

COMPRESSIBLE FLOW OVER AN AIRFOIL

INCOMPRESSIBLE FLOW: $\rho = \rho_\infty = \mathbf{const}$

: LIFT GENERATION, DRAG GENERATION, PRESSURE DISTRIBUTION ...
 $C_L(\alpha)$, $C_D(C_z)$, $C_{my}(C_L)$, aerodynamic center

COMPRESSIBLE FLOW: $\rho = \mathbf{var}$

?

COMPRESSIBLE FLOW OVER AN AIRFOIL

Continuity equation:
$$\frac{\partial(\rho U)}{\partial x} + \frac{\partial(\rho V)}{\partial y} = \nabla \cdot (\rho \vec{V}) = \mathbf{0}$$

INCOMPRESSIBLE FLOW: $\rho = \rho_{\infty} = \mathbf{const}$

$$\frac{\partial(\rho_{\infty} U)}{\partial x} + \frac{\partial(\rho_{\infty} V)}{\partial y} = \nabla \cdot (\rho_{\infty} \vec{V}) = \rho_{\infty} \mathbf{div} \vec{V} = \mathbf{0}$$

$$\mathbf{div} \vec{V} = \mathbf{0}$$

COMPRESSIBLE FLOW: $\rho = \mathbf{var}$

$$\frac{\partial(\rho U)}{\partial x} + \frac{\partial(\rho V)}{\partial y} = \nabla \cdot (\rho \vec{V}) = \rho \mathbf{div} \vec{V} + \vec{V} \cdot \mathbf{grad} \rho = \mathbf{0}$$

$$\mathbf{div} \vec{V} = -\frac{\vec{V} \cdot \mathbf{grad} \rho}{\rho} \neq \mathbf{0}$$

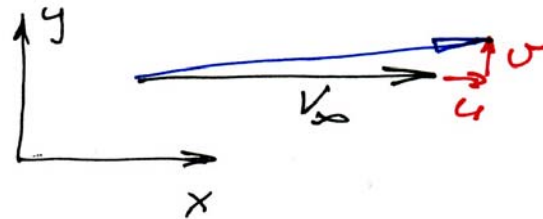
$\text{div}\vec{V}$:

$$\iiint_{\Omega} \text{div}\vec{V} \, d\Omega = \oiint_{\delta\Omega} \vec{V} \cdot \vec{n} \, dS$$

$$\text{div}\vec{V} = \frac{\text{VOLUME OUTFLOW RATE FROM THE CONTROL ELEMENT}}{\text{VOLUME OF THE CONTROL ELEMENT}}$$

RELATIVE GAS VOLUME RATE OF CHANGE IN CONTROL ELEMENT

SUBSONIC FLOW OVER AN AIRFOIL. SMALL DISTURBANCE THEORY



$$u, v \ll U_\infty$$

Potential flow: ϕ - perturbation velocity potential

$$U = U_\infty + u = U_\infty + \frac{\partial \phi}{\partial x}$$

$$V = \quad v = \quad \frac{\partial \phi}{\partial y}$$

PROCEDURE:

- Solve continuity eq. + boundary + additional conditions \rightarrow flow field***
- Find pressure field using Bernoulli eq. (momentum eq.!)***
- Find aerodynamic forces and moments (pressure forces)***

Continuity equation:

$$\frac{\partial(\rho U)}{\partial x} + \frac{\partial(\rho V)}{\partial y} = \mathbf{0} = (U_\infty + \mu) \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} + \nu \frac{\partial \rho}{\partial y} + \rho \frac{\partial v}{\partial y} \simeq$$

$$U_\infty \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} = U_\infty \frac{\partial \rho}{\partial x} + \rho \operatorname{div} \vec{v} =$$

$$|S = \text{const}| = U_\infty \left. \frac{\partial \rho}{\partial p} \right|_\infty \frac{\partial p}{\partial x} + \rho \operatorname{div} \vec{v} =$$

$$\left| \begin{aligned} \text{Euler eq. (x): } \rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial U}{\partial y} &= -\frac{\partial p}{\partial x} \\ \rho (U_\infty + \mu) \frac{\partial u}{\partial x} + \rho \nu \frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} \rightarrow \frac{\partial p}{\partial x} \simeq -\rho U_\infty \frac{\partial u}{\partial x} \end{aligned} \right|$$

$$= -\rho \frac{U_\infty^2}{a_\infty^2} \frac{\partial u}{\partial x} + \rho \operatorname{div} \vec{v} = -\rho \frac{U_\infty^2}{a_\infty^2} \frac{\partial u}{\partial x} + \rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} = \rho \left((1 - Ma_\infty^2) \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = \mathbf{0}$$

