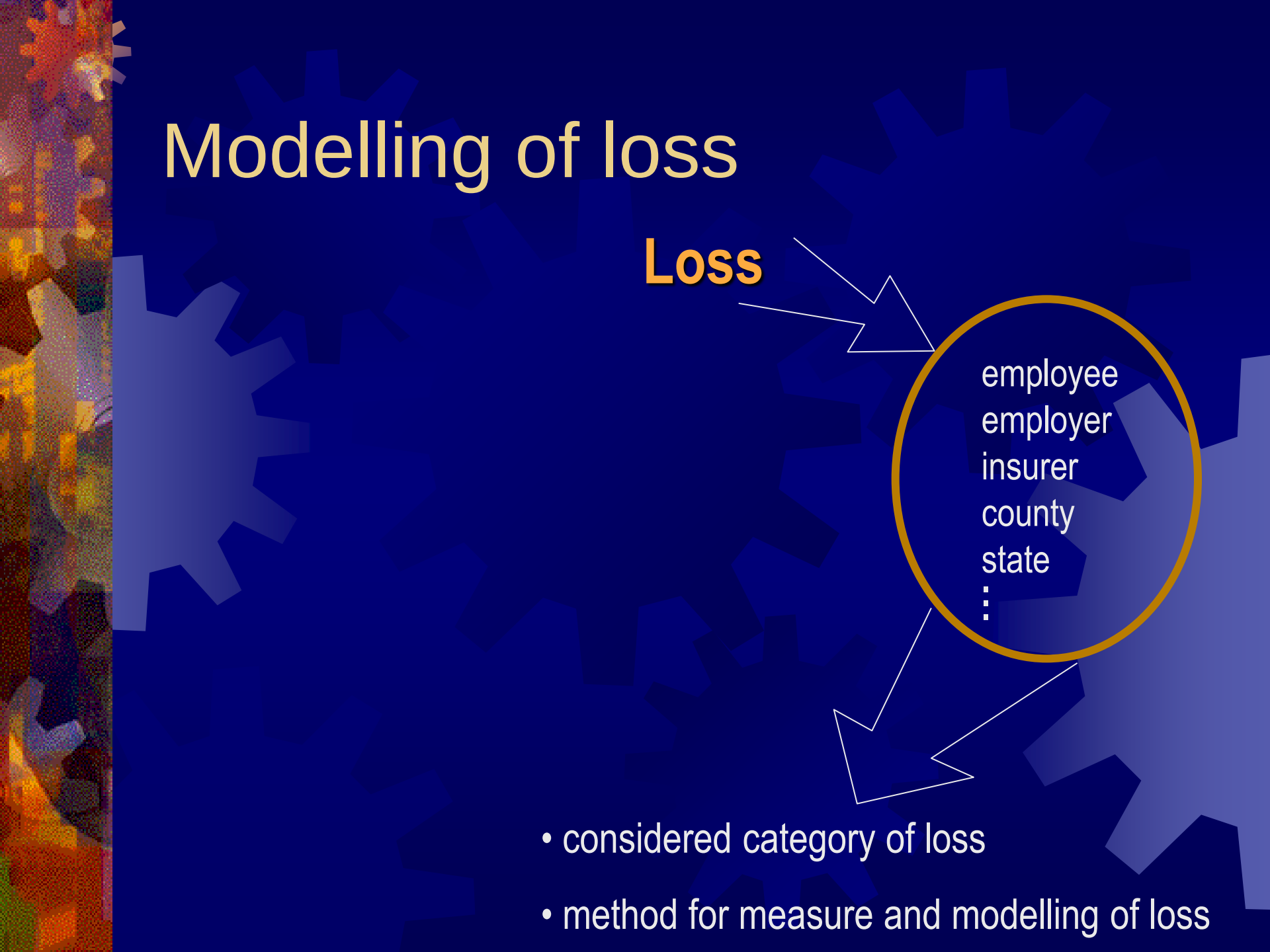
# Observed FAR

Number of fatalities per 100 million person-hours
for different activities, 1981-1986 Persons in the age range 15-69 years

| Activity | Observed FAR |
|---|---|
| ALL OCCUPATIONAL ACTIVITIES | 2.5 |
| ALL TRAVELLING | 27.0 |
| TIME SPENT IN THE HOME | 1.0 |
| ACTIVITIES ELSEWHERE | 8.2 |
| ALL DISEASES | 44.0 |
| SUICIDE | 2.0 |

| Activity | Observed FAR |
|---|---|
| ALL OCCUPATIONAL ACTIVITIES | 2.5 |
| Agriculture and forestry | 2.3 |
| Fishing | 63.0 |
| Oil activities | 19.0 |
| Industry | 1.1 |
| Service | 1.5 |
| Railway transport | 6.2 |
| Shipping | 11.0 |
| Aviation | 50.0 |
| ALL TRAVELLING | 27.0 |
| Travelling on roads | 27.0 |
| Pedestrians | 14 |
| Cyclists | 28.0 |
| Motorcyclists | 280.0 |
| Car drivers | 25.0 |
| Car passengers | 29.0 |
| Travelling by train | 4.7 |
| Travelling by ship | 9.1 |
| Domestic air services | 56.0 |

# Modelling of loss




- considered category of loss
- method for measure and modelling of loss

undesirable event

human losses

financial losses

fatalities

harmed

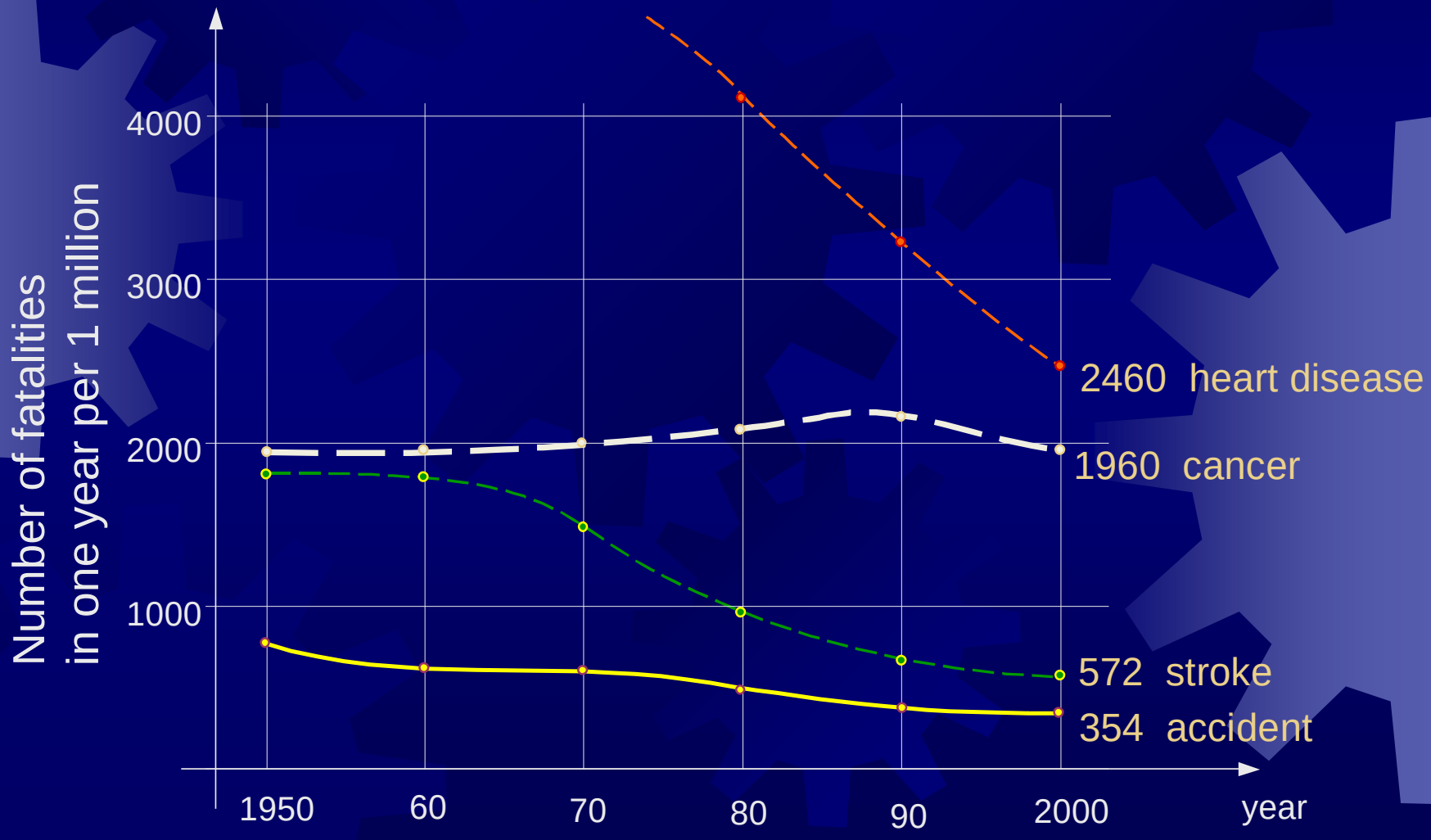
# Number of victims

| Kind of accident | Number of fatalities in 1 year per 1 million inhabitants | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Sweden | | Great Britain | | Germany | | Poland | | | |
| | 1985 | 1998 | 1984 | 1998 | 1987 | 1998 | 1989 | 1994 | 1999 | 2000 |
| At home | | | 100 | | 120 | | | | | |
| On roads | 98 | 33 | 100 | 59 | 120 | 92 | 152 | 178 | 174 | 163 |
| Travelling by air | 3,0 | | 1,2 | | | | 0,4 | 0,2 | | |
| Travelling by train | | | 1,7 | | | | | | | |
| At work | 11 | 10 | 23 | 3 | 18 | 16 | 26 | 25 | 20 | 18 |
| Suicides | | | 90 | | | | | 143 | 131 | 128 |
| Fires | 12 | | 13 | | | | | 14 | 14 | 13 |
| Murders | 7 | | | | | | | 30 | 28 | 33 |
| Drownings | 13 | | 6 | | | | 10 | 27 | 21 | 14 |
| Lightning struck | 0,1 | | 0,1 | | | | | | | |
| Disease from tobacco use | | | | | | | | | | 2200 |

# Average losses in 1992-2011

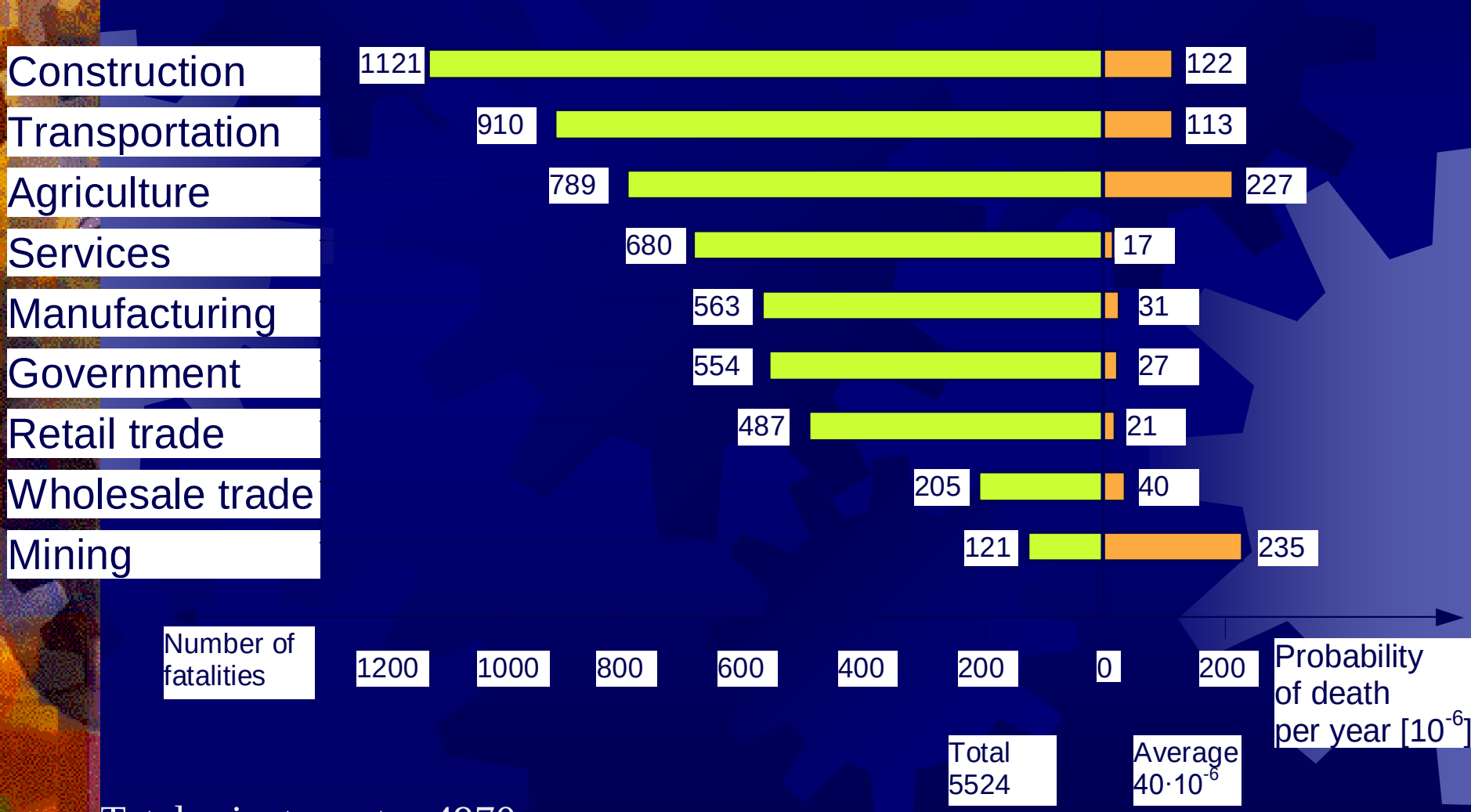
| Type of loss measure | Natural Disasters | | Technical Disasters | | Natural & Technical (combined) | |
|---|---|---|---|---|---|---|
| | World | Europe | World | Europe | USA | PL |
| Number of fatalities in one year per 1 million inhabitants | 13 | 10 | 1,4 | 1,0 | 1,8 | 2,7 |
| Number of victims in one year per 1 million inhabitants | **36 000** | 2120 | 17 | 8,6 | 85700 | 460 |
| Number of fatalities in 1 accident | 250 | 166 | 33 | 27 | | |
| Number of victims in 1 event | **700 000** | 30 000 | 415 | 200 | | |

