Mechanics of Thin-walled Structures

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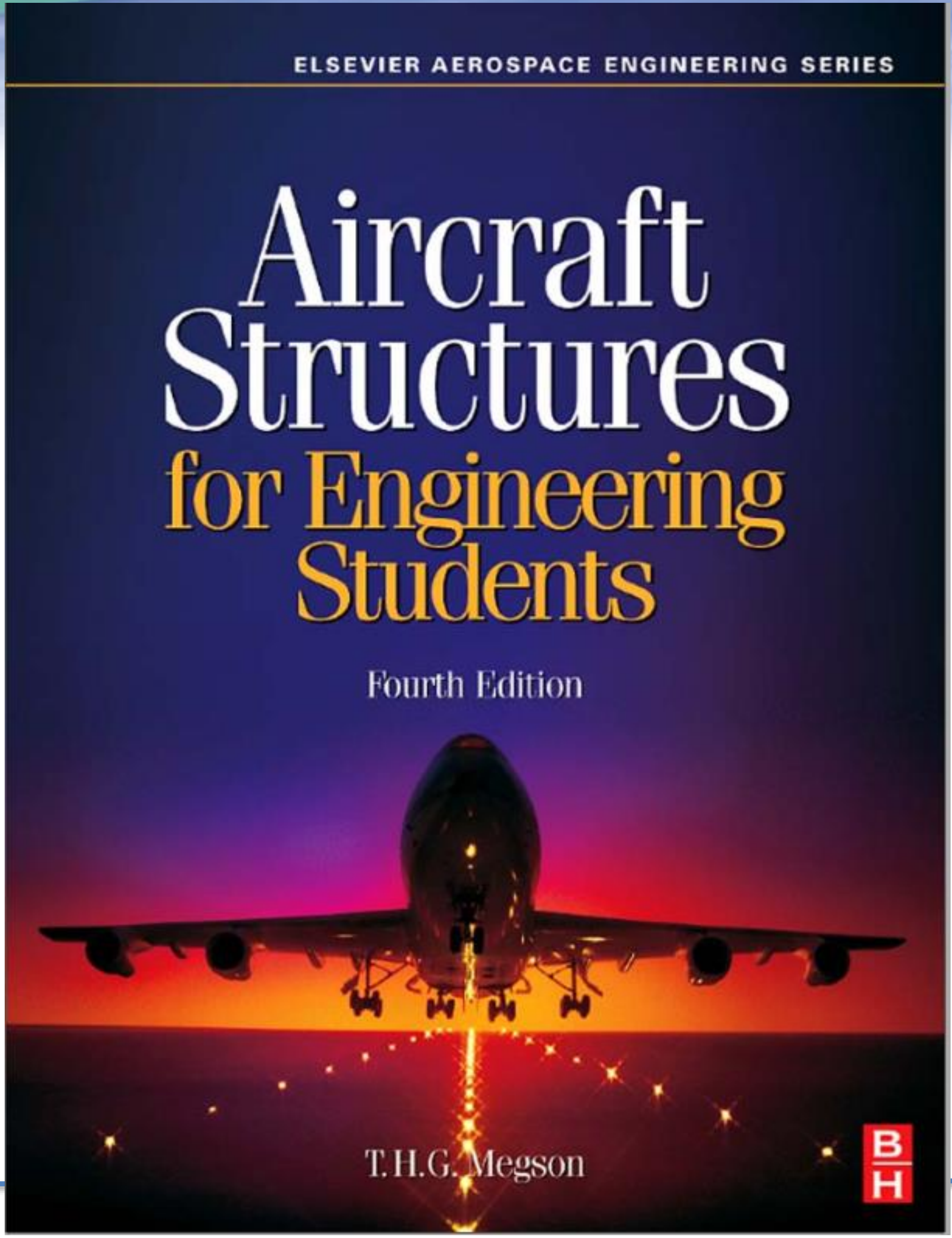
1. Basic concepts of mechanics of structures
    a. Stress
    b. Strain
    c. Moment of inertia
        i. First (static) moment of area (static)
        ii. Second moment of area (inertia)

2. Thin-walled structures introduction

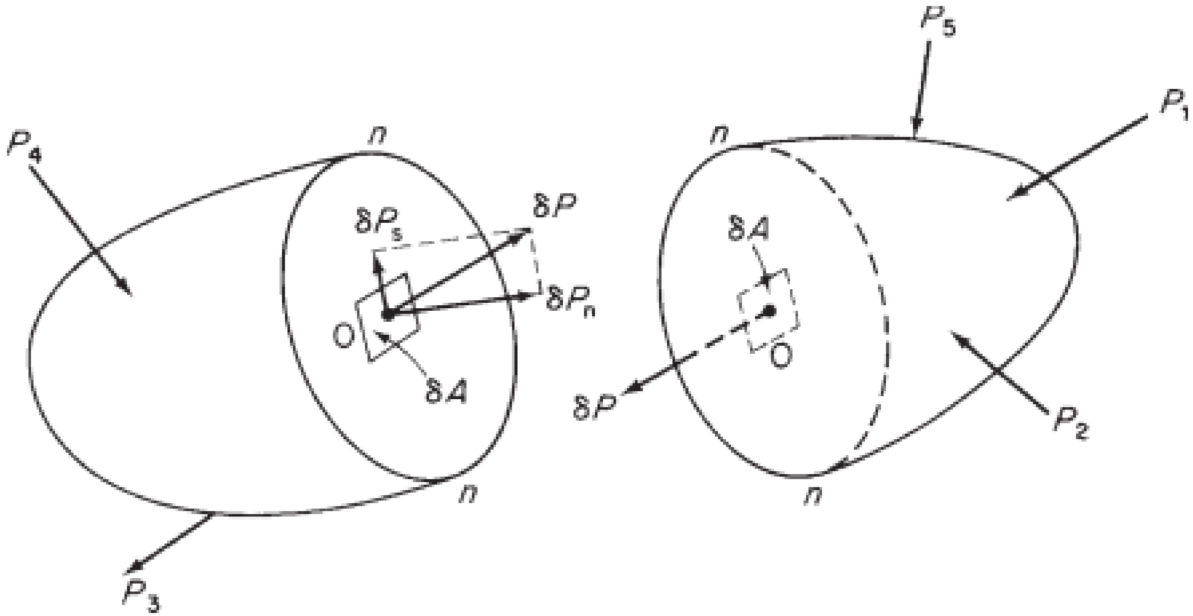
3. Beams
    a. Bending of beams
        i. Shear centre
        ii. Open section beams
            - First (static) moment of inertia approach
            - Function approach
        iii. Closed section beams
    b. Torsion of beams
        i. Free torsion
        ii. Constrained torsion

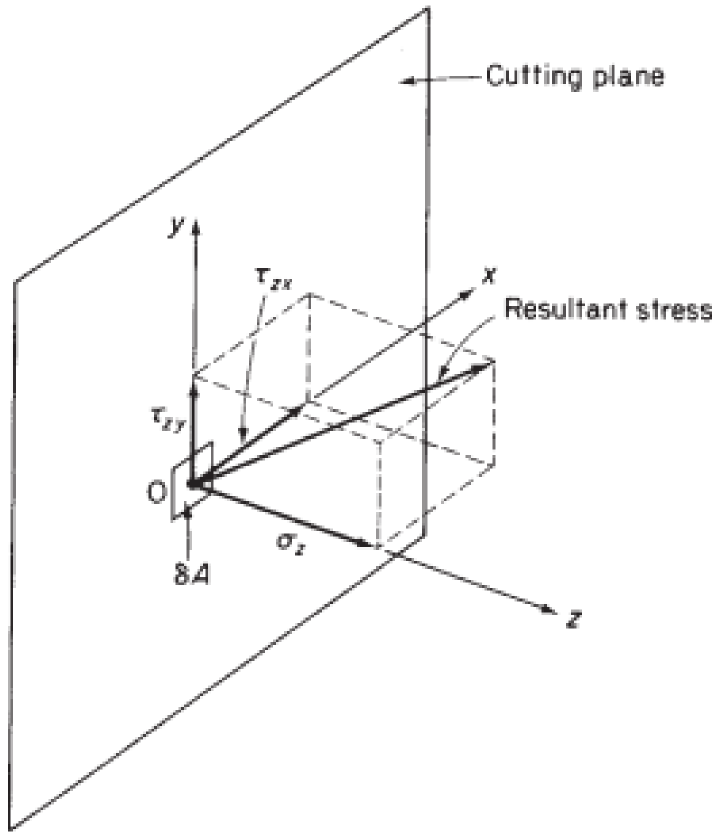
4. Plates and shells (2D structures)
5. Buckling
    a. Analytical approach
    b. Energy approach
    c. Buckling of columns
    d. Buckling of plates
    e. Buckling of shells


ELSEVIER AEROSPACE ENGINEERING SERIES
Aircraft Structures for Engineering Students
Fourth Edition
T.H.G. Megson

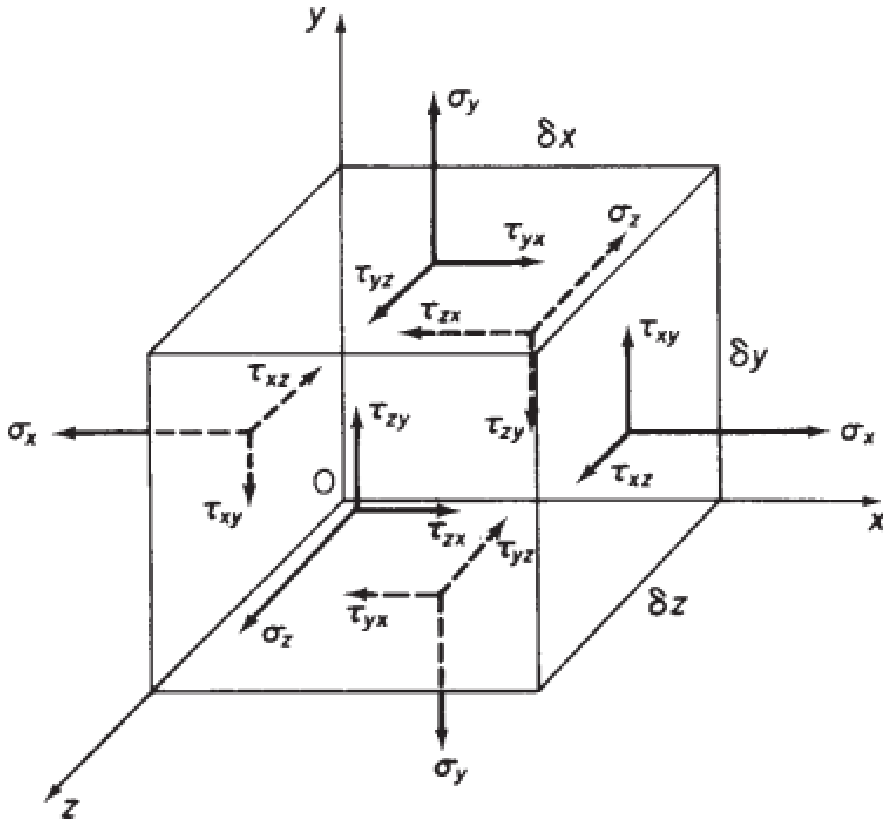
Stress, Strain, Hooke`s law moments of Area