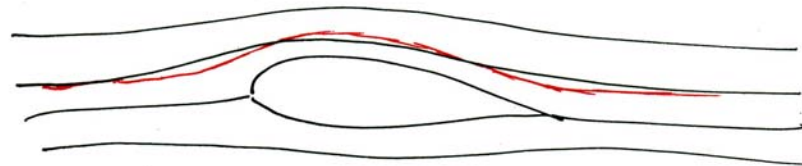
$$(1 - Ma_\infty^2) \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = (1 - Ma_\infty^2) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} =$$

$$\beta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 ; \quad \beta = \sqrt{1 - Ma_\infty^2}$$

Prandtl-Glauert eq. (..continuity)

or:

$$\text{div} \vec{V} = q = Ma_\infty^2 \frac{\partial u}{\partial x}$$



$$\begin{array}{cccc} \frac{\partial u}{\partial x} < 0 & > 0 & < 0 & > 0 \\ \leftarrow & \leftarrow & \leftarrow & \leftarrow \\ q < 0 & q > 0 & q < 0 & q > 0 \end{array}$$

BERNOULI EQUATION – SMALL DISTURBANCES

$$\text{Euler Eq.}(x): \rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial U}{\partial y} = -\frac{\partial p}{\partial x}$$

$$(\rho_\infty + \rho') (U_\infty + u) \frac{\partial u}{\partial x} + (\rho_\infty + \rho') v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} \rightarrow \frac{\partial p}{\partial x} \approx -\rho_\infty U_\infty \frac{\partial u}{\partial x}$$

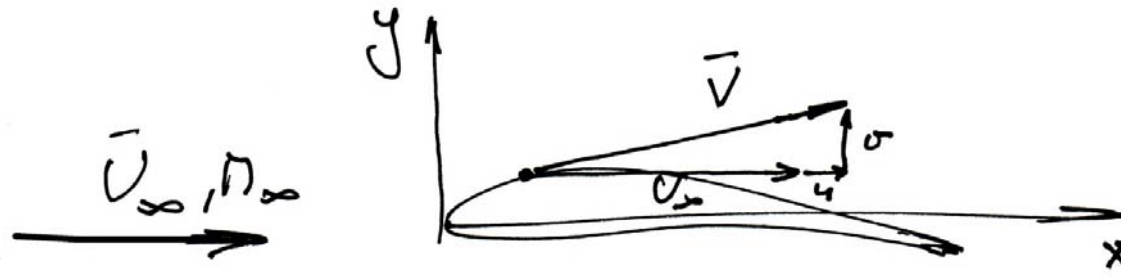
Integrating

$$\rho_\infty U_\infty \underbrace{\Delta U}_{U-U_\infty} = - \underbrace{\Delta p}_{p-p_\infty}$$

$$p - p_\infty = \rho_\infty U_\infty u$$

$$C_p = \frac{p - p_\infty}{\rho_\infty U_\infty^2 / 2} = \frac{\rho_\infty U_\infty u}{\rho_\infty U_\infty^2 / 2} = -2u / U_\infty$$

linearized Bernouli equation



$$y_B(x) = F(x)$$

$$\beta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 ; \quad \frac{\partial \phi}{\partial y} = v = (U_\infty + u) F' \approx U_\infty F'$$