# Number of fatalities




TOYOTA
KOMATSU
MATRIA 50

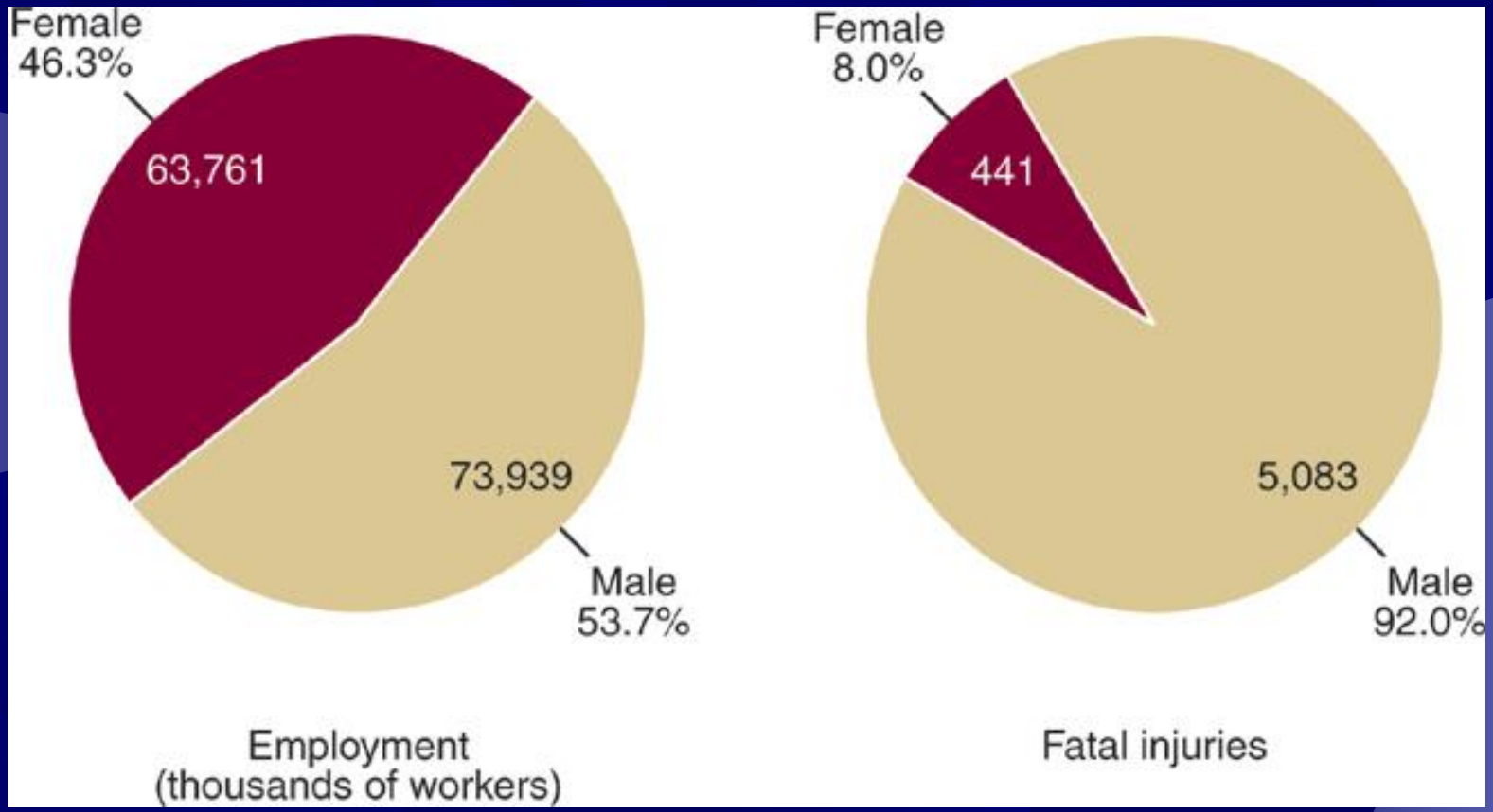
# Distribution and number of fatal occupational injuries by industry sector




Total private sector 4970

# Employment and fatality profiles by sex

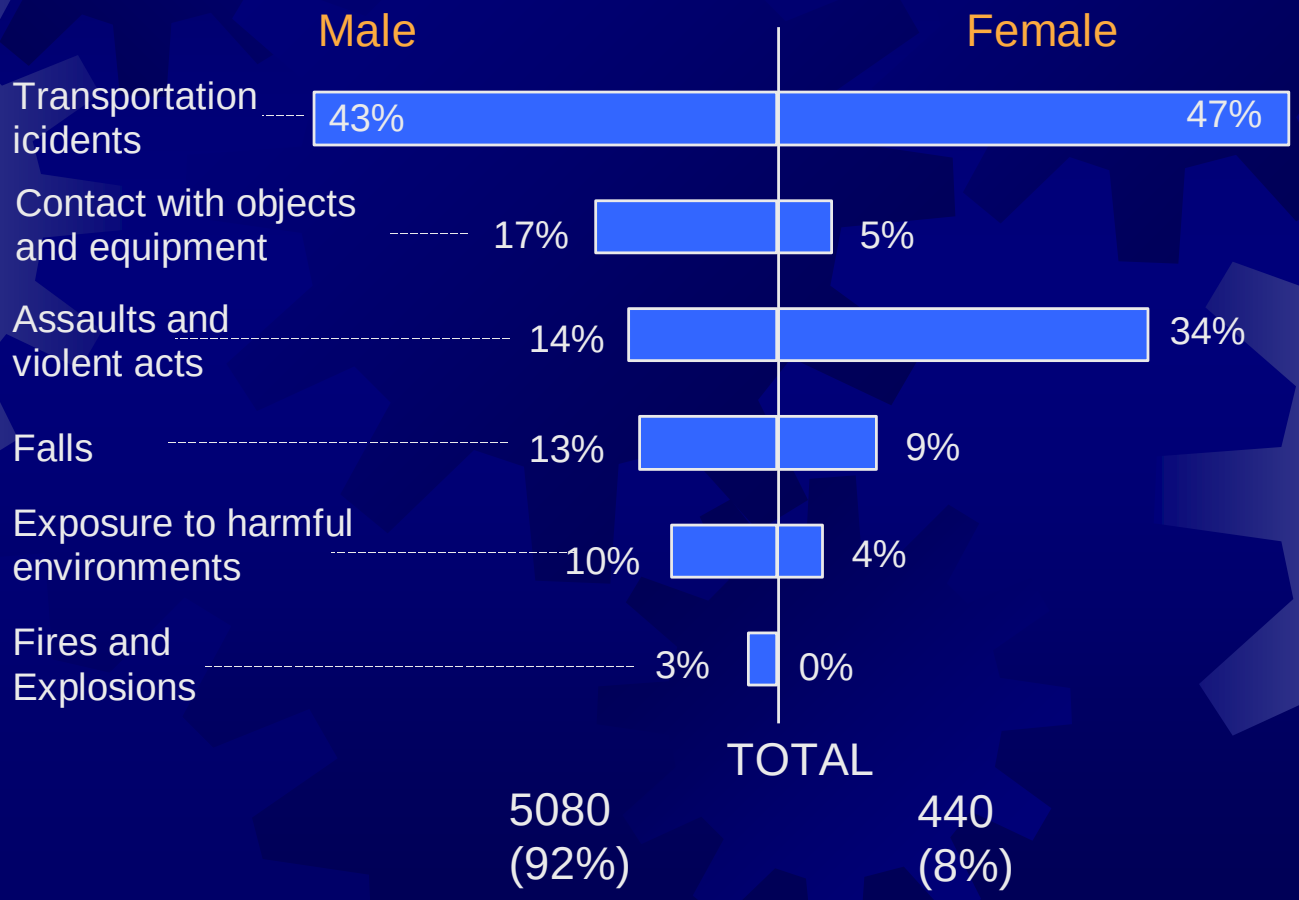
2002




estimated 138 million jobs

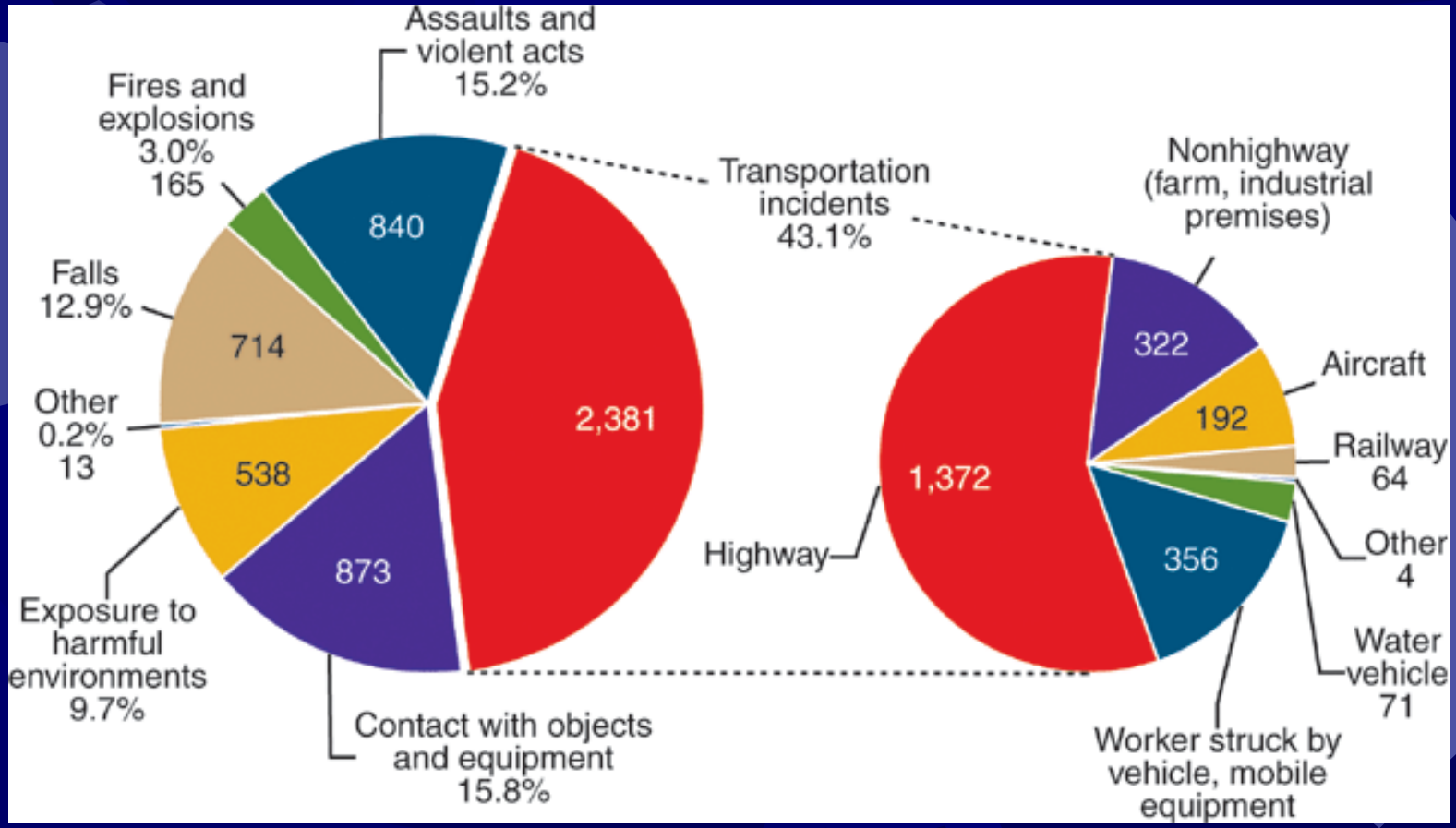
# Distribution of fatal occupational injuries by sex of worker and event or exposure

2002



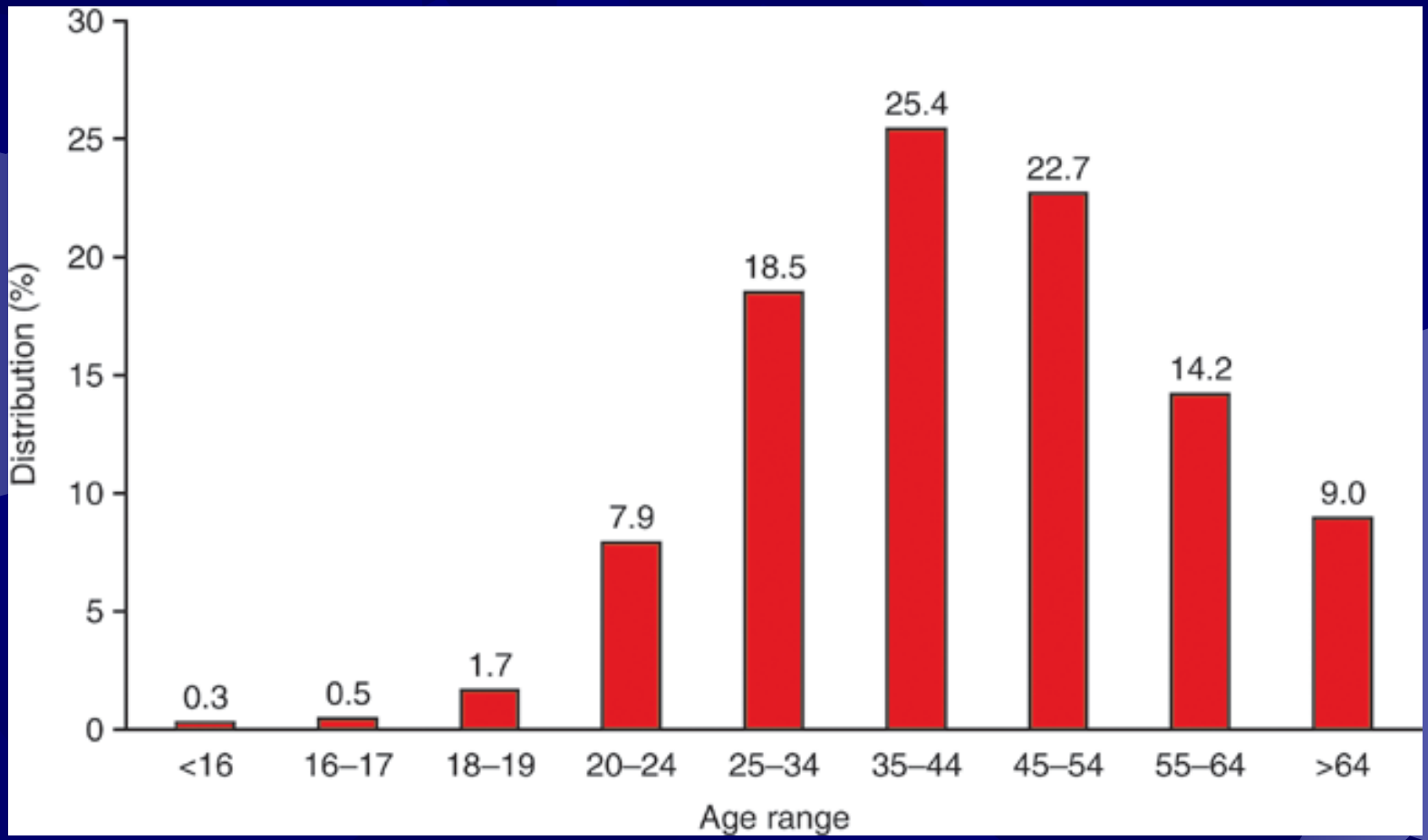

NIOSH National Institute for Occupational Safety and Health

# Distribution and number of fatal occupational injuries by event or exposure




highway fatalities accounted for 24.9% of all occupational fatalities

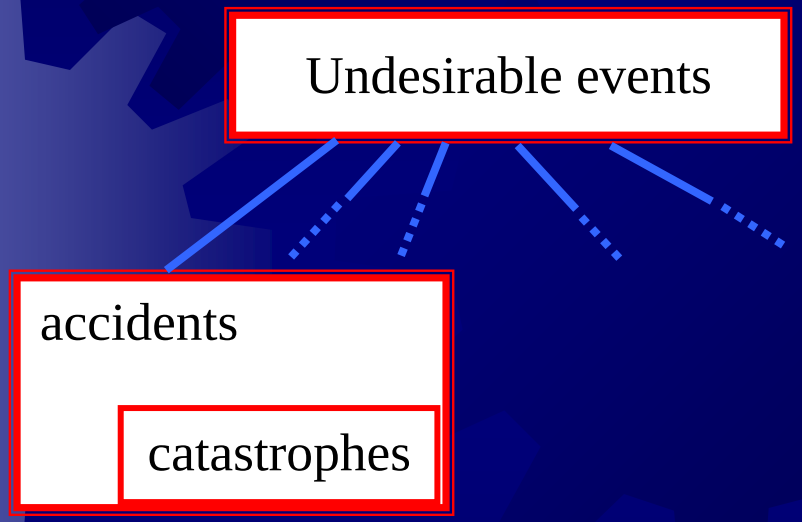
# Distribution of fatal occupational injuries by age of worker 2002