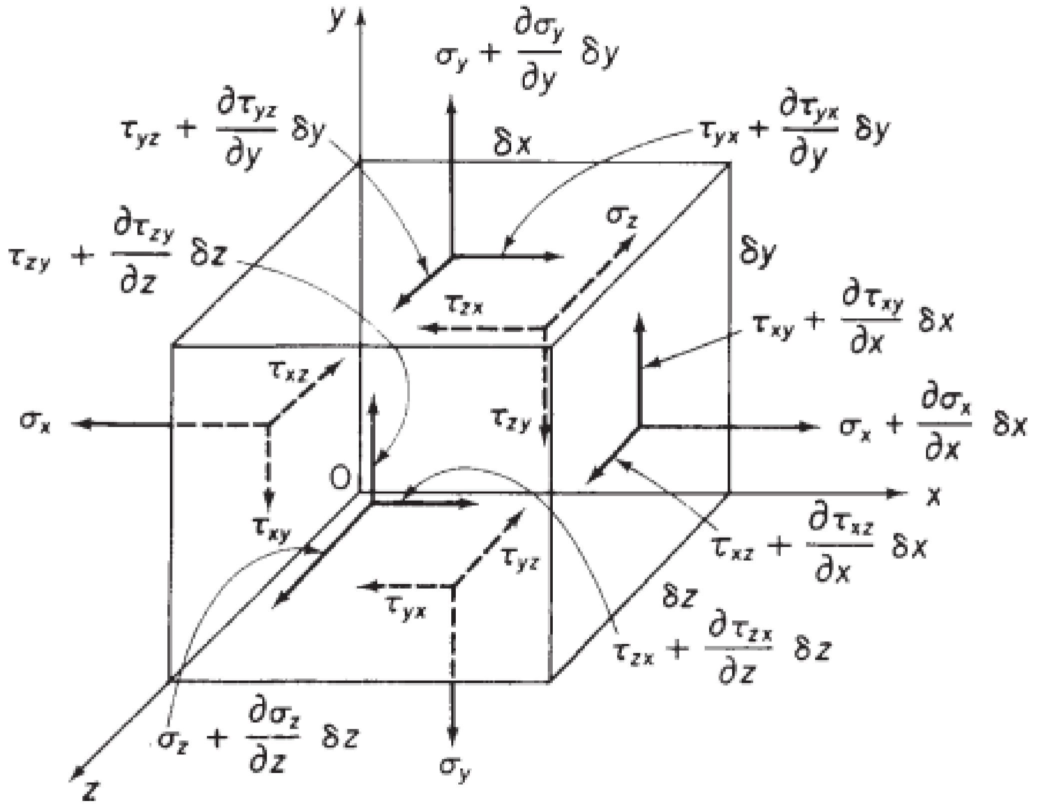
State of stress





State of stress



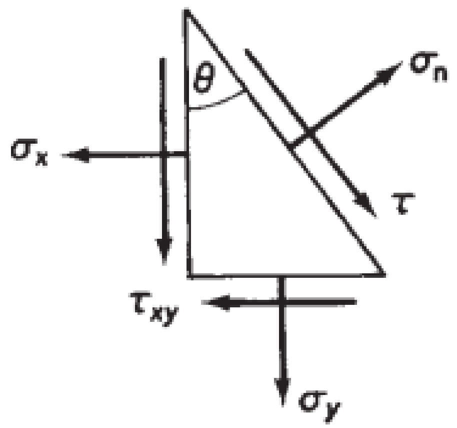


State of stress

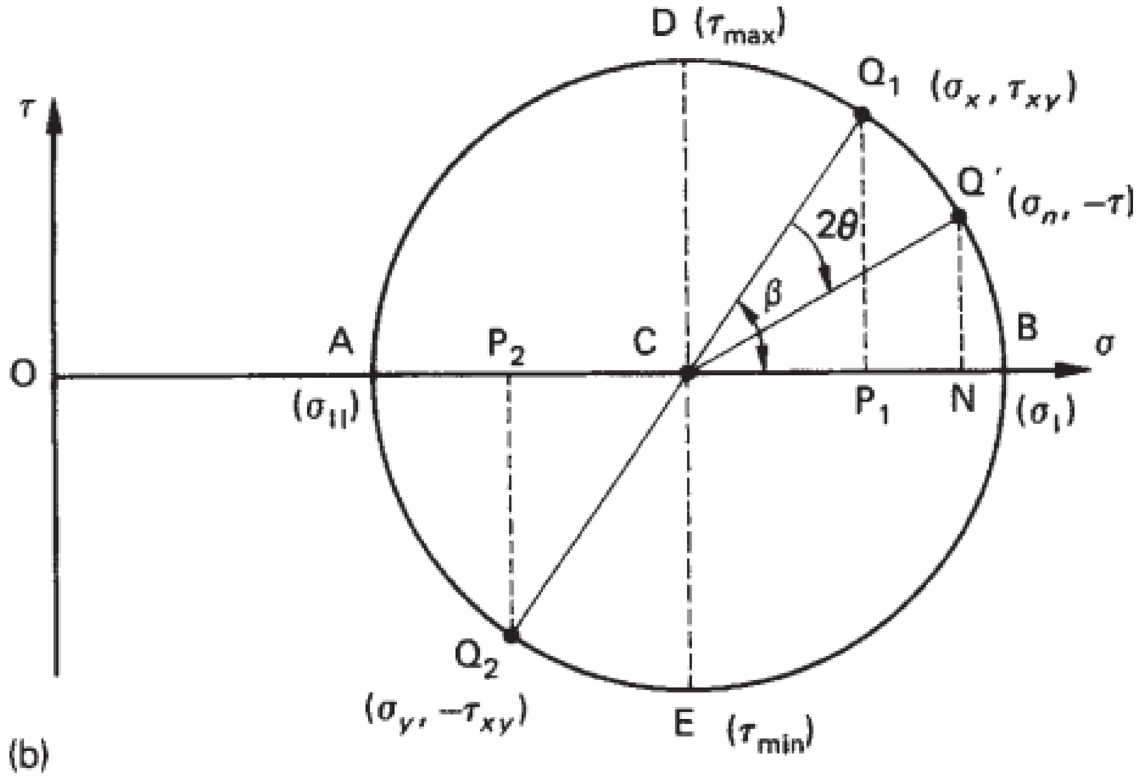
$$\left.\begin{aligned}
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + X = 0 \\
\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} + Y = 0 \\
\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + Z = 0
\end{aligned}\right\}$$

$$\left.\begin{aligned}
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X = 0 \\
\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + Y = 0
\end{aligned}\right\}$$

State of stress

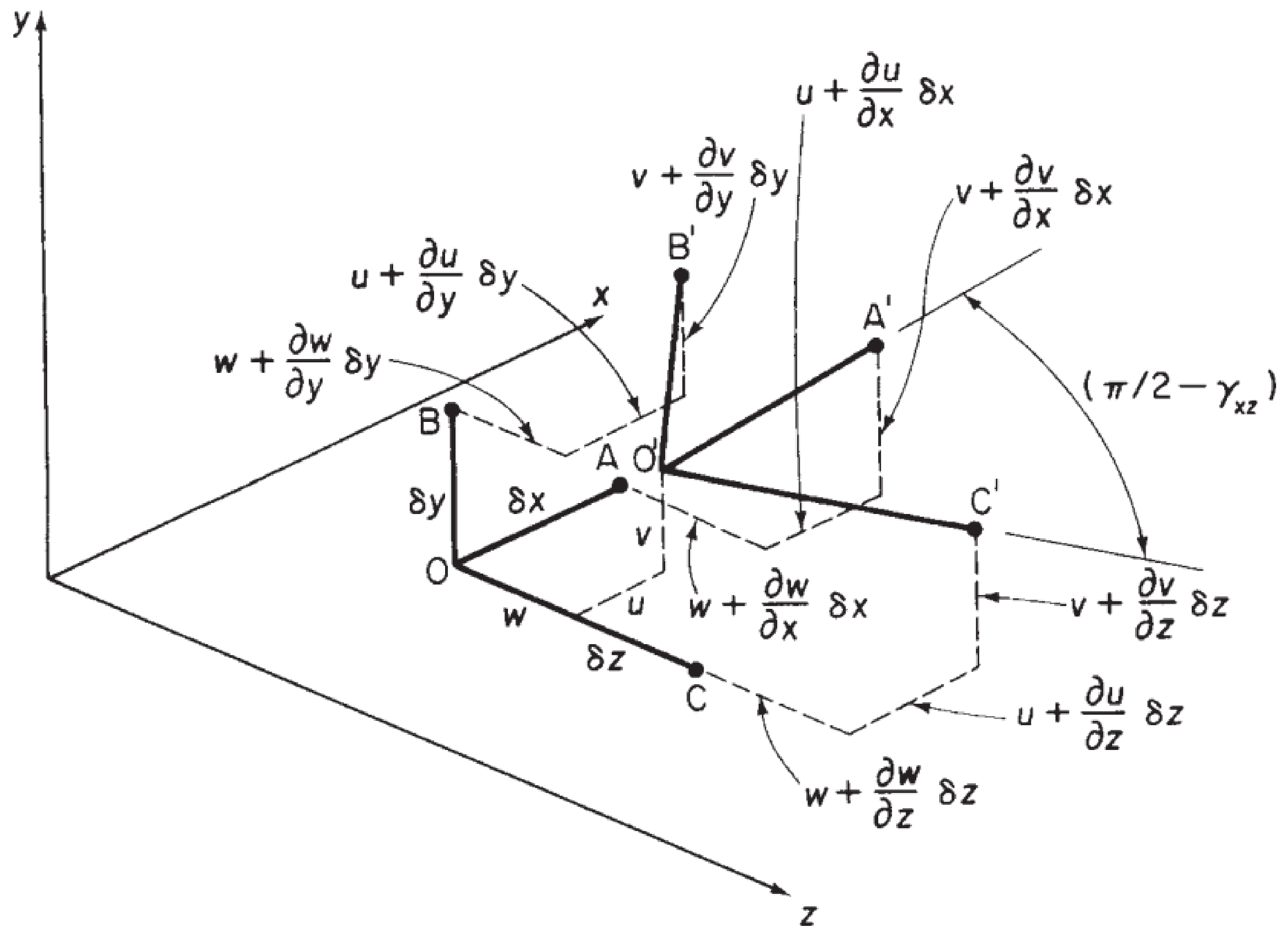


(a)



(b)

State of strain



State of strain

$$\varepsilon = \lim_{L \to 0} \frac{\Delta L}{L}$$

$$\left.\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} \\ \varepsilon_y &= \frac{\partial v}{\partial y} \\ \varepsilon_z &= \frac{\partial w}{\partial z} \end{aligned}\right\}$$

$$\left.\begin{aligned} \gamma_{xz} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \\ \gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\ \gamma_{yz} &= \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \end{aligned}\right\}$$

Stress – strain relations
1-D case

$$\varepsilon_x = \frac{\sigma_x}{E}$$

$$\varepsilon_y = -\nu \frac{\sigma_x}{E} \qquad \varepsilon_z = -\nu \frac{\sigma_x}{E}$$

Stress – strain relations
3-D case

$$\left.\begin{aligned}\varepsilon_x &= \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]\\ \varepsilon_y &= \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]\\ \varepsilon_z &= \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]\end{aligned}\right\}$$

$$\sigma_x = \frac{\nu E}{(1+\nu)(1-2\nu)}e + \frac{E}{(1+\nu)}\varepsilon_x$$

$$\sigma_y = \frac{\nu E}{(1+\nu)(1-2\nu)}e + \frac{E}{(1+\nu)}\varepsilon_y$$

$$\sigma_z = \frac{\nu E}{(1+\nu)(1-2\nu)}e + \frac{E}{(1+\nu)}\varepsilon_z$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Stress – strain relations
2-D case

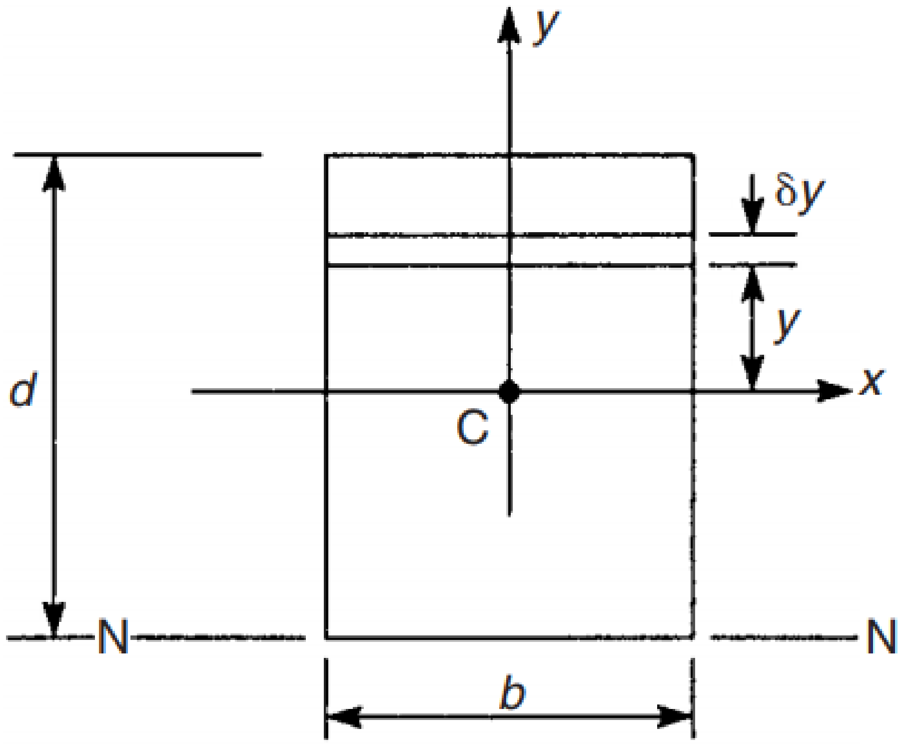
$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