↓

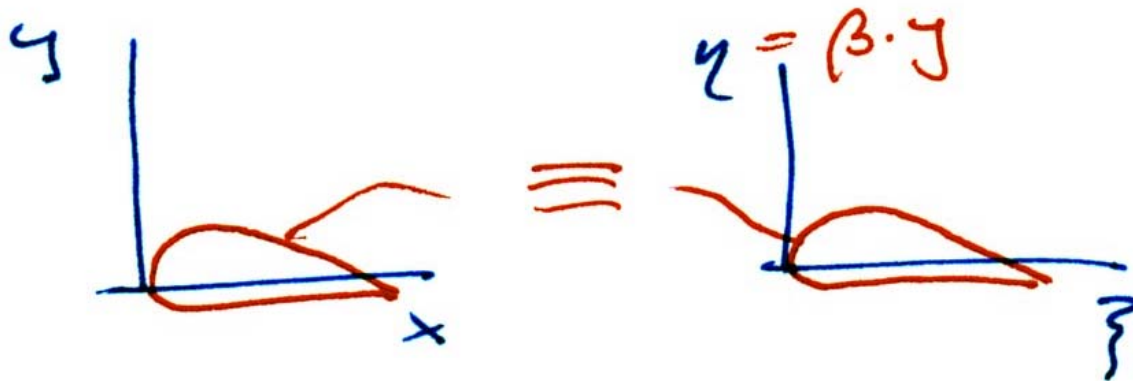
$$\phi \rightarrow \vec{V} = \vec{V}_\infty + \nabla \phi \rightarrow p \rightarrow C_p \rightarrow C_L, C_m$$

$$\xi = x, \quad \eta = \beta y, \quad \eta_B(\xi) = F(\xi)$$

$$\Phi = \lambda \varphi$$

$$\beta^2 \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \rightarrow \frac{\partial^2 \Phi}{\partial \xi^2} + \frac{\partial^2 \Phi}{\partial \eta^2} = 0$$

$$\frac{\partial \varphi}{\partial y} = (U_\infty + u) F' \simeq U_\infty F' \rightarrow \frac{\partial \Phi}{\partial \eta} = (U_\infty + \tilde{u}) F' \simeq U_\infty F'$$



$$\frac{\partial \varphi}{\partial y} = (U_\infty + u) F' \simeq U_\infty F' \quad \rightarrow \quad U_\infty F' \simeq \frac{\partial \Phi}{\partial \eta} = \frac{\partial(\lambda \varphi)}{\partial(\beta y)} = \underbrace{\left(\frac{\lambda}{\beta}\right)}_{=1} \frac{\partial \varphi}{\partial y} \simeq U_\infty F'$$

$$\lambda = \beta$$

$$\varphi = \Phi / \beta$$

$$C_p = -2 \frac{u}{U_\infty} = -2 \frac{\partial \varphi}{\partial x} / U_\infty = -2 \frac{\partial(\Phi / \beta)}{\partial \xi} / U_\infty = \frac{1}{\beta} (-) 2 \frac{\partial \Phi}{\partial \xi} / U_\infty = \frac{1}{\beta} C_{p_{INC}}$$

$C_p = \frac{1}{\beta} C_{p_{INC}}$	$C_L = \frac{1}{\beta} C_{L_{INC}}$	$C_m = \frac{1}{\beta} C_{m_{INC}}$
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XFOIL
V 6.97