Two-thirds of all fatally injured workers were aged 25–54

NIOSH National Institute for Occupational Safety and Health

# Consequences

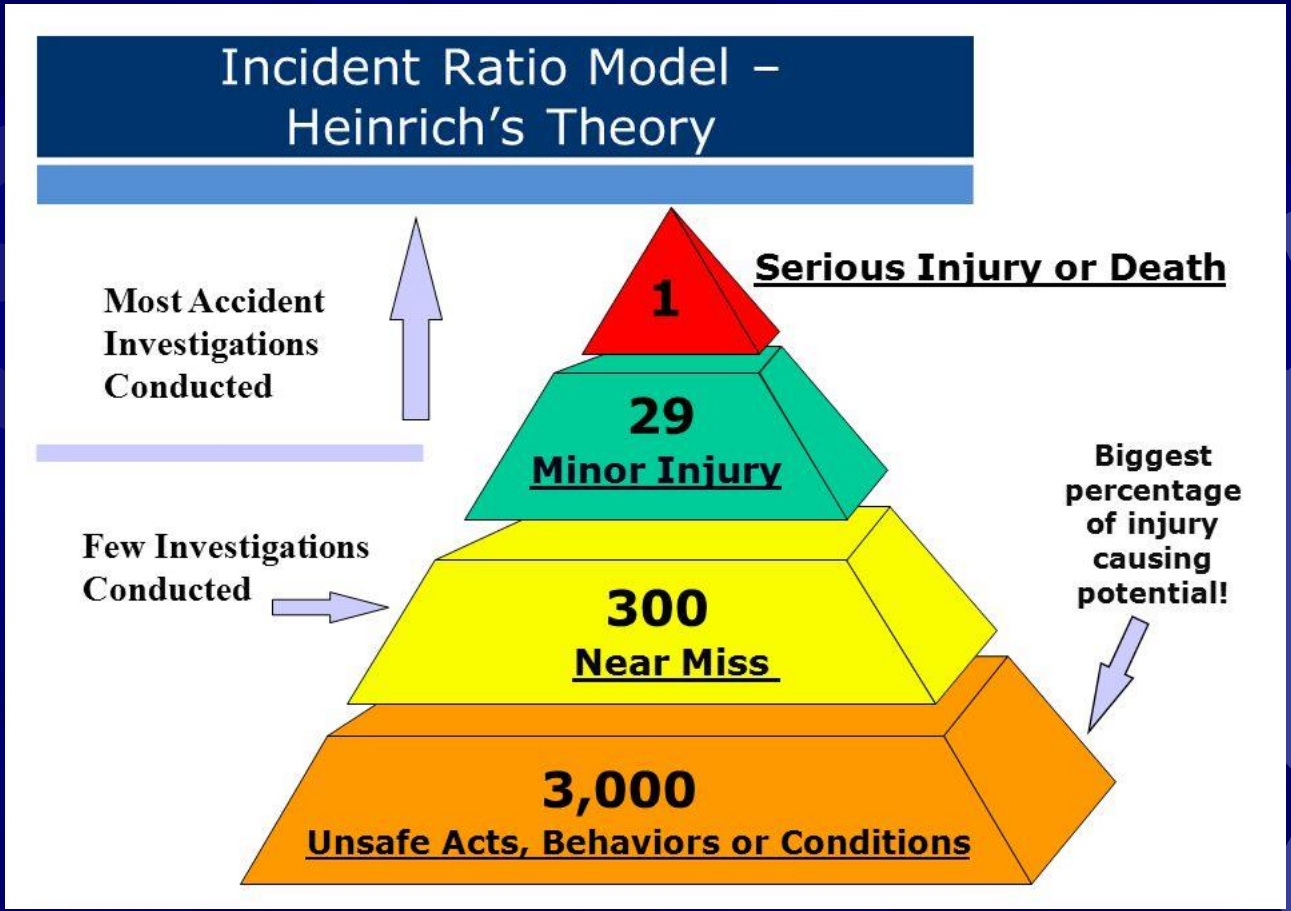




**Accidents** – frequent events, small loss

**Disasters** – rare events, big loss

# Heinrich's Pyramid

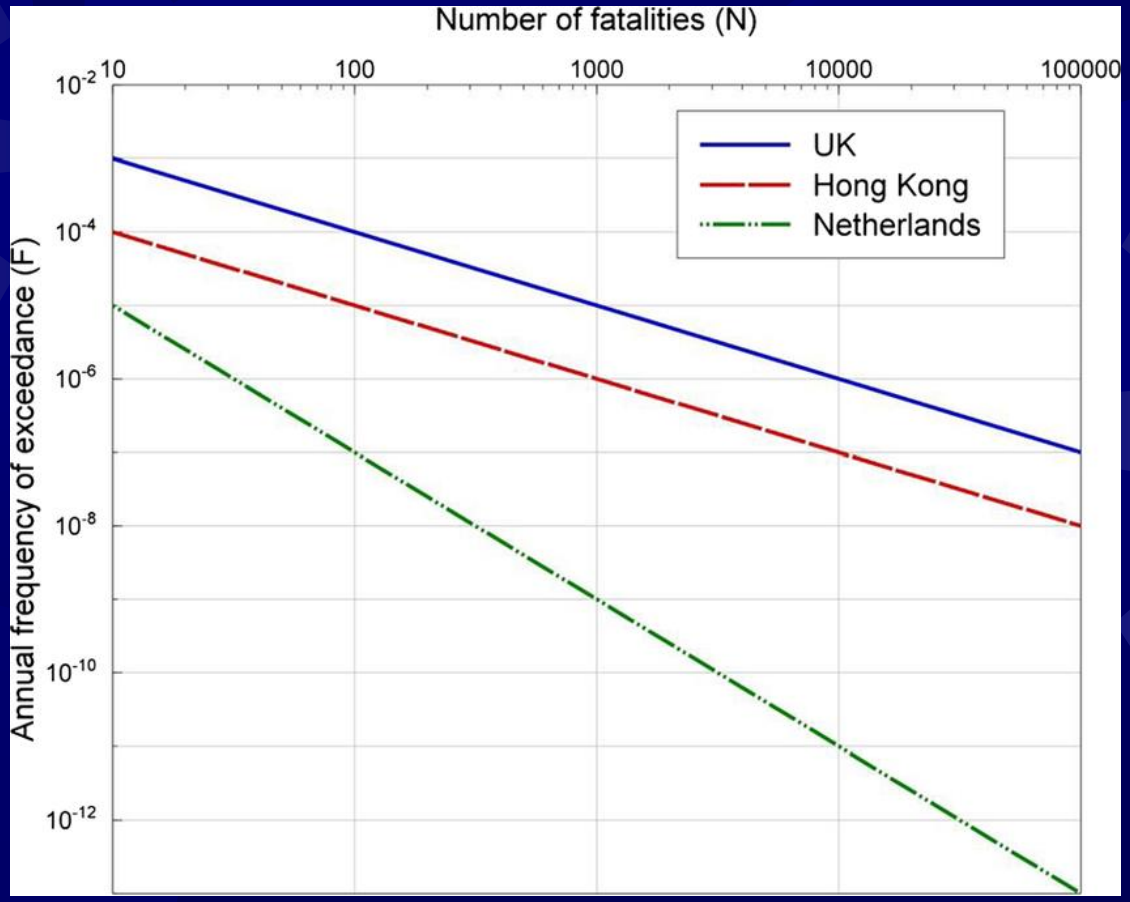


# Measure of human losses

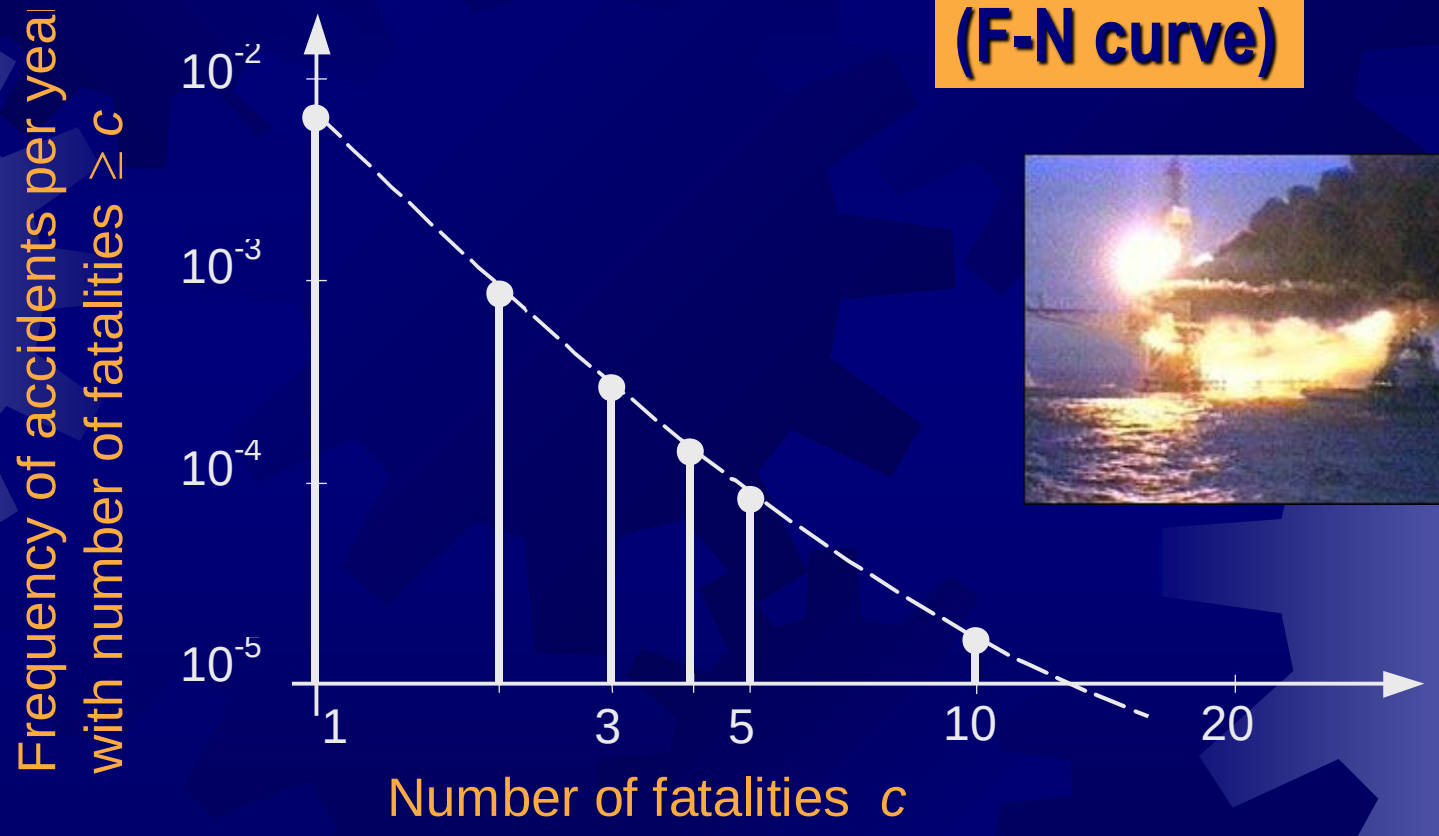
- mean values
- values taking into account a scatter of random losses

# F-N curve

A plot of cumulative frequency versus consequences.
Often expressed as number of fatalities.



# Loss of life on an offshore platform



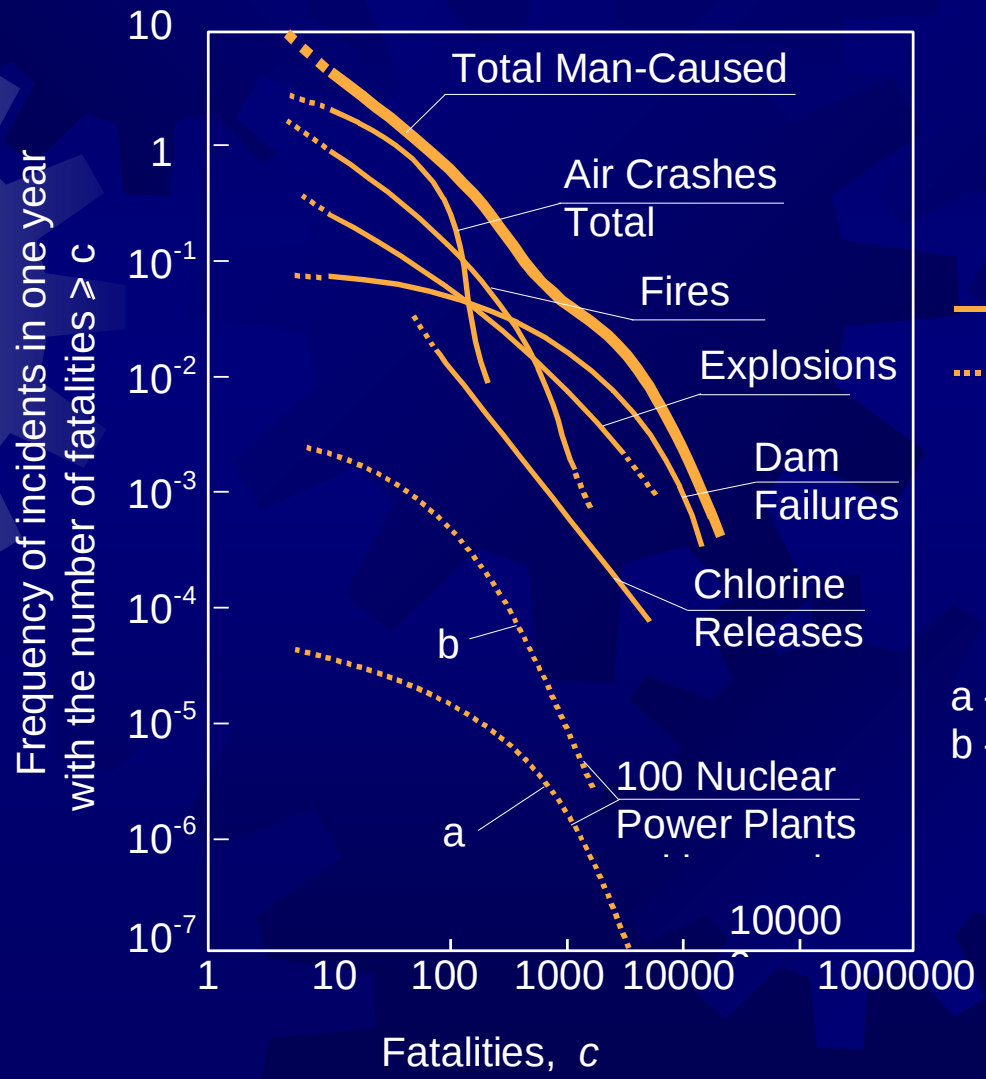

**(F-N curve)**



**F** – accident frequency

**N** – loss of life

# Human loss due to technical disasters



# Workplace Injury in 1996

**Rates of Fatal and of Over 3 day injury per 100,000 workers**

| Country | Rate of Fatal Injury | Rate of Over 3 day injury |
|---|---|---|
| **USA** | **2.7** | **3000** |
| **EU Average** | **3.6** | **4200** |
| Belgium | 5.5 | 5100 |
| Austria | 5.4 | 3600 |
| Italy | 4.1 | 4200 |
| Greece | 3.7 | 3800 |
| France | 3.6 | 5000 |
| Germany | 3.5 | 5100 |
| Ireland | 3.3 | 1500 |
| Denmark | 3.0 | 2700 |
| Netherlands | 2.7 | 4300 |
| Sweden | 2.1 | 1200 |
| Great Britain | 1.9 | 1600 |
| Finland | 1.7 | 3400 |

undesirable event

human loss

financial loss

employer

insurer

accident victim

taxes, prices, ...

loss sustained by society

**Social costs of accident** (event)

# Costs of job accidents

**(1÷3)% GNP** (Gross National Product)

- **in Poland**
  (7÷21) billion zł (PLN)
  115 thousand accidents (2001 r.)
  average cost of an accident: ~ 87,000 zł
- **in USA** → $27,000
- wide range of financial loss
- cost of fatal accident >> cost of non-fatal accident

