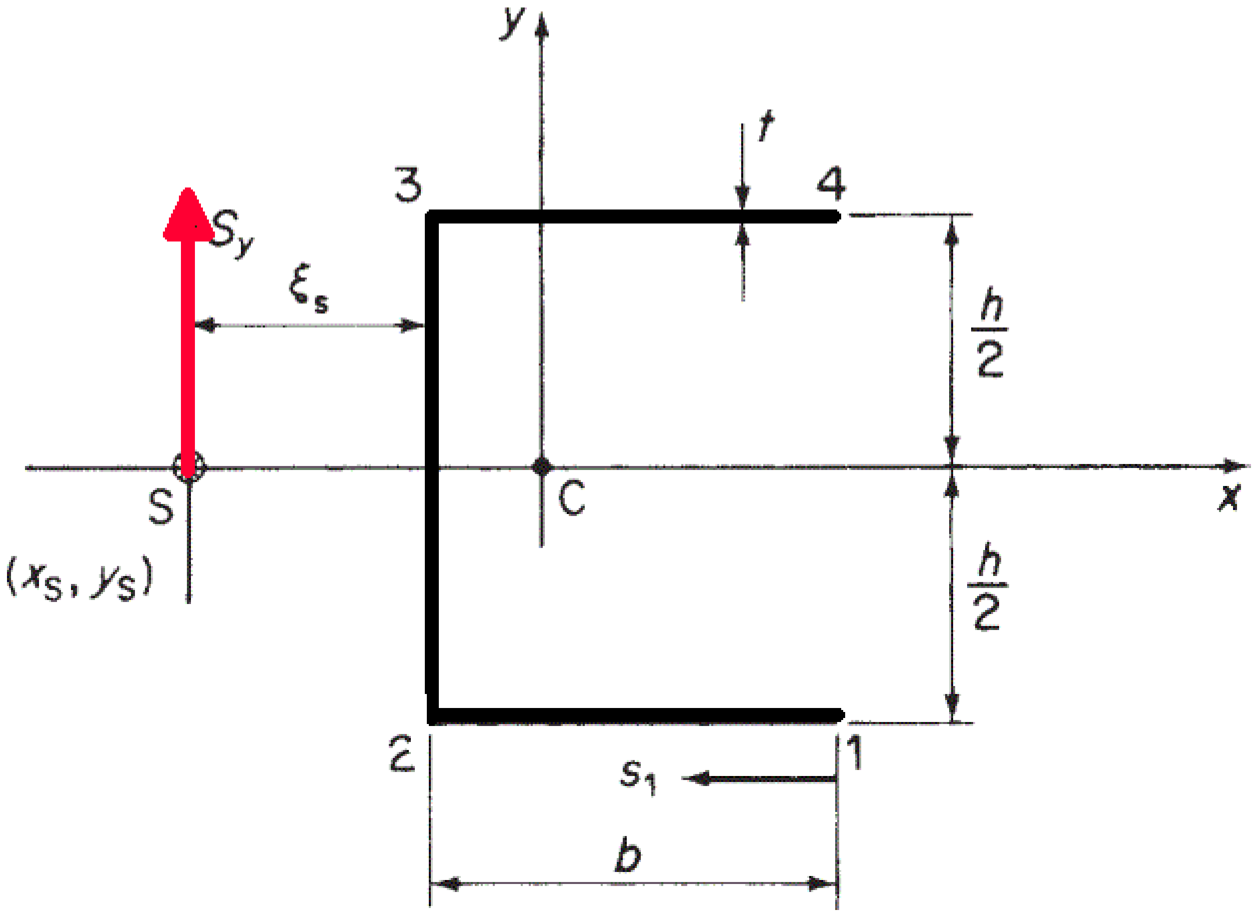
Shear Center of open thin-walled beams



EQUIVALENCE of:

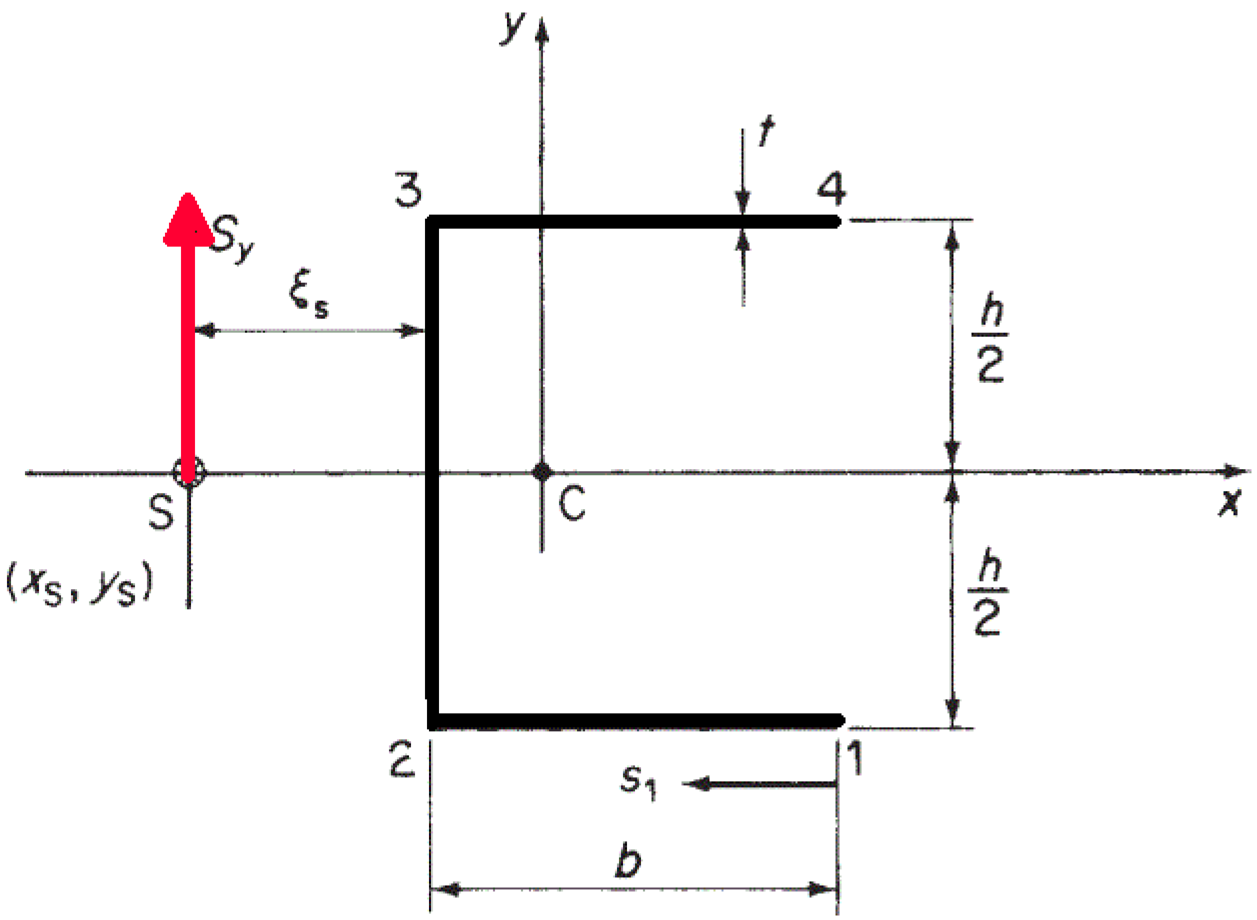
***Cause** (external load, S_x and S_y)*