$$\gamma = \tau/G$$

$$\gamma = \frac{2(1+\nu)}{E}\tau$$

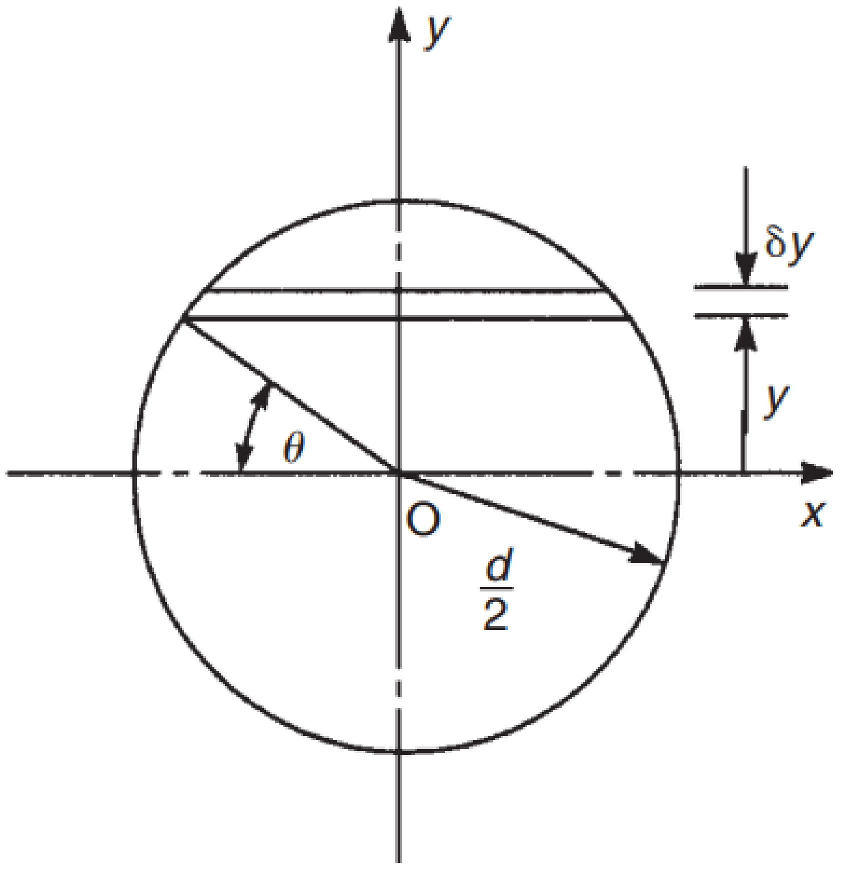
Second moment of area



$$I_{xx} = \int_A y^2 \mathrm{d}A = \int_{-d/2}^{d/2} by^2 \mathrm{d}y = b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$I_{xx} = \frac{bd^3}{12}$$

Second moment of area



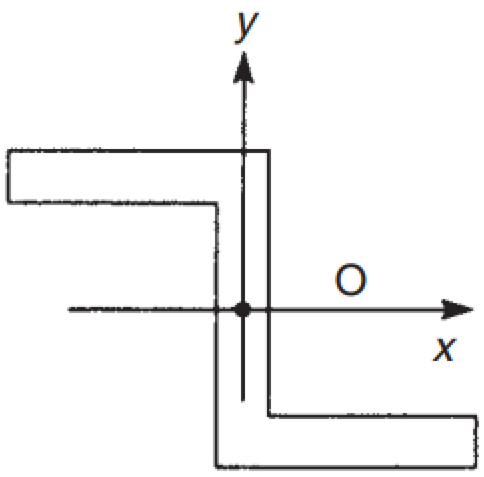
$$I_{xx} = \int_A y^2 \mathrm{d}A = \int_{-d/2}^{d/2} 2\left(\frac{d}{2}\cos\theta\right) y^2 \mathrm{d}y$$

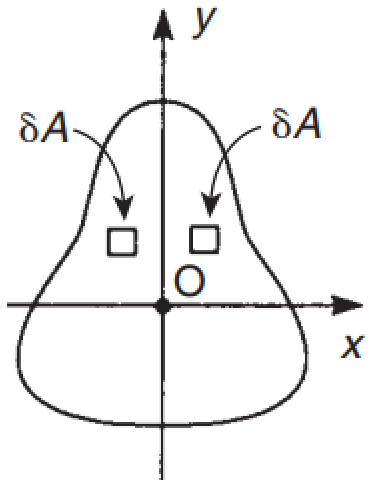
$$I_{xx} = \int_{-\pi/2}^{\pi/2} d\cos\theta \left(\frac{d}{2}\sin\theta\right)^2 \frac{d}{2}\cos\theta\,\mathrm{d}\theta$$

$$I_{xx} = \frac{d^4}{8}\int_{-\pi/2}^{\pi/2} \cos^2\theta\,\sin^2\theta\,\mathrm{d}\theta$$

$$I_{xx} = \frac{\pi d^4}{64}$$

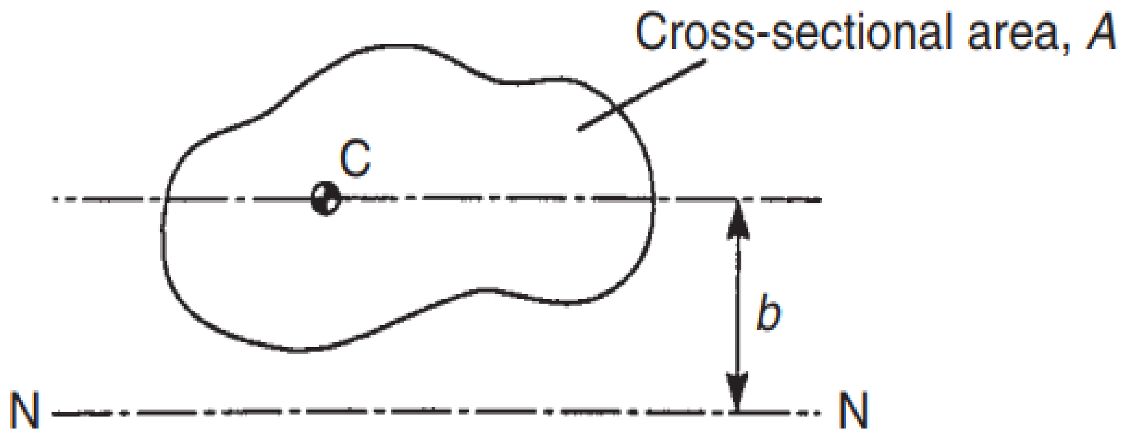
Product second moment of area





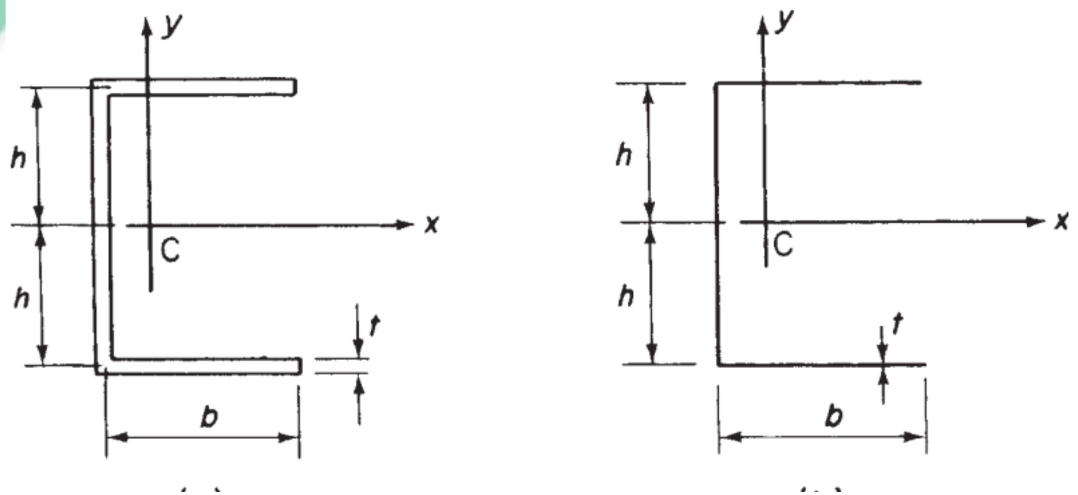
$$I_{xy} = \int_A xy\,\mathrm{d}A$$

Parallel axes theorem (Steiner principle)



$$I_N = I_C + Ab^2$$

Aproximations for Thin-Walled sections

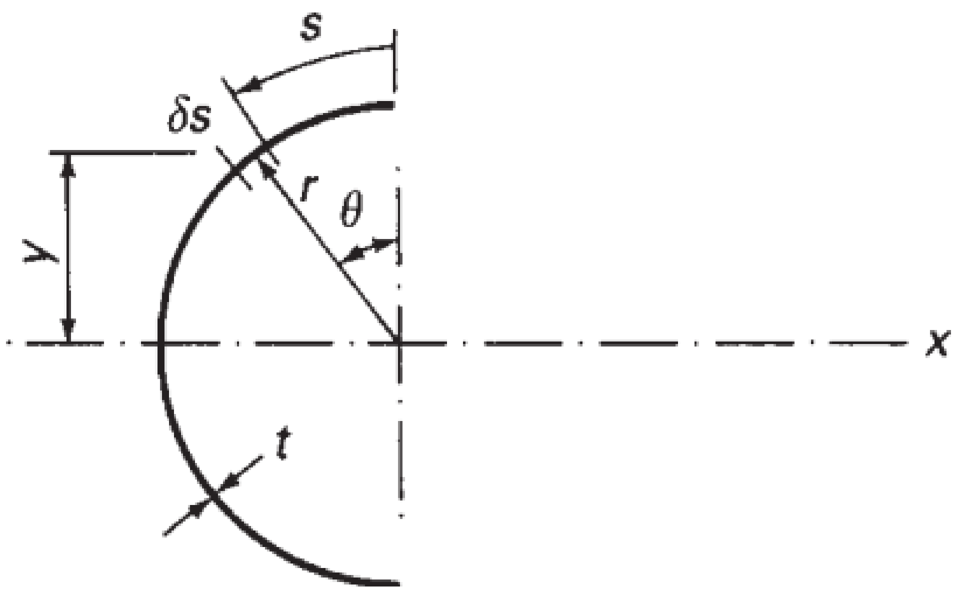


$$I_{xx} = 2\left[\frac{(b+t/2)t^3}{12} + \left(b+\frac{t}{2}\right)th^2\right] + t\frac{[2(h-t/2)]^3}{12}$$

$$I_{xx} = 2\left[\frac{(b+t/2)t^3}{12} + \left(b+\frac{t}{2}\right)th^2\right] + \frac{t}{12}\left[(2)^3\left(h^3 - 3h^2\frac{t}{2} + 3h\frac{t^2}{4} - \frac{t^3}{8}\right)\right]$$

