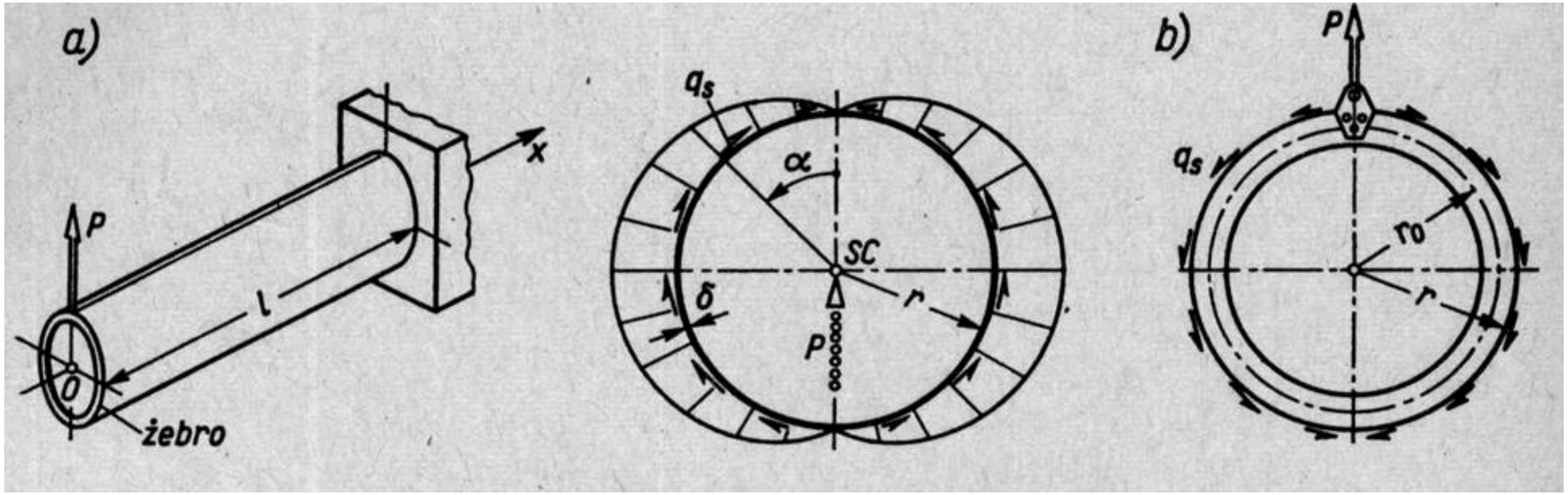
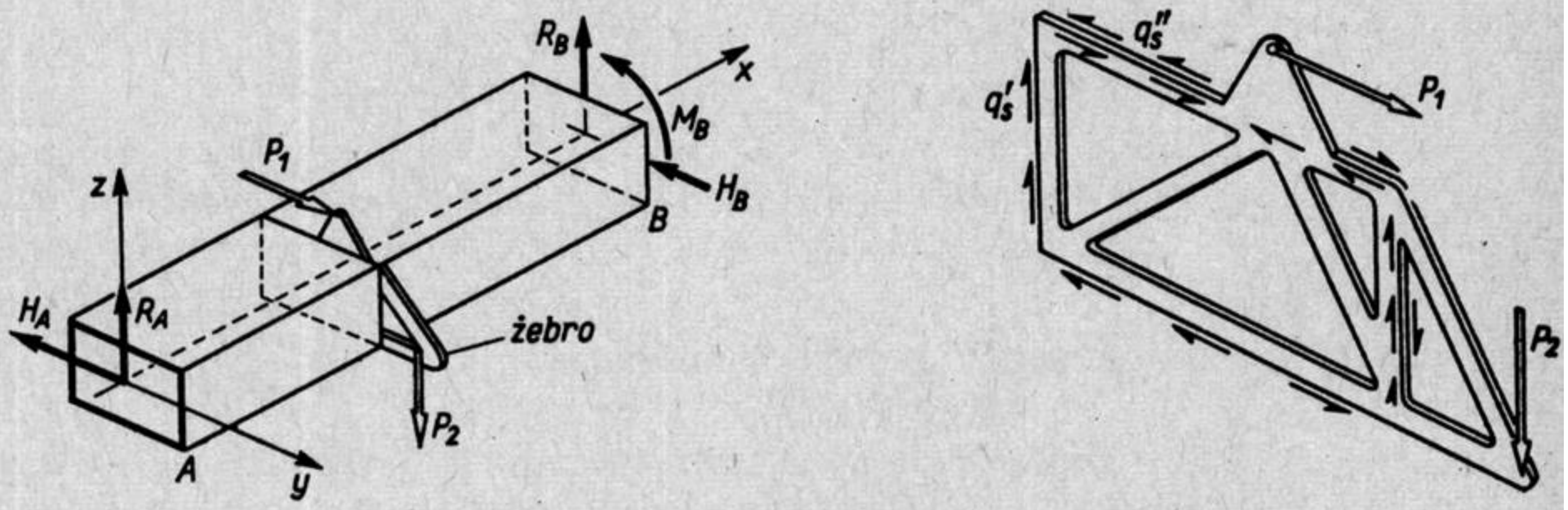
Constrained torsion of thin-walled beams