and

***Result** (created shear stress)*

Logical choice – <u>equivalence of moment</u> w.r.t. arbitrary chosen location

Shear Center of open thin-walled beams



$$q_s = -\frac{S_y}{I_{xx}} \int_0^s ty \, ds$$

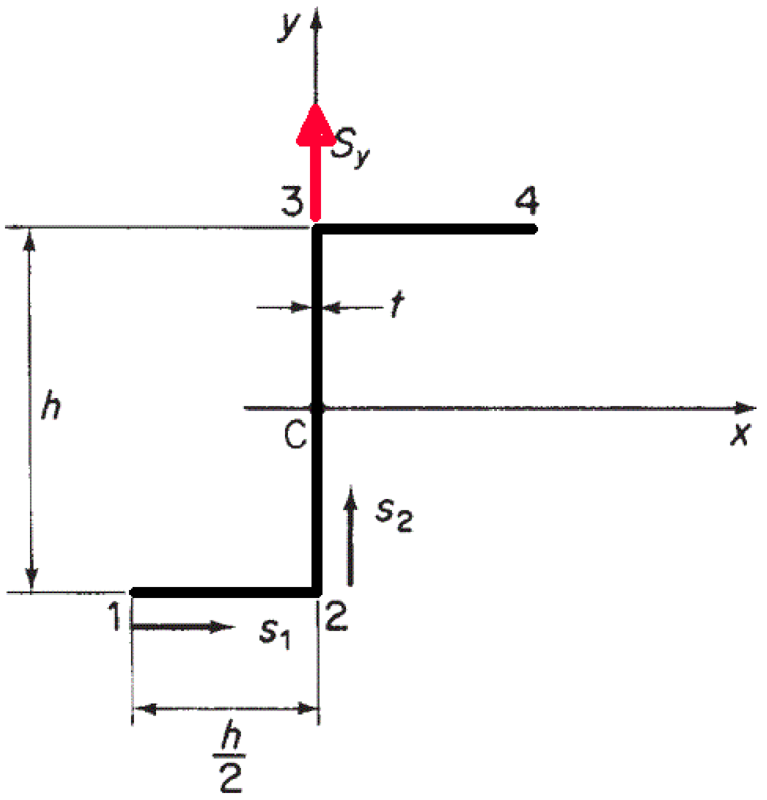
$$I_{xx} = 2bt\left(\frac{h}{2}\right)^2 + \frac{th^3}{12} = \frac{h^3 t}{12}\left(1 + \frac{6b}{h}\right)$$

$$q_s = \frac{-12S_y}{h^3(1+6b/h)} \int_0^s y \, ds$$

$$q_{12} = \frac{6S_y}{h^2(1+6b/h)} s_1$$

$$\xi_s = \frac{3b^2}{h(1+6b/h)}$$

Shear of open thin-walled beams – Example 1



$$I_{xx} = \frac{h^3 t}{3}, \quad I_{yy} = \frac{h^3 t}{12}, \quad I_{xy} = \frac{h^3 t}{8}$$

$$q_s = \frac{S_y}{h^3} \int_0^s (10.32x - 6.84y)\mathrm{d}s$$

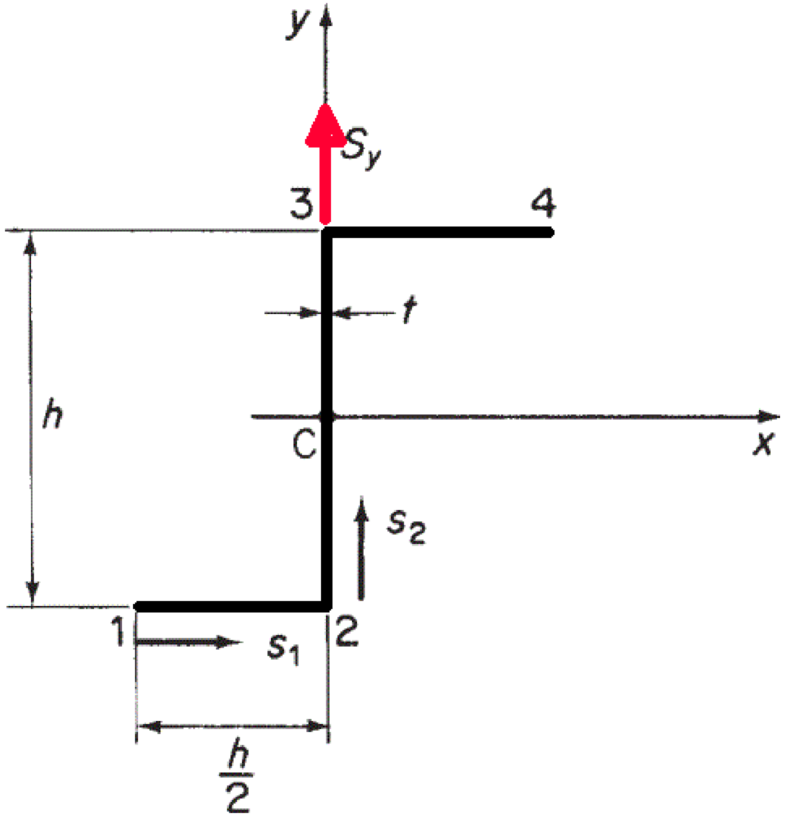
On the bottom flange 12, $y = -h/2$ and $x = -h/2 + s_1$, where $0 \leq s_1 \leq h/2$.