$$I_{xx} = 2bth^2 + t\frac{(2h)^3}{12}$$

Aproximations for Thin-Walled sections

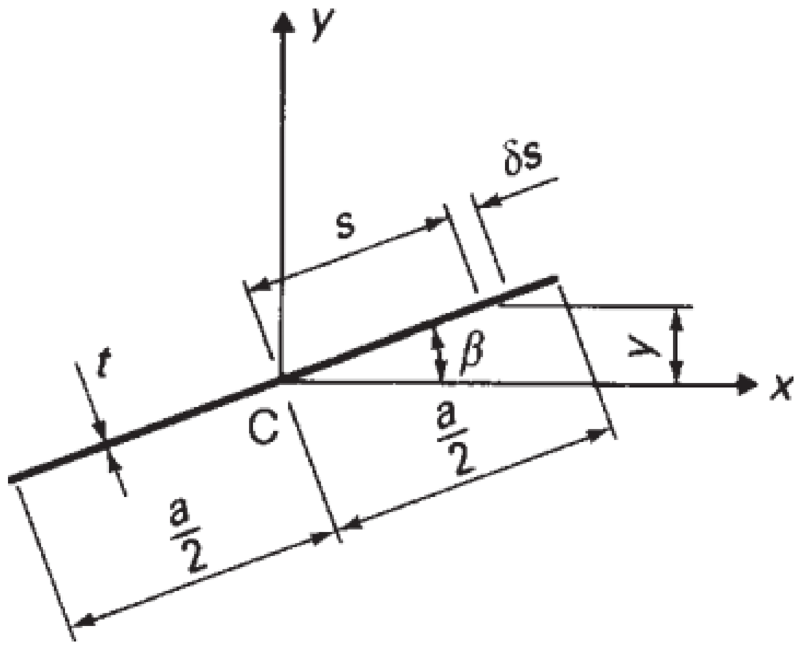


$$I_{xx} = \int_0^{\pi r} t y^2 \, ds$$

$$I_{xx} = \int_0^{\pi} t (r \cos\theta)^2 r \, d\theta$$

$$I_{xx} = \frac{\pi r^3 t}{2}$$

Thin-Walled sections – inclined walls



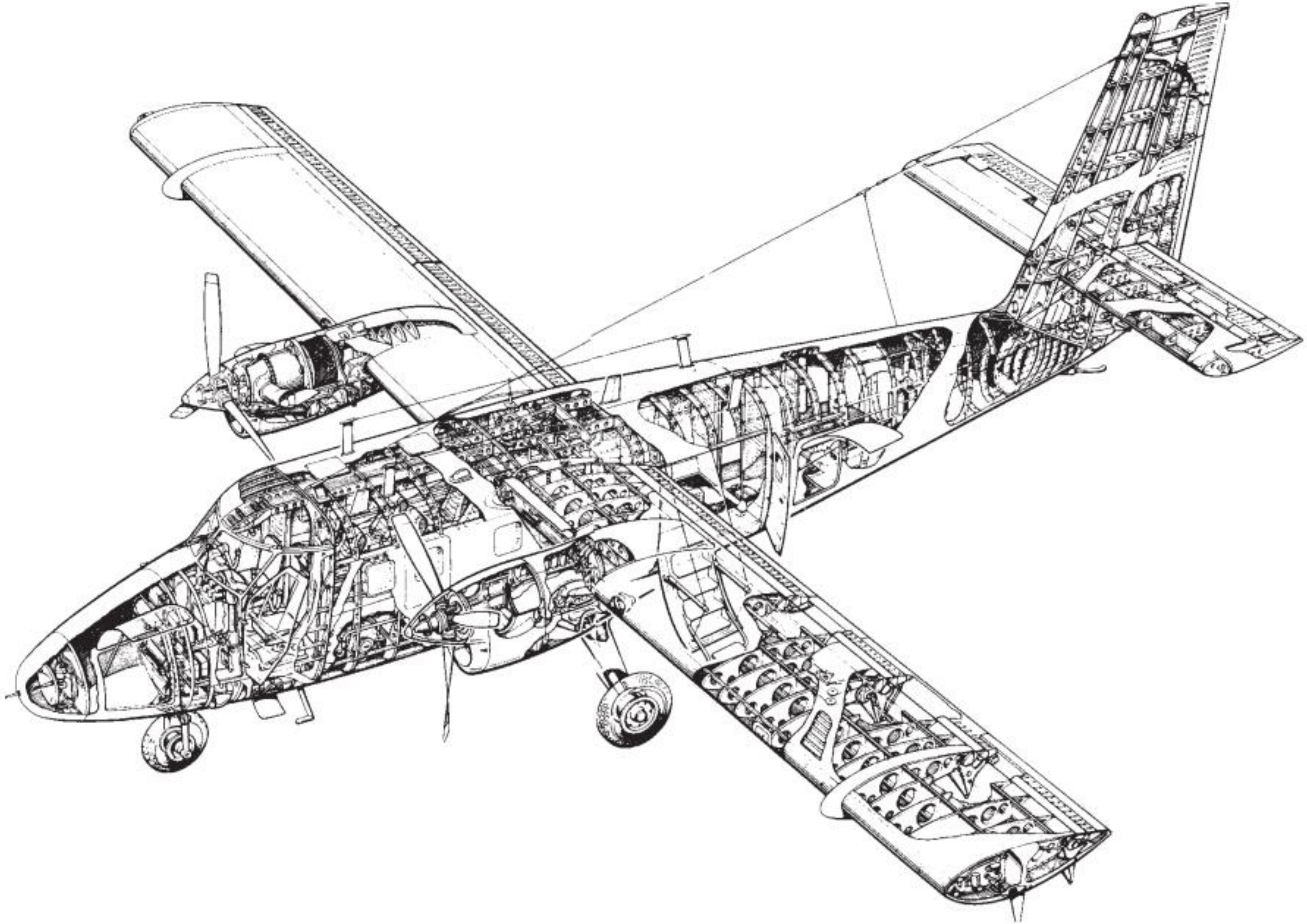
$$I_{xx} = 2\int_0^{a/2} ty^2\, ds = 2\int_0^{a/2} t(s\sin\beta)^2\, ds$$

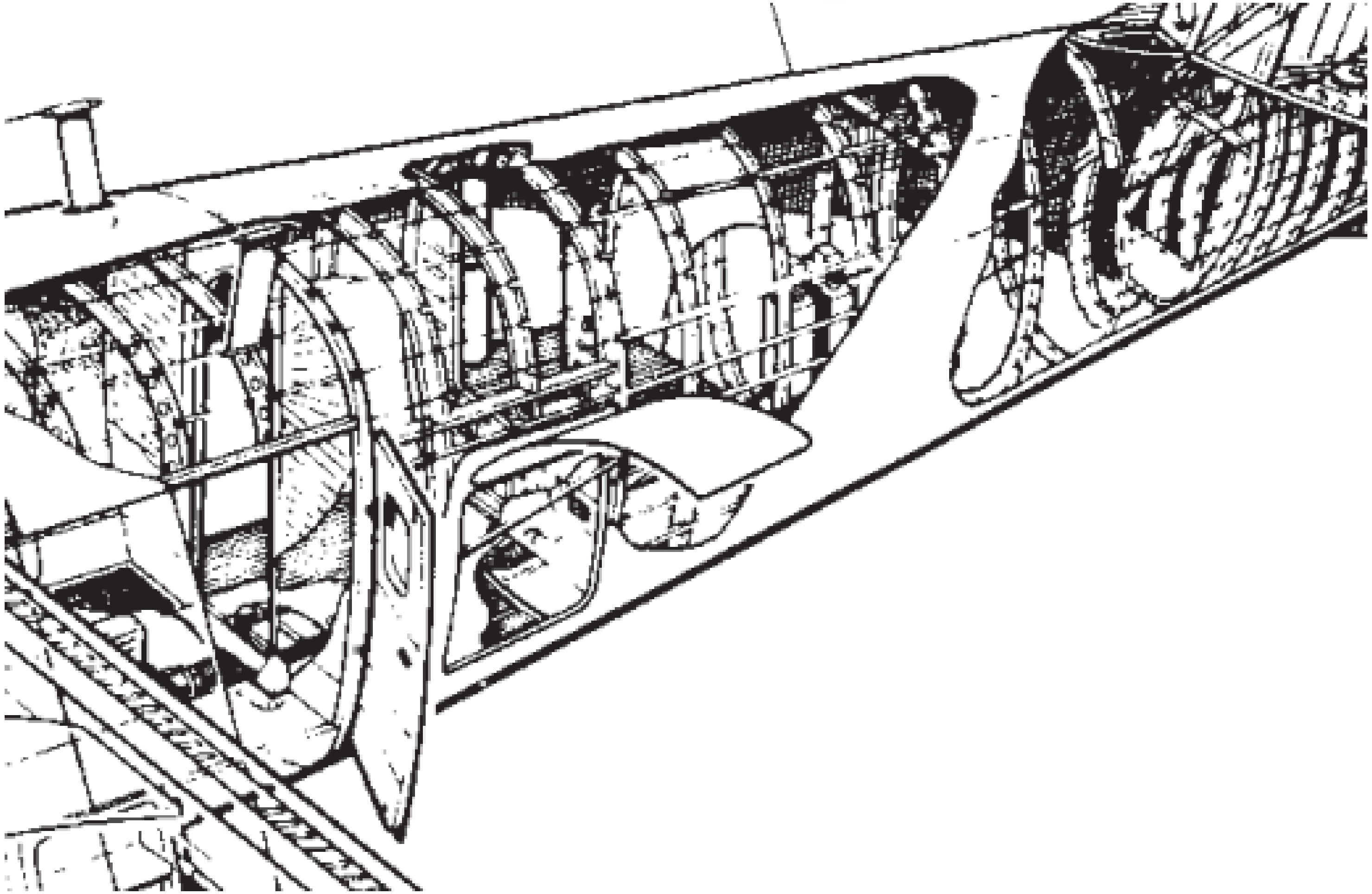
$$I_{xx} = \frac{a^3 t \sin^2\beta}{12}$$

$$I_{yy} = \frac{a^3 t \cos^2\beta}{12}$$

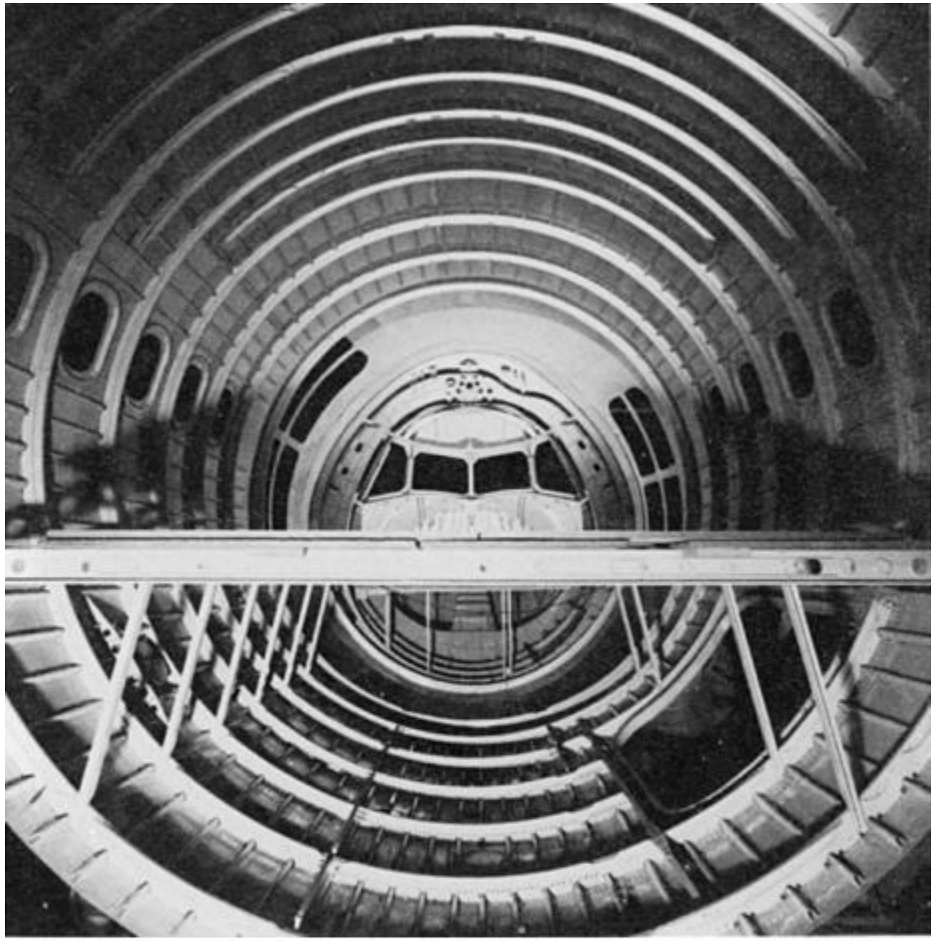
$$I_{xy} = \frac{a^3 t \sin 2\beta}{24}$$

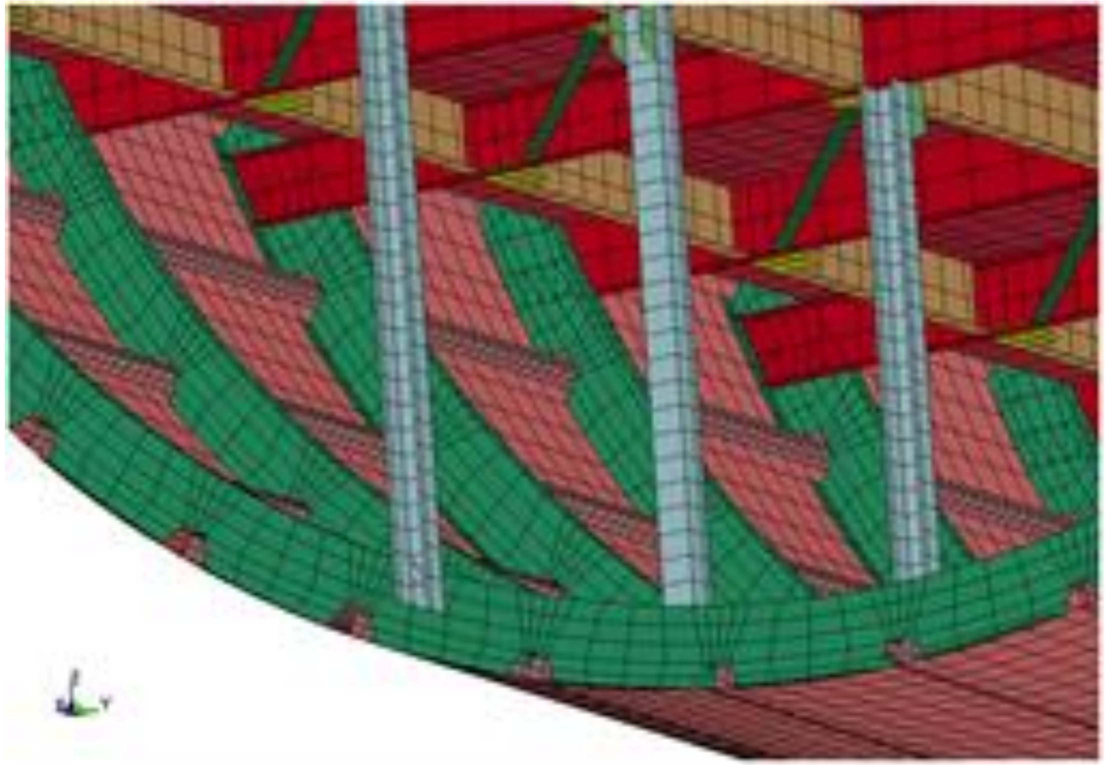
Thin-walled Structures
Idealization and modelling



