Politechnika Warszawska | Wydział Mechaniczny Energetyki i Lotnictwa | Zakład Wytrzymałości Materiałów i Konstrukcji

Wykład 9

Pręty zginane – belki

Wyznaczanie rozkładu sił wewnętrznych

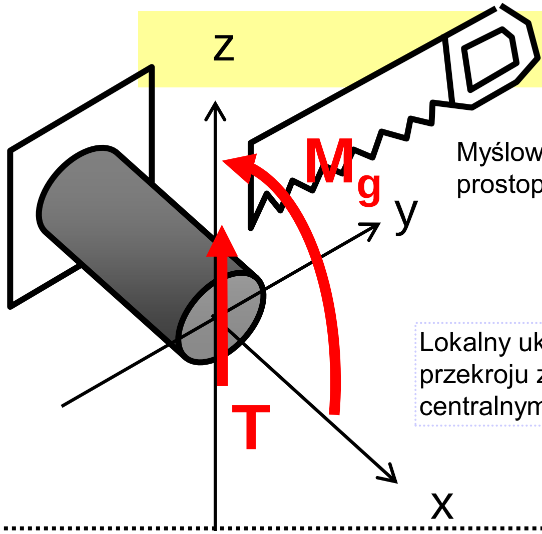









Zginanie proste



Myślowe przecięcie prostopadłe do osi pręta

Lokalny układ współrzędnych przekroju związany z jego osiami centralnymi głównymi

Występują składowe wysiłku przekroju:

T – Siła tnąca

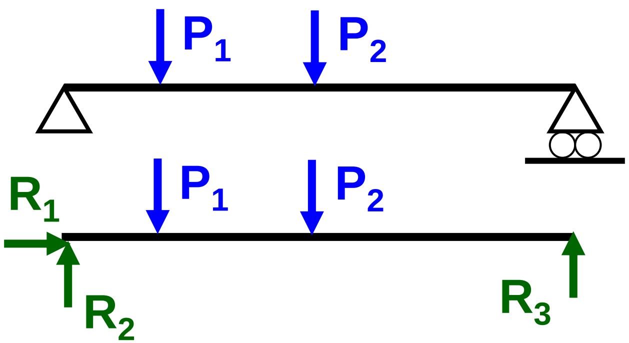
M_g – Moment gnący

Jeśli **T** = 0 to mamy zginanie czyste

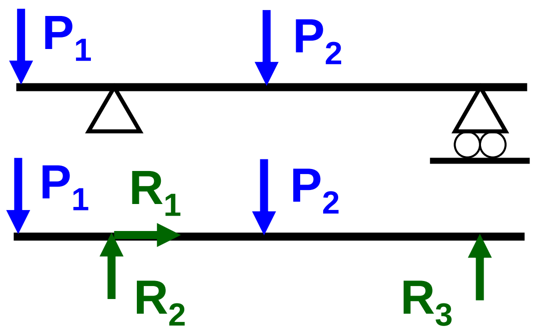
Jeśli **T** $\neq$ 0 to mamy zginanie poprzeczne

Podstawowe schematy belek statycznie wyznaczalnych:

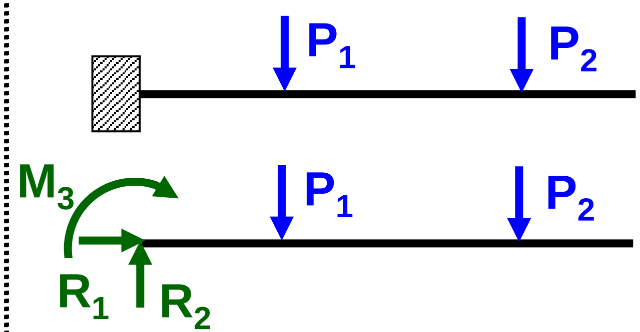
a) dwupodporowa



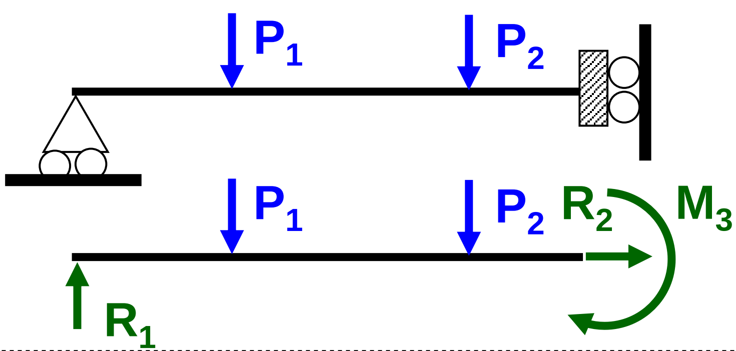
b) dwupodporowa z wysięgnikiem



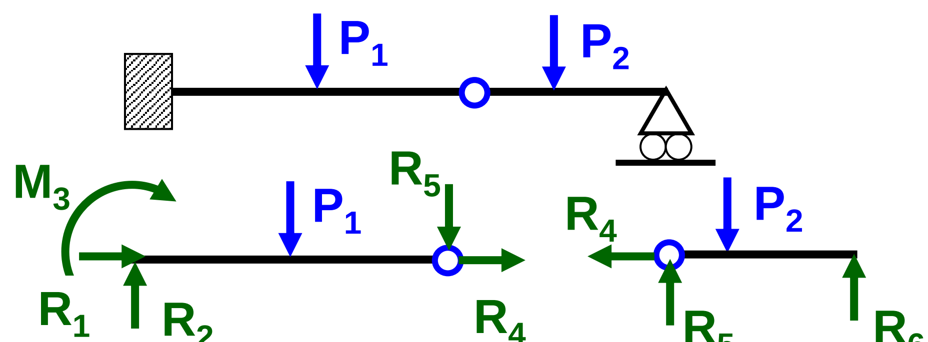
c) wspornikowa



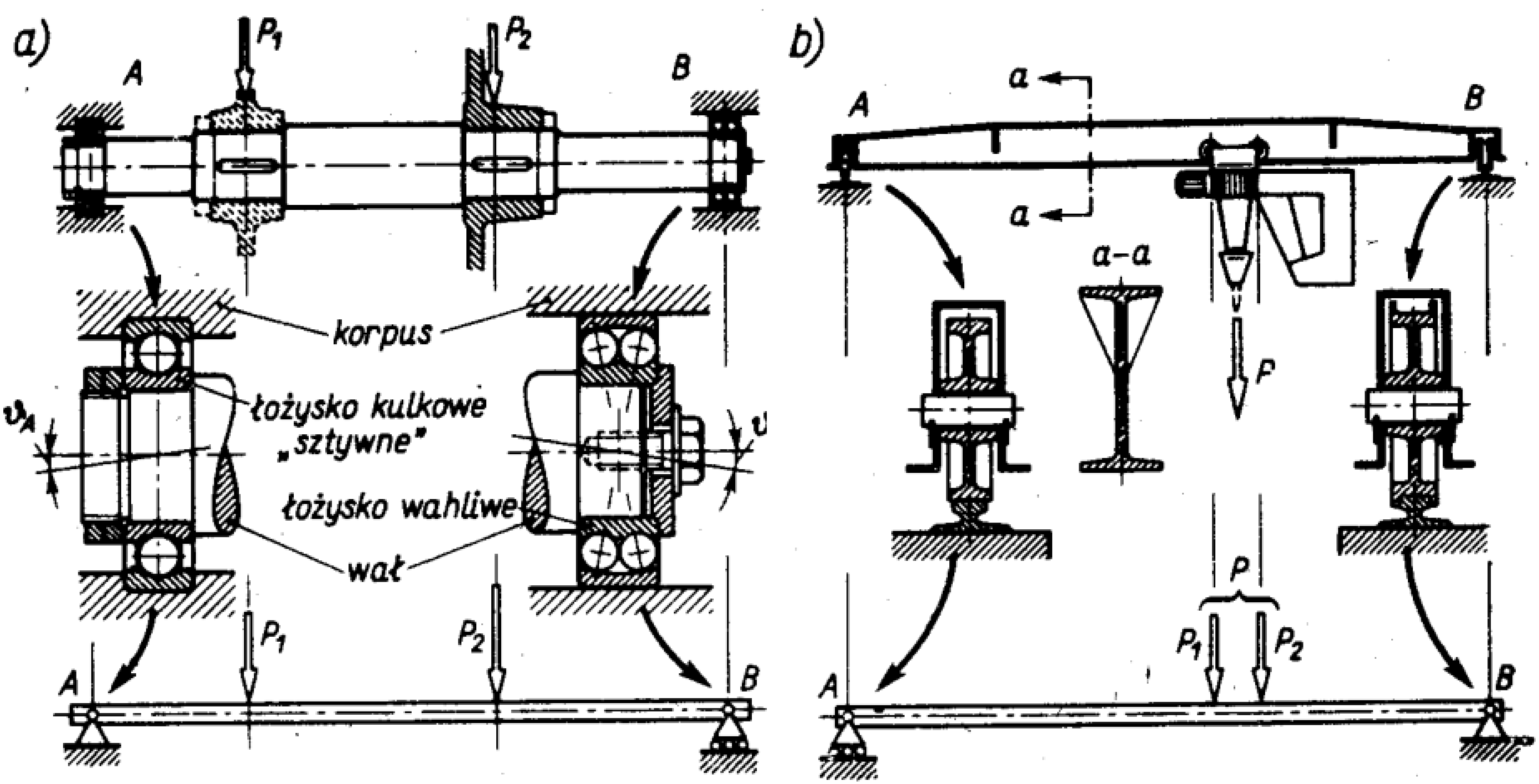
d) z wózkiem odbierającym moment



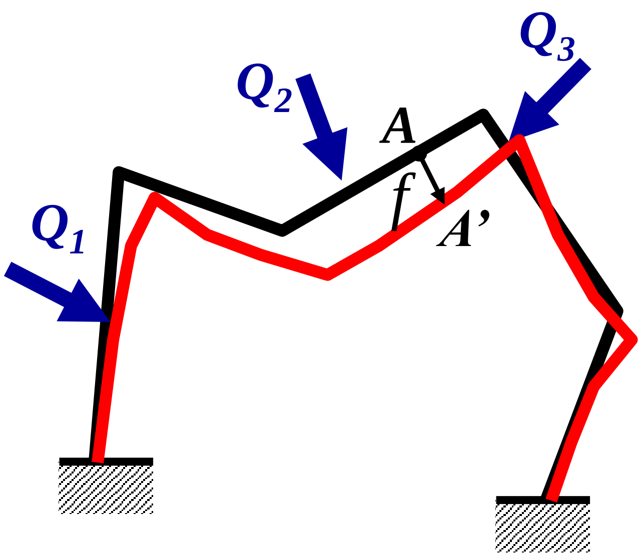
e) z przegubem



Przykłady tworzenia schematu obliczeniowego



Zasada superpozycji



Konstrukcja liniowa : Przemieszczenia i naprężenia są liniowymi funkcjami obciążeń

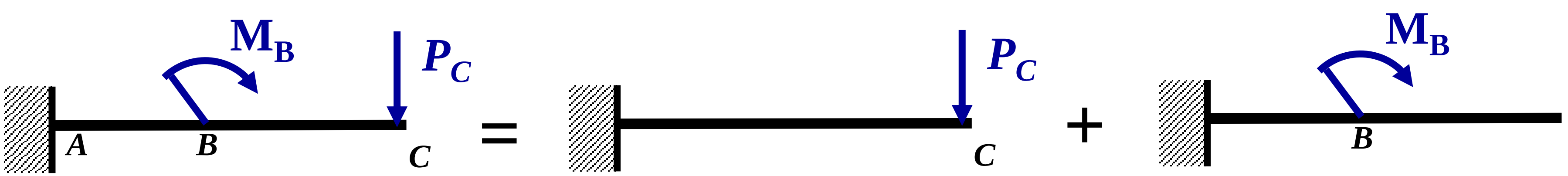
$$f = \sum_{j=1}^{n} \alpha_j Q_j \qquad \sigma = \sum_{j=1}^{n} \beta_j Q_j$$

α_j , β_j - współczynniki liniowe

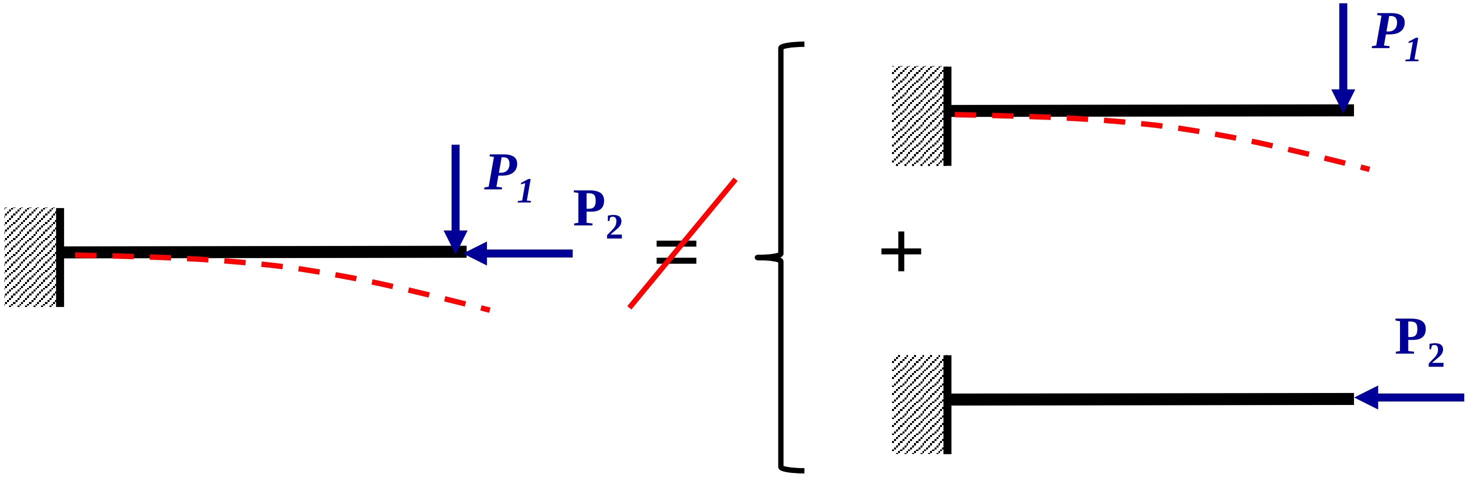
Warunki: 1) Materiał liniowo-sprężysty
2) Odkształcenia są małe
3) Przemieszczenia są małe

Zasada zesztywnienia (wymiary początkowe)

Zasada superpozycji *(niezależności działania obciążeń)*
W przypadku, gdy na konstrukcję liniową działa złożony układ obciążeń, to skutek działania tego układu jest równy sumie skutków obciążeń składowych

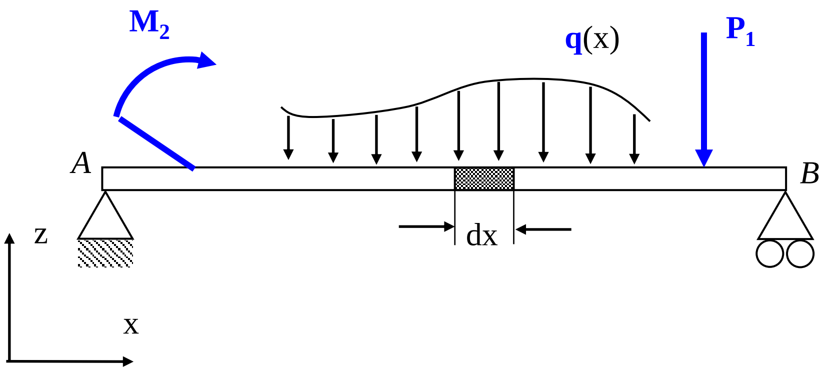


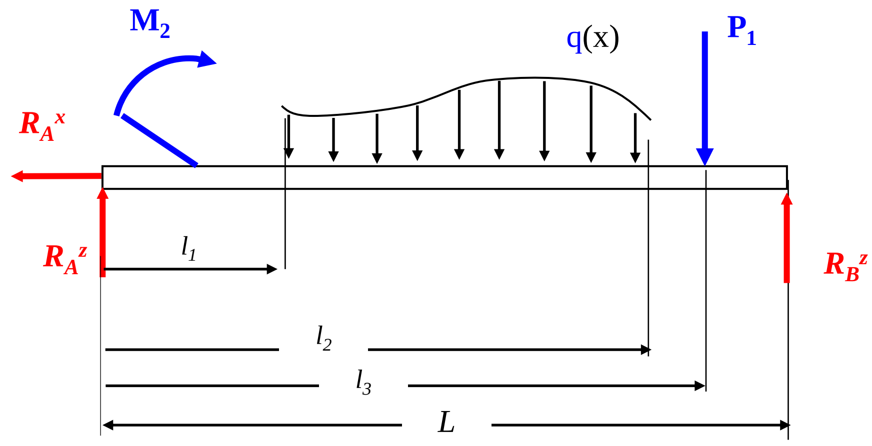
Nieraz nie można zastosować zasady superpozycji



Wyznaczanie składowych wysiłku przekroju w prostym zginaniu

Zginanie to taki przypadek obciążenia pręta, w którym występuje siła tnąca T i moment gnący Mg





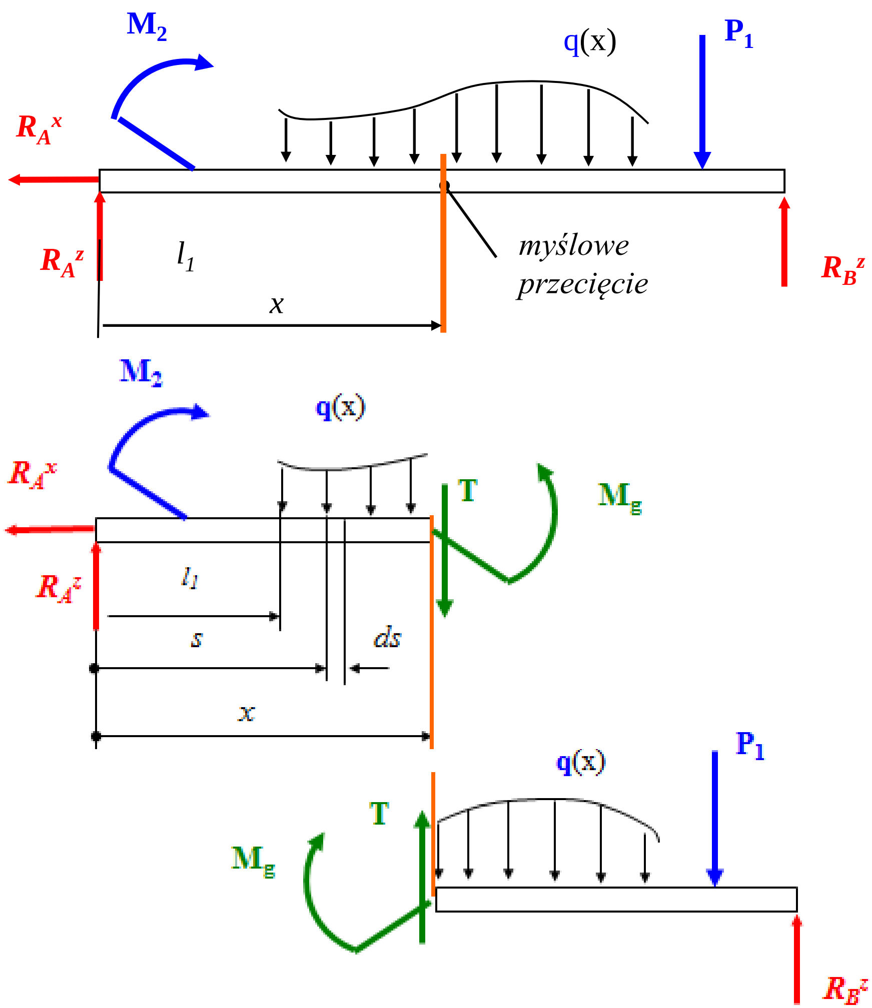
Zwykle zadanie zaczynamy od uwolnienia belki od więzów i wyznaczenia reakcji:

$$\sum F_x = 0: \quad R_A^x = 0$$

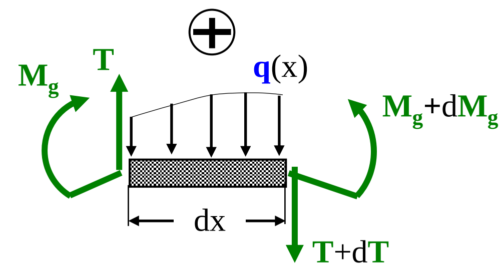
$$\sum M_A = 0: \quad R_B^z \cdot L - P_1 \cdot l_3 - \int_{l_1}^{l_2} q(x) \cdot x \, dx - M_2 = 0$$

$$\sum F_z = 0: \quad R_A^z + R_B^z - P_1 - \int_{l_1}^{l_2} q(x) \, dx = 0$$

Wyznaczanie składowych wysiłku przekroju w prostym zginaniu



Konwencja dodatnich znaków



Poszukiwane funkcje Mg(x) i T(x) możemy wyznaczyć z warunku równowagi myślowo wyciętej części (np. lewej):

$$\sum F_z = 0: \quad R_A^z - T(x) - \int_{l_1}^{x} q(s)\, ds = 0$$

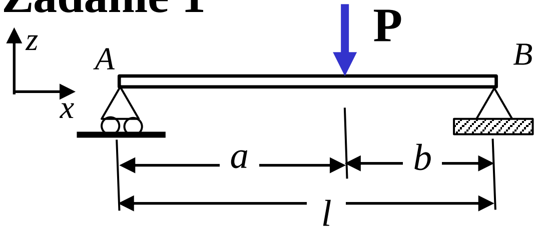
$$\sum M_{pp} = 0:$$

$$-R_A^z \cdot x + \int_{l_1}^{x} q(s) \cdot (x - s)\, ds - M_2 + M_g(x) = 0$$

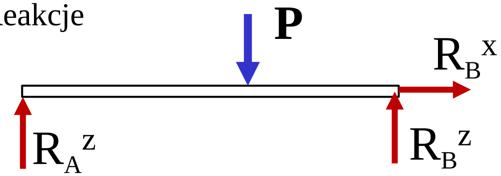
Warto pamiętać, że pomiędzy funkcjami Mg(x), T(x) i q(x) zachodzą związki:

$$\boxed{T(x) = \frac{dM_g(x)}{dx}} \qquad \boxed{q(x) = -\frac{dT(x)}{dx}}$$

Zadanie 1



Reakcje



Równania równowagi:

$\sum F_x = 0 \rightarrow$ $R_B^x = 0$

$\sum M_A = 0 \rightarrow R_B^z \cdot l - P \cdot a = 0$

$R_B^z = P \cdot a/l$

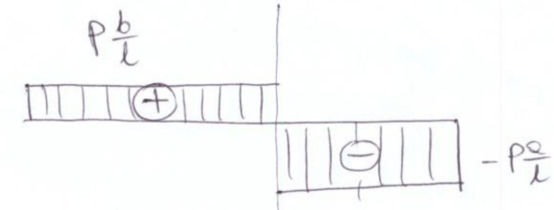
$\sum M_B = 0 \rightarrow -R_A^z \cdot l + P \cdot b = 0$

$R_A^z = P \cdot b/l$

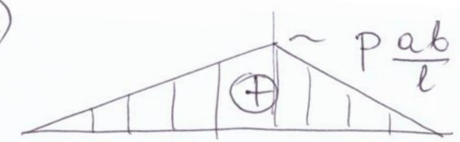
Wyniki:

$\boxed{T = \frac{dM_g}{dx}}$

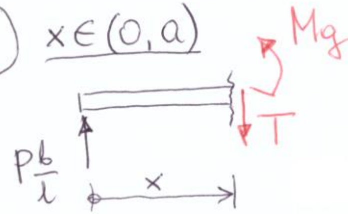
$T(x)$



$M_g(x)$

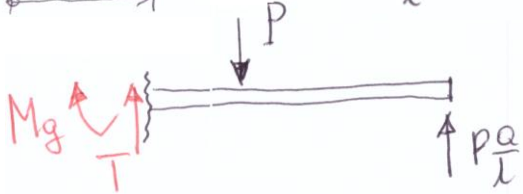


Ⅰ $x \in (0, a)$



r-nia r-gi: $\boxed{\Sigma F_z = 0:}$

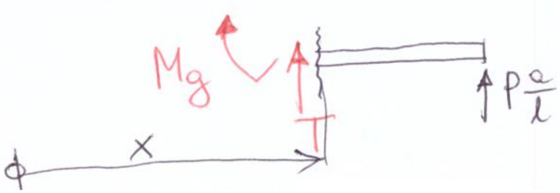
$P\frac{b}{l} - T = 0 \rightarrow \boxed{T = P\frac{b}{l}}$



$\boxed{\Sigma M_{pp} = 0:}$ $-P\frac{b}{l} \cdot x + M_g = 0$

$\boxed{M_g = P\frac{b}{l} \cdot x}$

Ⅱ $x \in (a, l)$

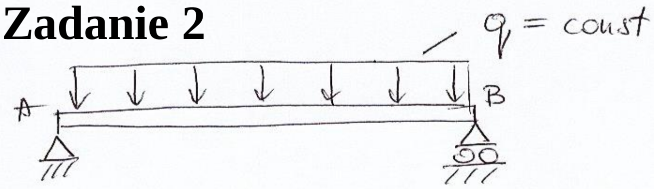


$\boxed{\Sigma F_z = 0:}$ $T + P\frac{a}{l} = 0 \Rightarrow \boxed{T = -P\frac{a}{l}}$

$\boxed{\Sigma M_{pp} = 0:}$ $P\frac{a}{l} \cdot (l - x) - M_g = 0$

$\boxed{M_g = P\frac{a}{l}(l - x)}$

Zadanie 2



Reakcje:

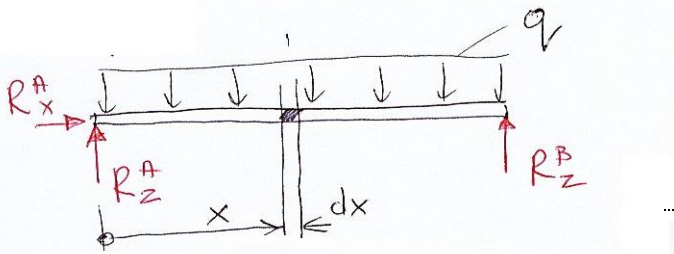
$\Sigma M_A = 0: \quad R_B^z \cdot l - \int_0^l q\,dx \cdot x = 0$

$R_B^z = \frac{1}{l} \cdot q \frac{1}{2} x^2 \Big|_0^l \qquad \boxed{R_B^z = \frac{ql}{2}}$

$\Sigma F_z = 0: \quad R_z^A + R_z^B - \int_0^l q\,dx = 0$

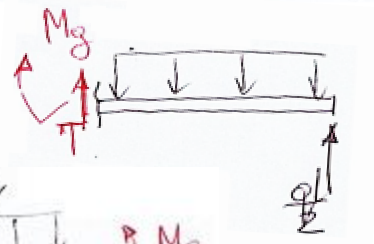
$R_z^A = qx\Big|_0^l - \frac{ql}{2}$

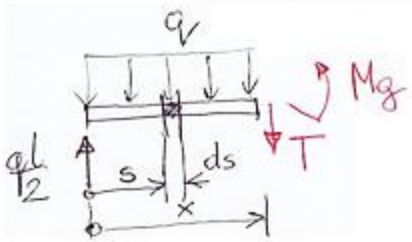
$\boxed{R_z^A = \frac{ql}{2}}$



r-nia r-gi: $\Sigma F_z = 0$: $\quad \frac{ql}{2} - \int_0^x q\,ds - T = 0$

$\boxed{T = \frac{ql}{2} - q\,s\Big|_0^x = q\left(\frac{l}{2} - x\right)}$





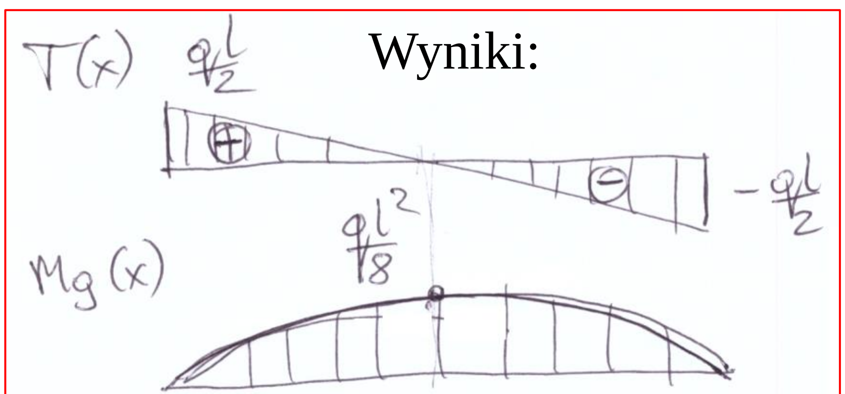
$\Sigma M_{pp} = 0$:

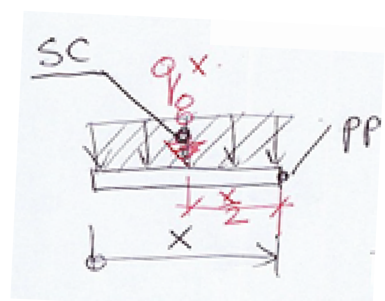
$-\frac{ql}{2} \cdot x + \int_0^x q\,ds \cdot (x-s) + M_g = 0 \rightarrow M_g = \frac{ql}{2} \cdot x - q \int_0^x (x-s)\,ds$

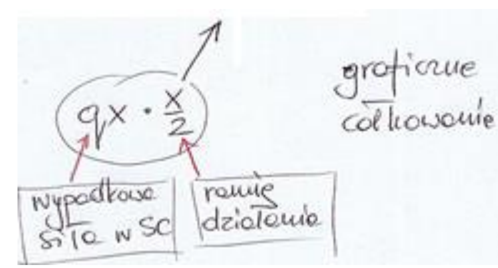
$M_g = \frac{ql}{2} x - q\left(xs - \frac{1}{2}s^2\right)\Big|_0^x \rightarrow M_g = \frac{ql}{2} x - q\left(x^2 - \frac{1}{2}x^2\right)$

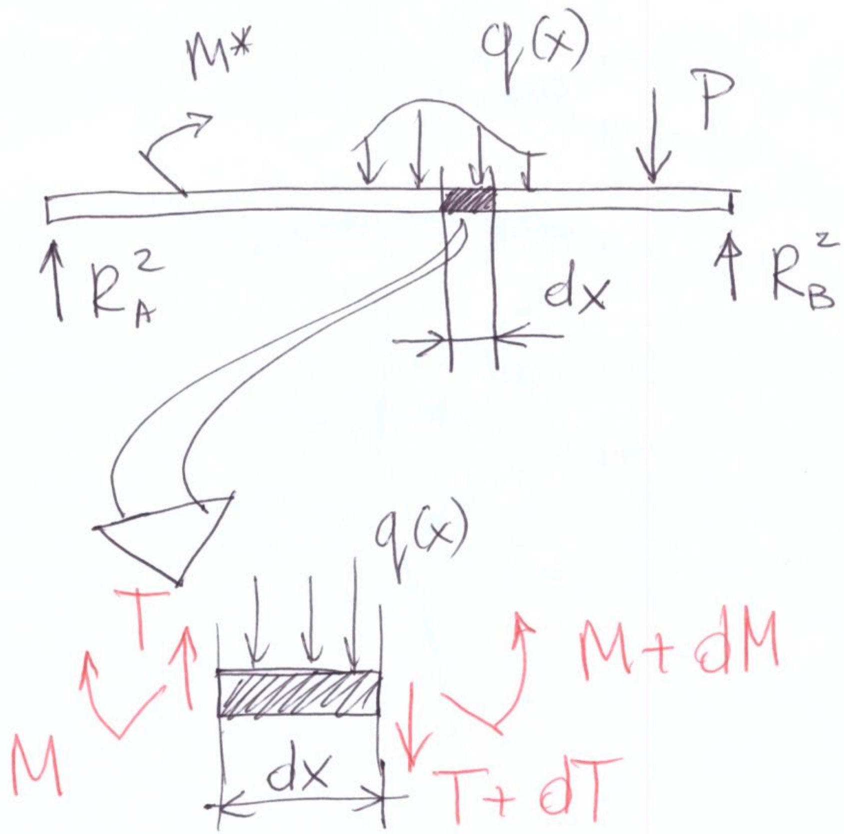
$$\boxed{M_g = \frac{ql}{2} x - \frac{1}{2} q x^2}$$

Wyniki:





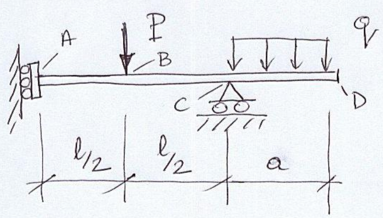




$$q(x) = -\frac{dT(x)}{dx}$$

$$T(x) = \frac{dMg(x)}{dx}$$

Zadanie 3



$P = 2\,kN$

$q = 10\,\frac{kN}{m}$

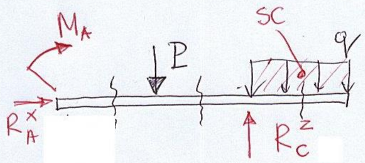
$l = 1\,m$

$a = 0.4\,m$

$T(x)$

$M_g(x)$

?



Reakcje:

r-nie r-gi:

$\Sigma F_x = 0: \quad \boxed{R_A^x = 0}$

$$\Sigma M_c = 0: \quad -M_A + P\cdot\frac{l}{2} - qa\cdot\frac{a}{2} = 0$$

$$\boxed{M_A = \frac{Pl}{2} - \frac{qa^2}{2}}$$

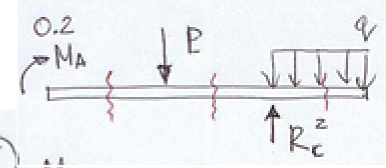
$$M_A = \frac{2\cdot 1}{2} - \frac{10\cdot 0.4^2}{2}$$

$$M_A = 1 - 0.8 = 0.2\ kNm$$

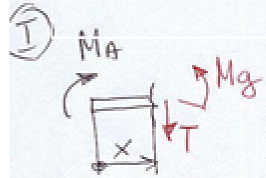
$$\Sigma F_z = 0 \quad -P + R_c^z - qa = 0$$

$$\boxed{R_c^z = qa + P}$$

$$R_c^z = 10\cdot 0.4 + 2 = 6\ kN$$



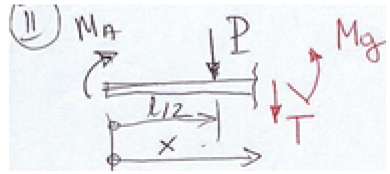
Myślowe przecięcia:



$\Sigma F_z = 0: \quad -T = 0 \quad \boxed{T = 0}$

$\Sigma M_{pp} = 0: \quad -M_A + M_g = 0$

$$\boxed{M_g(x) = M_A}$$



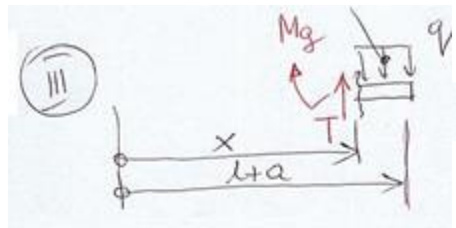
$\Sigma F_z = 0: \quad -P - T = 0$

$$\boxed{T(x) = -P}$$

$\Sigma M_{pp} = 0: \quad -M_A + P\left(x - \frac{l}{2}\right) + M_g = 0$

$$\boxed{M_g(x) = M_A + P\left(\frac{l}{2} - x\right)}$$

SC



$\Sigma F_z = 0: \quad T - q\cdot(l+a-x) = 0$

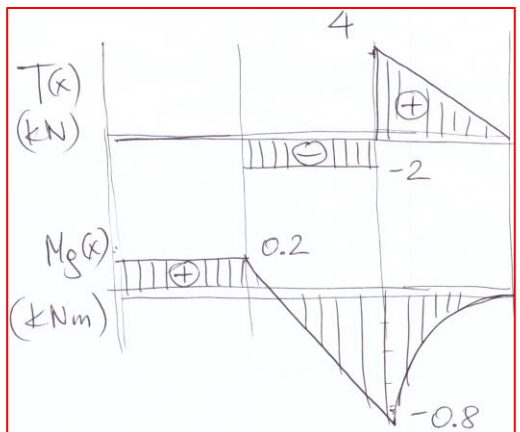
$$\boxed{T(x) = q\,(l+a-x)}$$

$\Sigma M_{pp} = 0:$

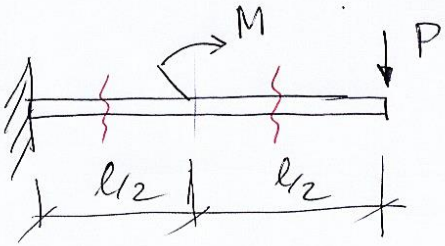
$$-M_g - \frac{q\cdot(l+a-x)^2}{2} = 0$$

$$\boxed{M_g(x) = -q\,\frac{(l+a-x)^2}{2}}$$

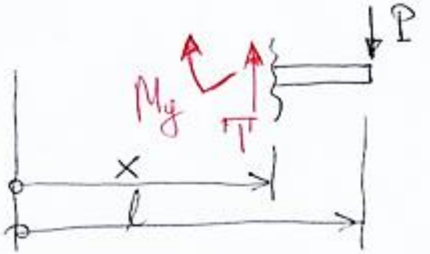
Wyniki:



Zadanie 4



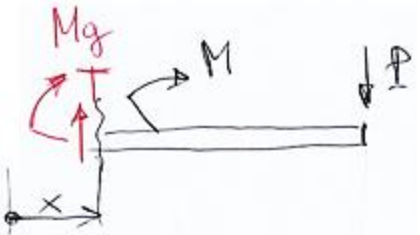
$M = 0.5\ kNm$ | $M_g(x)$
$P = 1\ kN$ | $T(x)$
$l = 1\ m$ | 2



r-nie r-gi:

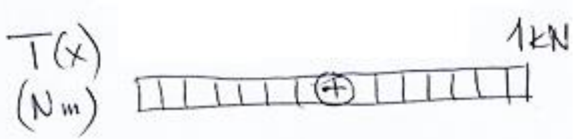
$\Sigma F_z = 0: \quad T - P = 0$ $\qquad \boxed{T(x) = P}$

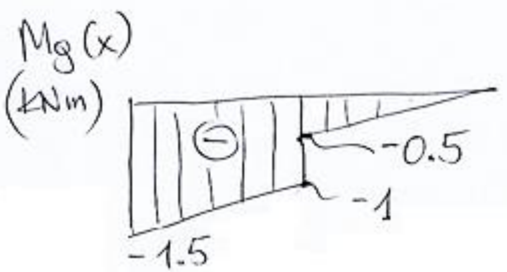
$\Sigma M_{PP} = 0: \quad -M_g - P\cdot(l-x) = 0$ $\qquad \boxed{M_g(x) = -P(l-x)}$



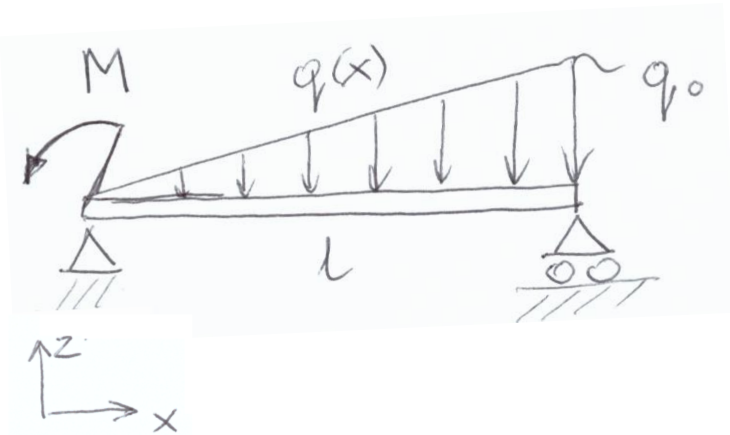
$\Sigma F_z = 0: \quad T - P = 0$ $\qquad \boxed{T(x) = P}$

$\Sigma M_{PP} = 0: \quad -M_g - M - P(l-x) = 0$ $\qquad \boxed{M_g(x) = -P(l-x) - M}$





Zadanie 5

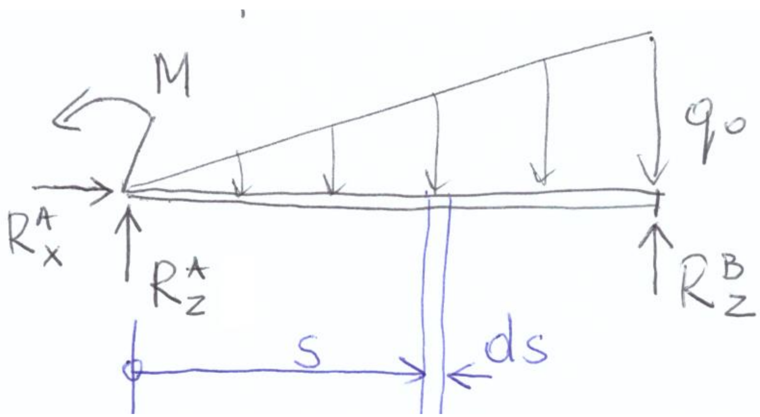


$$q_0 = 9\ \frac{kN}{m}$$

$$M = 1{,}25\ kNm$$

$$l = 1\ m$$

$$\boxed{q(x) = \frac{q_0}{l}\cdot x}$$



Reakcje: $\sum M_A = 0$:

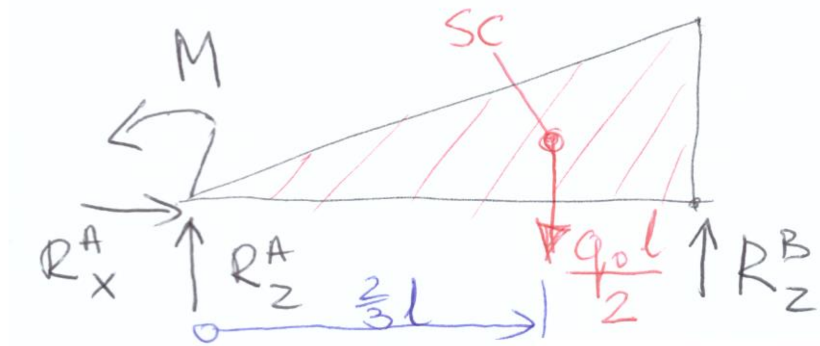
$$-\int_0^l s\cdot q(s)\cdot ds + R_z^B\cdot l + M = 0$$

$$R_z^B = \int_0^l \frac{q_0}{l^2}\cdot s^2 ds - \frac{M}{l}$$

$$R_z^B = \frac{q_0}{l^2}\,\frac{1}{3}\,l^3 - \frac{M}{l}$$

$$\boxed{R_z^B = \frac{q_0 l}{3} - \frac{M}{l}}$$

Prościej (graficznie)



$\Sigma M_A = 0:$

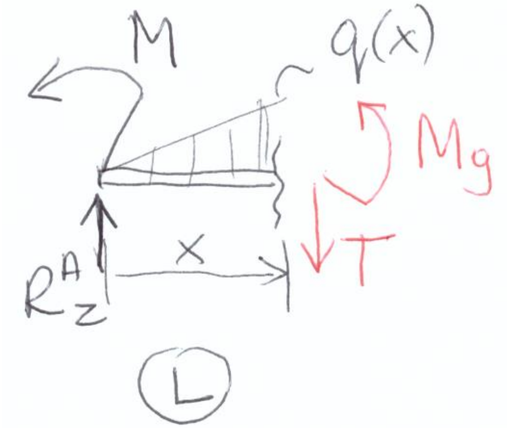
$$-\boxed{\frac{q_0 l}{2}} \cdot \boxed{\frac{2}{3} l} + R_z^B \cdot l + M = 0$$

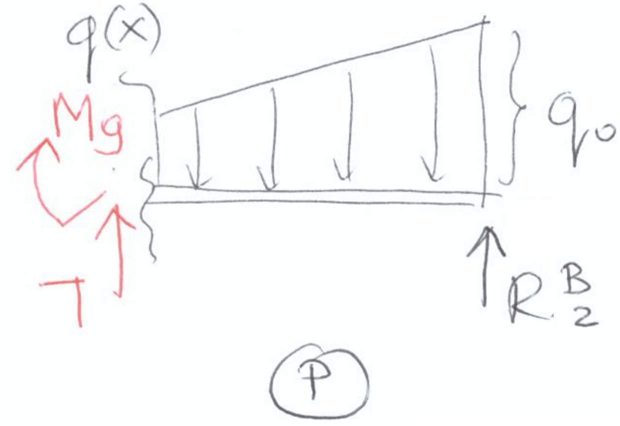
$$\boxed{R_z^B = \frac{q_0 l}{3} - \frac{M}{l}} = 1{,}75 \text{ kN}$$

$$\Sigma M_B = 0: \quad \boxed{\frac{q_0 l}{2}} \cdot \boxed{\frac{1}{3} l} - R_z^A \cdot l + M = 0$$

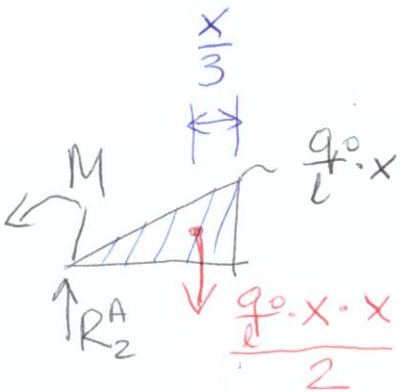
$$\boxed{R_z^A = \frac{q_0 l}{6} + \frac{M}{l}} = 2{,}75 \text{ kN}$$

$$\Sigma F_x = 0: \quad \boxed{R_x^A = 0}$$





$$\Sigma F_z = 0: \quad -T + R_z^A - \int_0^x q(s)\,ds = 0$$



$$T = \boxed{\frac{q_0 l}{6} + \frac{M}{l}} - \boxed{\frac{\frac{q_0}{l}\cdot x \cdot x}{2}} = \boxed{\frac{q_0 l}{6} + \frac{M}{l} - \frac{q_0 x^2}{2l}}$$

graficznie

$$\Sigma M_{pp} = 0: \quad M_g + \int_0^x (x-s)\, q(s)\cdot ds - R_z^A \cdot x + M = 0$$