$$q_{12} = \frac{S_y}{h^3} \int_0^{s_1} (10.32s_1 - 1.74h)\mathrm{d}s_1$$

$$q_{12} = \frac{S_y}{h^3}(5.16s_1^2 - 1.74hs_1)$$

Hence at 1 ($s1 = 0$), $\mathbf{q_1 = 0}$
and at 2 ($s1 = h/2$), $\mathbf{q_2 = 0.42S_y/h}$.

Shear of open thin-walled beams – Example 1



$$I_{xx} = \frac{h^3 t}{3}, \quad I_{yy} = \frac{h^3 t}{12}, \quad I_{xy} = \frac{h^3 t}{8}$$

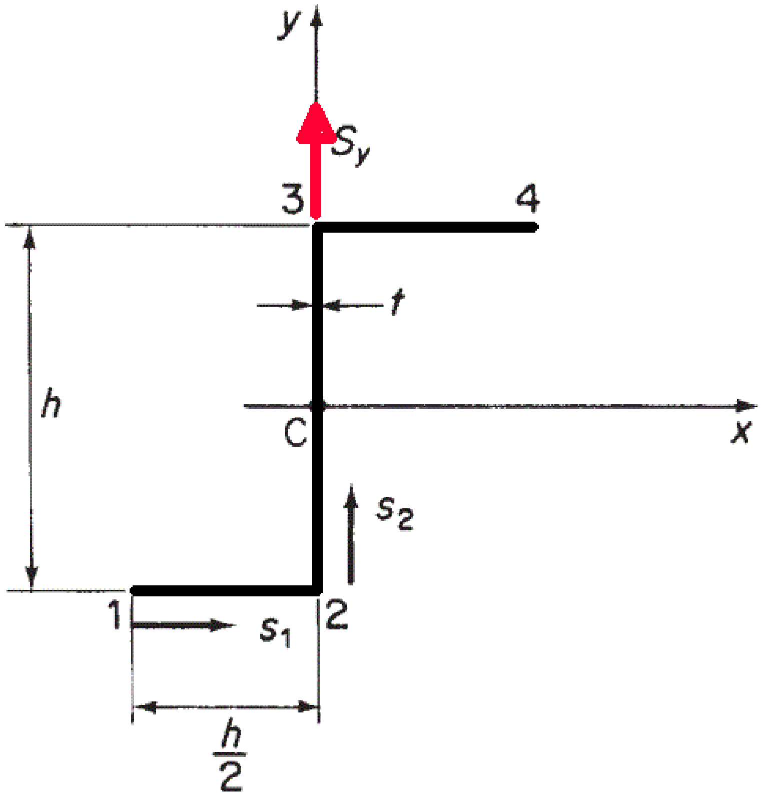
In the web 23, $y = -h/2 + s_2$, where $0 \leq s_2 \leq h$ and $x = 0$. Then

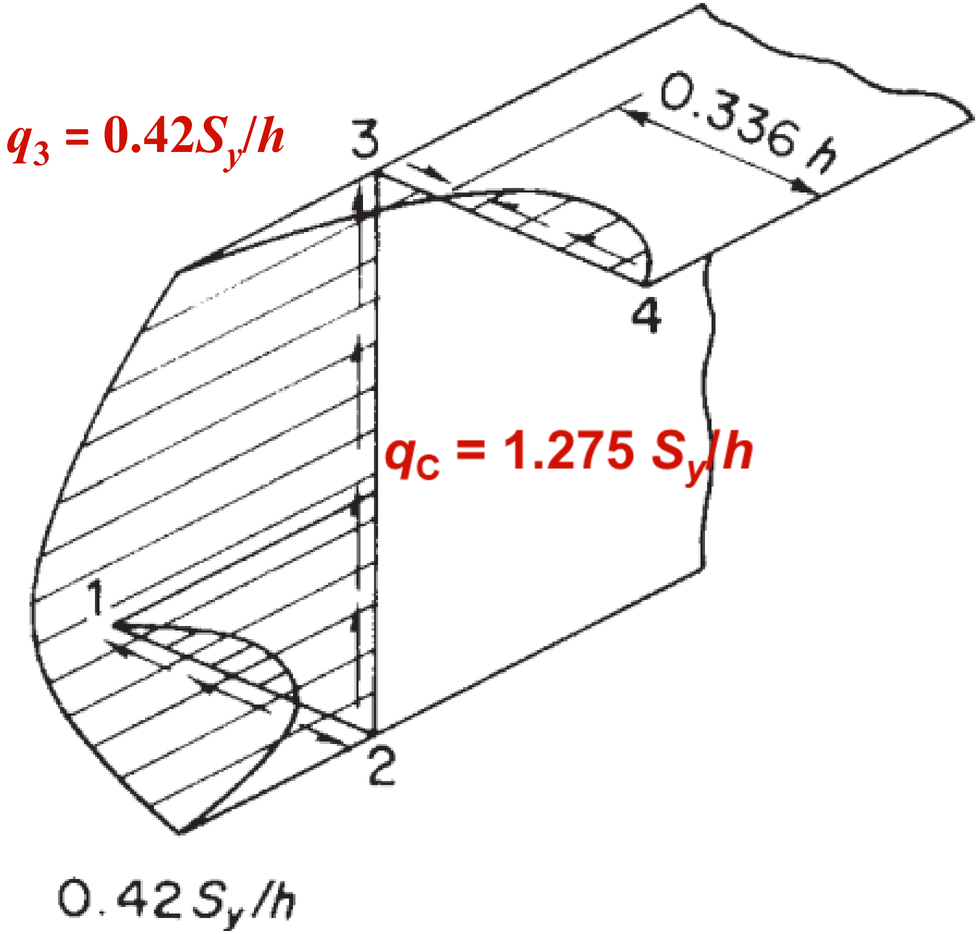
$$q_{23} = \frac{S_y}{h^3} \int_0^{s_2} (3.42h - 6.84s_2)\mathrm{d}s_2 + q_2$$