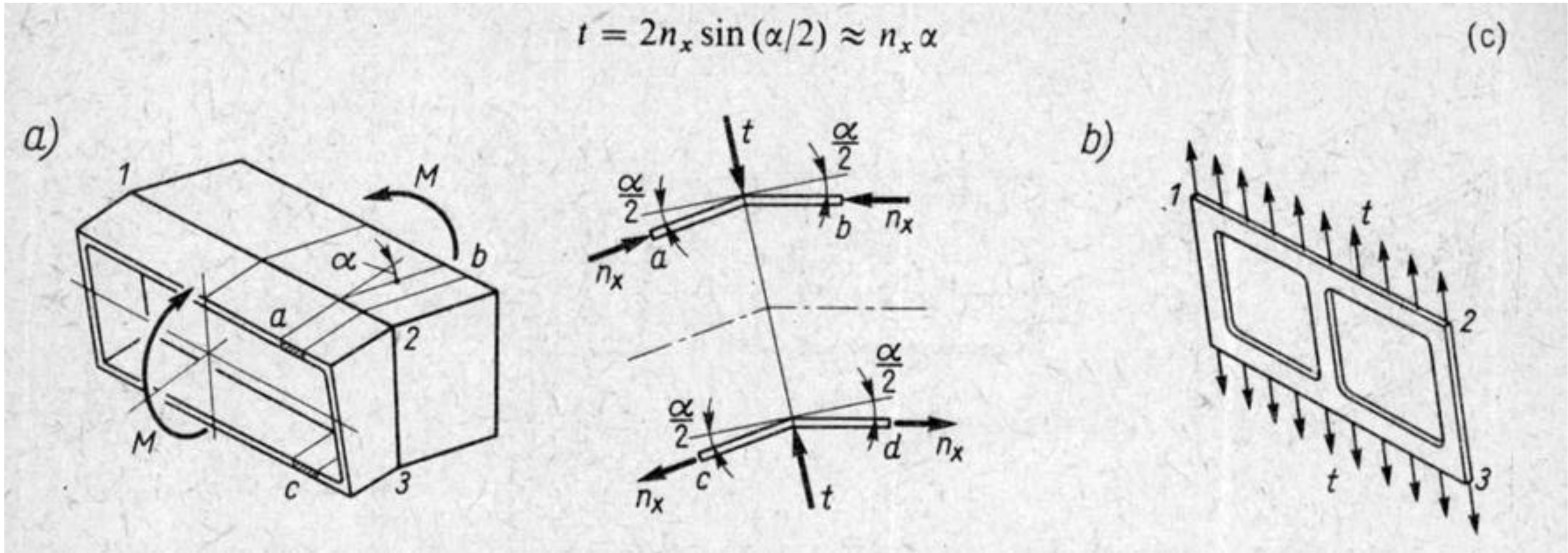


Constrained torsion of thin-walled beams

$$\Delta q_s = q_s'' - q_s',$$

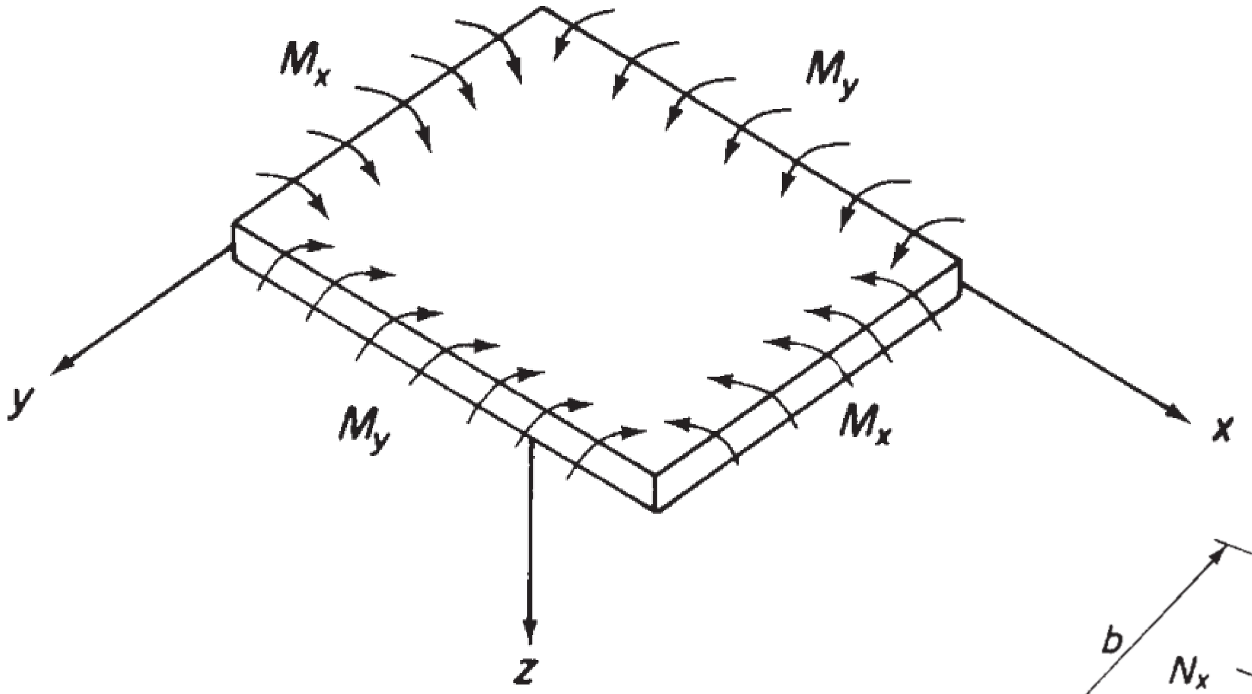


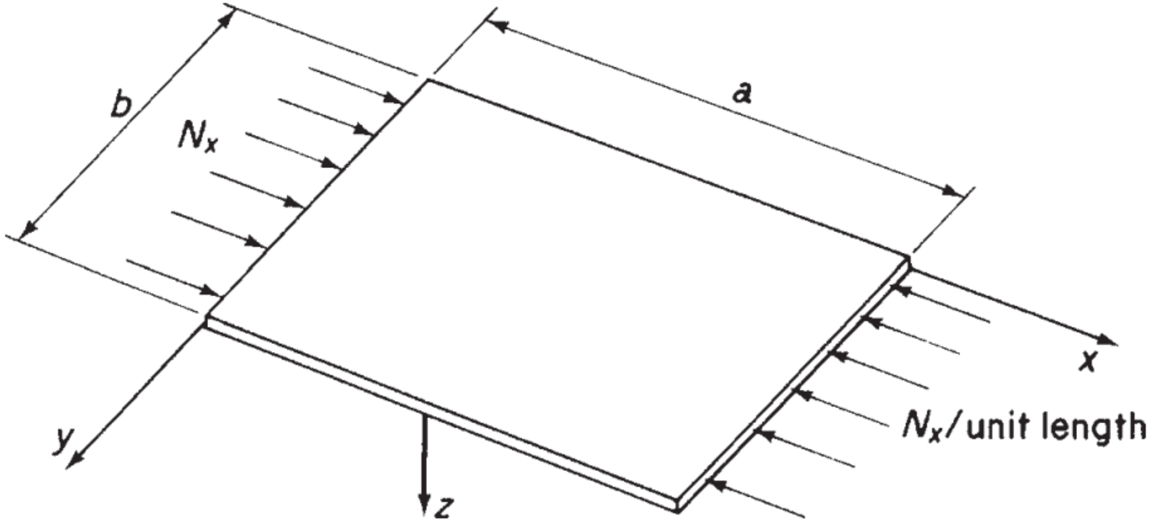