graficznie

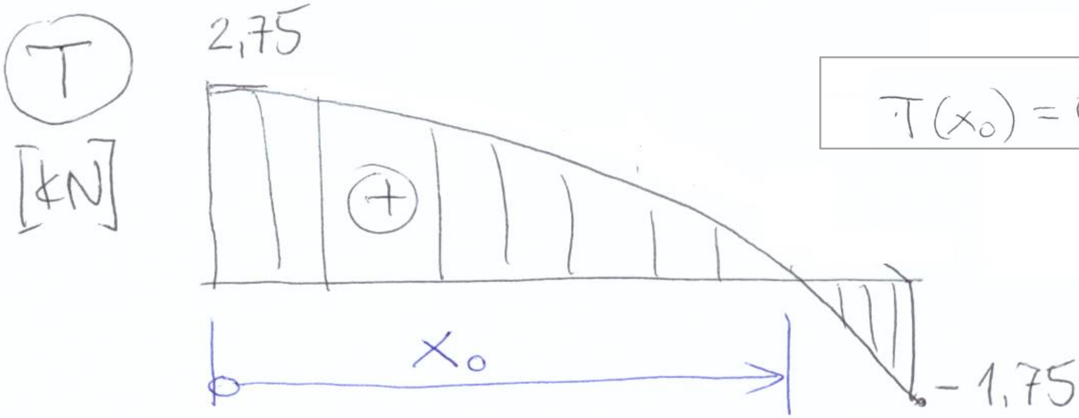
$$M_g = \boxed{\left(\frac{q_0 l}{6} + \frac{M}{l}\right)} \cdot x - M - \boxed{\frac{\frac{q_0}{l} x \cdot x}{2} \cdot \frac{x}{3}}$$

$$\boxed{M_g = -\frac{q_0 x^3}{6l} + \left(\frac{q_0 l}{6} + \frac{M}{l}\right)\cdot x - M}$$

$$\boxed{T = \frac{q_0 l}{6} + \frac{M}{l} - \frac{q_0 x^2}{2l}}$$

$$T(0) = \frac{q_0 l}{6} + \frac{M}{l} = \frac{9 \cdot 1}{6} + \frac{1{,}25}{1} = 2{,}75 \text{ kN} = R_z^A$$

$$T(l) = \frac{q_0 l}{6} + \frac{M}{l} - \frac{q_0 l}{2} = -1{,}75 \text{ kN} = -R_z^B$$



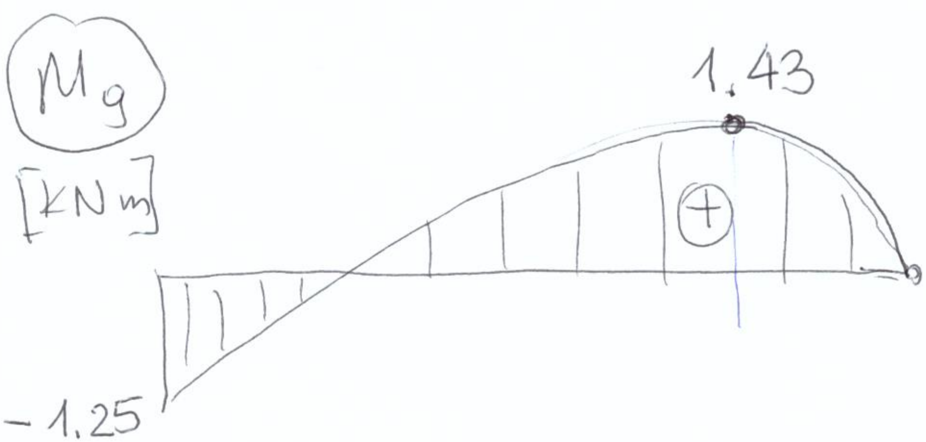
$$\boxed{T(x_0) = 0} \rightarrow \frac{q_0 l}{6} + \frac{M}{l} - \frac{q_0 x_0^2}{2l} = 0$$

$$x_0 = \pm\sqrt{\left(\frac{q_0 l}{6} + \frac{M}{l}\right) \cdot \frac{2l}{q_0}}$$

$$x_0 = \pm\sqrt{\left(\frac{9 \cdot 1}{6} + \frac{1{,}25}{1}\right) \cdot \frac{2 \cdot 1}{9}}$$

$$x_0 = 0{.}782 \text{ m}$$

$$\boxed{M_g = -\frac{q_0 x^3}{6l} + \left(\frac{q_0 l}{6} + \frac{M}{l}\right) \cdot x - M}$$

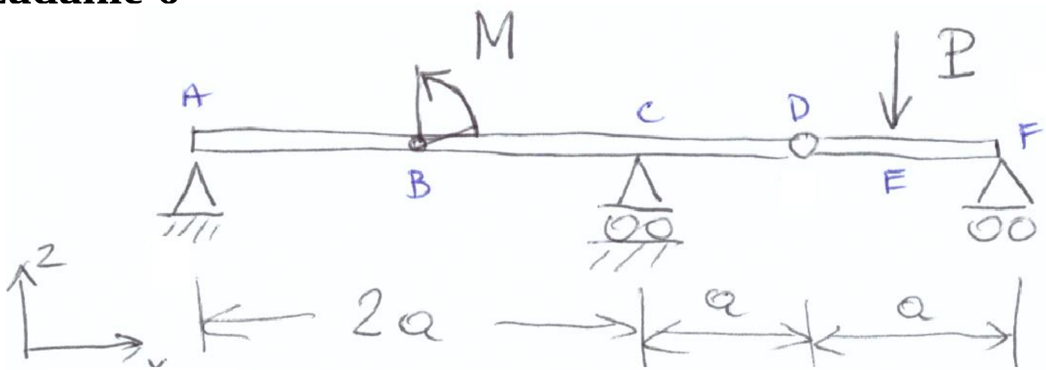


$$M_g(0) = -M = -1{,}25 \text{ kNm}$$

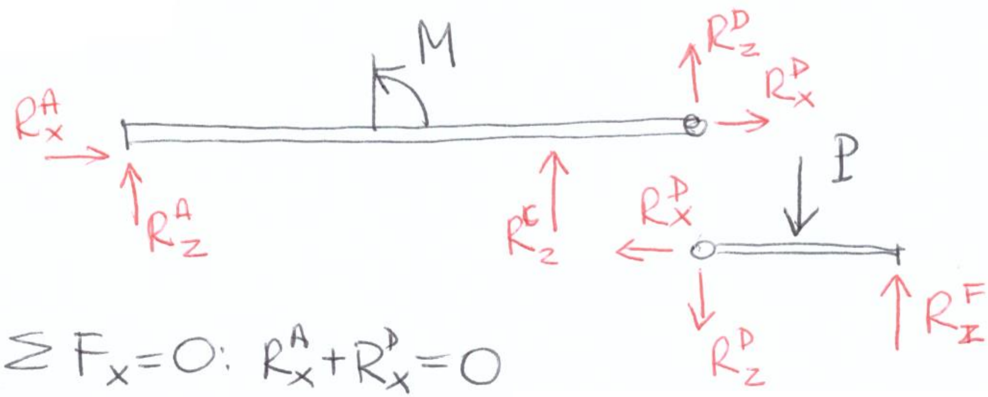
$$M_g(l) = 0$$

$$M_g(x_0) = 1{,}43 \text{ kNm}$$

Zadanie 6



$M = 1\,kNm$
$P = 1\,kN$
$a = 1\,m$



$$\Sigma F_x = 0: \quad R_x^A + R_x^D = 0$$

$$R_x^A = 0$$

$$\Sigma M_A = 0: \quad M + R_z^C \cdot 2a + R_z^D \cdot 3a = 0$$

$$\boxed{R_z^C = -\frac{M}{2a} + \frac{3}{4}P}$$

$$\Sigma F_x = 0: \quad R_x^D = 0$$

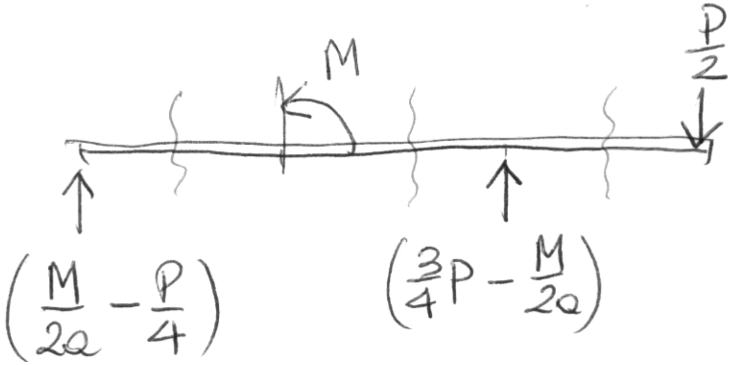
$$\Sigma M_D = 0: \quad -P \cdot \frac{a}{2} + R_z^F \cdot a = 0$$