$$q_{23} = \frac{S_y}{h^3}(0.42h^2 + 3.42hs_2 - 3.42s_2^2)$$

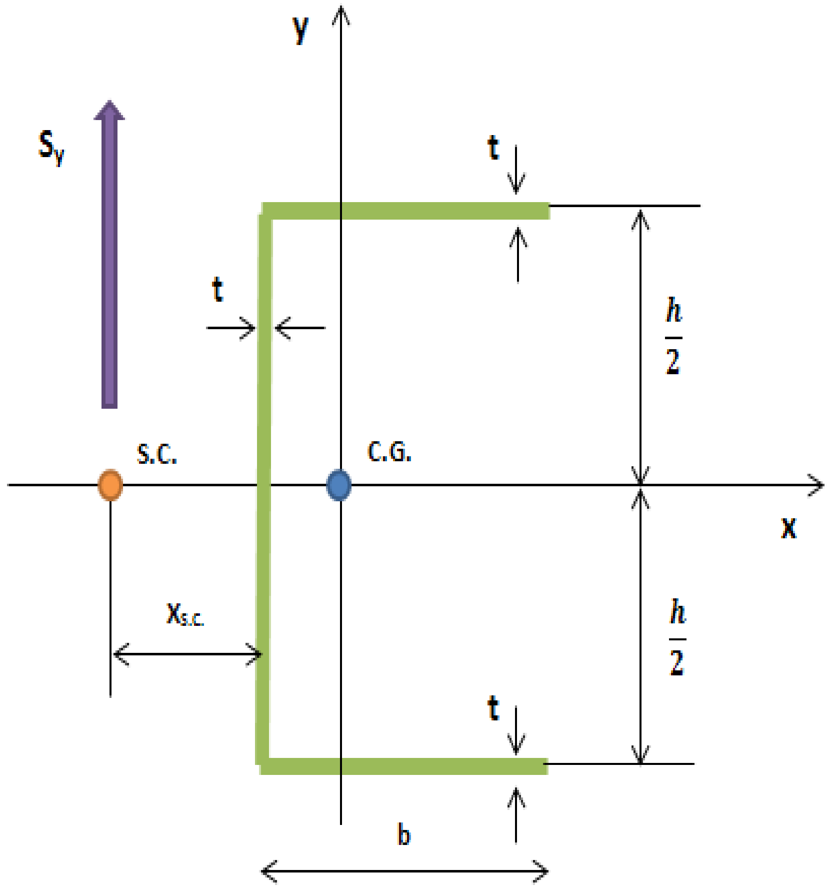
At C point ($s_2 = h/2$) there is maximum,
and $q_C = 1.275\, S_y/h$.

Shear of open thin-walled beams – Example 1





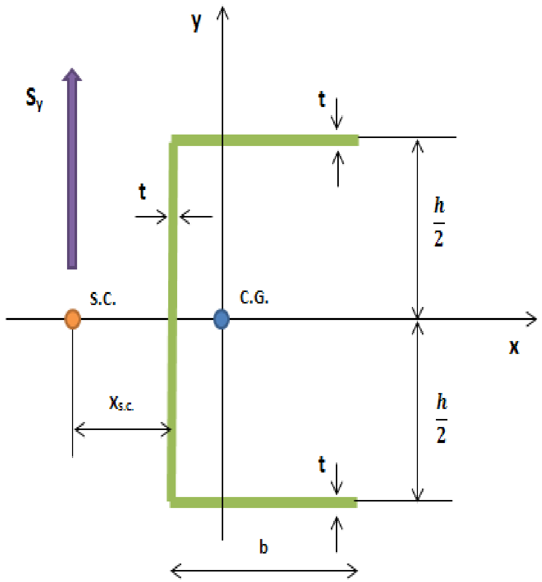
Shear of open thin-walled beams – Example 2



$t = 0{,}5$ [cm]
$h = 16$ [cm]
$b = 8$ [cm]
$S_y = 20$ [kN] $= 2 \cdot 10^4$ [N]
$S_x = 0$ [N]

S.C. – shear center
C.G. – center of gravity

Shear of open thin-walled beams – Example 2



$$I_{xx} = \frac{t \cdot h^3}{12} + 2 \cdot \left[\frac{b \cdot t^3}{12} + t \cdot b \cdot \left(\frac{h}{2}\right)^2\right]$$

$$= \frac{0{,}5 \cdot 16^3}{12} + 2 \cdot \left[0 + 0{,}5 \cdot 8 \cdot \left(\frac{16}{2}\right)^2\right]$$