Constrained torsion of thin-walled beams



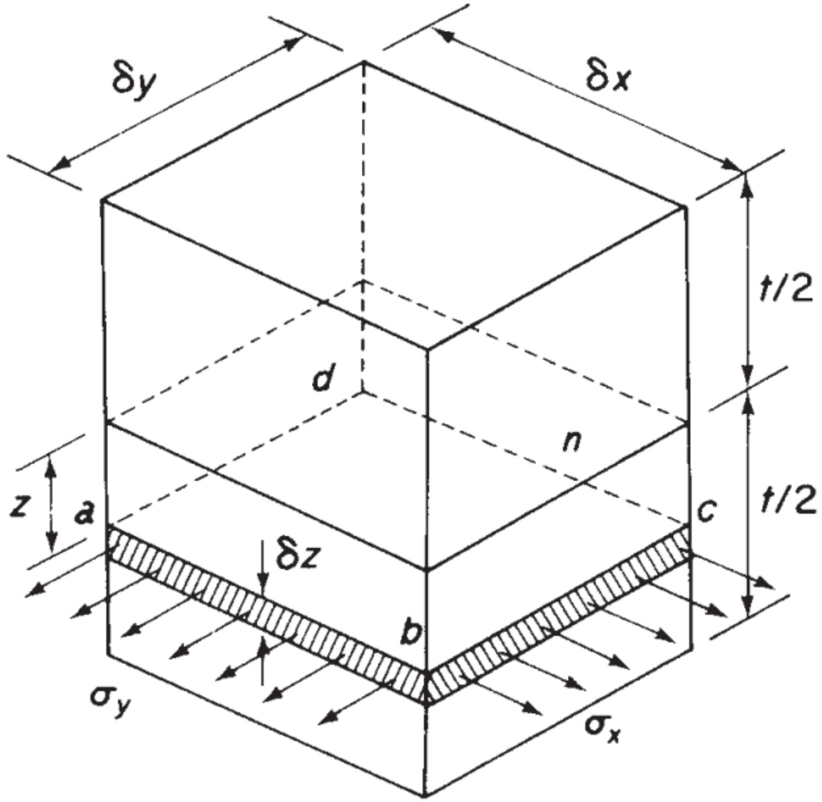
Plates and Shells

Plates

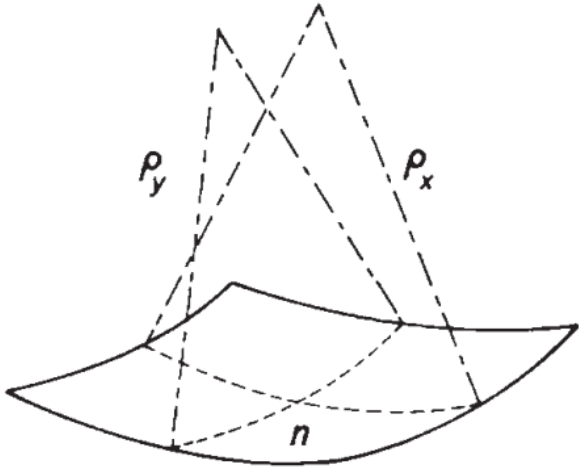




Plates



(a)