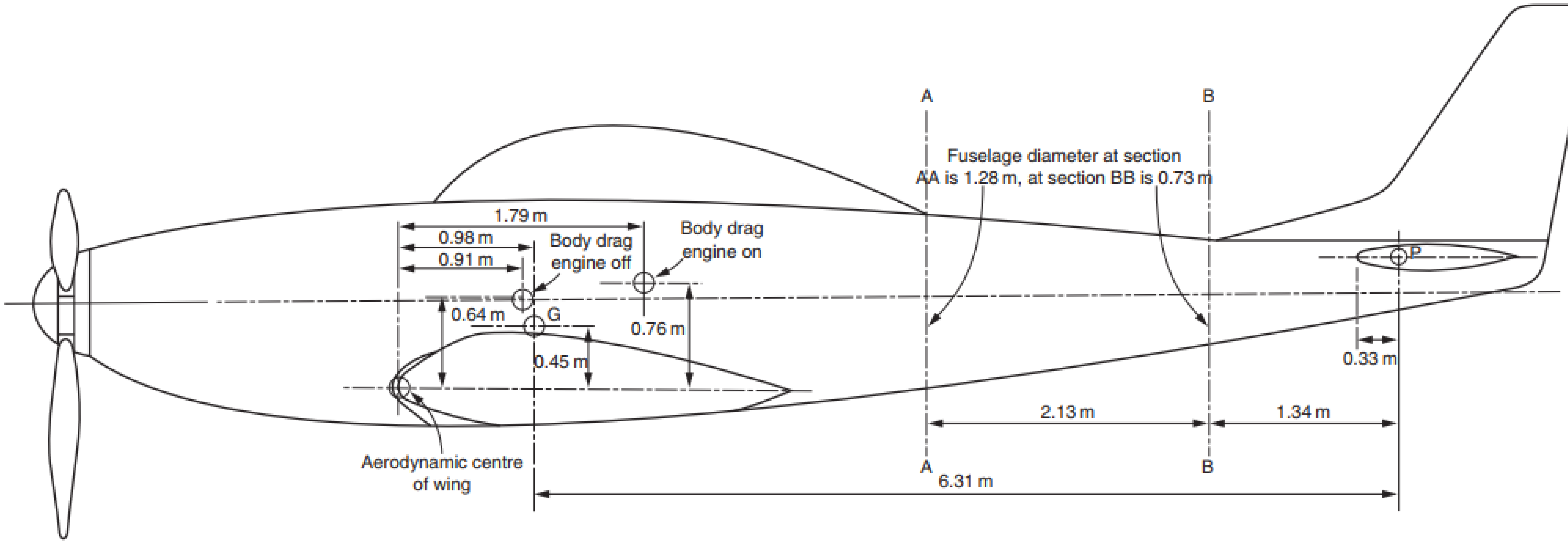


Thin-Walled structures – idealization and modeling

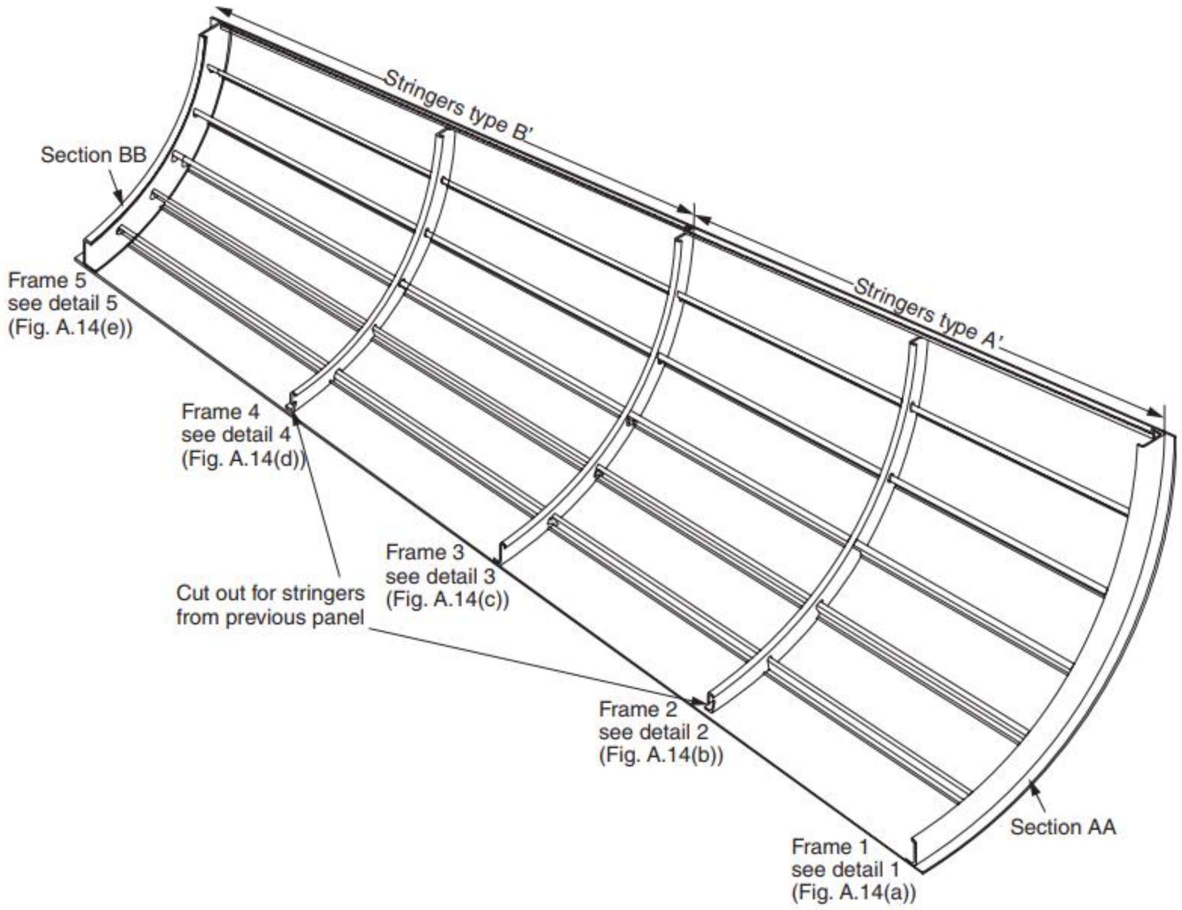


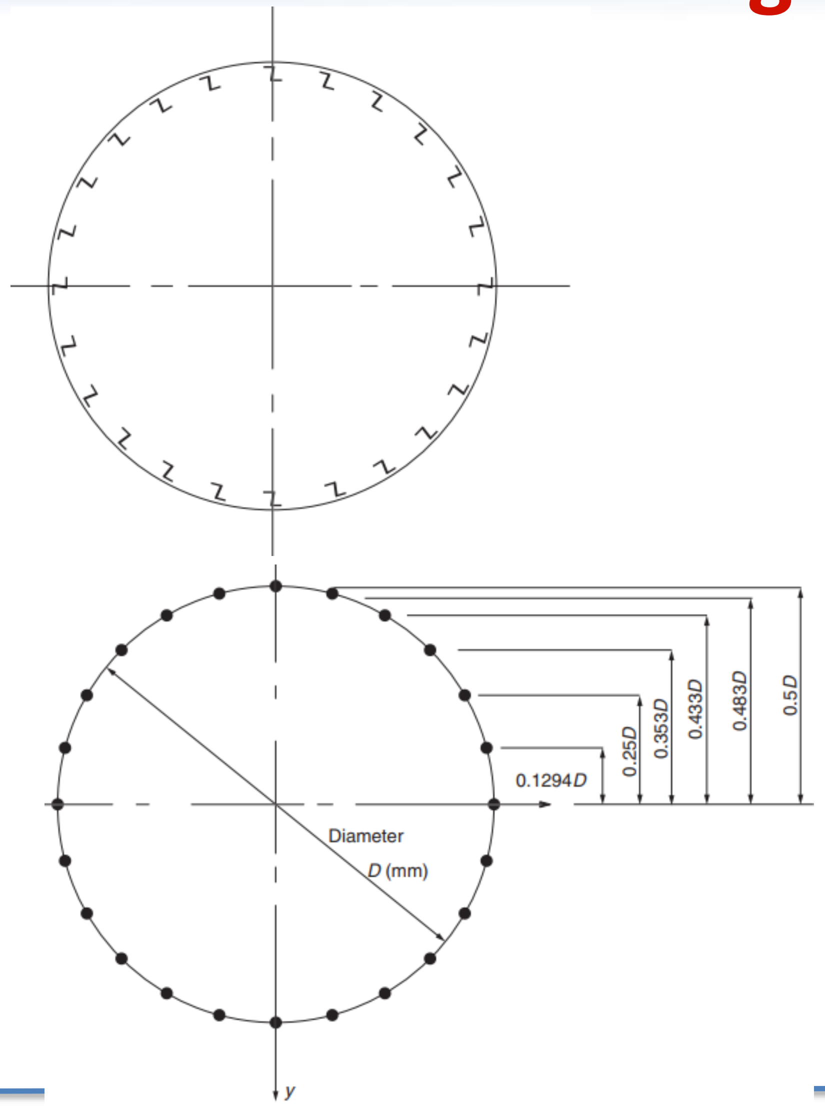


Thin-Walled structures – idealization and modeling

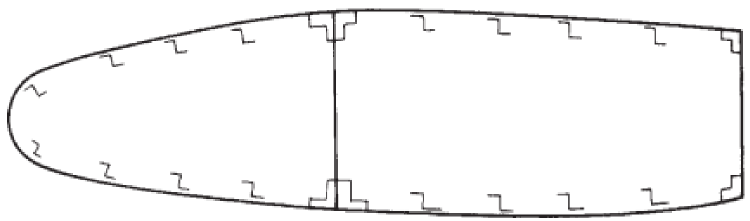


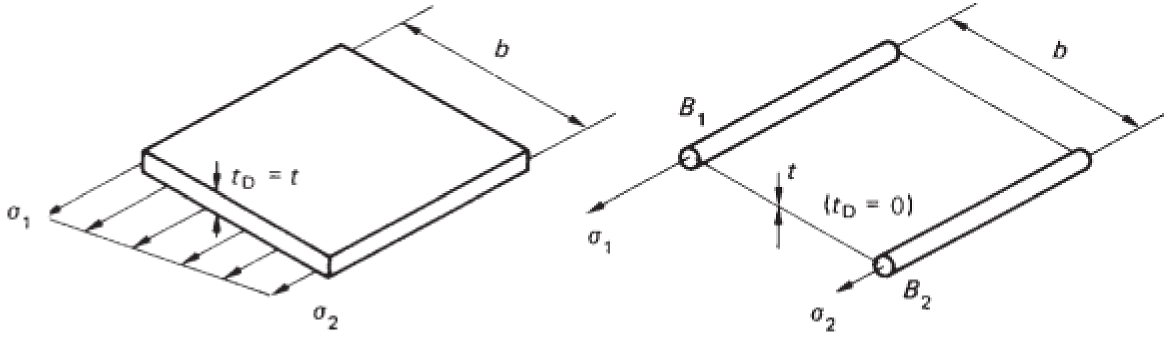
Thin-Walled structures – idealization and modeling