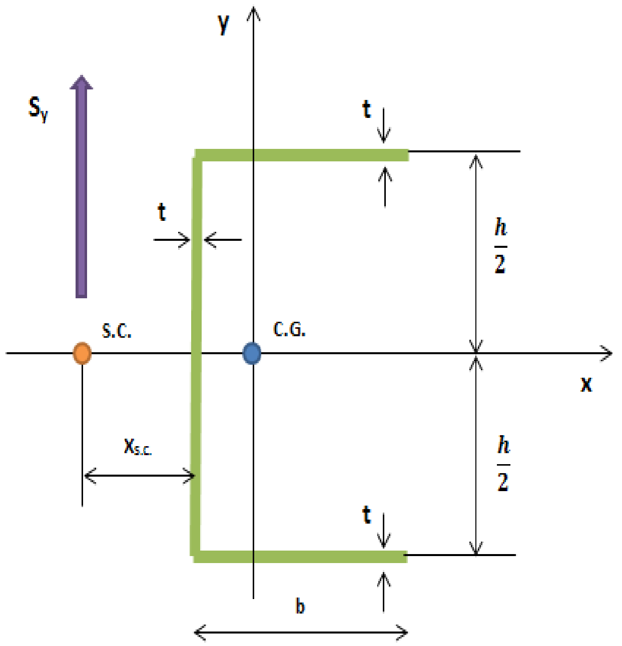
where the following assumption can be written:

$$\frac{b \cdot t^3}{12} \approx 0$$

thus

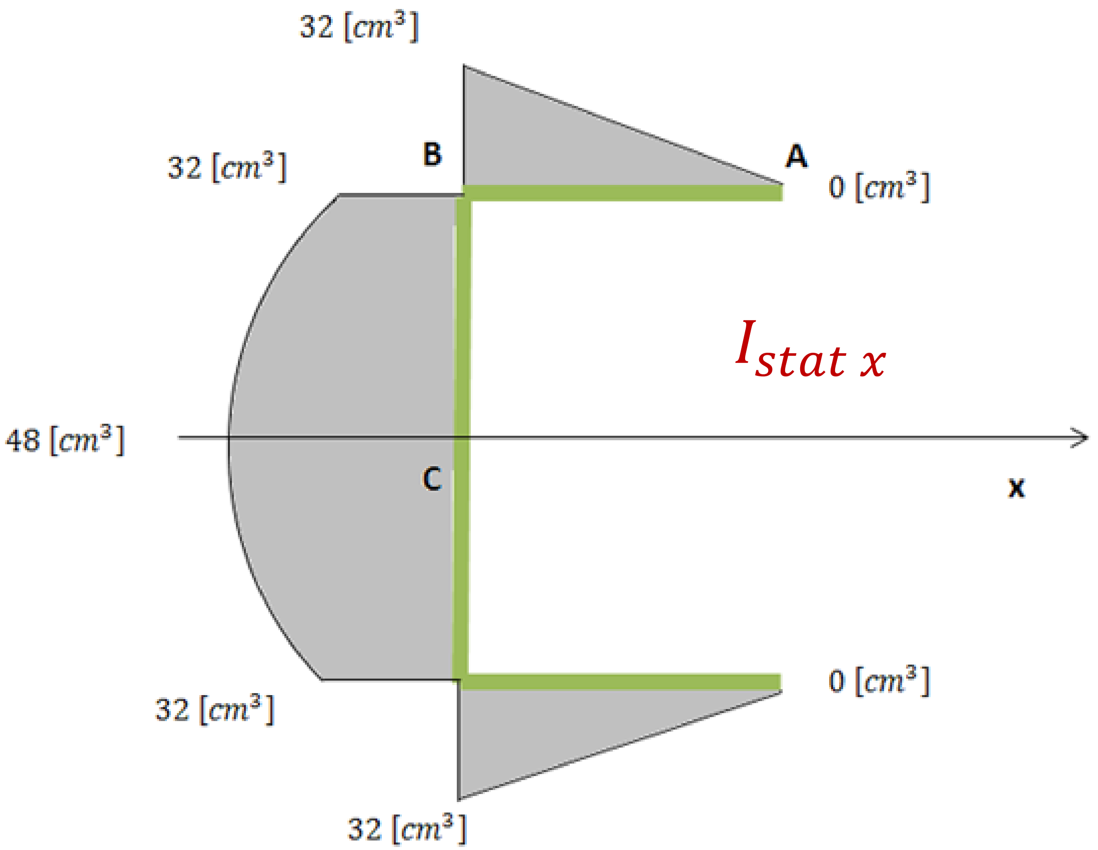
$$I_{xx} = 170{,}66 + 512 = 682{,}67\ [cm^4] = 682{,}67\ \cdot 10^4\ [mm^4]$$