$$\varepsilon_x = \frac{z}{\rho_x} \quad \varepsilon_y = \frac{z}{\rho_y}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \quad \varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

Plates

$$M_x \delta y = \int_{-t/2}^{t/2} \sigma_x z \delta y \, \mathrm{d}z$$

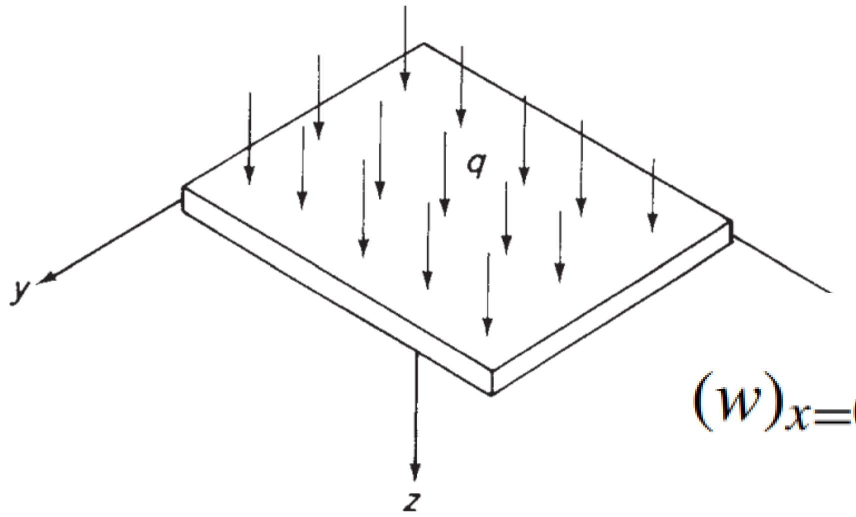
$$M_x = \int_{-t/2}^{t/2} \frac{Ez^2}{1-\nu^2} \left(\frac{1}{\rho_x} + \frac{\nu}{\rho_y} \right) \mathrm{d}z$$

$$M_x = D \left(\frac{1}{\rho_x} + \frac{\nu}{\rho_y} \right)$$

$$M_x = -D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right)$$

$$M_y = -D \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right)$$

Plates



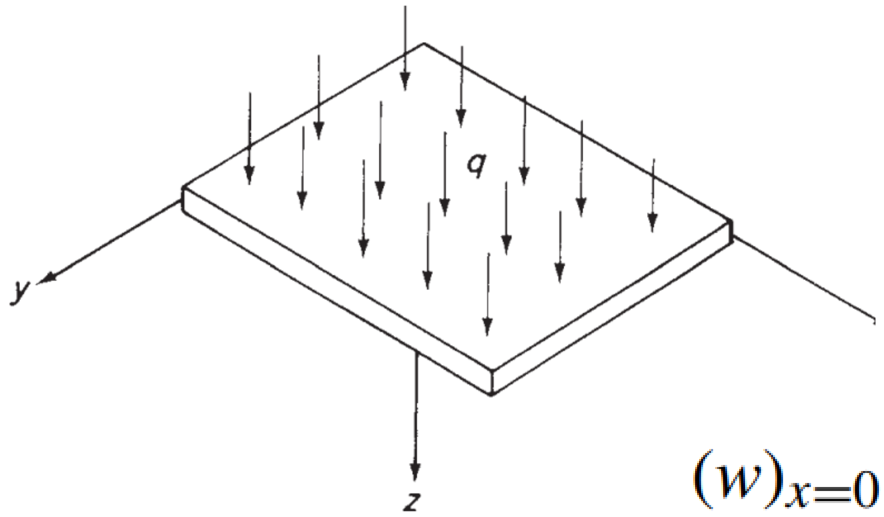
$$(\nabla^2)^2 w = \frac{q}{D}$$

$$(w)_{x=0} = 0 \quad \text{and} \quad (M_x)_{x=0} = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)_{x=0} = 0$$

$$(w)_{x=0} = 0 \quad \left(\frac{\partial w}{\partial x}\right)_{x=0} = 0$$

$$(M_x)_{x=0} = 0 \quad (M_{xy})_{x=0} = 0 \quad (Q_x)_{x=0} = 0$$

Plates



$$(\nabla^2)^2 w = \frac{q}{D}$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q(x,y)}{D}$$

$$(w)_{x=0} = 0 \quad \text{and} \quad (M_x)_{x=0} = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)_{x=0} = 0$$