# Estimated Number and Medical Costs of Occupational Injuries 2007

| Category | Number and Percentage (of column) of Injuries | Total Medical Costs (in $billions) and Percentage (of column) | Average Medical Costs per Injury |
|---|---|---|---|
| Nonfatal injuries | | | |
| Injuries with no days away from work | 6,084,086 (71.0%) | $5.69 (12.3%) | $935 |
| Injuries with 1 to 4 days away from work | 934,049 (10.9%) | $0.87 (1.9%) | $935 |
| Temporary total disabilities | 1,020,181 (11.9%) | $8.21 (17.7%) | $8,046 |
| Permanent partial disabilities | 512,438 (6.0%) | $25.58 (55.3%) | $49,925 |
| Permanent total disabilities | 8208 (<0.1%) | $5.59 (12.1%) | $681,615 |
| Total for nonfatal injuries | 8,558,962 (99.9%) | $45.95 (99.3%) | $5,369 |
| Fatal injuries | 5657 (<0.1%) | $0.31 (0.7%) | $55,595 |
| Total for nonfatal and fatal injuries | 8,564,619 | $46.26 | $5,401 |

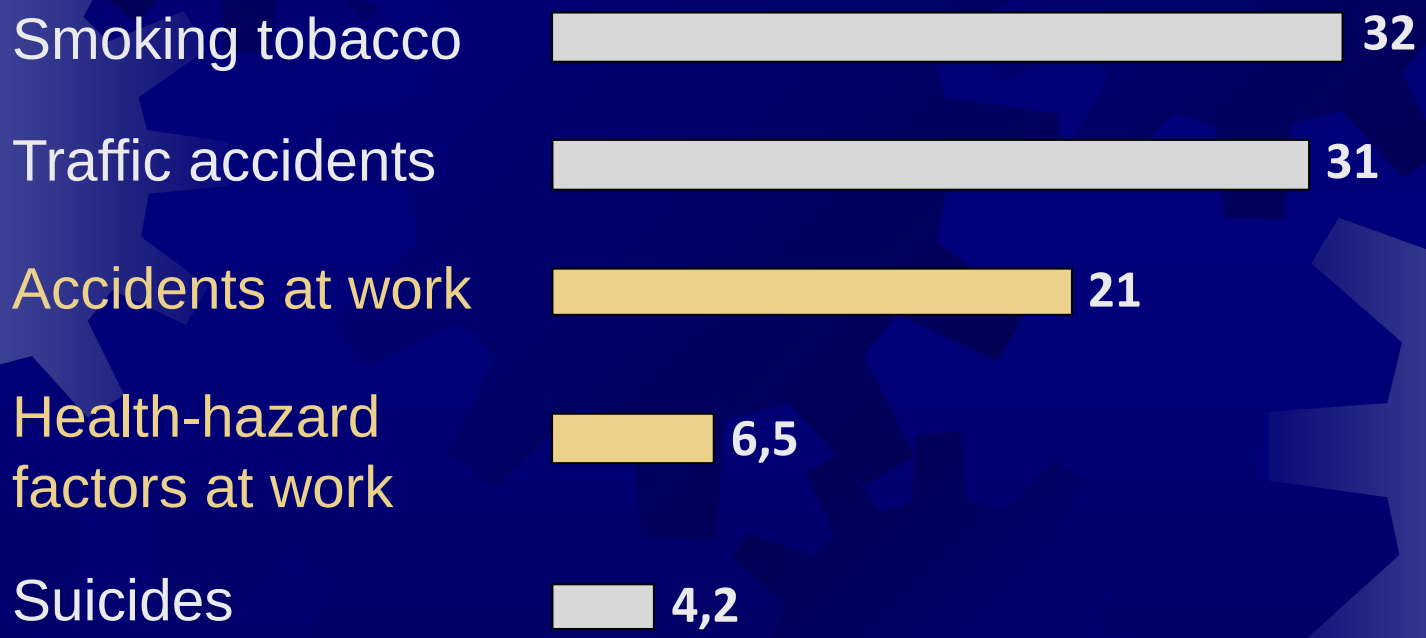
*Sources:* Biddle 2009; NCCI 2008; U.S. Bureau of Labor Statistics 2010a, 2010b, 2010c, 2010d, 2011a.

# Accidents costs

In USA costs of all accidents exceed

**10% of GNP**

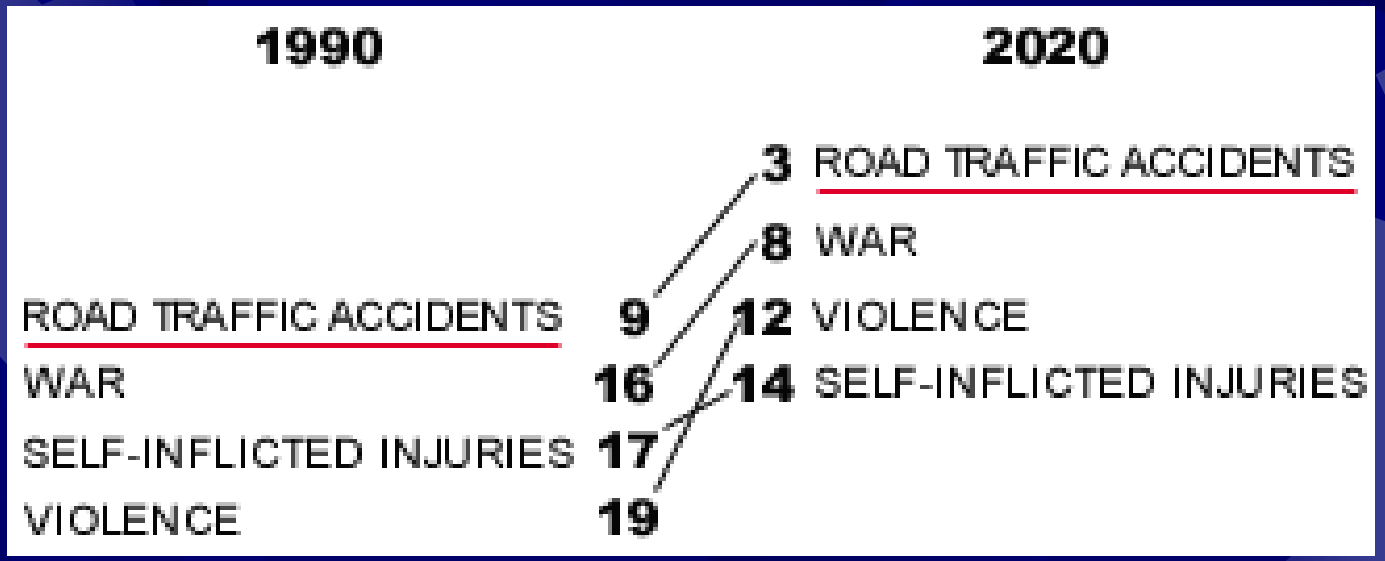
# Social financial losses in Poland (social costs) in billion PLN



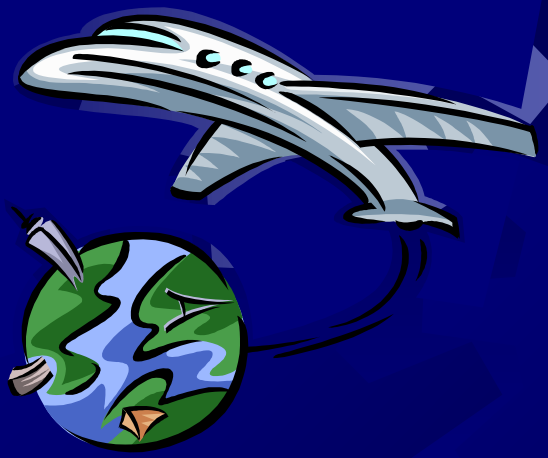

Cost estimates based on US indicators

# The Transport Apocalypse

DALYs (disability-adjusted life years)



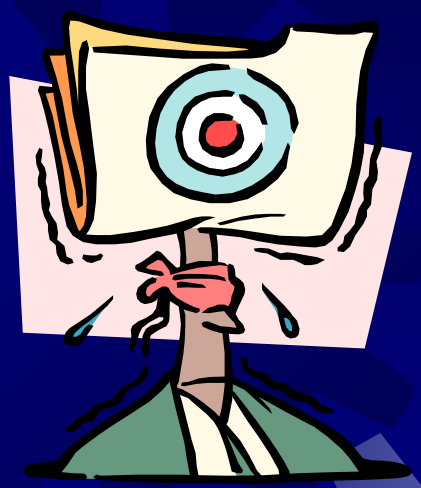

**Projected change in the ranking of the 15 leading causes of death and disease (DALYs) worldwide, 1990-2020** - source WHO "The Global Burden of Disease"



# RISK AND RELIABILITY IN AVIATION

Definitions

# Basic terminology



**Human imperfection**

Limited knowledge

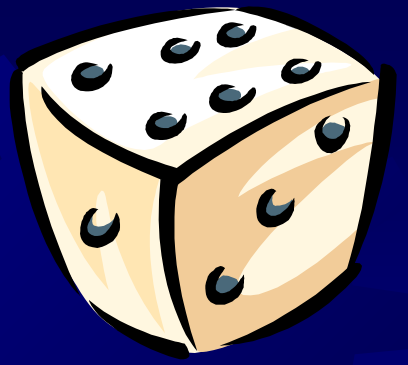
**Randomness** of incidents and undesirable events

**Randomness** of injuries and losses

Safety science

Concept of risk

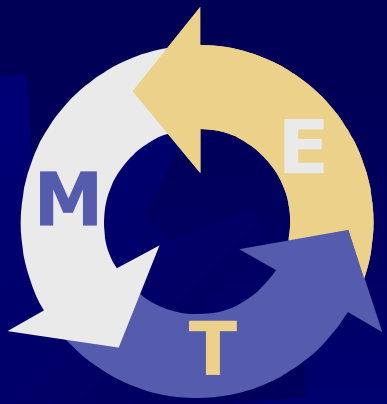
# Basic terminology



**To be conscious of variability (randomness)**
**To take variability into consideration**

- in risk analysis
- in accident investigation
- in safety management
- in definition of terms and their interpretation

# Undesirable events




T ⇒ failures, breakdowns, malfunctions

M ⇒ errors

E ⇒ globally – natural disasters:
hurricanes, earthquakes, floods, etc.

climate change (greenhouse effect)

**in a workplace:** noise, dust, etc.

# Undesirable events

(abnormal events)

(initiating events)

**Undesirable event** is an event, which occurrence, in the considered **M-T-E** system, could result in hazard exposure for humans or property

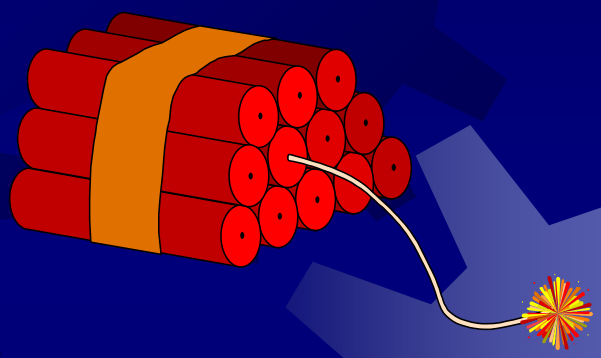
***Zdarzenie niepożądane*** *jest to zdarzenie, którego zajście w rozpatrywanym systemie C-T-O wywołuje w efekcie zagrożenie dla chronionych dóbr*

Occurrence of an **undesirable event**, if not correctly responded to, may lead to loss or injury

# Harmful consequences

**Main areas of loss and damage:**

- human health and life,
- natural environment,
- public or private property.



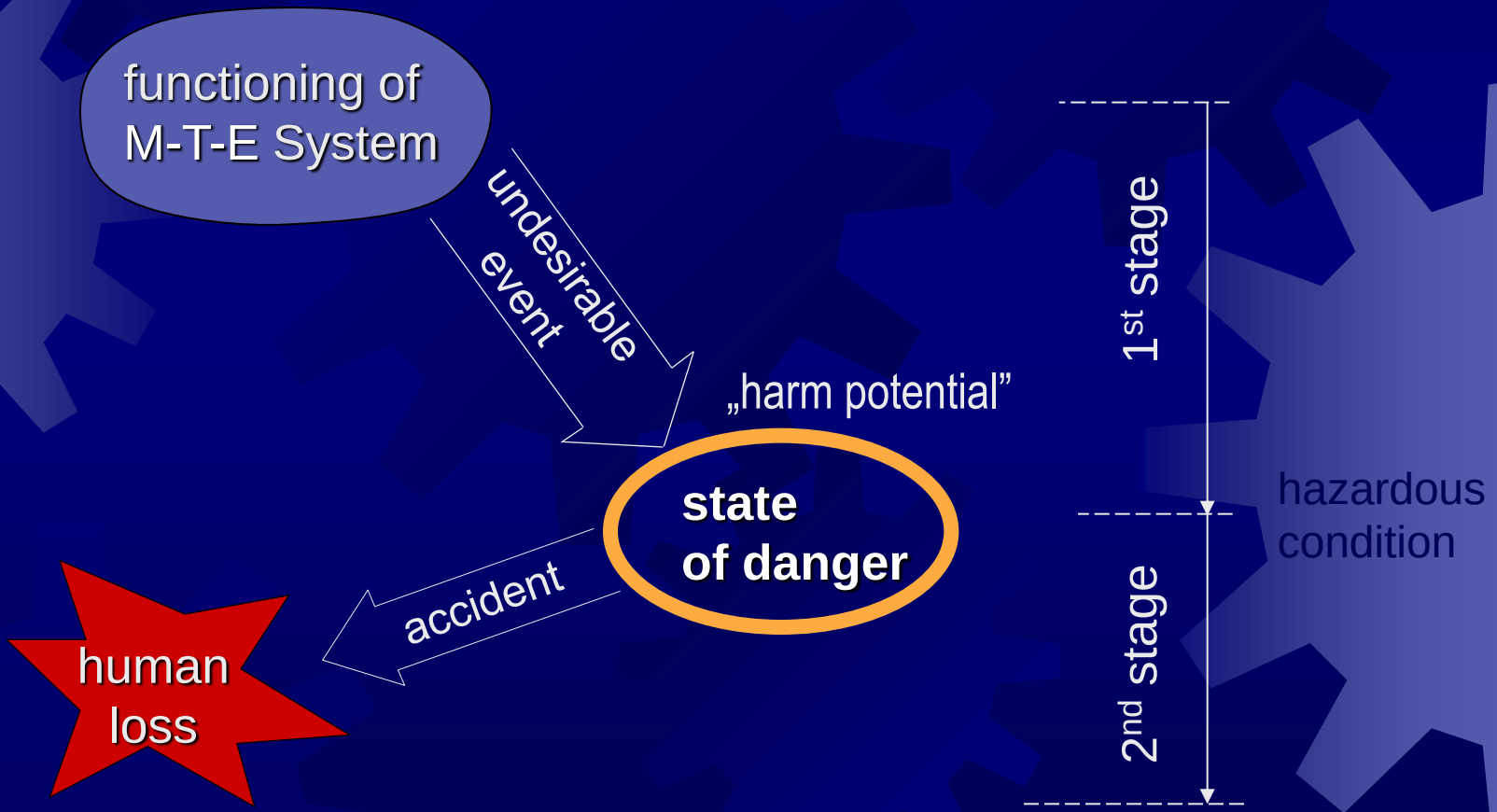
**Main categories of loss:**

- **human loss**, i.e. death or injury,
- **financial loss**, costs, loss of, or damage to property.