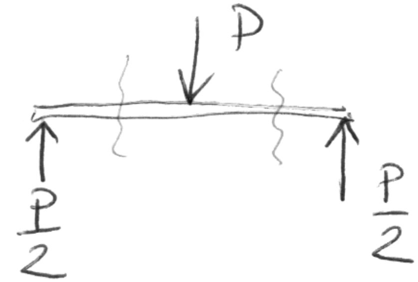
$$\boxed{R_z^F = \frac{P}{2}}$$

$$\Sigma F_z = 0: \quad R_z^D = -P + R_z^F$$

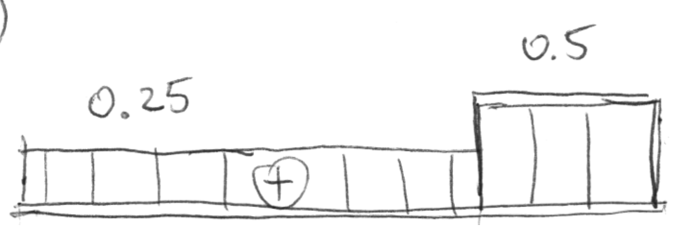
$$\boxed{R_z^D = -\frac{P}{2}}$$

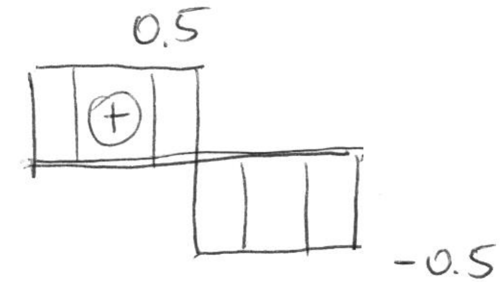
$$\Sigma F_z = 0: \quad R_z^A + R_z^C + R_z^D = 0 \rightarrow R_z^A = \frac{M}{2a} - \frac{3}{4}P + \frac{P}{2} \rightarrow \boxed{R_z^A = \frac{M}{2a} - \frac{P}{4}}$$


M
$\frac{P}{2}$
$\left(\frac{M}{2a} - \frac{P}{4}\right)$
$\left(\frac{3}{4}P - \frac{M}{2a}\right)$