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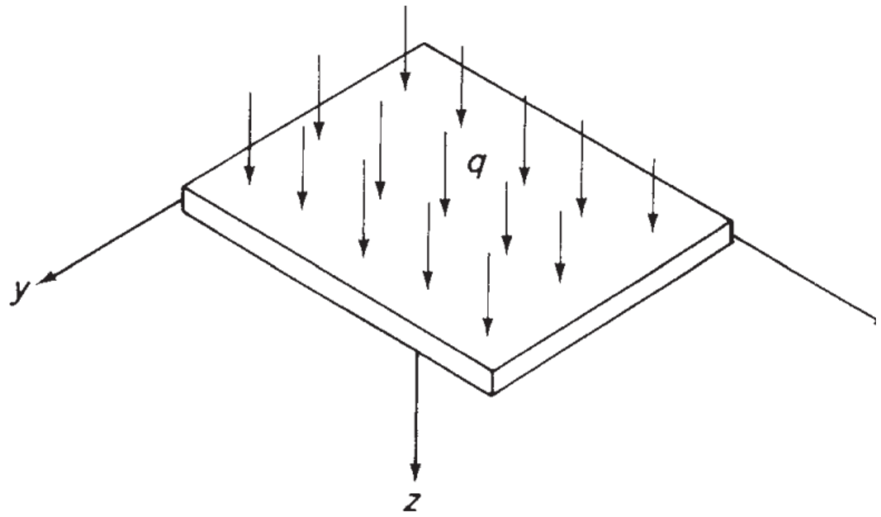
Plates

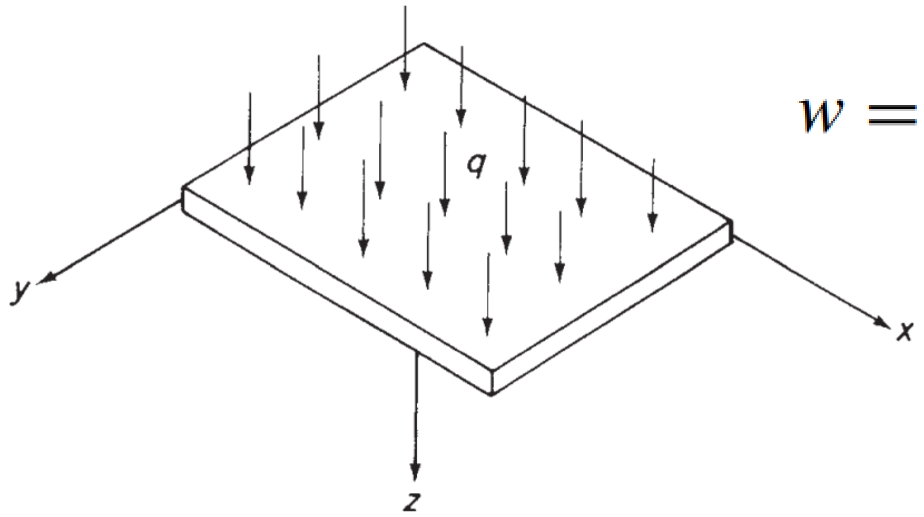


$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q(x,y)}{D}$$

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$q(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

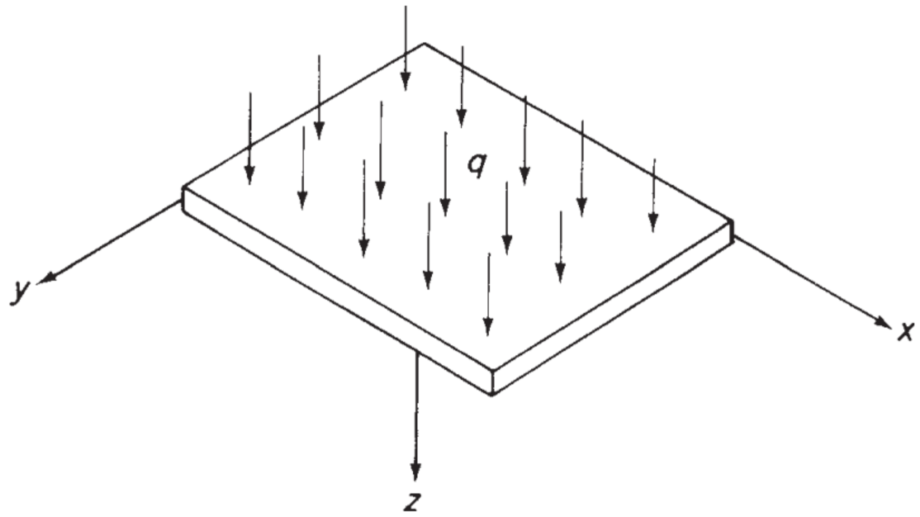
Plates



$$w = \frac{1}{\pi^4 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_{mn}}{[(m^2/a^2) + (n^2/b^2)]^2} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

$$a_{m'n'} = \frac{4}{ab} \int_0^a \int_0^b q(x, y) \sin \frac{m'\pi x}{a} \sin \frac{n'\pi y}{b} \, \mathrm{d}x \, \mathrm{d}y$$