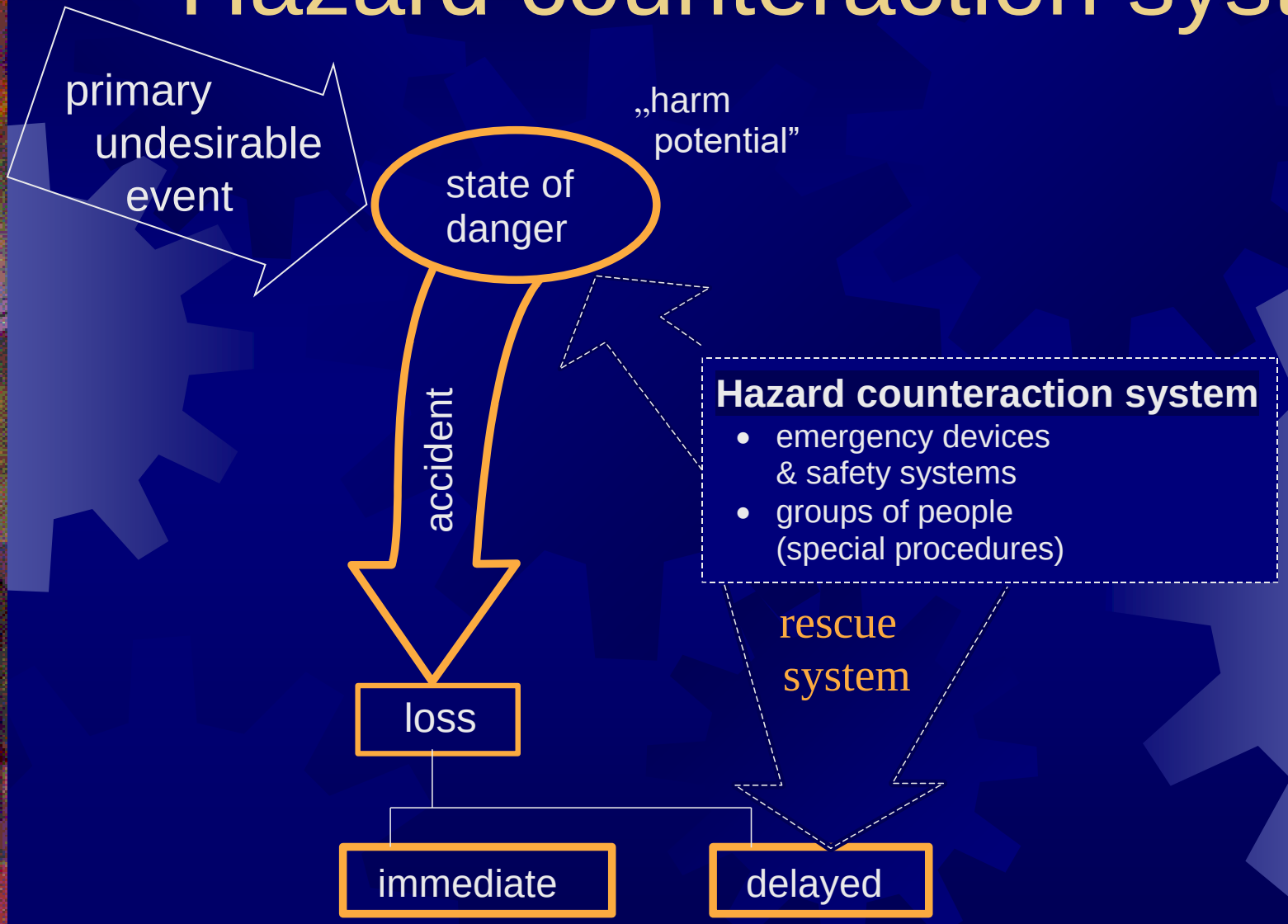
immediate loss – delayed loss

individual loss – collective loss

# Stages in development of the loss process



# Hazard counteraction system



# Hazard counteraction devices

(emergency devices)

**Personal fall-arrest system**

**Automatic Fire Extinction devices**

**Residual-Current Device (RCD)**

# Definition of risk

**Risk** is a possibility of incurring specified loss (damage)
- in a definite period of life or
- during a particular activity

due to various kinds of incidents or undesirable events, that may occure in the man-technology-environment system

***Ryzyko*** *jest to możliwość doznania przez człowieka określonych strat (szkód)*
- *podczas określonego czasu jego życia lub*
- *podczas podjętego przez niego określonego działania*

*wskutek różnego rodzaju zjawisk i zdarzeń niepożądanych, które mogą wystąpić w rozpatrywanym systemie człowiek-technika-otoczenie*

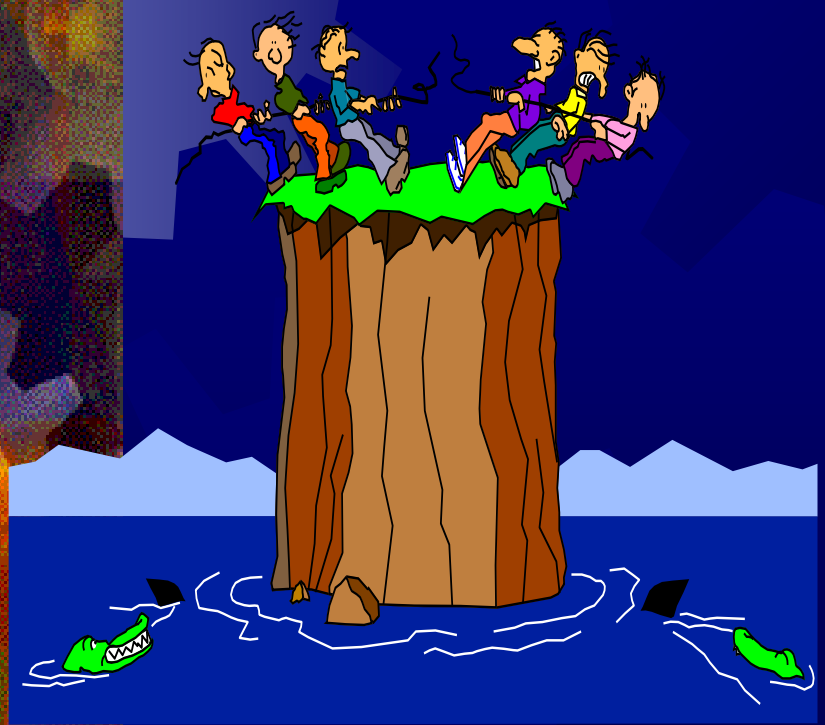
# Other Risk descriptions

**"the probability of a future loss”**

Risk is the likelihood that a harmful consequence (death, injury or illness) might result when exposed to the hazard

# Risk

Risk of human loss
Risk of financial loss
⋮



Individual risk, collective risk

**Risk measures**

**Total risk, partial risk**

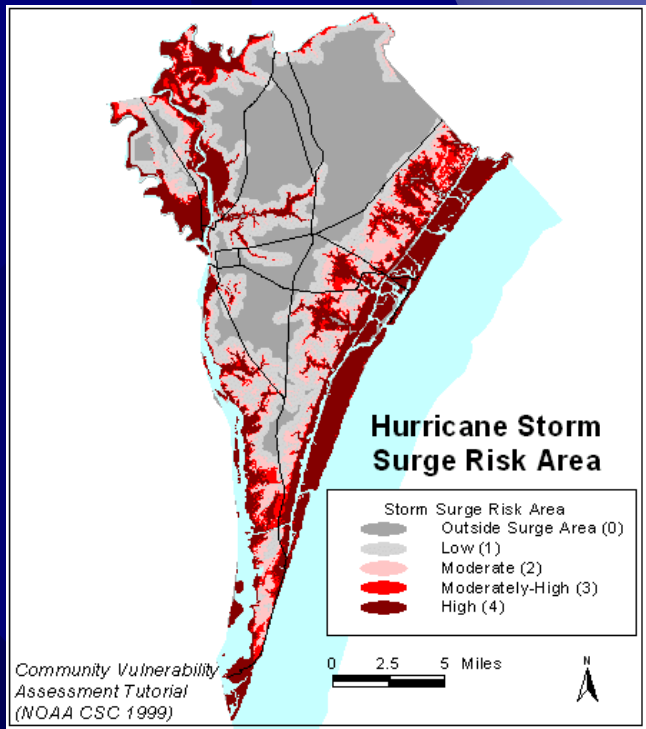
# Concept of hazard

Risk (conditional), specified for a state of the system caused by an undesirable event

**hazard**

**Hazard** is the potential to cause harm

Hazard: a condition, associated with design, operation or environment of a system that has the potential for harmful consequences



# Hazards grouped by type

- Slipping/tripping hazards
- Fire
- Chemicals
- Moving parts of machinery
- Work at height
- Ejection of material
- Pressure systems
- Vehicles
- Biological hazards
- Electricity
- Repetitive work
- Dust
- Fumes
- Manual handling
- Noise
- Poor lighting
- Extreme temperatures

# Definition of hazard

**Hazard** is a possibility of making specified loss, assigned to a situation developed after occurrence of an undesirable event in the man-technology-environment system

***zagrożenie*** *jest to możliwość powstania określonych strat, ustalana dla sytuacji powstałej po zajściu pojedynczego zdarzenia niepożądanego w rozpatrywanym systemie człowiek-technika-środowisko*

# Hazard (danger)

**hazard** (level of hazard)

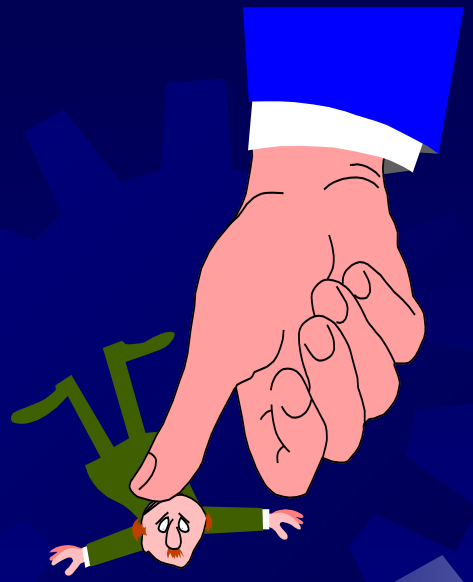
**hazards** (mechanical, fire, acoustic, etc.)

source of danger (**hazard**)

state of danger (**hazardous** condition)

(hazardous event)

# Safety ???



**Safety** ⟷ **man**

***Safety*** *is a contrary notion (an opposite concept) to the notion of* ***human loss risk***

Occupational safety ⟷ protection of human health and life in the workplace

# Safety management

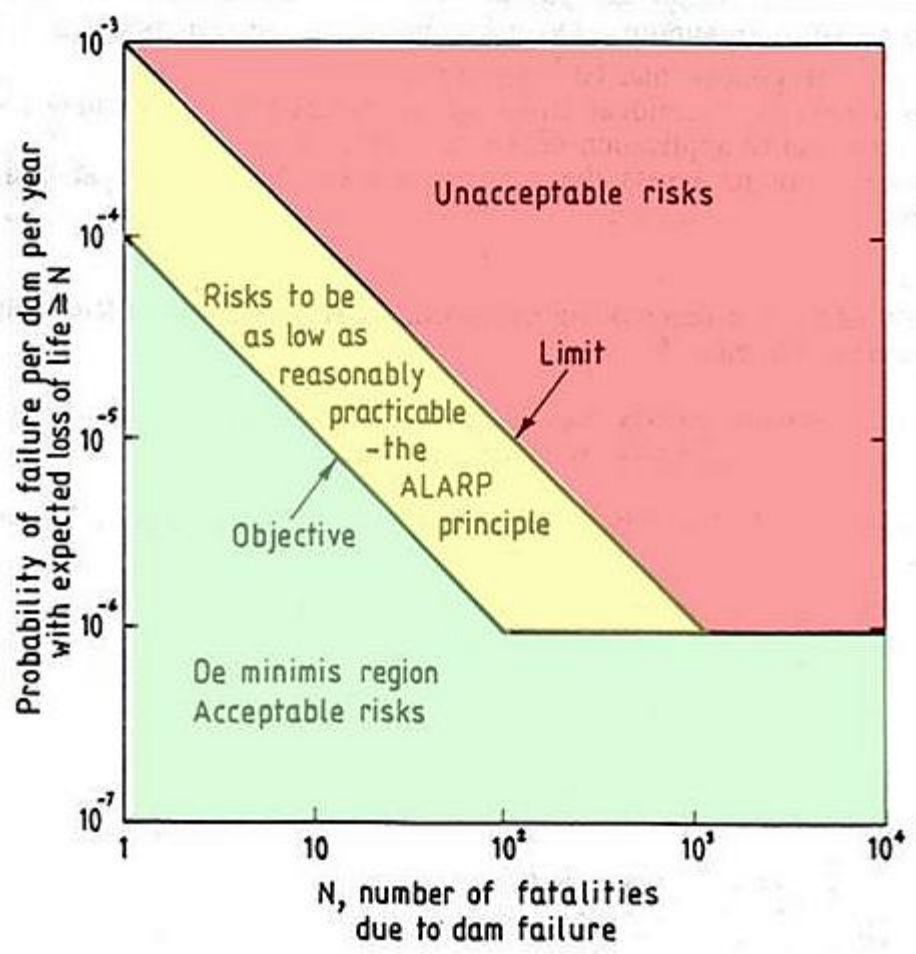
**decisions**

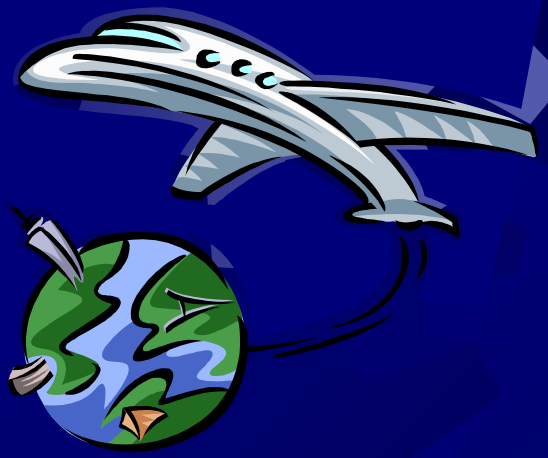
Risk analysis

Cost-benefit analysis

Risk analysis – a  basis for rational activities in safety domain

# Safety management





# RELIABILITY and SAFETY

Measures

# Measure of Risk

Choice of risk measure

Risk level

Safety level

# Measure of Risk

$$\Lambda_c(t) = P\{\boldsymbol{C}(t) \geq c\}$$

$\boldsymbol{C}(t)$ – symbol of loss, that may occur over a period $t$ of considered system functioning

$c$ – value of $\boldsymbol{C}(t)$

in **boldface** a random variable

# Measure of Risk

$$\Lambda_c(t) = P\{C(t) \geq c\}$$

**Risk measure** is a probability $\Lambda_c(t)$ of loss $C$ not less then $c$ in a period $t$ of functioning of considered system M-T-E

# Measure of Risk

Usually $t$ = 1 (1 year, 1 day, 1 hour, 1 task etc.)

