Equation of continuity: $\dfrac{\partial \rho}{\partial t} + \dfrac{1}{R}\dfrac{\partial}{\partial R}(R\rho u_R) + \dfrac{1}{R}\dfrac{\partial}{\partial \varphi}(\rho u_\varphi) + \dfrac{\partial}{\partial z}(\rho u_z) = 0$

Navier-Stokes equations with constant ρ, constant ν, and no body force:

$$\frac{\partial u_R}{\partial t} + (\mathbf{u}\cdot\nabla)u_R - \frac{u_\varphi^2}{R} = -\frac{1}{\rho}\frac{\partial p}{\partial R} + \nu\left(\nabla^2 u_R - \frac{u_R}{R^2} - \frac{2}{R^2}\frac{\partial u_\varphi}{\partial \varphi}\right),$$

$$\frac{\partial u_\varphi}{\partial t} + (\mathbf{u}\cdot\nabla)u_\varphi + \frac{u_R u_\varphi}{R} = -\frac{1}{\rho R}\frac{\partial p}{\partial \varphi} + \nu\left(\nabla^2 u_\varphi + \frac{2}{R^2}\frac{\partial u_R}{\partial \varphi} - \frac{u_\varphi}{R^2}\right),$$

$$\frac{\partial u_z}{\partial t} + (\mathbf{u}\cdot\nabla)u_z = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 u_z,$$

where: $\mathbf{u}\cdot\nabla = u_R\dfrac{\partial}{\partial R} + \dfrac{u_\varphi}{R}\dfrac{\partial}{\partial \varphi} + u_z\dfrac{\partial}{\partial z}$ and $\nabla^2 = \dfrac{1}{R}\dfrac{\partial}{\partial R}\left(R\dfrac{\partial}{\partial R}\right) + \dfrac{1}{R^2}\dfrac{\partial^2}{\partial \varphi^2} + \dfrac{\partial^2}{\partial z^2}$.