Plates – example (q=q*=const)



$$w = \frac{16q_0}{\pi^6 D} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{\sin(m\pi x/a)\sin(n\pi y/b)}{mn[(m^2/a^2)+(n^2/b^2)]^2}$$

$$w_{\max} = 0.0443 q_0 \frac{a^4}{Et^3}$$

$$M_{x,\max} = M_{y,\max} = 0.0479 q_0 a^2$$

$$\sigma_x = \frac{12 M_x z}{t^3} \qquad \sigma_y = \frac{12 M_y z}{t^3}$$

$$\sigma_{x,\max} = \sigma_{y,\max} = 0.287 q_0 \frac{a^2}{t^2}$$

Plates – transverse and in-plane loads

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D}\left(q + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y}\right)$$

$$q = \frac{16q_0}{\pi^2}\sum_{m=1,3,5}^{\infty}\sum_{n=1,3,5}^{\infty}\frac{1}{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$w = \frac{16q_0}{\pi^6 D}\sum_{m=1,3,5}^{\infty}\sum_{n=1,3,5}^{\infty}\frac{1}{mn\left[\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2 + \frac{N_x m^2}{\pi^2 D a^2}\right]}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$w = \frac{16q_0}{\pi^6 D}\sum_{m=1,3,5}^{\infty}\sum_{n=1,3,5}^{\infty}\frac{\sin(m\pi x/a)\sin(n\pi y/b)}{mn[(m^2/a^2)+(n^2/b^2)]^2}$$

Plates – Energy aproach

$$U = \frac{D}{2}\int_0^a\int_0^b\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\nu\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right]\mathrm{d}x\,\mathrm{d}y$$

$$V = -\int_0^a\int_0^b wq\,\mathrm{d}x\,\mathrm{d}y$$

$$V = -\frac{1}{2}\int_0^a\int_0^b\left[N_x\left(\frac{\partial w}{\partial x}\right)^2 + N_y\left(\frac{\partial w}{\partial y}\right)^2 + 2N_{xy}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right]\mathrm{d}x\,\mathrm{d}y$$

Plates – Energy aproach

$$U + V = \frac{D}{2} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} A_{mn}^2 \frac{\pi^4 ab}{4} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 - q_0 \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} A_{mn} \frac{4ab}{\pi^2 mn}$$

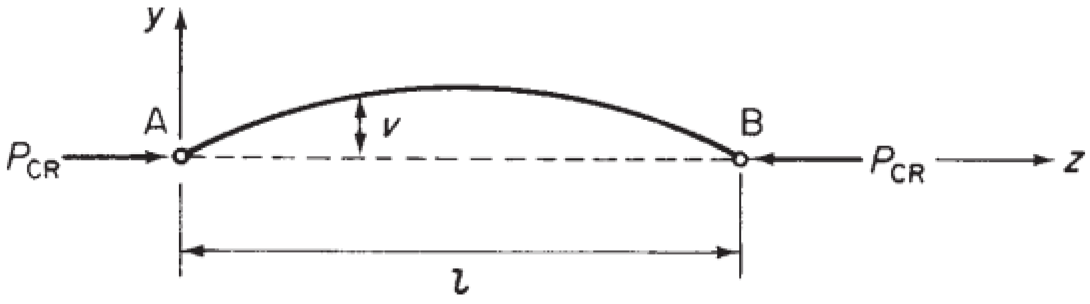
$$\frac{\partial(U + V)}{\partial A_{mn}} = \frac{D}{2} 2A_{mn} \frac{\pi^4 ab}{4} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 - q_0 \frac{4ab}{\pi^2 mn} = 0$$

$$A_{mn} = \frac{16 q_0}{\pi^6 D mn[(m^2/a^2) + (n^2/b^2)]^2}$$

$$w = \frac{16 q_0}{\pi^6 D} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{\sin(m\pi x/a) \sin(n\pi y/b)}{mn[(m^2/a^2) + (n^2/b^2)]^2}$$

Buckling

Buckling / stability

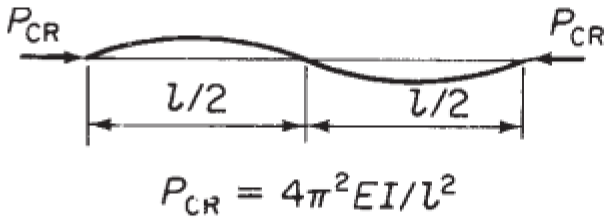


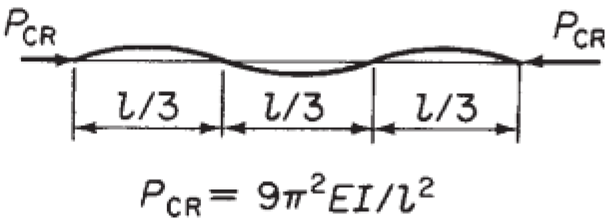
$$EI\frac{d^2v}{dz^2} = -P_{CR}v$$

$$v = A\cos\mu z + B\sin\mu z \qquad \mu^2 = P_{CR}/EI$$

$$\sin\mu l = 0 \quad \text{or} \quad \mu l = n\pi \quad \text{where } n = 1, 2, 3, \ldots$$