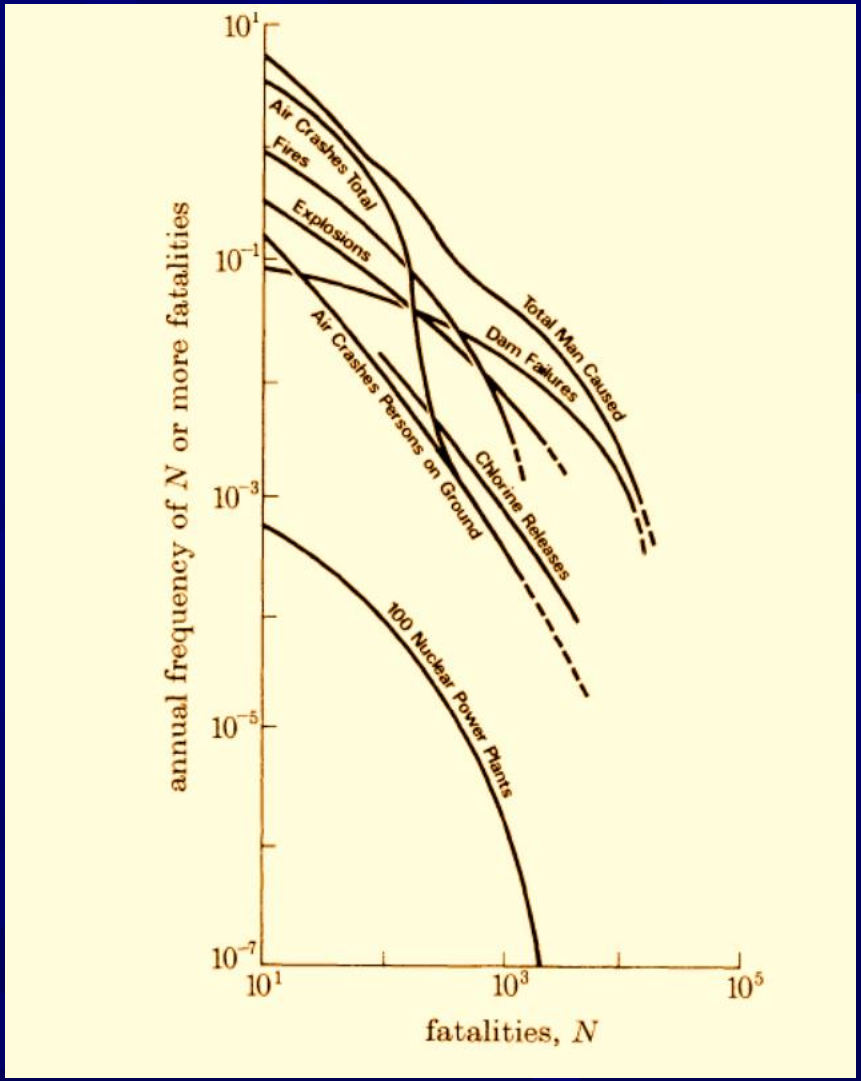
$$\Lambda_c(1) = P\{\mathbf{C}(1) \geq c\}$$

# Risk estimator

$$\hat{\Lambda}_c(1) = \frac{N_{C_1 \geq c}(1)}{N}$$

$N_{C_1 \geq c}(1)$ – number of undesirable events, that occurred in time $t$ =1 in the population of $N$ considered M-T-E systems and caused (each) a loss $C_1 \geq c$

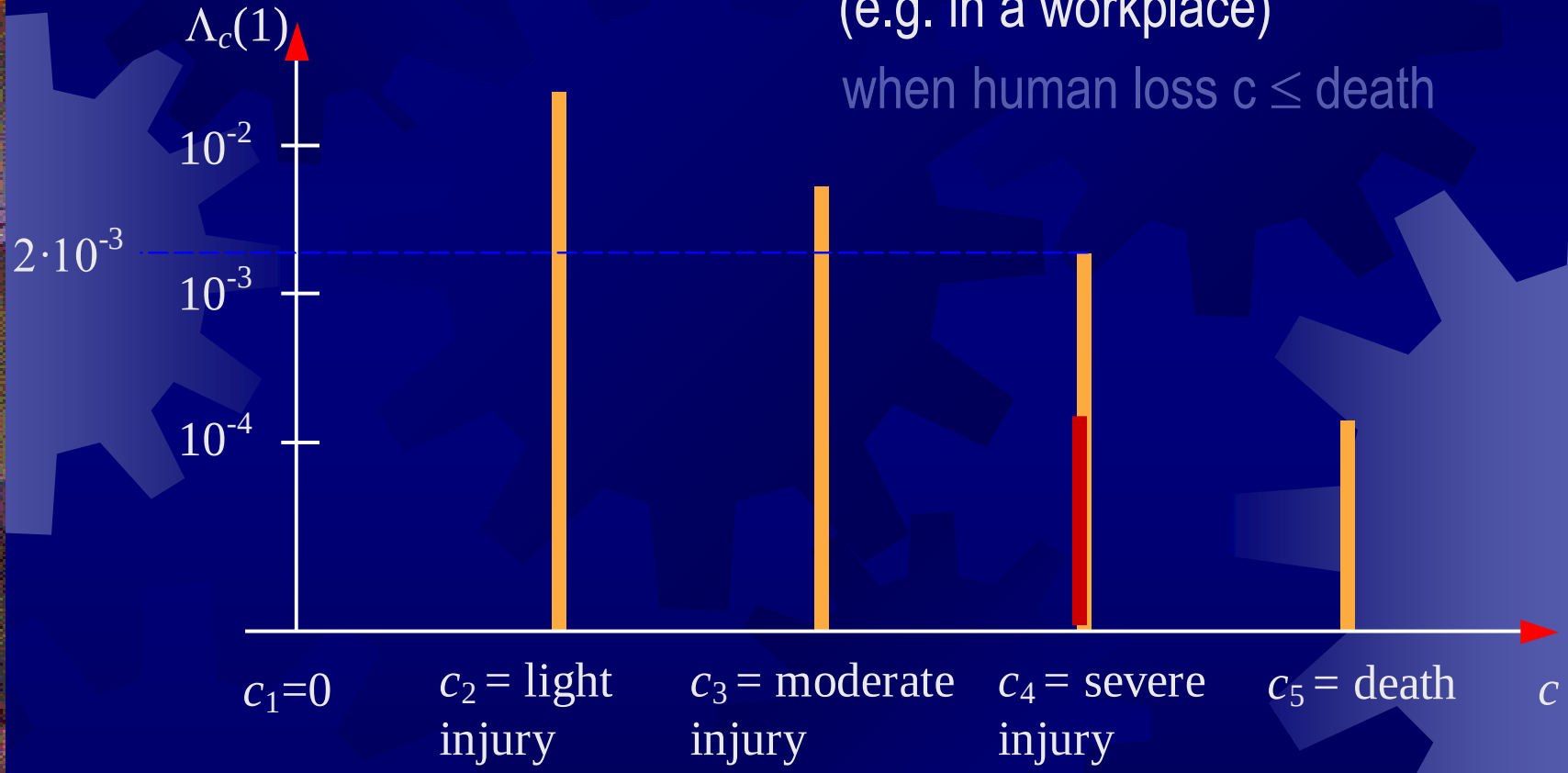
# Example of risk measure



# Measure of individual risk

(e.g. in a workplace)

when human loss c ≤ death




2 workers for 1000 employed sustain at least severe injury

per 1 year

severe injury = fatal injury

# Human loss in Poland (2001)

number of fatal occupational injuries by industry sector

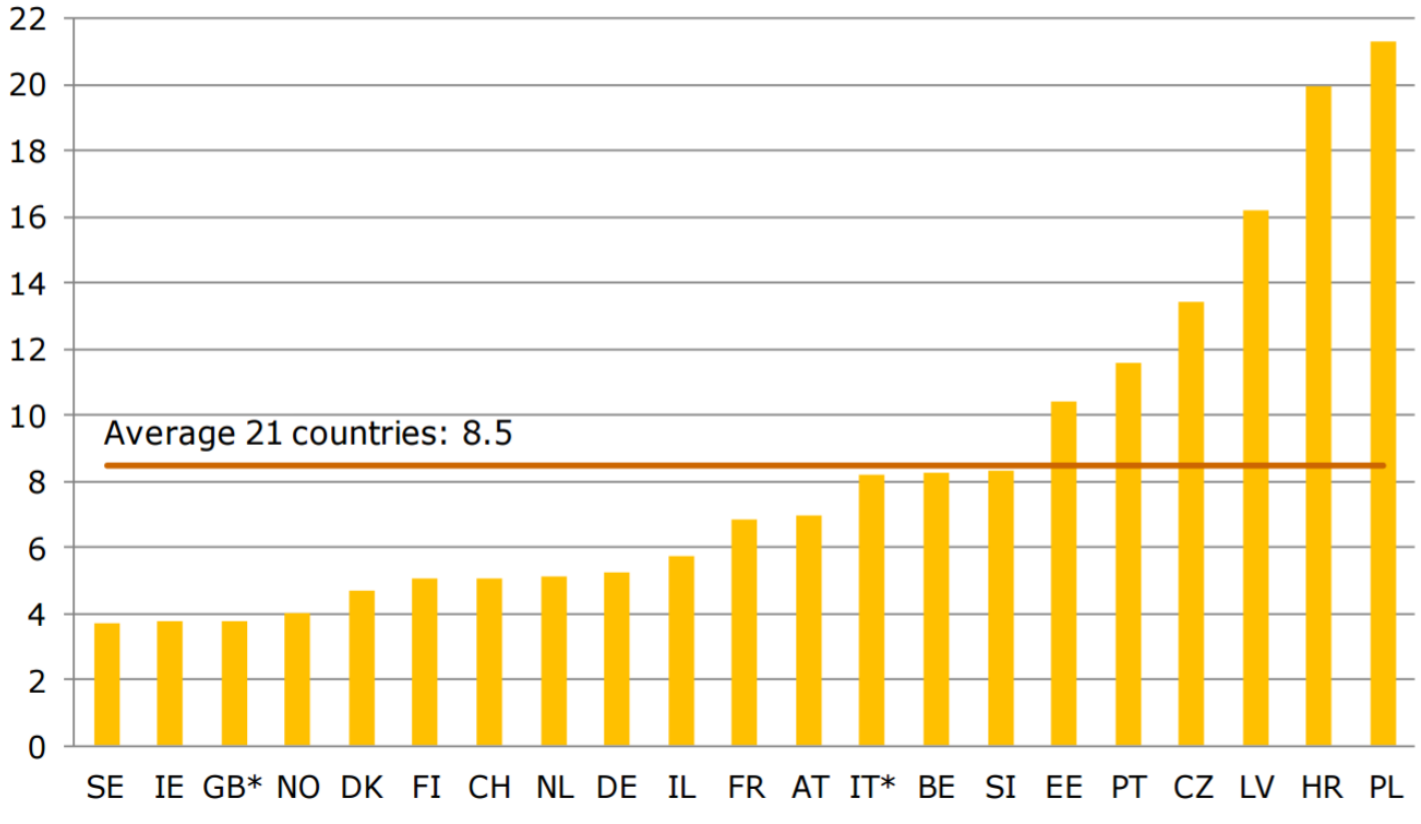
# Road fatalities refered to driven distace

**Fig. 6: Road deaths per billion vehicle-kilometres. Average for the latest three years for which both the road deaths and the estimated number of vehicle-kilometres are available.**

2011-2013 (SE, IE, GB, CH, IL, IT, PT, LV), 2010-2012 (NO, DK, FI, NL, DE, FR, AT, BE, EE, CZ, HR), 2009-2011 (SI, PL).

*Provisional figures for road deaths in 2013.

Vehicle-km travelled are not available or available on part of the network only in Bulgaria, Greece, Spain, Cyprus, Lithuania, Luxembourg, Hungary, Malta, Romania, Slovakia and Serbia.



# Another measure of Risk beside $\Lambda_c(1)$

Expected (predicted) value of loss in time $t = 1$

$$c_o(1) = E\left[\boldsymbol{C}(1)\right] = \overline{\boldsymbol{C}}(1)$$

the most likely loss in time $t = 1$

the most probable category of loss in time $t = 1$

$$c_o(1) = \tilde{\boldsymbol{C}}(1)$$

# Most likely loss in time

the most probable category of loss in time $t = 1$

$$c_o(1) = \tilde{C}(1)$$

For a preliminary or crude risk analysis
(including qualitative risk analysis)

e.g. in the Risk Score method

# Risk measure related to benefit

utility

for comparison of similar systems

$$\Lambda_c^* = \frac{\Lambda_c(1)}{KR(1)}$$

$$c_o^* = \frac{c_o(1)}{KR(1)}$$

$KR(1)$ – benefit, profit from the system functioning

# Number of fatalities per billion $

Number of fatal occupational injuries
per 1 billion dollars GNP

| Country | Value of risk measure $\hat{c}_o^*$ | | | |
|---|---|---|---|---|
| | 1993 | 1996 | 1998 | 2000 |
| Russia | - | 15,20 | 11,33 | 16,95 |
| Poland | 11,00 | 6,67 | 5,98 | 4,72 |
| Czech Rep. | 8,07 | 5,46 | 4,46 | 4,49 |
| Spain | 2,33 | 1,70 | 1,93 | 1,82 |
| Germany | 1,36 | - | 0,69 | 0,62 |
| France | 0,84 | 0,77 | 0,55 | 0,55 |
| Sweden | 0,43 | 0,35 | 0,30 | 0,25 |
| G. Britain | 0,27 | 0,20 | 0,14 | 0,15 |

# Risk Based on Exposure

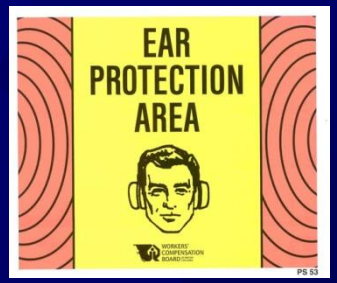
| Type | 5 Yr. Average | General Population | Risk Based on Exposure |
|---|---|---|---|
| | | Risk Per Year | deaths per 100 million vehicle miles |
| Motor Vehicle | 36,676 | 1 out of 7,700 | 1.3 |
| Large Trucks | 5,15 | 1 out of 55,000 | 2.5 |
| Motorcycles | 3,112 | 1 out of 91,500 | 31.3 |
| Railroads | 931 | 1 out of 306,000 | 1.3 |
| Air Carriers | 138 | 1 out of 2,067,000 | 1.9 aircraft miles |
| HAZMAT Transportation | 12 | 1 out of 23,350,000 | 4.2 100 million shipments |

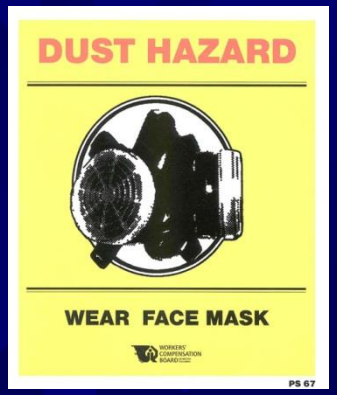
# Health-hazard factors

**Measure of** (individual) **risk** in case of loss due to long time exposure to health-hazard factors leading to occupational disease

**Insidious hazards**: noise, vibration, toxic substances (e.g. tobacco)

**Noxious agents**



# Industrial hygiene

The **level of health loss** increases with the exposure time $\tau$

(and is not identical in each unit of time)

Other risk measures

# Risk measure for an occupational disease

$$\Lambda_c(\tau_H) = P\{C(\tau_H) \geq c\}$$

probability of health loss in the degree higher or equal $c$

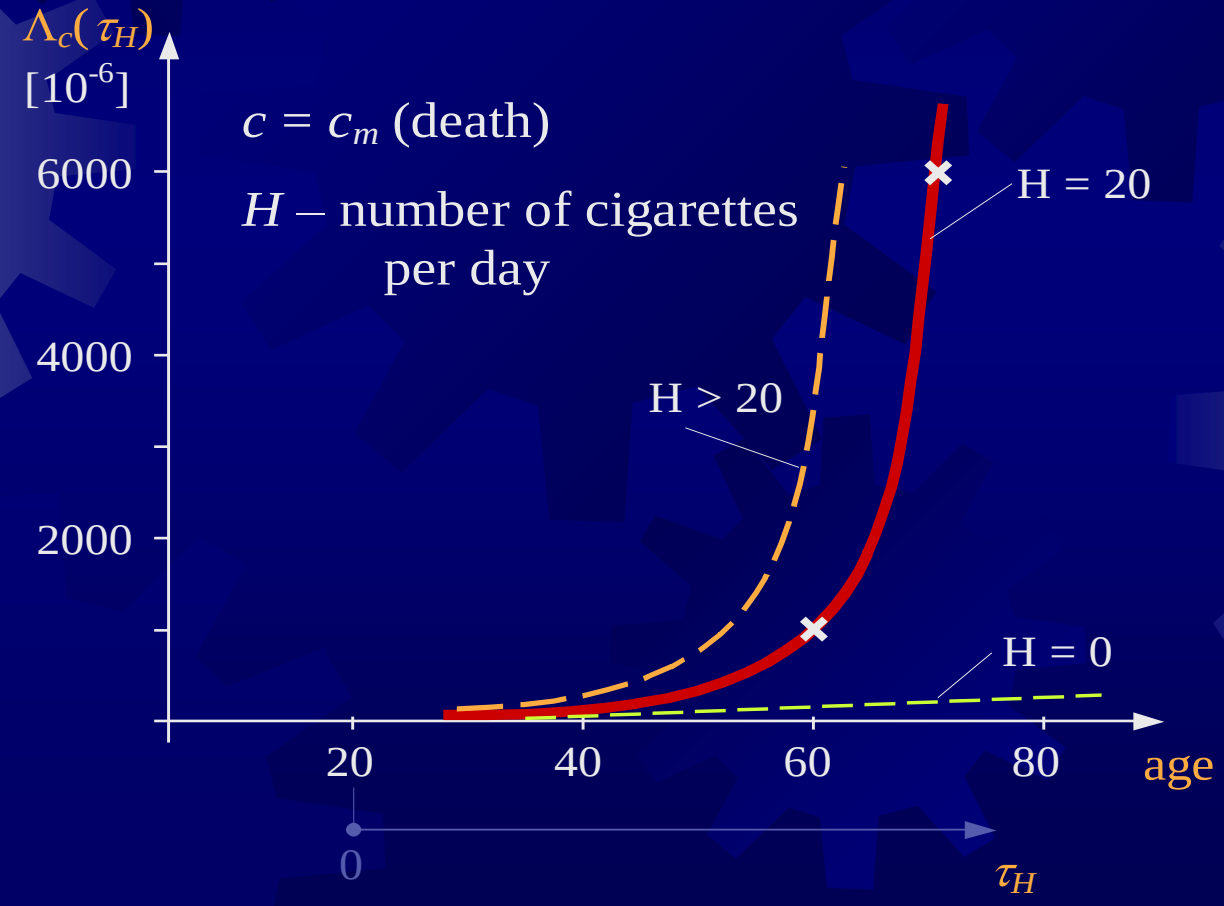
$\tau_H$ – exposure time

$H$ – intensity of the noxious agent (hazard)