Shear of open thin-walled beams – Example 2

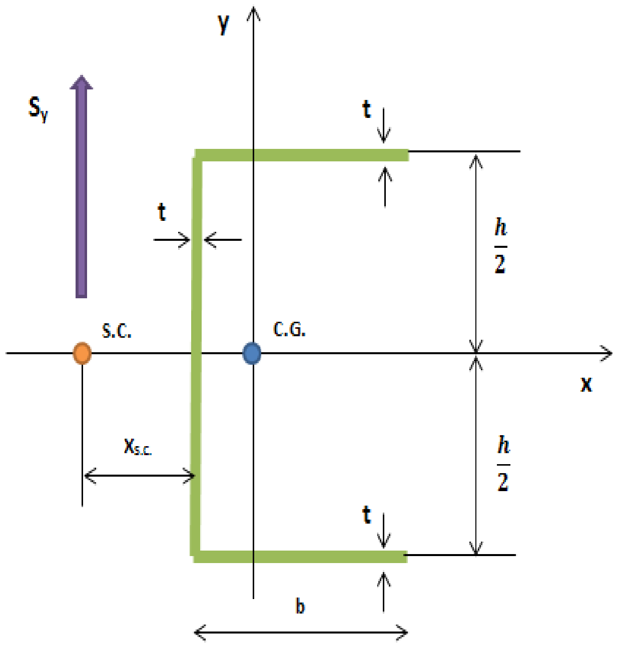


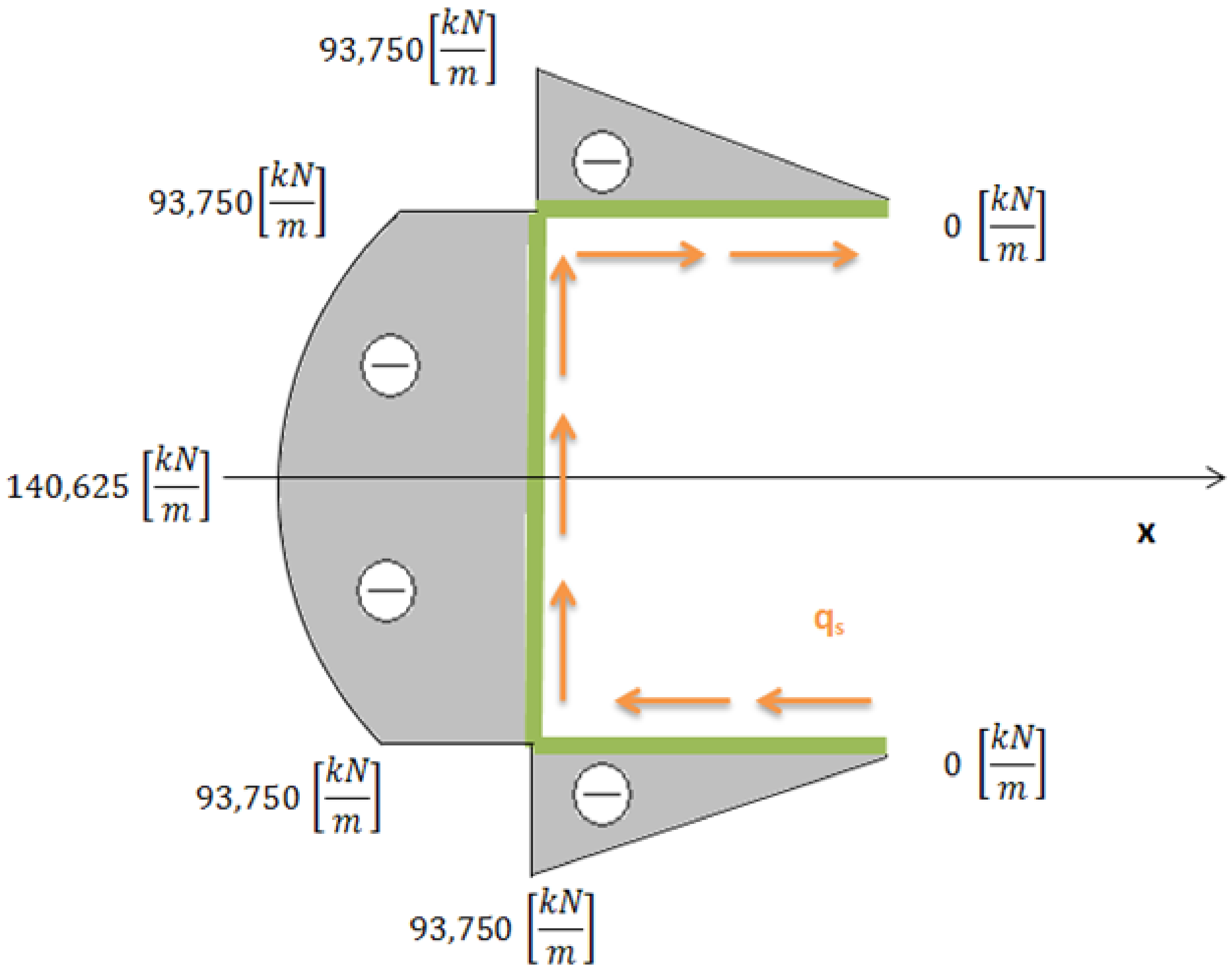
$$q_s = -\frac{S_y}{I_{xx}} \cdot I_{stat\,x}$$

S_y=20 kN
I_{xx}=682.67 cm^4



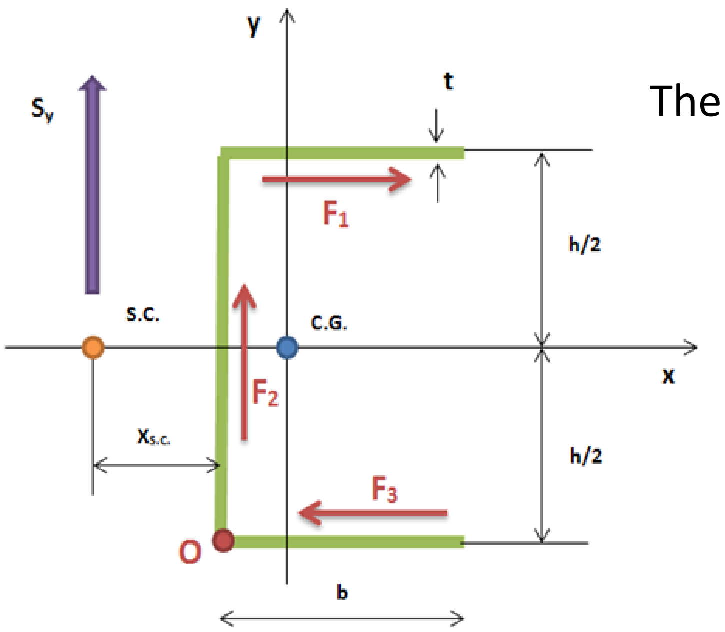
Shear of open thin-walled beams – Example 2





Shear of open thin-walled beams – Example 2

$$S_y \cdot x_{S.C.} = F_1 \cdot h$$



The force F_1 generates the moment about "O" point. Thus, this force is calculated:

$$F_1 = \int_0^{s_1} q_s\, ds_1 = \int_0^{b} q_s\, ds_1 = \int_0^{b} \left(-\frac{S_y}{I_{xx}} \cdot I_{stat\ x}\right) ds_1 = -\frac{S_y}{I_{xx}} \int_0^{b} (\, I_{stat\ x}) \, ds_1 =$$