$$P_{CR} = \frac{n^2\pi^2 EI}{l^2}$$



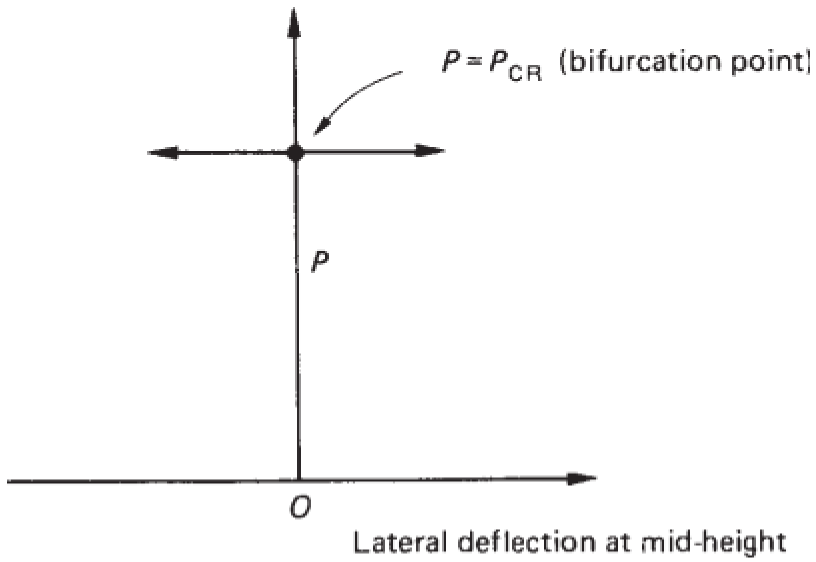


Buckling / stability

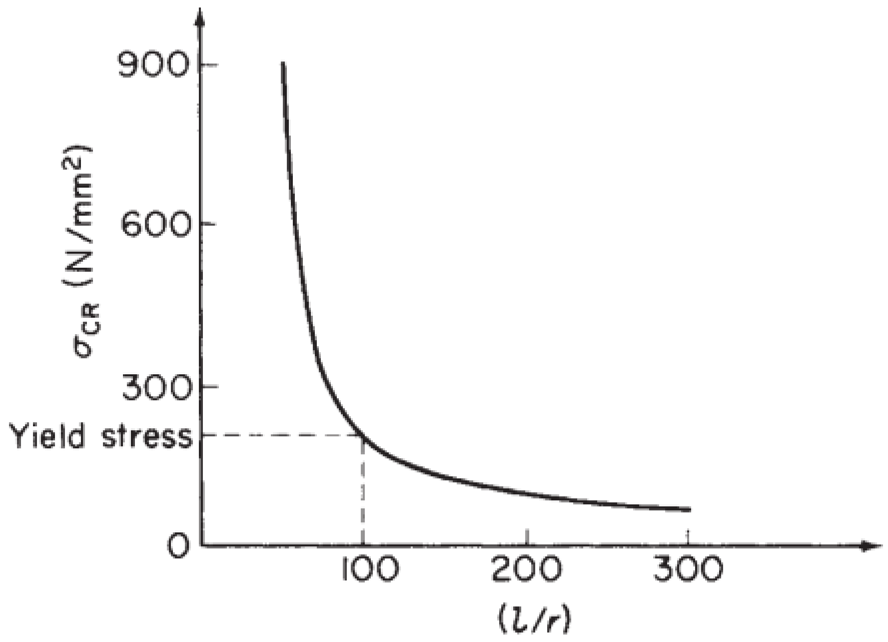
Ends	l_e/l	Boundary conditions
Both pinned	1.0	$v = 0$ at $z = 0$ and l
Both fixed	0.5	$v = 0$ at $z = 0$ and $z = l$, $dv/dz = 0$ at $z = l$
One fixed, the other free	2.0	$v = 0$ and $dv/dz = 0$ at $z = 0$
One fixed, the other pinned	0.6998	$dv/dz = 0$ at $z = 0$, $v = 0$ at $z = l$ and $z = 0$