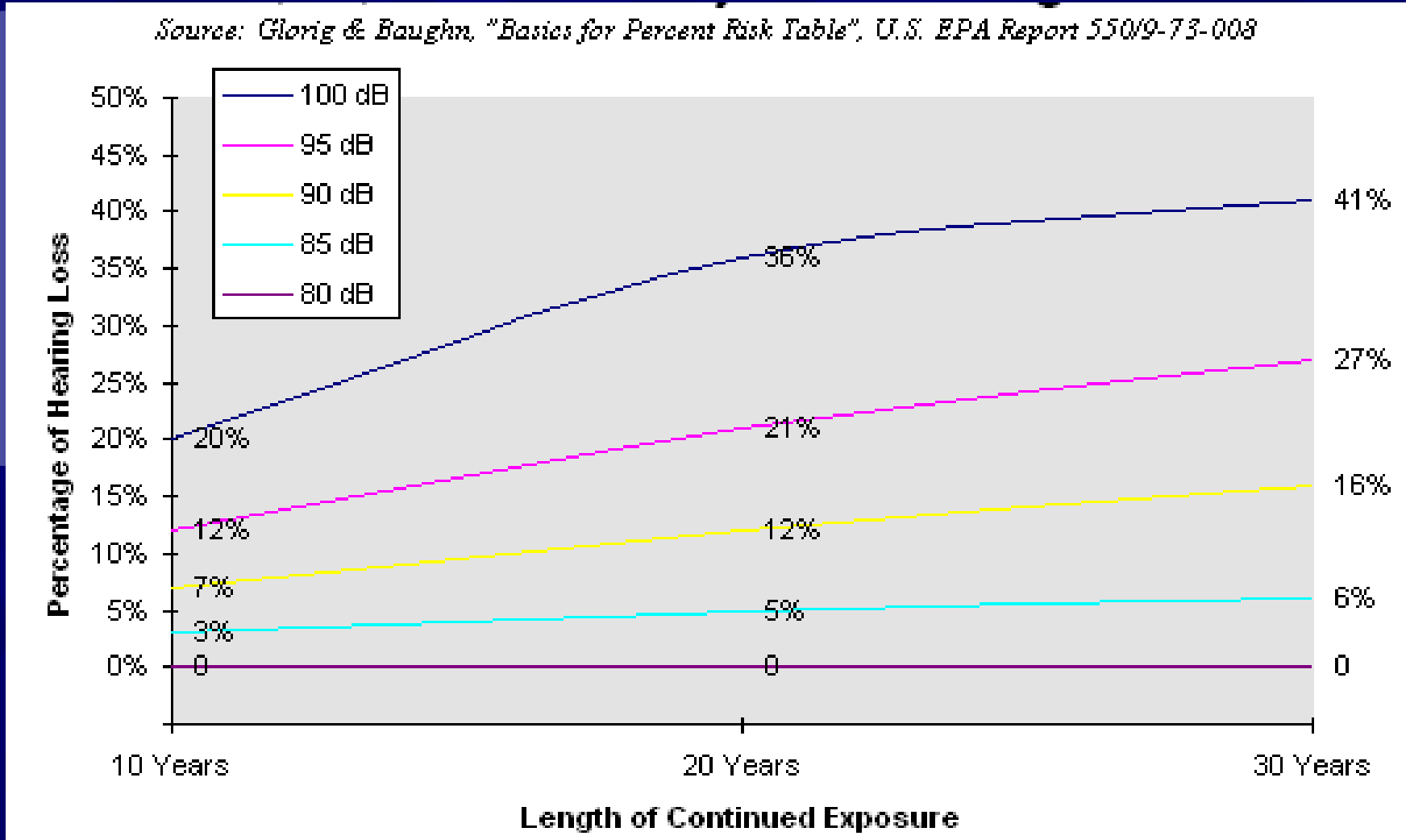
e.g. $C(t)$ – percentage of hearing loss

# Loss of life probability $\Lambda_c(\tau_H)$

caused by the lung cancer, as an effect of smoking




# Hearing loss

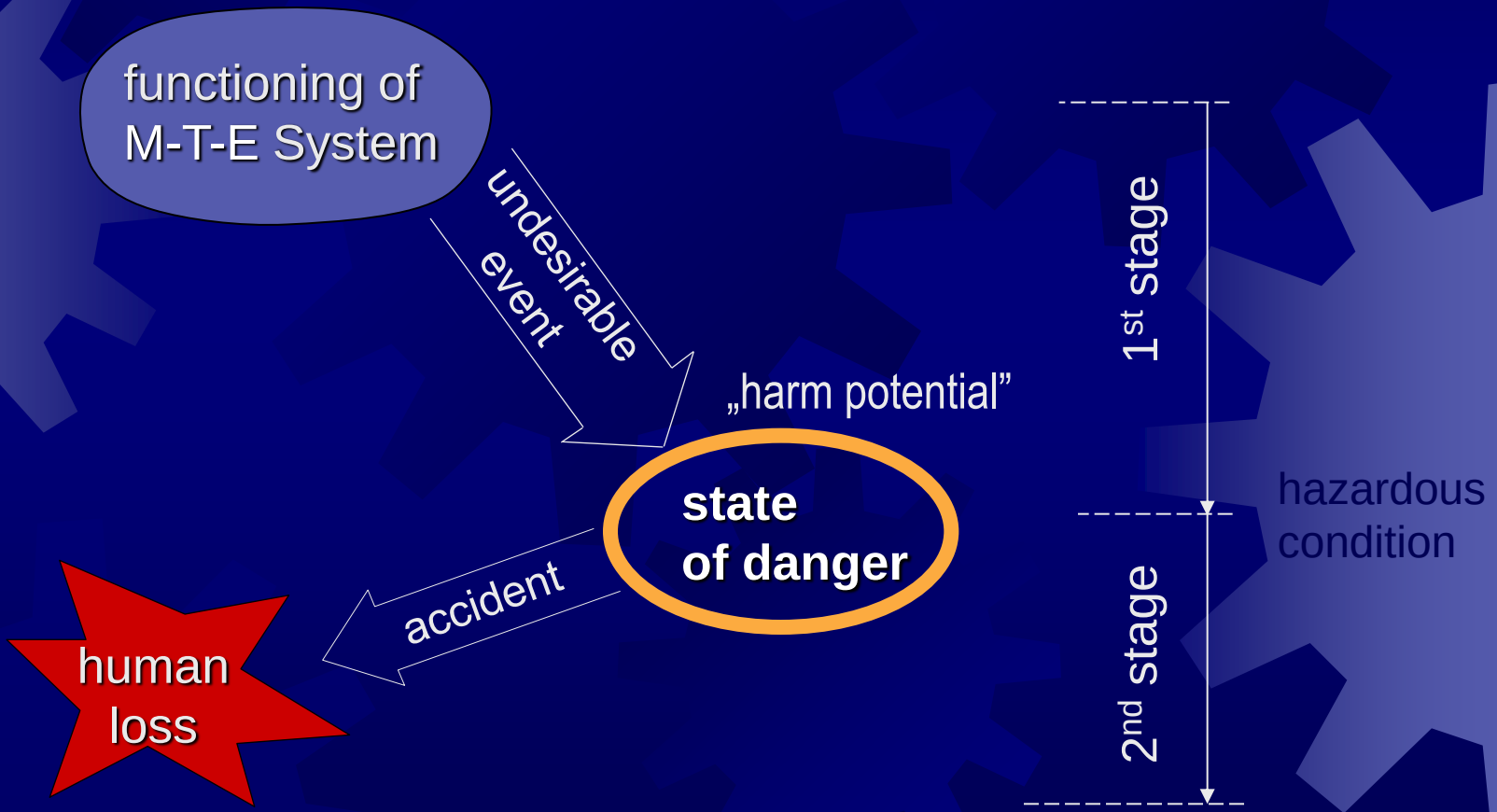


# The concept of risk

three elements:

- The perception that something could happen
- The likelihood of something happening
- The consequences if it does happen

# Stages in development of the loss process



# Correlation of **Risk Measure** with **Measures** of **Reliability** and **Hazard**

$$\begin{pmatrix} risk \\ measure \end{pmatrix} = \begin{pmatrix} unreliability \\ measure \end{pmatrix} \cdot \begin{pmatrix} hazard \\ measure \end{pmatrix}$$

risk level — likelihood of UE — probability of loss

**CONSEQUENCE**

$$\begin{pmatrix} \text{miara} \\ \text{ryzyka} \end{pmatrix} = \begin{pmatrix} \text{miara} \\ \text{zawodności} \end{pmatrix} \cdot \begin{pmatrix} \text{miara} \\ \text{zagrożenia} \end{pmatrix}$$

# Correlation of **Risk Measure** with **Measures** of **Reliability** and **Hazard**

$$\begin{pmatrix}\text{risk}\\ \text{measure}\end{pmatrix} = \begin{pmatrix}\text{unreliabil ity}\\ \text{measure}\end{pmatrix} \cdot \begin{pmatrix}\text{hazard}\\ \text{measure}\end{pmatrix}$$

the fundamental formula for risk analysis and safety improvement

# Measure of Reliability

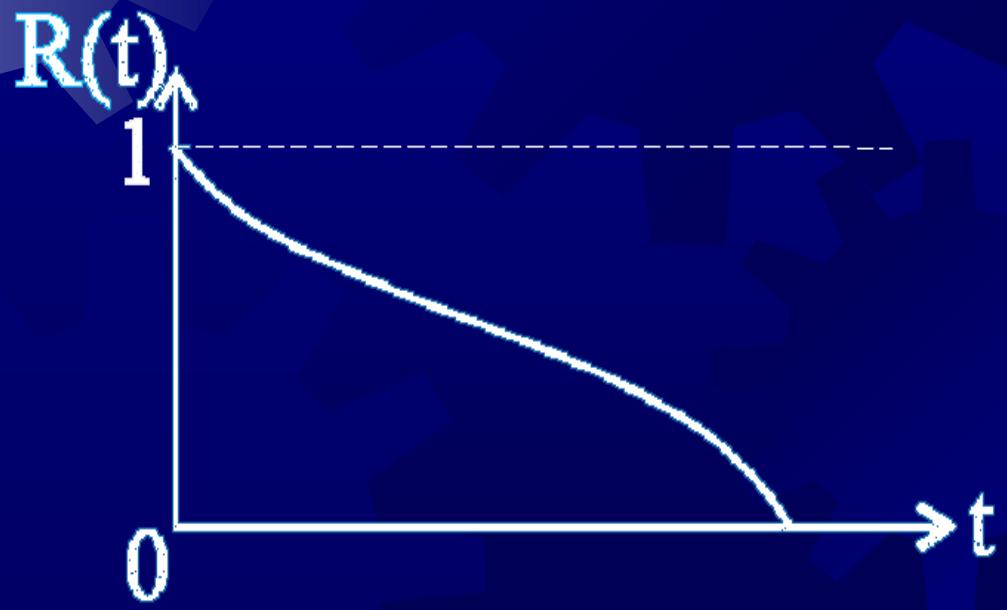
$$\Lambda_c(1) = Q(1) \cdot Z(c)$$

$Q(1)$ – the probability of un undesirable event occurrence in period $t$ = 1 (for systems

M-T $\Rightarrow$ **unreliability measure** (failure function)

# Reliability function

$$R(1) = 1 - Q(1)$$



# Measure of Hazard

$$\Lambda_c(1) = Q(1) \cdot Z(c)$$

$$Z(c) = P\{C_1 \geq c \mid A\}$$

$A$ – a symbol of an undesirable event

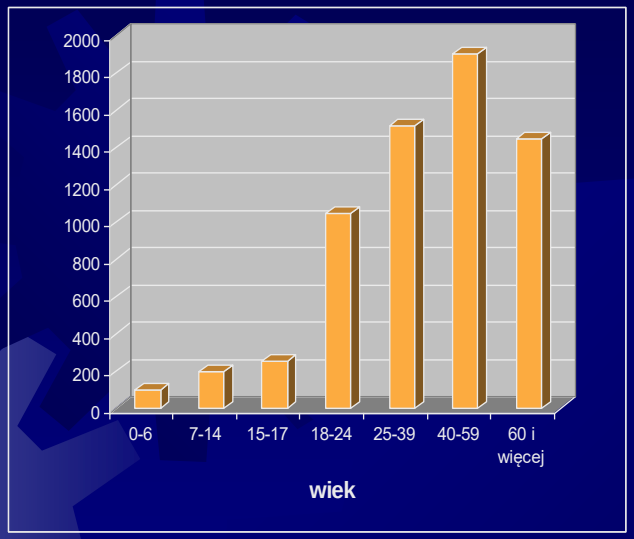
$C_1$ – the loss caused by occurrence of the event $A$

# Hazard in traffic accidents

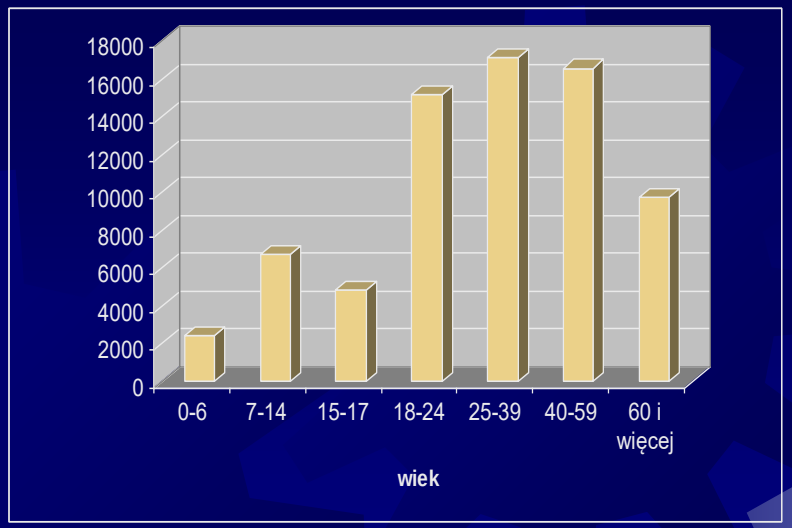
Values of life hazard in traffic accidents