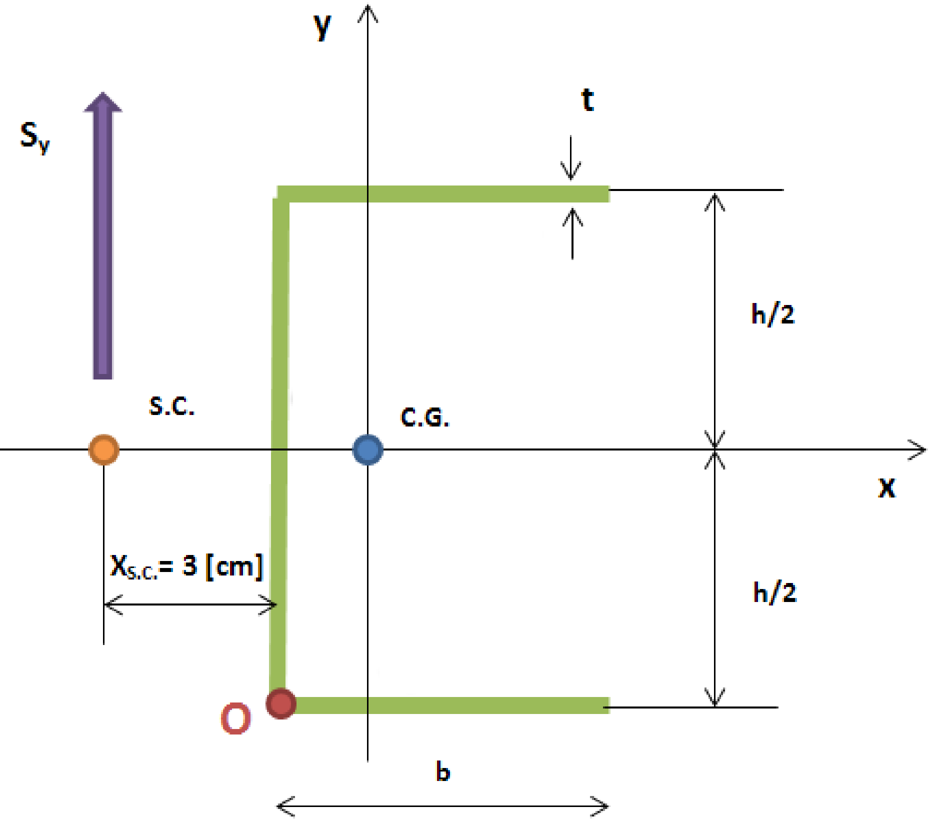
$$= \int_0^{b} \left(-29{,}30 \cdot t \cdot \frac{h}{2} \cdot s_1\right) ds_1 = -29{,}30 \cdot t \cdot \frac{h}{2} \int_0^{b} s_1\, ds_1 =$$

$$= -29{,}30 \cdot t \cdot \frac{h}{2} \cdot \left[\frac{{s_1}^2}{2}\right|_0^b = -29{,}30 \cdot t \cdot \frac{h}{2} \cdot \frac{b^2}{2}$$

The range is $s_1 \ \in < 0\, ; b >$where $b = 8[cm]$, thus:

$$F_1 = -29{,}30 \frac{[N]}{[cm^4]} \cdot 0{,}5\,[cm] \cdot \frac{16}{2}\,[cm] \cdot \frac{(8\,[cm])^2}{2} = -29{,}30 \cdot 0{,}5 \cdot 4 \cdot 32\,[N] = -3750\,[N]$$

Shear of open thin-walled beams – Example 2



Calculation of the coordinate of the shear center

$$x_{S.C.} = \frac{F_1 \cdot h}{S_y} = \frac{-3750\ [N] \cdot 0{,}16\ [m]}{2 \cdot 10^4\ [N]} = -0{,}03\ [m] = -3\ [cm]$$

The minus sign means that the shear center position lies opposite to the "*X*" axis, so to the left from the "O" point.

Thin-walled beams – bending / shear

NOW generalized for closed section

Recall - Shear of open thin-walled beams

$$\frac{\partial q}{\partial s} + t\frac{\partial \sigma_z}{\partial z} = 0$$

$$\frac{\partial \sigma_z}{\partial z} = \frac{[(\partial M_y/\partial z)I_{xx} - (\partial M_x/\partial z)I_{xy}]}{I_{xx}I_{yy} - I_{xy}^2}x + \frac{[(\partial M_x/\partial z)I_{yy} - (\partial M_y/\partial z)I_{xy}]}{I_{xx}I_{yy} - I_{xy}^2}y$$

$$\frac{\partial q}{\partial s} = -\frac{(S_x I_{xx} - S_y I_{xy})}{I_{xx}I_{yy} - I_{xy}^2}tx - \frac{(S_y I_{yy} - S_x I_{xy})}{I_{xx}I_{yy} - I_{xy}^2}ty$$

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)\int_0^s tx\,\mathrm{d}s - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)\int_0^s ty\,\mathrm{d}s$$

$$q_s = -\frac{S_x}{I_{yy}}\int_0^s tx\,\mathrm{d}s - \frac{S_y}{I_{xx}}\int_0^s ty\,\mathrm{d}s$$