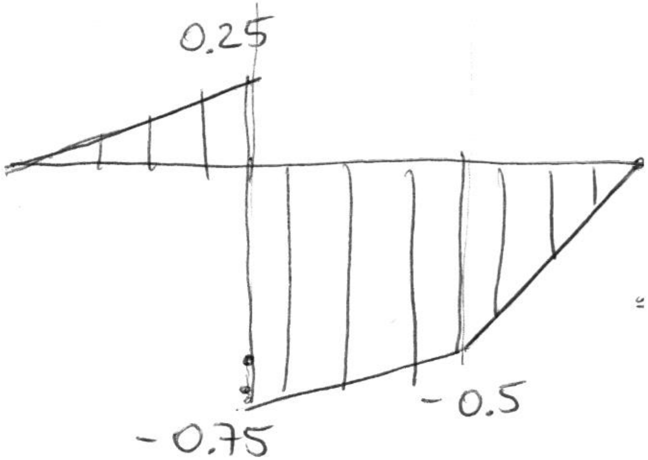

P
$\frac{P}{2}$
$\frac{P}{2}$

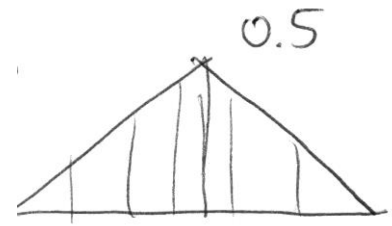
T


0.25
0.5
+

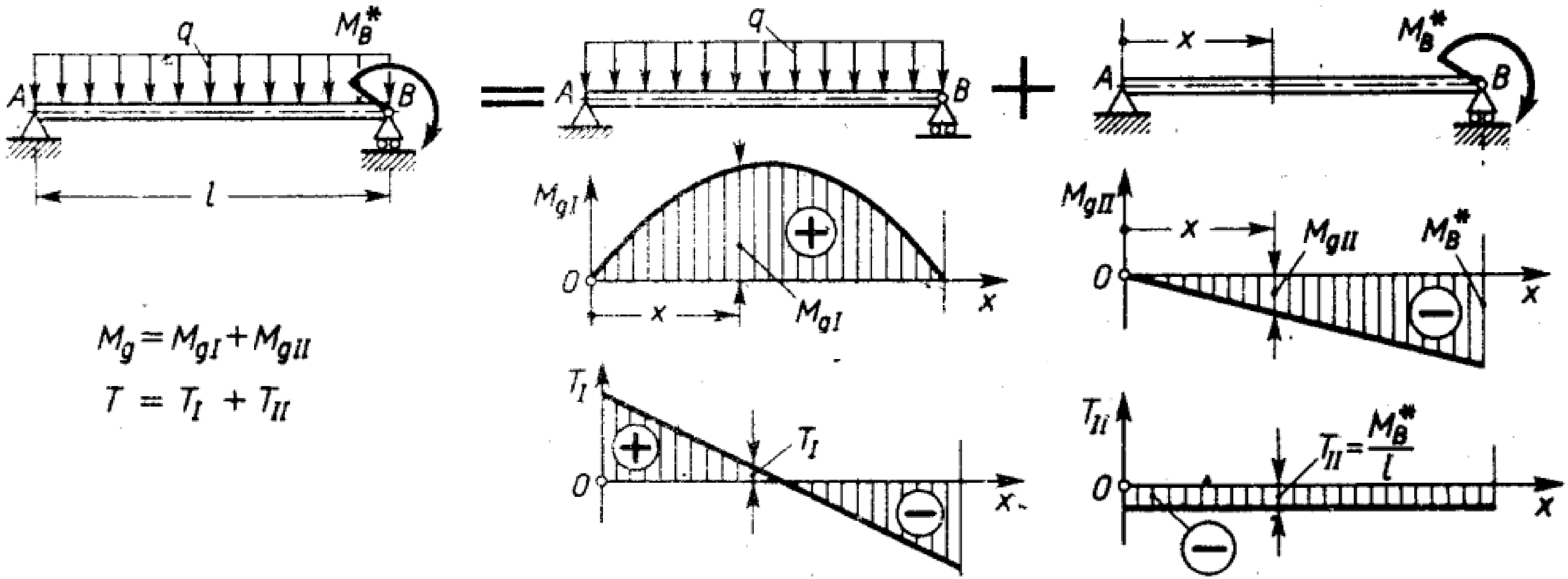

0.5
+
-0.5

Mg

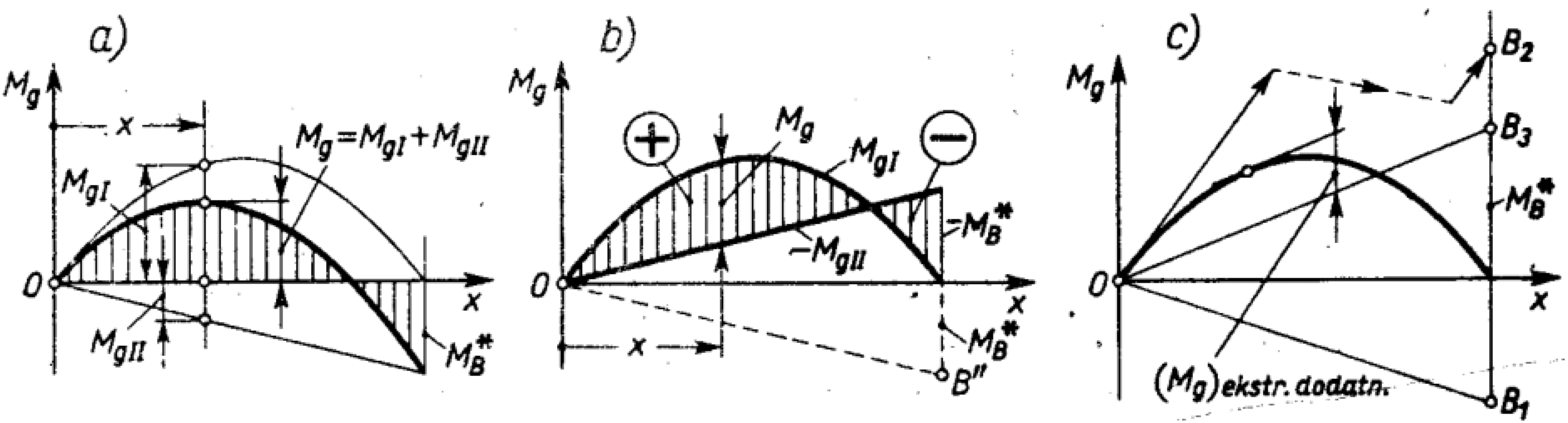

0.25
-0.5
-0.75


0.5

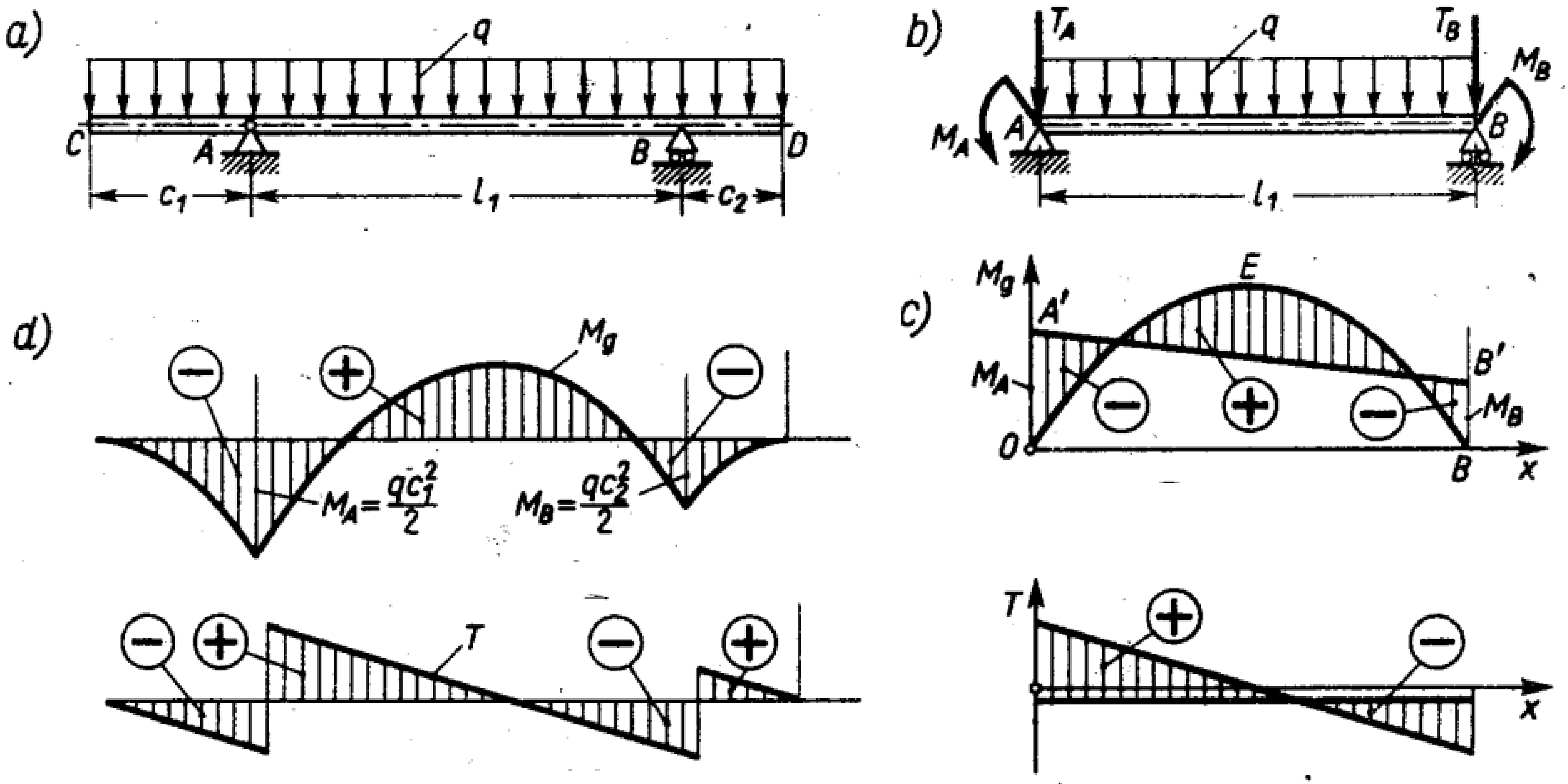
Niektóre ułatwienia obliczeń



Zastosowanie zasady superpozycji do określania M_g i T

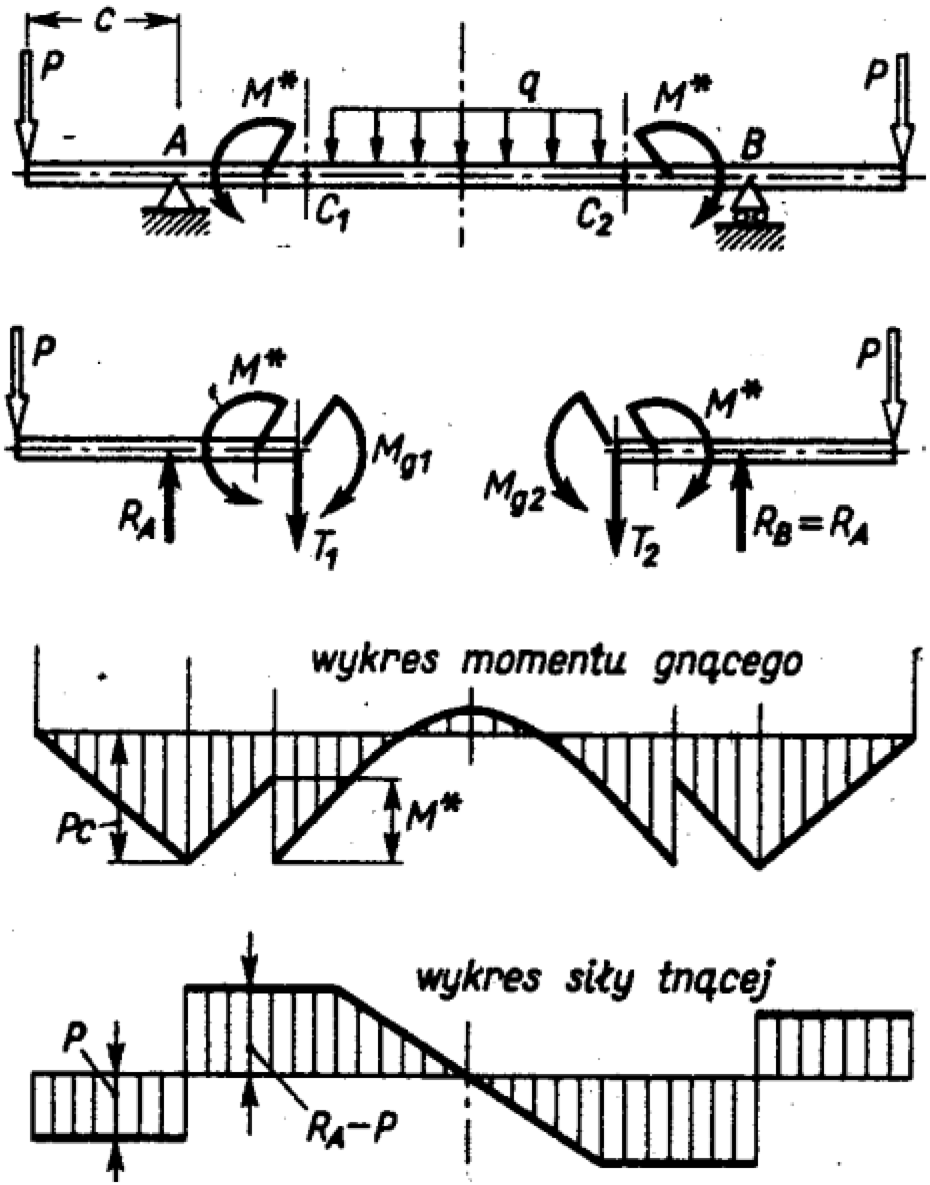


Niektóre ułatwienia obliczeń

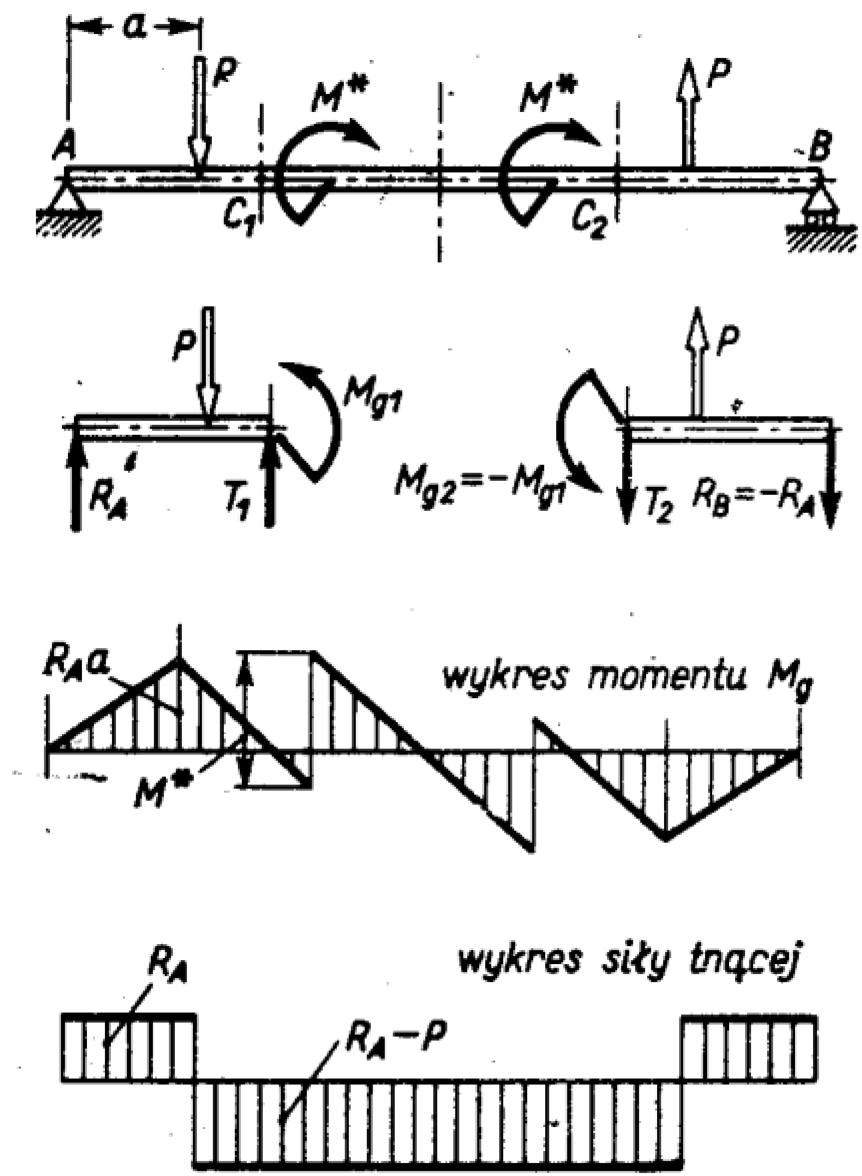


Przykład zastosowania zasady superpozycji

Niektóre ułatwienia obliczeń



Wykresy T i M_g przy obciążeniu symetrycznym



Wykresy T i M_g przy obciążeniu antysymetrycznym