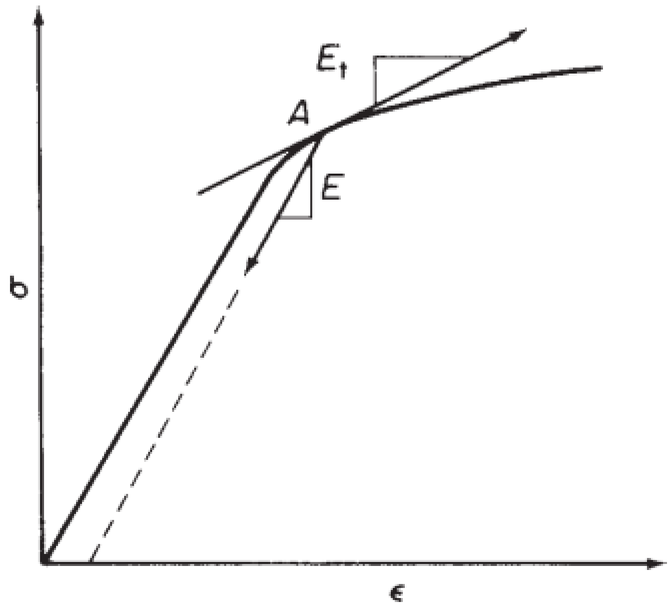
$$P_{CR} = \frac{\pi^2 EI}{l_e^2}$$

$$\sigma_{CR} = \frac{\pi^2 E}{(l_e/r)^2}$$

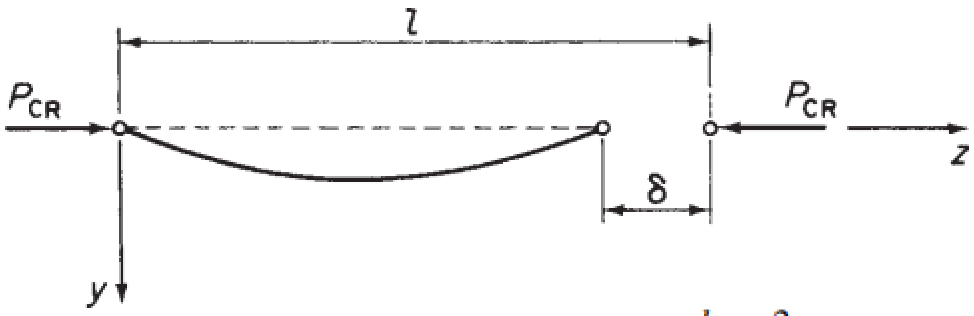


Buckling / stability





Buckling / stability
Energy approach

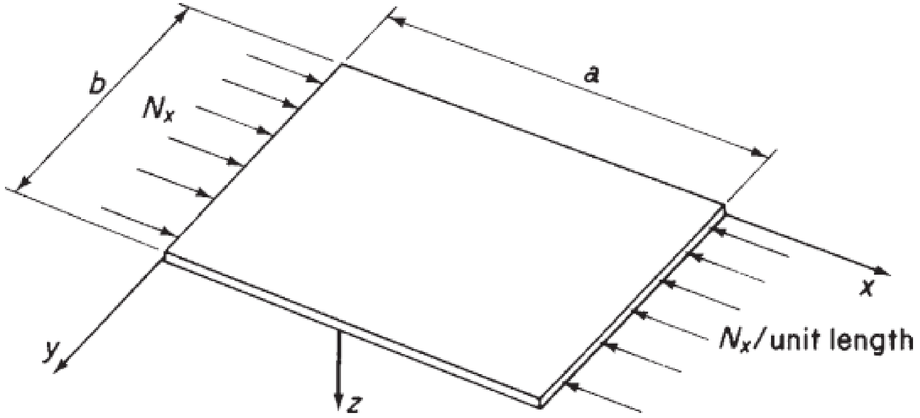


$$U = \int_0^l \frac{M^2}{2EI}\,dz \qquad U = \frac{EI}{2}\int_0^l \left(\frac{d^2v}{dz^2}\right)^2 dz$$

$$V = -\frac{P_{CR}}{2}\int_0^l \left(\frac{dv}{dz}\right)^2 dz$$

$$U + V = \int_0^l \frac{M^2}{2EI}\,dz - \frac{P_{CR}}{2}\int_0^l \left(\frac{dv}{dz}\right)^2 dz \qquad U + V = \frac{EI}{2}\int_0^l \left(\frac{d^2v}{dz^2}\right)^2 dz - \frac{P_{CR}}{2}\int_0^l \left(\frac{dv}{dz}\right)^2 dz$$

Buckling / stabilityof plates
Energy approach



$$w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

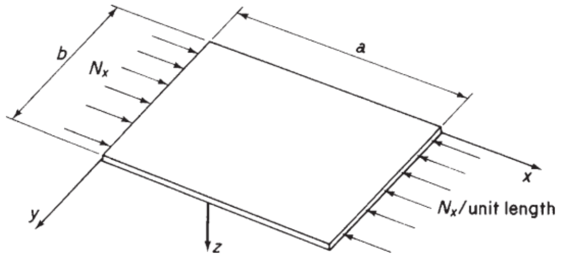
$$U+V=\frac{1}{2}\int_0^a\int_0^b\left[D\left\{\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2 -2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right]\right\}-N_x\left(\frac{\partial w}{\partial x}\right)^2\right]\mathrm{d}x\,\mathrm{d}y$$

$$U+V=\frac{\pi^4 abD}{8}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}^2\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)-\frac{\pi^2 b}{8a}N_x\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}m^2A_{mn}^2$$

$$\frac{\partial(U+V)}{\partial A_{mn}}=\frac{\pi^4 abD}{4}A_{mn}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2-\frac{\pi^2 b}{4a}N_{x,\mathrm{CR}}m^2A_{mn}=0$$

$$N_{x,\mathrm{CR}}=\pi^2a^2D\frac{1}{m^2}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2$$

Buckling / stability of plates
Energy approach



$$N_{x,CR} = \pi^2 a^2 D \frac{1}{m^2}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2$$

$$N_{x,CR} = \pi^2 a^2 D \frac{1}{m^2}\left(\frac{m^2}{a^2}+\frac{1}{b^2}\right)^2$$

$$N_{x,CR} = \frac{k\pi^2 D}{b^2}$$

$$k = \left(\frac{mb}{a}+\frac{a}{mb}\right)^2$$