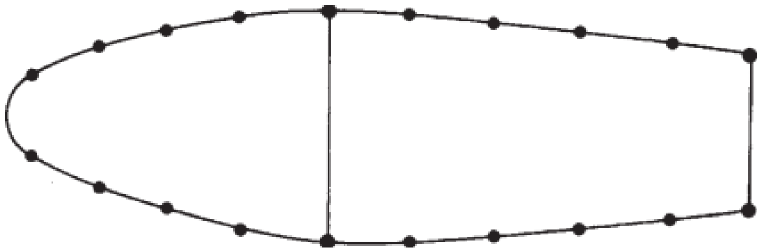


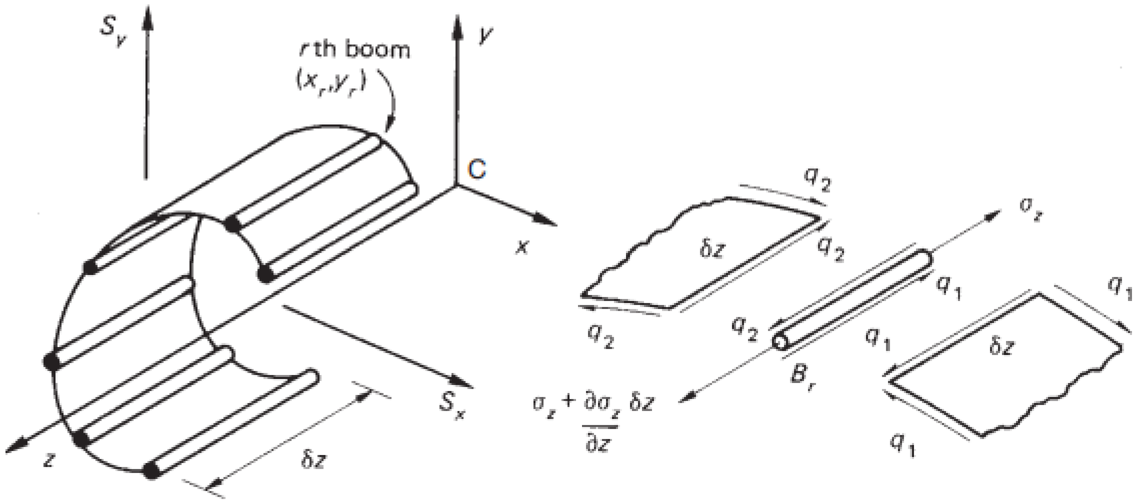
Thin-Walled structures – idealization and modeling



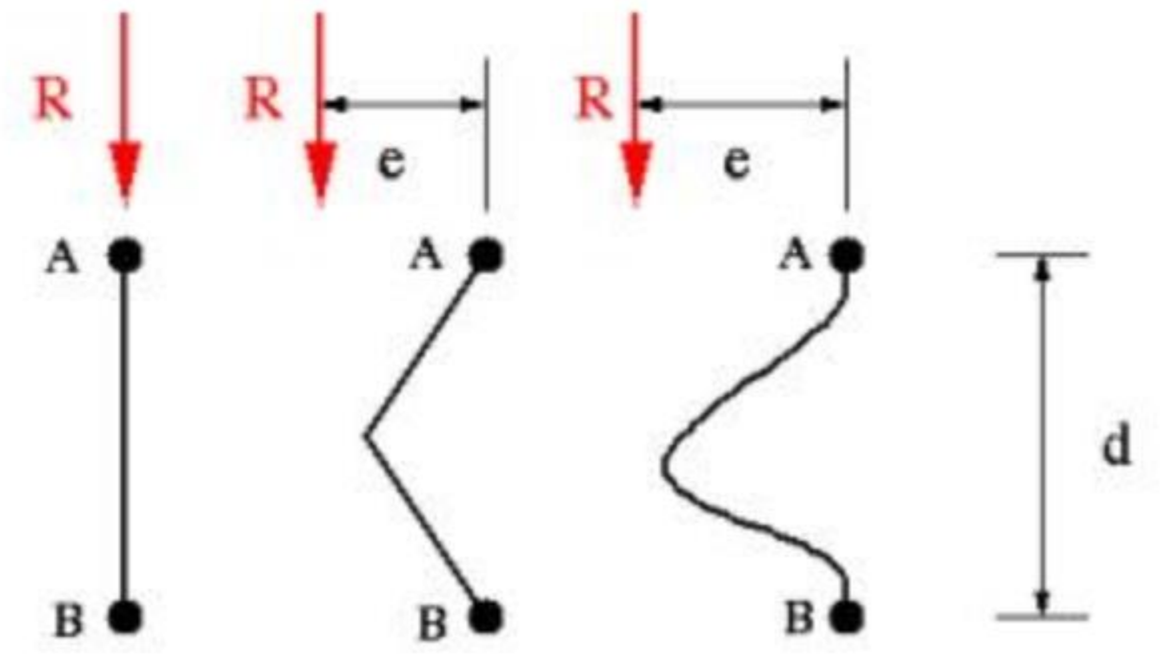


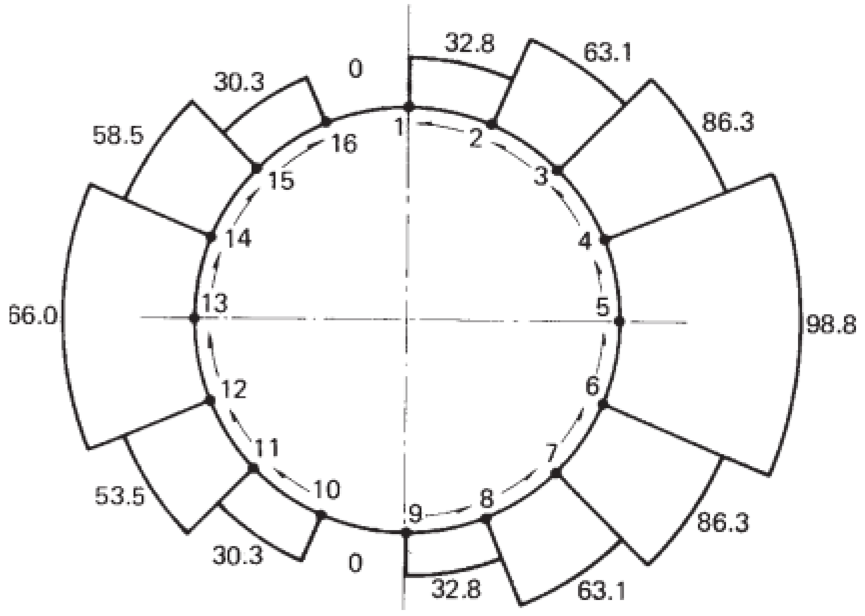
(a) Actual (b) Idealized





Thin-Walled structures – idealization and modeling

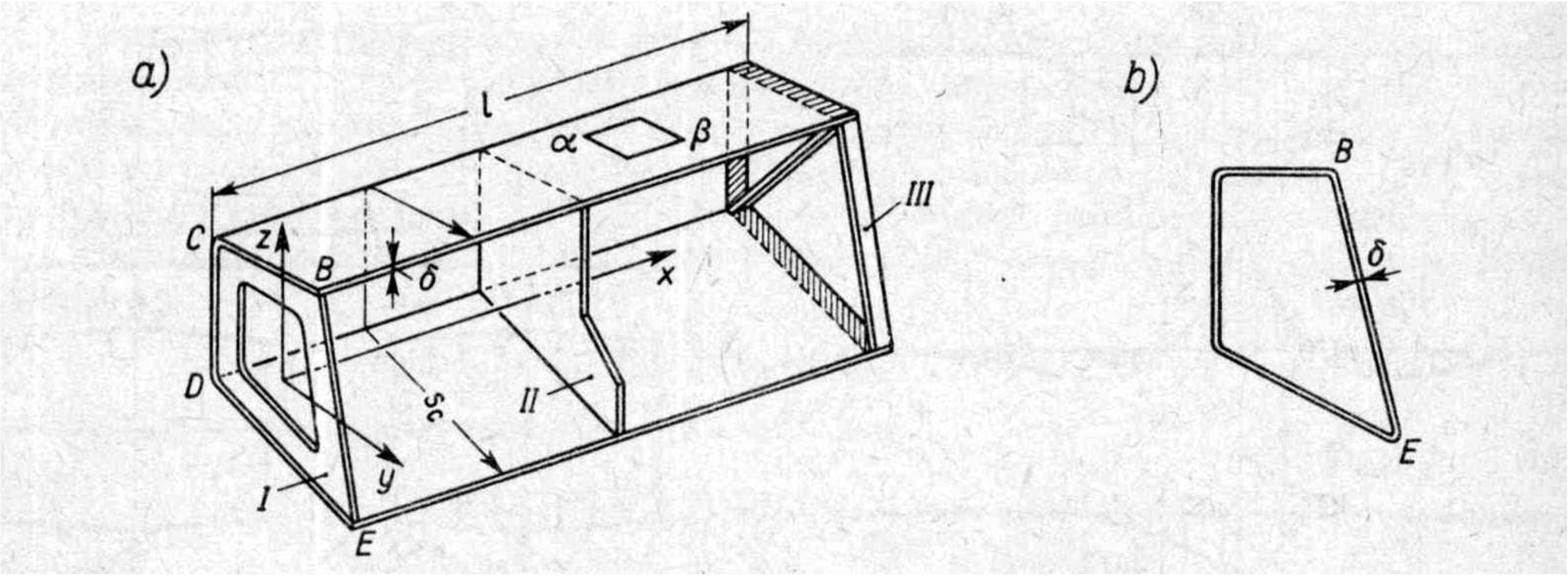




Thin-Walled structures – idealization and modeling

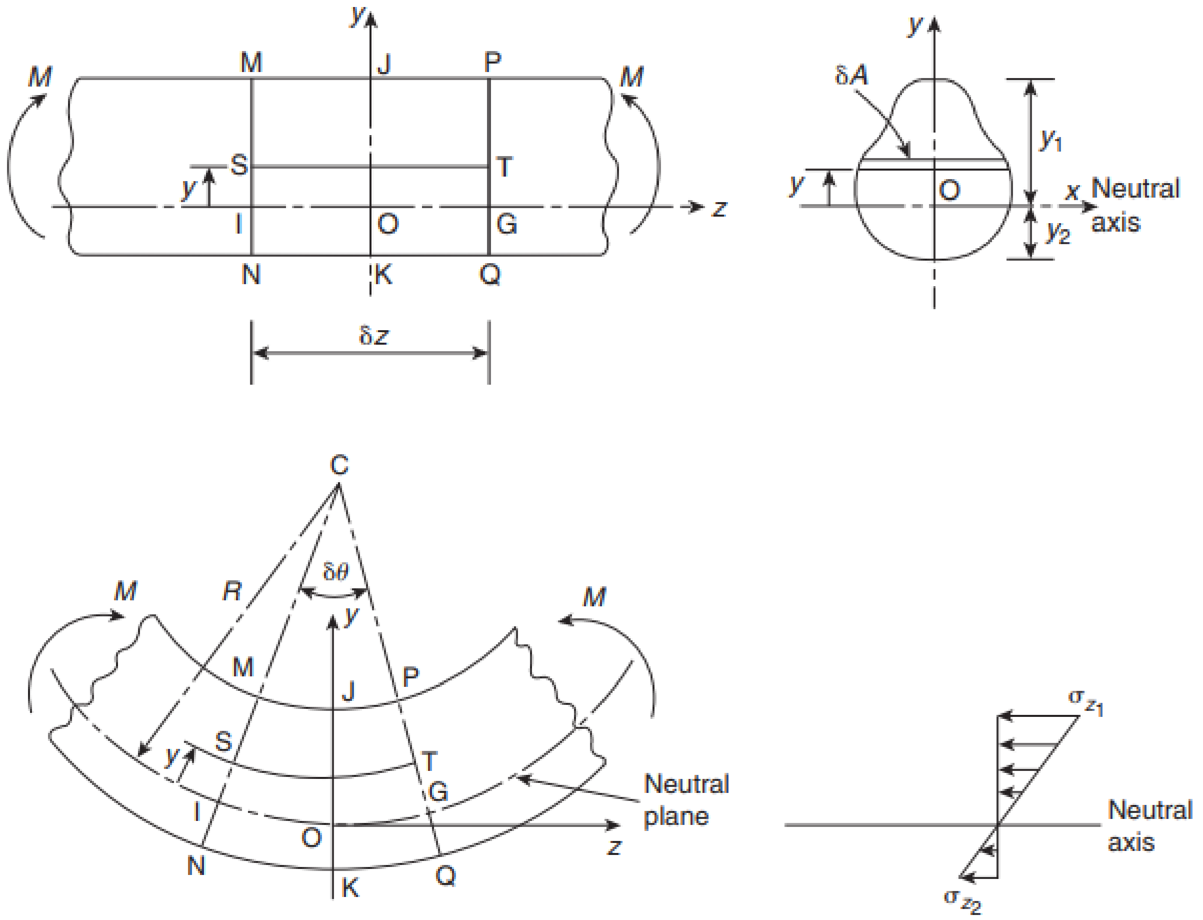
Assumptions:

- beam
- thin-walled
- CSRD



Beams – bending
(any section, any shape, any thickness)

Bending of open and closed, thin-walled beams

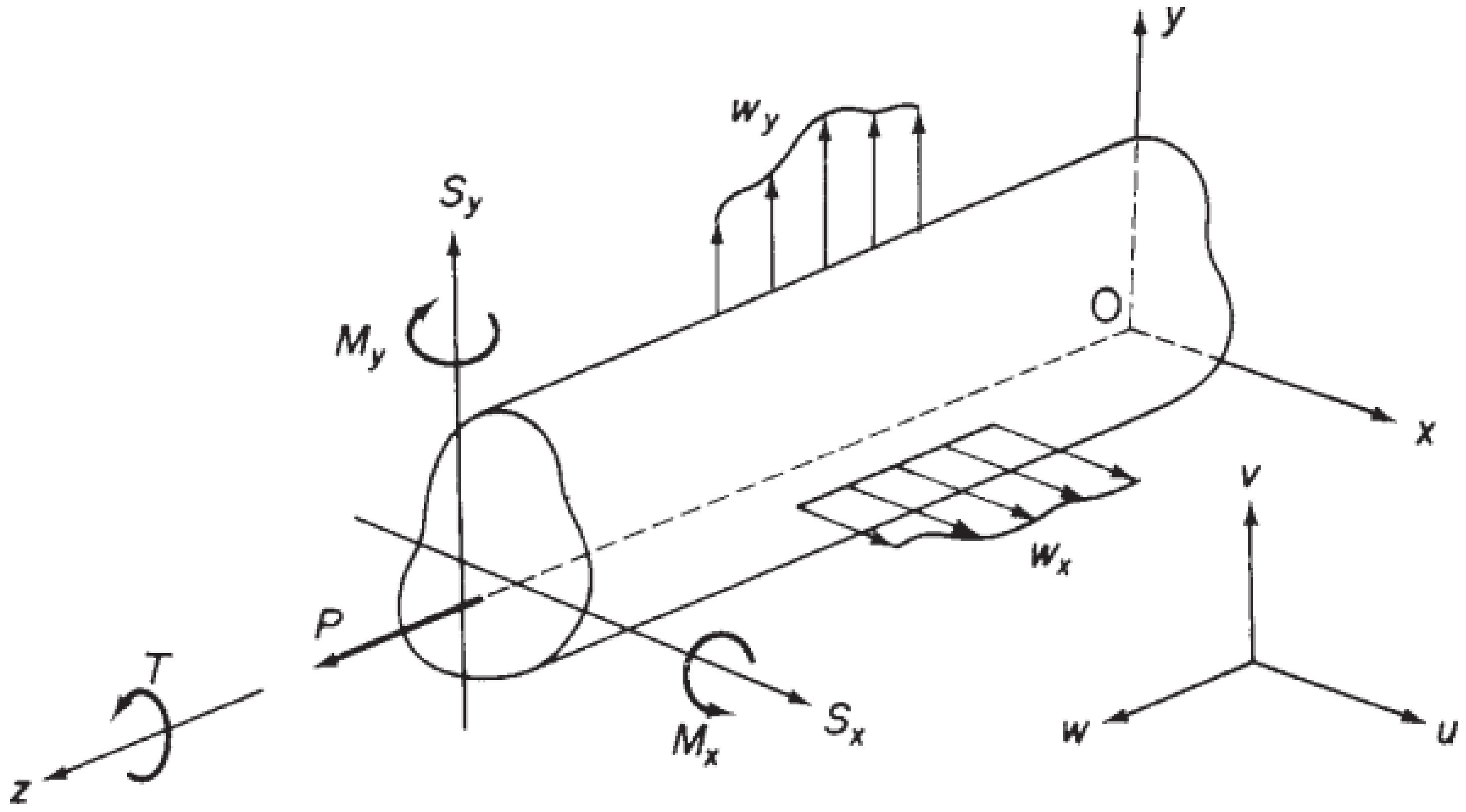


Assumptions:

Plane section

Kirchhoff assumption

Bending of open and closed, thin-walled beams
SIGN convention

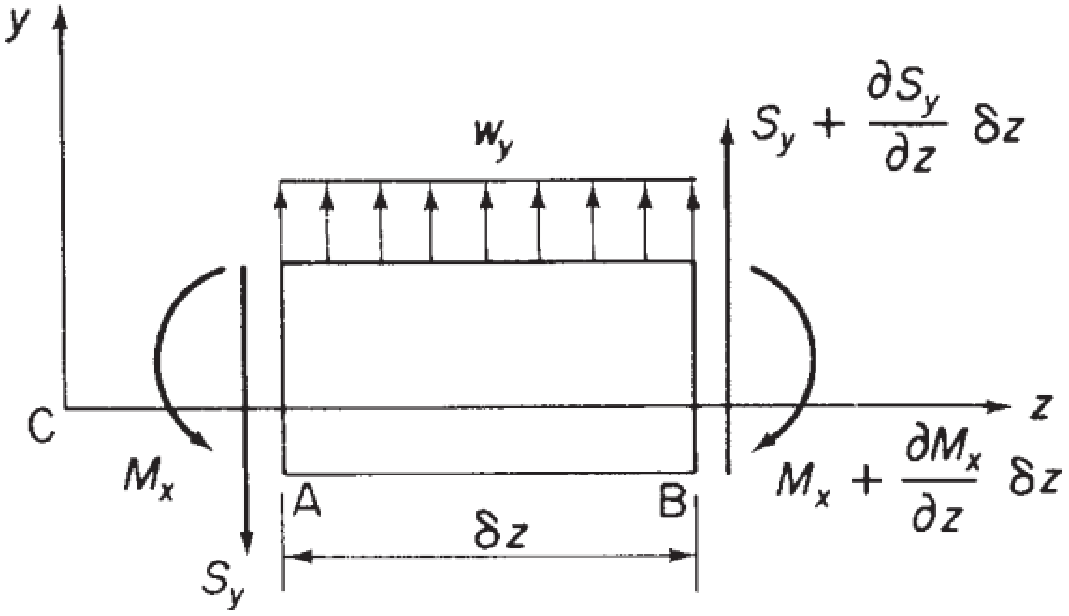


Bending of open and closed, thin-walled beams

$$\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) y$$

$$\sigma_z = \frac{M_x}{I_{xx}} y + \frac{M_y}{I_{yy}} x$$

Bending of open and closed, thin-walled beams



$$-w_y = \frac{\partial S_y}{\partial z} = \frac{\partial^2 M_x}{\partial z^2}$$

$$-w_x = \frac{\partial S_x}{\partial z} = \frac{\partial^2 M_y}{\partial z^2}$$

Bending of open and closed, thin-walled beams deflections thru curvatures

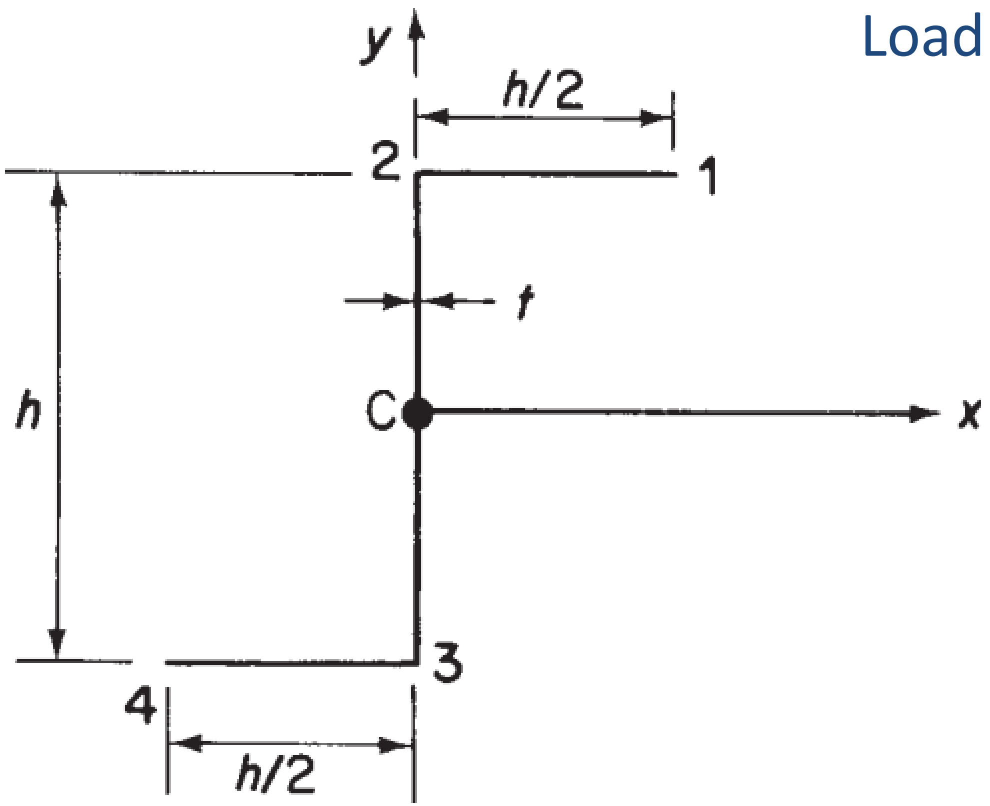
$$\begin{Bmatrix} u'' \\ v'' \end{Bmatrix} = \frac{-1}{E(I_{xx}I_{yy} - I_{xy}^2)} \begin{bmatrix} -I_{xy} & I_{xx} \\ I_{yy} & -I_{xy} \end{bmatrix} \begin{Bmatrix} M_x \\ M_y \end{Bmatrix}$$

$$\left.\begin{aligned} M_x &= -EI_{xy}u'' - EI_{xx}v'' \\ M_y &= -EI_{yy}u'' - EI_{xy}v'' \end{aligned}\right\}$$