Shear of closed section thin-walled beams

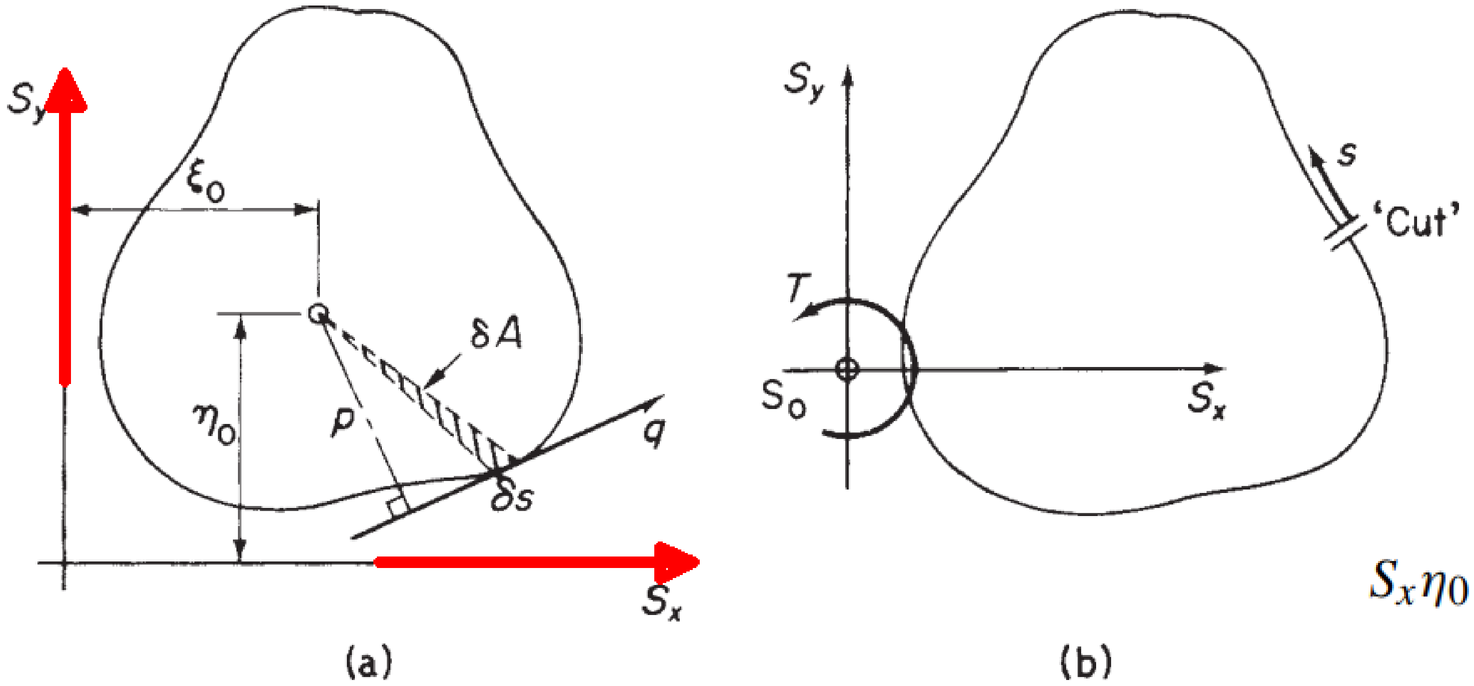
$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s tx \, \mathrm{d}s - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s ty \, \mathrm{d}s$$

$$q_s - q_{s,0} = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s tx \, \mathrm{d}s - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s ty \, \mathrm{d}s$$

$$q_s = q_b + q_{s,0}$$

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s tx \, \mathrm{d}s - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s ty \, \mathrm{d}s + q_{s,0}$$

Shear of closed section thin-walled beams



EQUIVALENCE of:

***Cause** (external load, S_x and S_y)*

and

***Result** (created shear stress)*

$$S_x\eta_0 - S_y\xi_0 = \oint pq\,\mathrm{d}s = \oint pq_\mathrm{b}\,\mathrm{d}s + q_{s,0}\oint p\,\mathrm{d}s$$

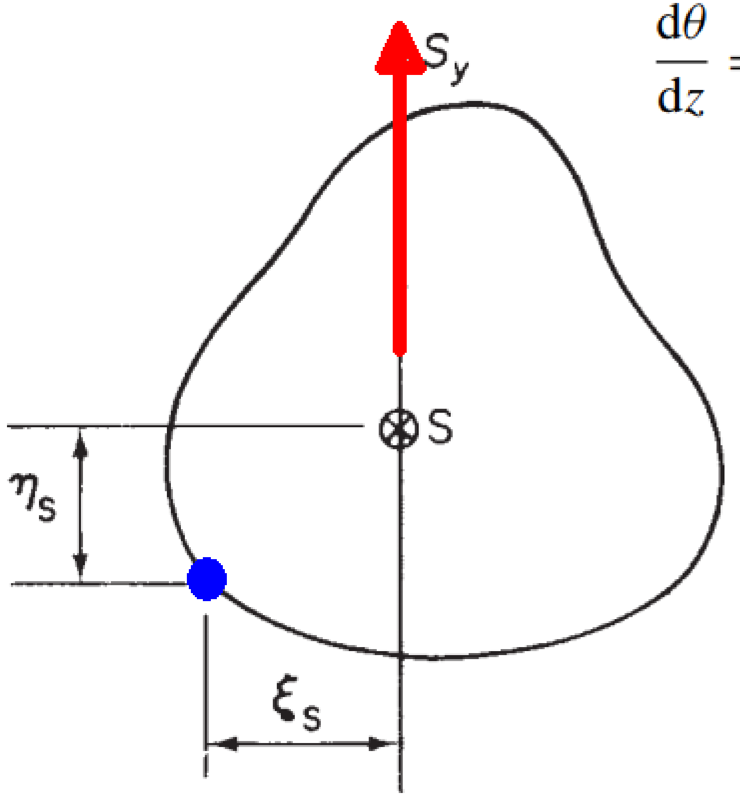
$$S_x\eta_0 - S_y\xi_0 = \oint pq_\mathrm{b}\mathrm{d}s + 2Aq_{s,0}$$

Shear Center of closed section thin-walled beams

It is necessary to use the ADDITIONAL information: condition that a shear load acting through the shear centre of a section produces zero twist.

$$\frac{d\theta}{dz} = \frac{1}{2A} \oint \frac{q_s}{Gt} ds$$

Shear Center of closed section thin-walled beams



$$\frac{d\theta}{dz} = \frac{1}{2A}\oint \frac{q_s}{Gt}ds \quad \Rightarrow \quad 0 = \oint \frac{q_s}{Gt}ds$$

$$0 = \oint \frac{1}{Gt}(q_b + q_{s,0})ds$$

$$q_{s,0} = -\frac{\oint (q_b/Gt)ds}{\oint ds/Gt}$$

$$q_{s,0} = -\frac{\oint q_b\, ds}{\oint ds}$$