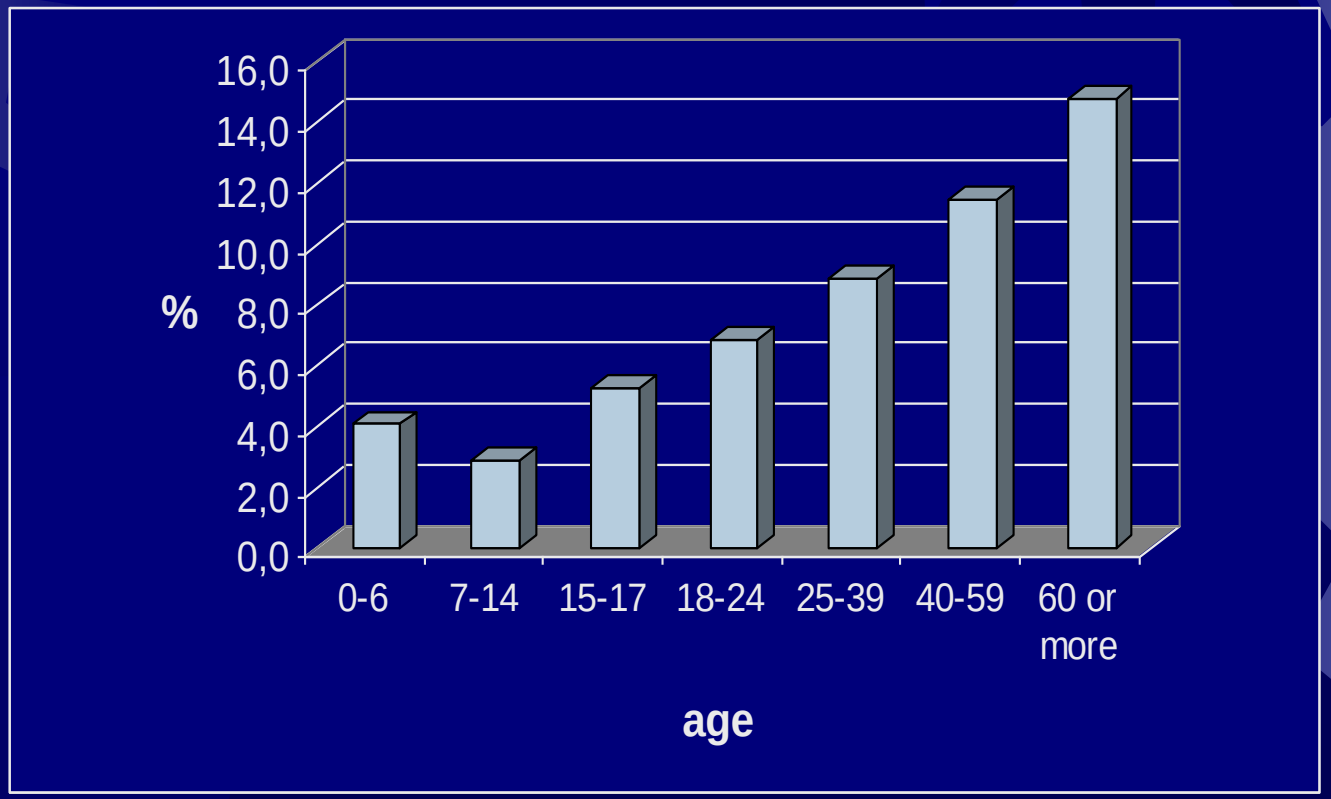
| Country | Value of hazard measure $Z(c_m)$ | | | | |
|---|---|---|---|---|---|
| | 1993 | 1995 | 1997 | 1998 | 2000 |
| Russia | 0,200 | 0,196 | 0,137 | 0,138 | 0,148 |
| Poland | 0,140 | 0,122 | 0,104 | 0,105 | 0,110 |
| Hungary | 0,086 | 0,080 | 0,070 | 0,066 | 0,075 |
| Spain | 0,078 | 0,068 | 0,061 | 0,056 | 0,049 |
| France | 0,064 | 0,075 | 0,066 | 0,067 | 0,060 |
| Czech Rep. | 0,063 | 0,057 | 0,050 | 0,045 | 0,056 |
| Austria | 0,031 | 0,031 | 0,027 | 0,024 | 0,024 |
| Germany | 0,026 | 0,024 | 0,021 | 0,020 | 0,018 |
| G. Britain | 0,017 | 0,015 | 0,014 | 0,014 | 0,014 |
| USA | 0,0006 | 0,019 | 0,017 | 0,019 | 0,019 |

killed




injured



# Hazard at work

Values of life hazard in occupational accidents in Poland

| Industry sector | Value of hazard measure $Z(c_m)$ | | | |
|---|---|---|---|---|
| | 1995 | 1997 | 1999 | 2002 |
| **Total** | 0,0062 | 0,0064 | 0,0060 | 0,0065 |
| Industry | 0,0039 | 0,0038 | 0,0037 | 0,0047 |
| including: | | | | |
| Mining | 0,0047 | 0,0038 | 0,0061 | 0,0135 |
| Energy and utilities | 0,0122 | 0,0062 | 0,0109 | 0,0065 |
| Manufacturing | 0,0032 | 0,0037 | 0,0029 | 0,0037 |
| Construction | 0,0089 | 0,0132 | 0,0102 | 0,0122 |
| Agriculture & forestry | 0,0080 | 0,0080 | 0,0080 | 0,0071 |
| Transport, storage | 0,0104 | 0,0103 | 0,0081 | 0,0112 |
| Fishing | 0,0080 | 0,0571 | 0,0230 | 0,0102 |

# Another hazard measure

$$c_o(1) = Q(1) \cdot Z_o$$

$Z_o$ – the expected (predicted) value or the most likely loss caused by occurrence of the event $A$

# Problem 1

In a company there have been 6 accidental deaths in the last 10 years. In this period of time altogether 10000 man-years (1 man-year = 1600 hours) have been registered. Calculate observed FAR value for this period of time.

**F**atal **A**ccident **R**ate is statistically expected number of accidental deaths per 100 million ($\mathbf{10^8}$) exposed hours

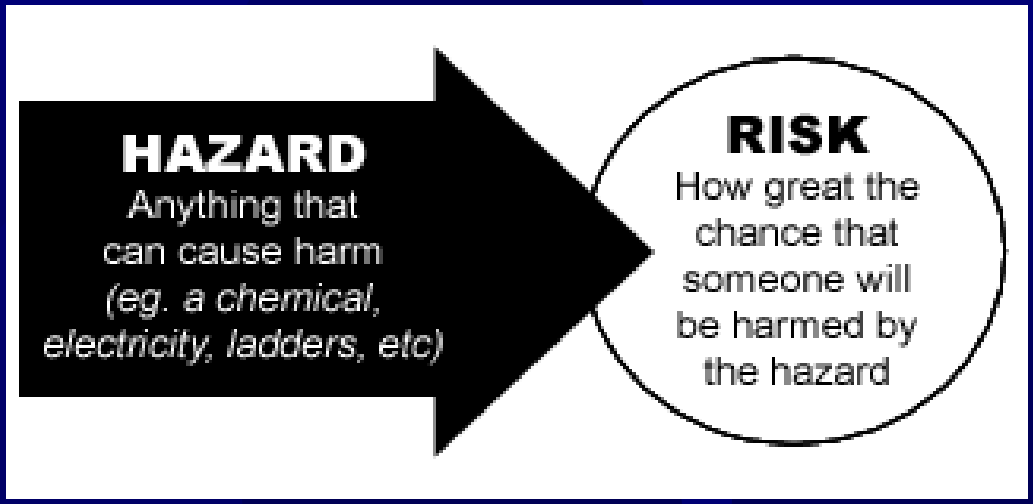
$$P = \frac{6}{10^4 \cdot 1.6 \cdot 10^3} = \frac{6}{1.6} \cdot 10^{-7} = 37.5 \cdot 10^{-8}$$

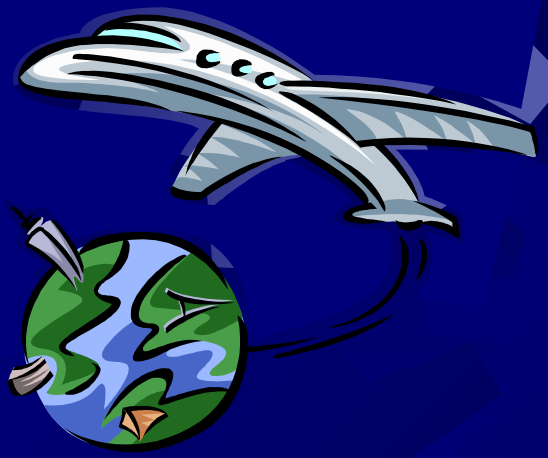
FAR $\approx$ 38

# WHAT IS THE DIFFERENCE BETWEEN A 'HAZARD' AND A 'RISK'?

A **hazard** is something that can cause harm, e.g. electricity, chemicals, working up a ladder, noise, a keyboard, a bully at work, stress, etc.

A **risk** is the chance, high or low, that any hazard will actually cause somebody harm.

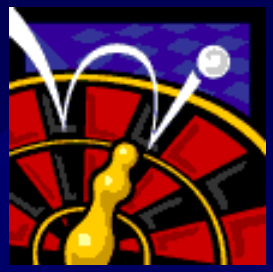




# RELIABILITY and SAFETY

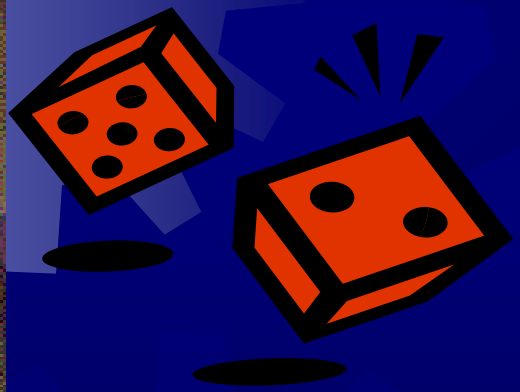
Technical statistics

# Probability



An experiment is performed $n$ times
the event $A$ occurs $n_A$ times

$$P(A) = \lim_{n \to \infty} \frac{n_A}{n}$$



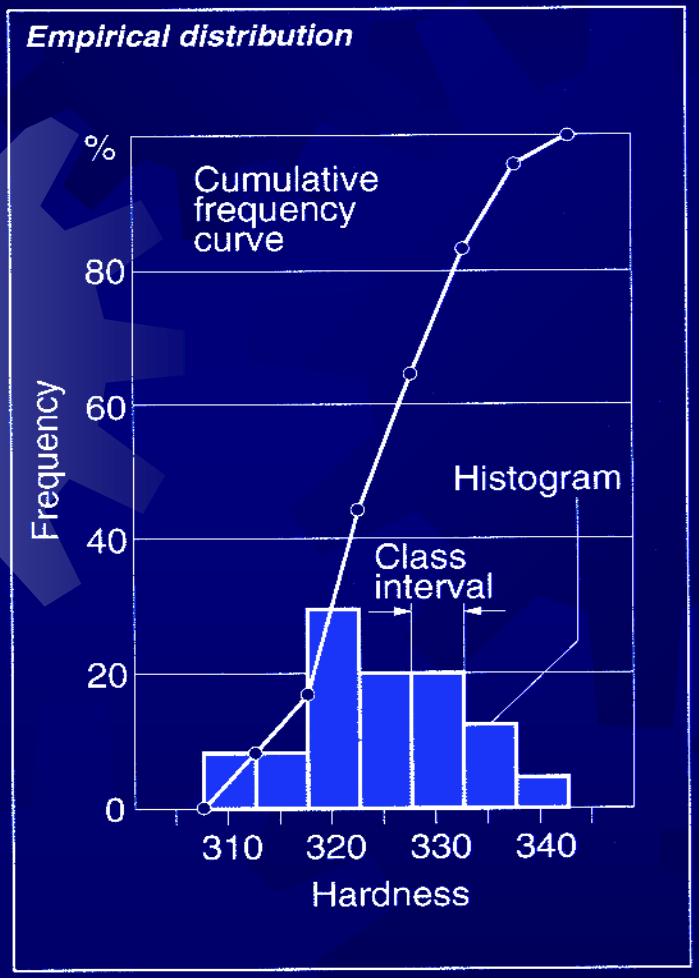
## Probabilities by argument

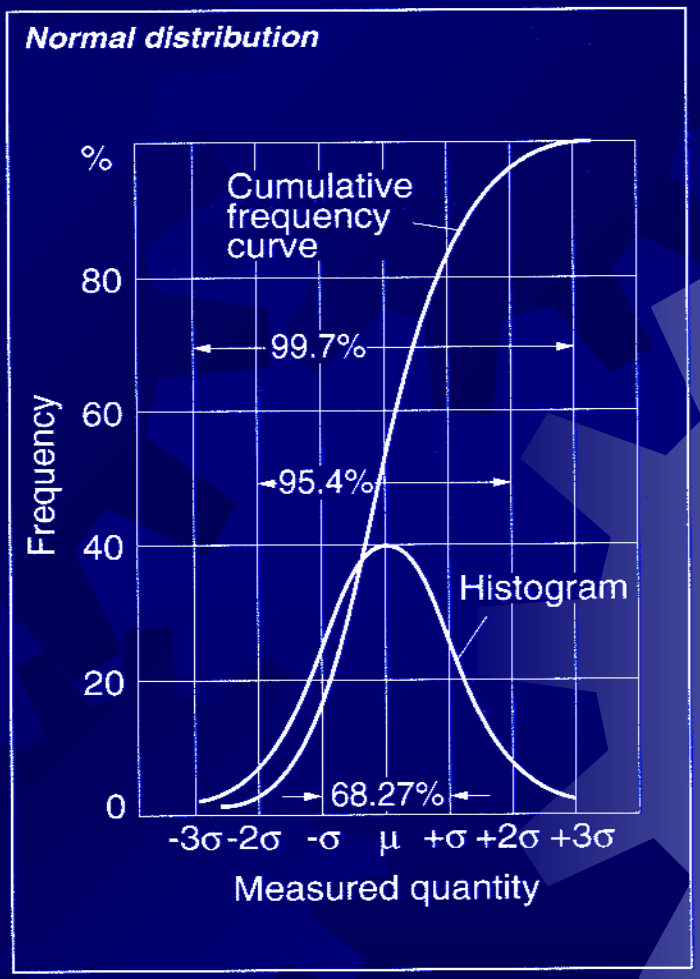
outcomes equally likely to appear

## Probability – a measure of belief

The probability that Poland will qualify for the next Europe championship in football is 40%

# Histogram and cumulative frequency




frequency-density function, distribution function

# Random variables (stochastic variables)

$X$ – random variable

$x$ – one of the values $X$ can take (individual measured value)

$F(x)$ – distribution function, probability for values $\leq x$

$$F(x) = P(X \leq x)$$

$f(x)$ – probability density function

$$f(x) = \frac{\mathrm{d}\,F(x)}{\mathrm{d}\,x}$$

$T$ – unit lifetime (from installation to failure)

$t$ – time $(t \geq 0)$

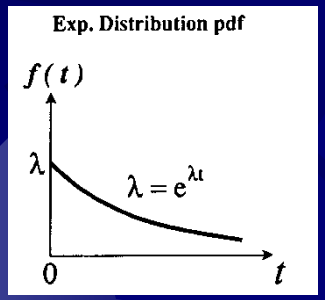
$Q(t)$ – probability of failure up to time $t$ (service life distribution)

$$Q(t) = P(T \leq t)$$

$$Q(t) = \int_0^t f(\tau)\,d\tau$$

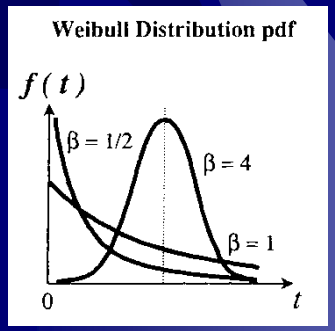
probability density of $T$

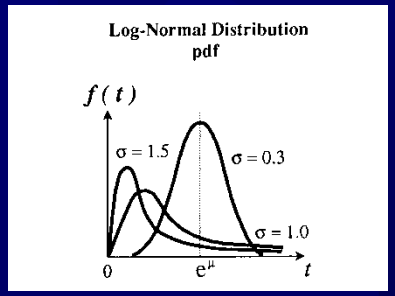
# Common lifetime distributions




$$Q(t) = 1 - e^{-\lambda t} \qquad f(t) = \lambda e^{-\lambda t}$$

$$Q(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}} \qquad f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$




Log-normal distribution

ln($t$) follows normal distribution