$$u'' = -\frac{M_y}{EI_{yy}}, \quad v'' = -\frac{M_x}{EI_{xx}}$$

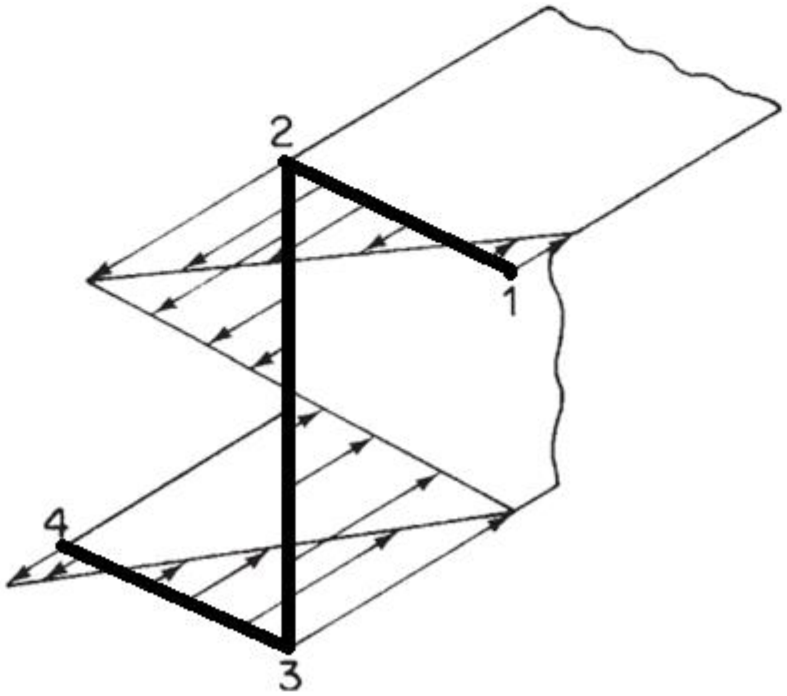
$$M_y = -EI_{yy}u''$$

Bending of open and closed, thin-walled beams



Load: positive M_x

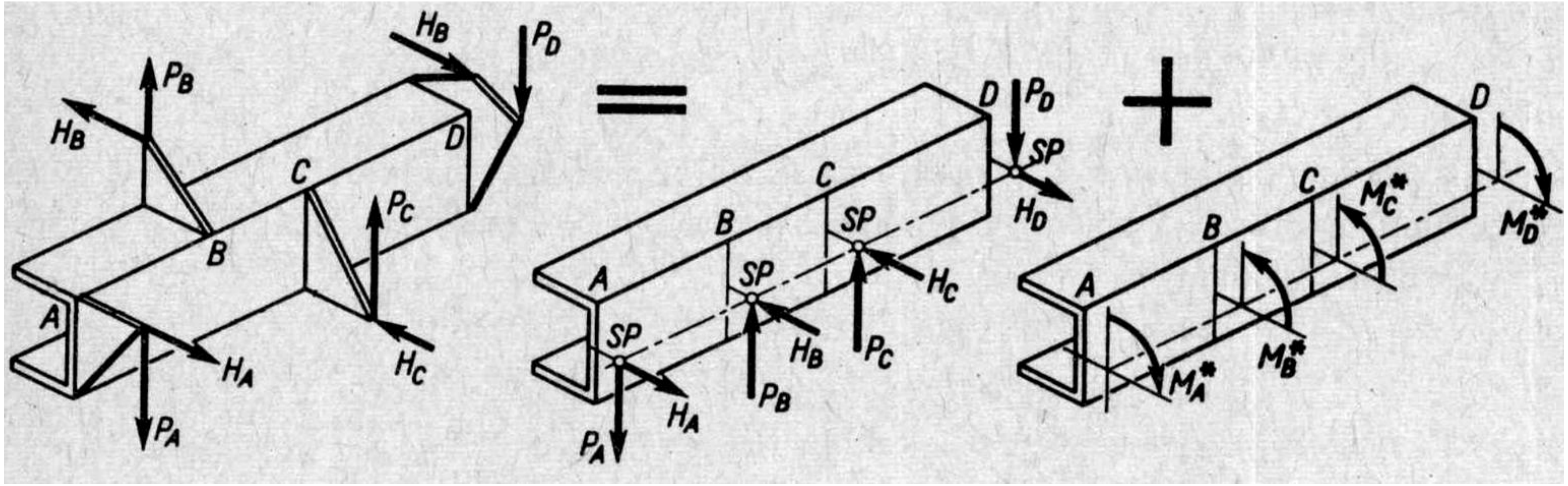
$$\sigma_z = \frac{M_x(I_{yy}y - I_{xy}x)}{I_{xx}I_{yy} - I_{xy}^2}$$



Thin-walled beams – bending / shear

open section – later generalized for closed section

Shear of open thin-walled beams

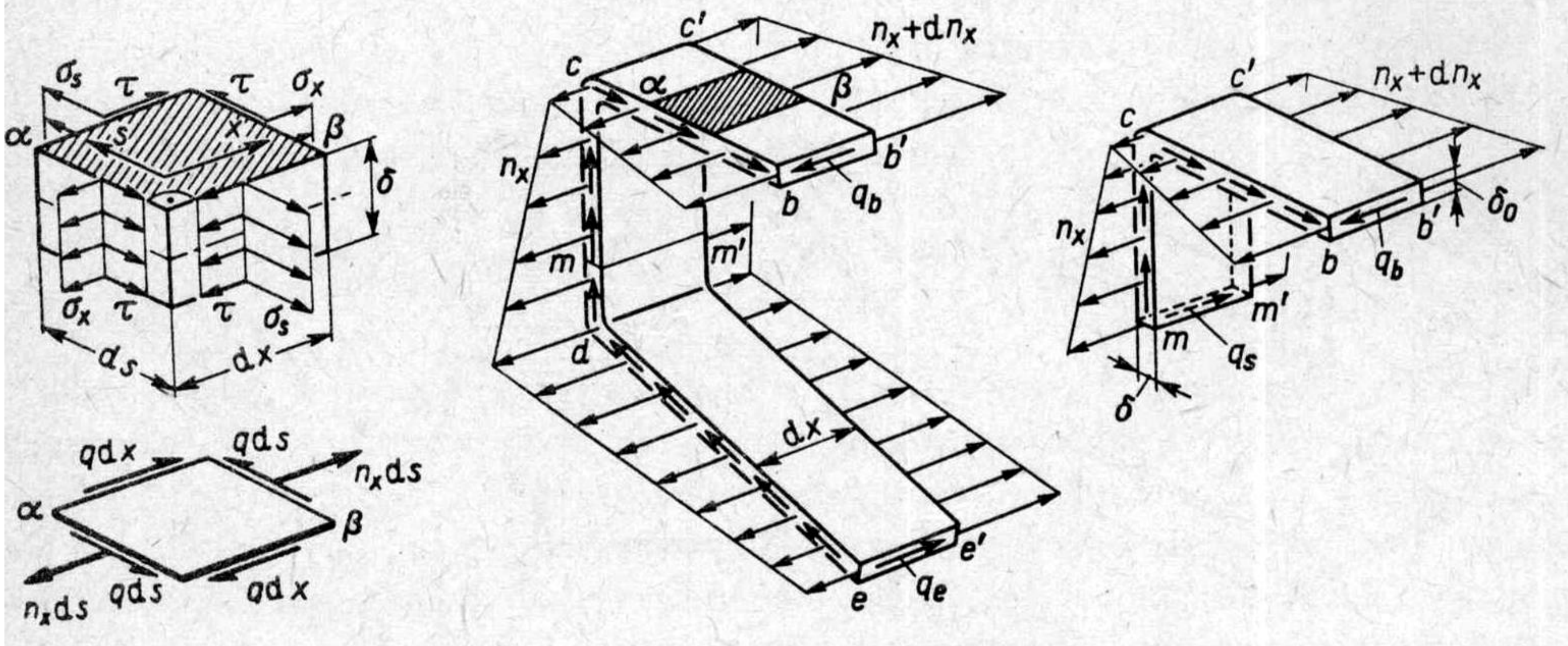


Arbitrary load split into bending/shear PLUS torsion

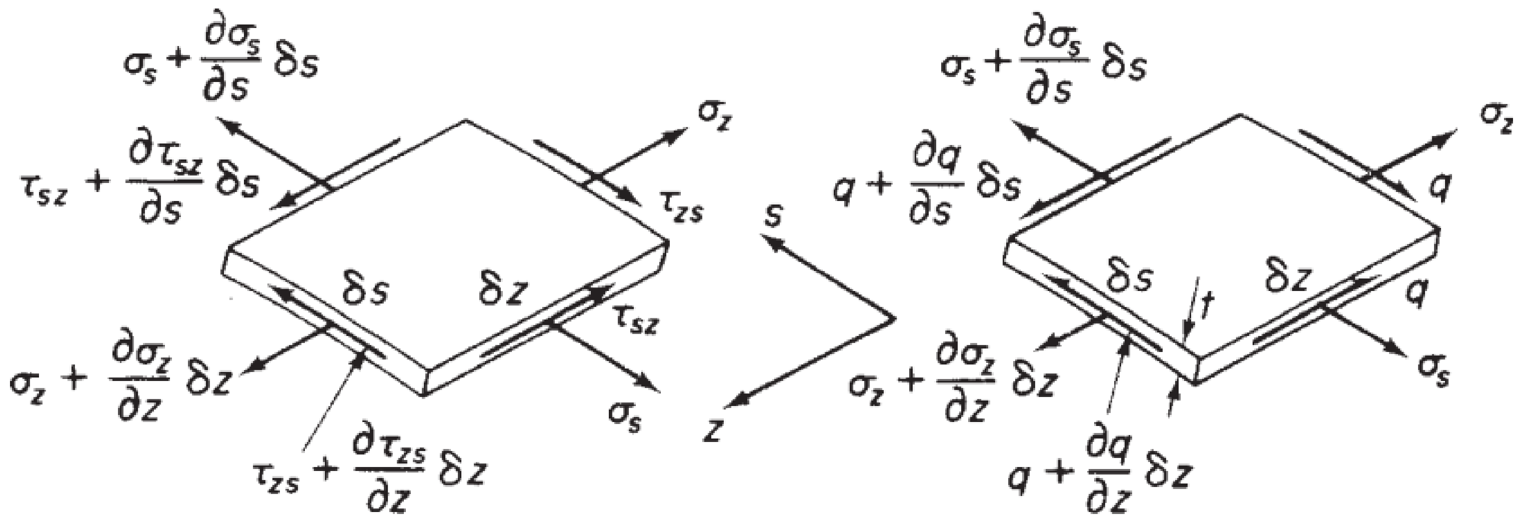
No torsion = Shear center

Shear of open thin-walled beams

$$n_x = \sigma_x \delta, \qquad q = \tau\delta. \tag{13.1}$$



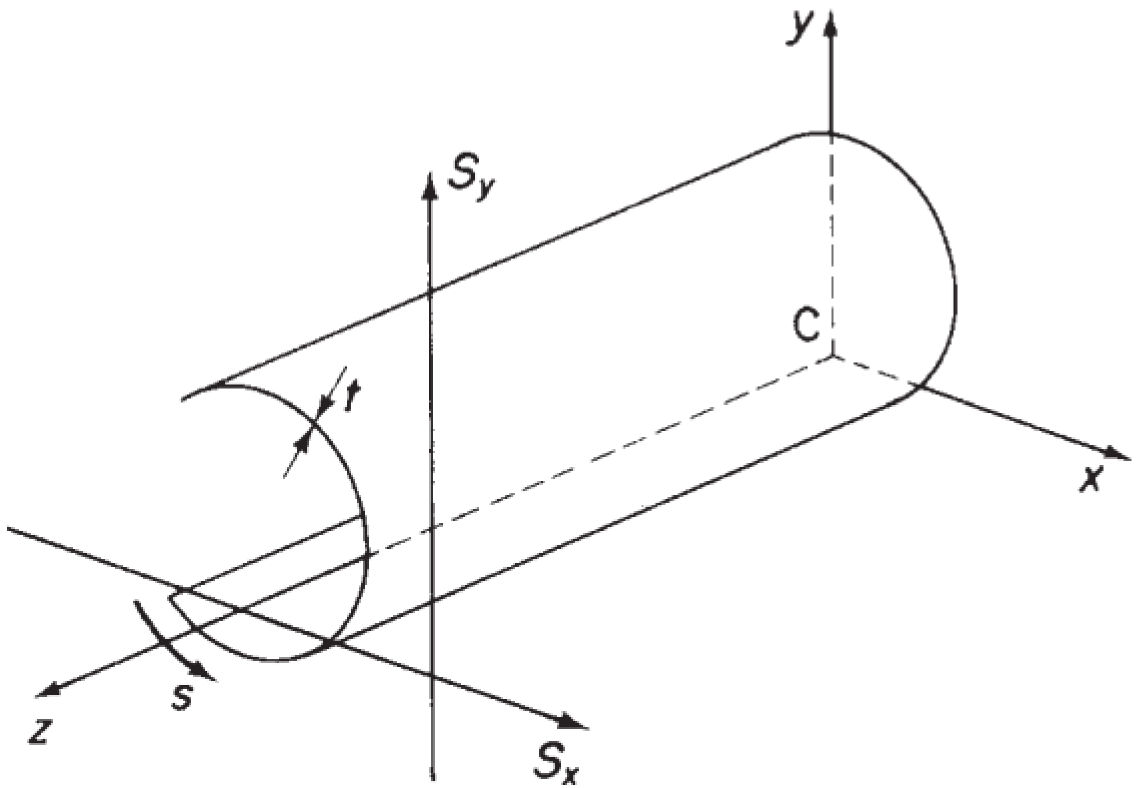
Shear of open thin-walled beams



$$\frac{\partial q}{\partial s} + t\frac{\partial \sigma_z}{\partial z} = 0$$

$$\frac{\partial q}{\partial z} + t\frac{\partial \sigma_s}{\partial s} = 0$$

Shear of open thin-walled beams



No torsion = Shear center

Shear of open thin-walled beams

$$\frac{\partial q}{\partial s} + t\frac{\partial \sigma_z}{\partial z} = 0$$

$$\frac{\partial \sigma_z}{\partial z} = \frac{[(\partial M_y/\partial z)I_{xx} - (\partial M_x/\partial z)I_{xy}]}{I_{xx}I_{yy} - I_{xy}^2}x + \frac{[(\partial M_x/\partial z)I_{yy} - (\partial M_y/\partial z)I_{xy}]}{I_{xx}I_{yy} - I_{xy}^2}y$$

$$\frac{\partial q}{\partial s} = -\frac{(S_x I_{xx} - S_y I_{xy})}{I_{xx}I_{yy} - I_{xy}^2}tx - \frac{(S_y I_{yy} - S_x I_{xy})}{I_{xx}I_{yy} - I_{xy}^2}ty$$

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)\int_0^s tx\,\mathrm{d}s - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)\int_0^s ty\,\mathrm{d}s$$

$$q_s = -\frac{S_x}{I_{yy}}\int_0^s tx\,\mathrm{d}s - \frac{S_y}{I_{xx}}\int_0^s ty\,\mathrm{d}s$$