NACA 4415

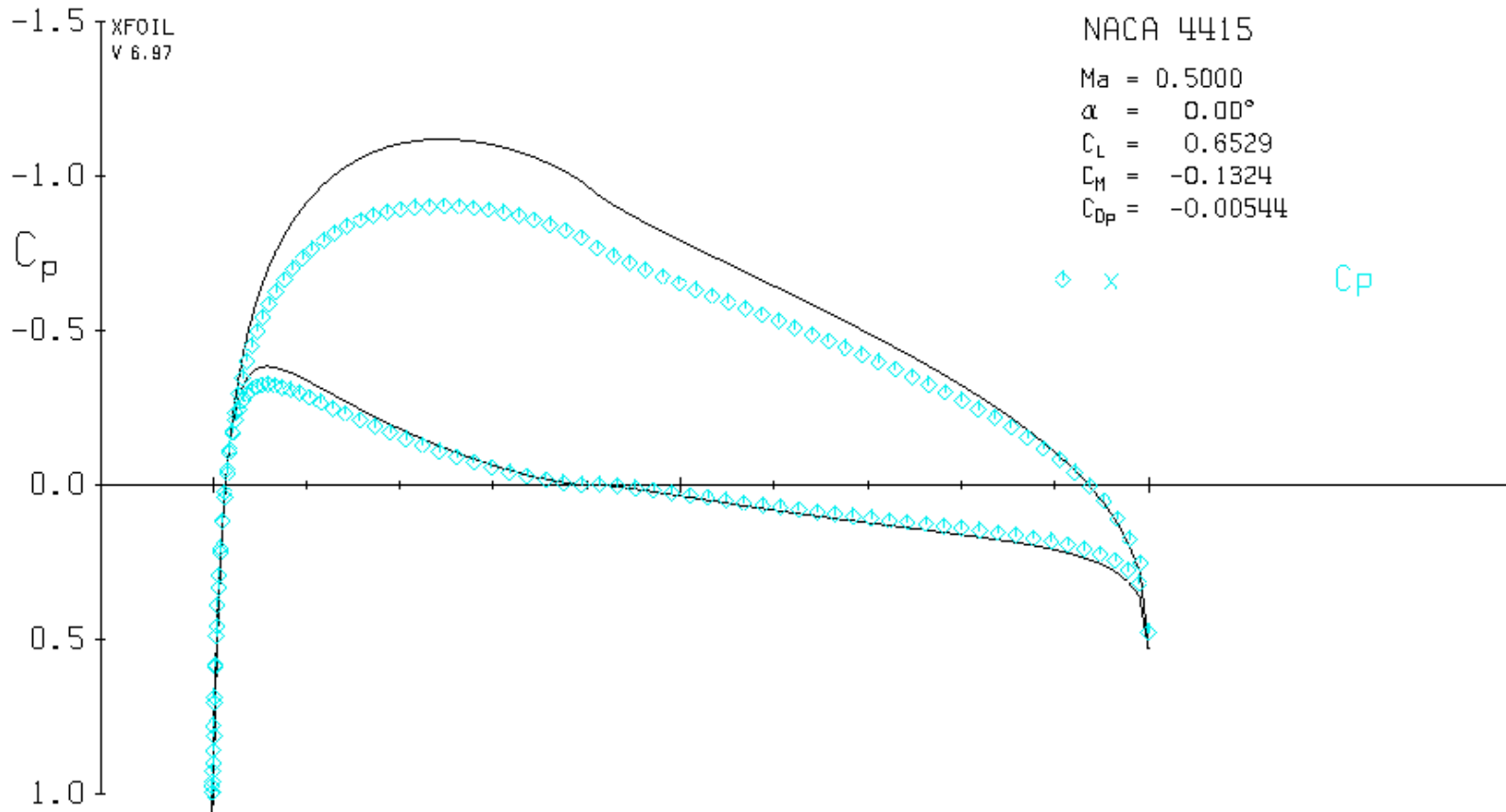
Ma = 0.5000

α = 0.00°

C_L = 0.6529

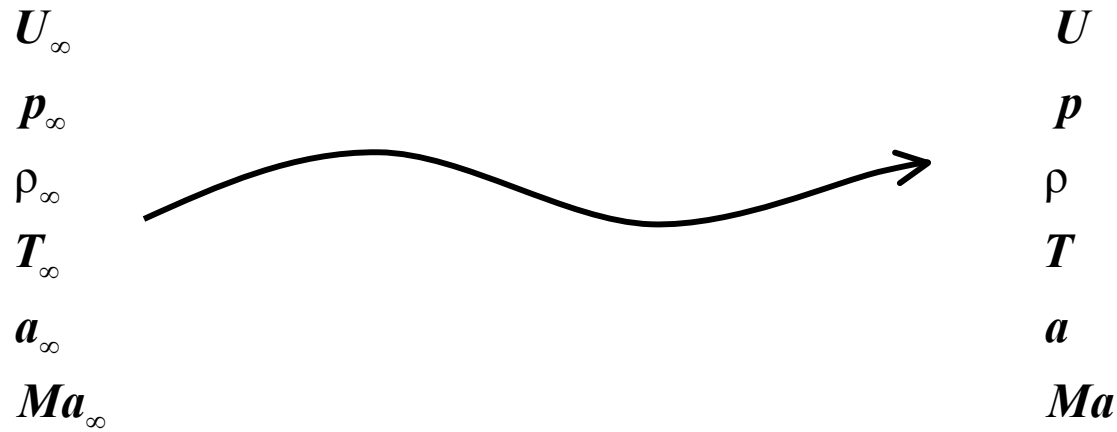
C_H = -0.1324

C_{Dp} = -0.00544



◇ x Cp

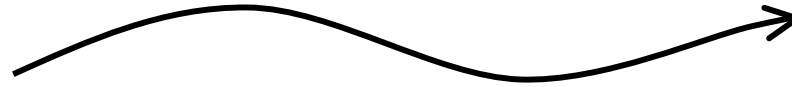




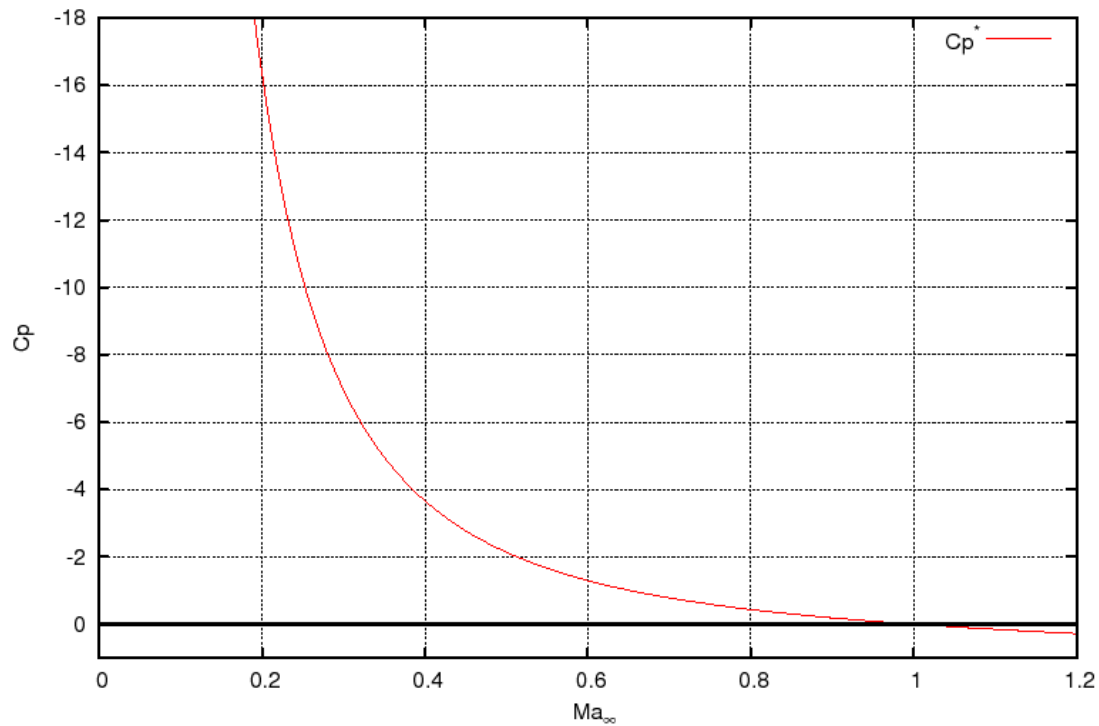
$$S = \text{const.} \quad p \left(1 + \frac{k-1}{2} Ma^2 \right)^{\frac{k}{k-1}} = p_0 = \text{const.}$$

$$p \left(1 + \frac{k-1}{2} Ma^2 \right)^{\frac{k}{k-1}} = p_\infty \left(1 + \frac{k-1}{2} Ma_\infty^2 \right)^{\frac{k}{k-1}} \rightarrow \frac{p}{p_\infty} = \left(\frac{1 + \frac{k-1}{2} Ma_\infty^2}{1 + \frac{k-1}{2} Ma^2} \right)^{\frac{k}{k-1}}$$

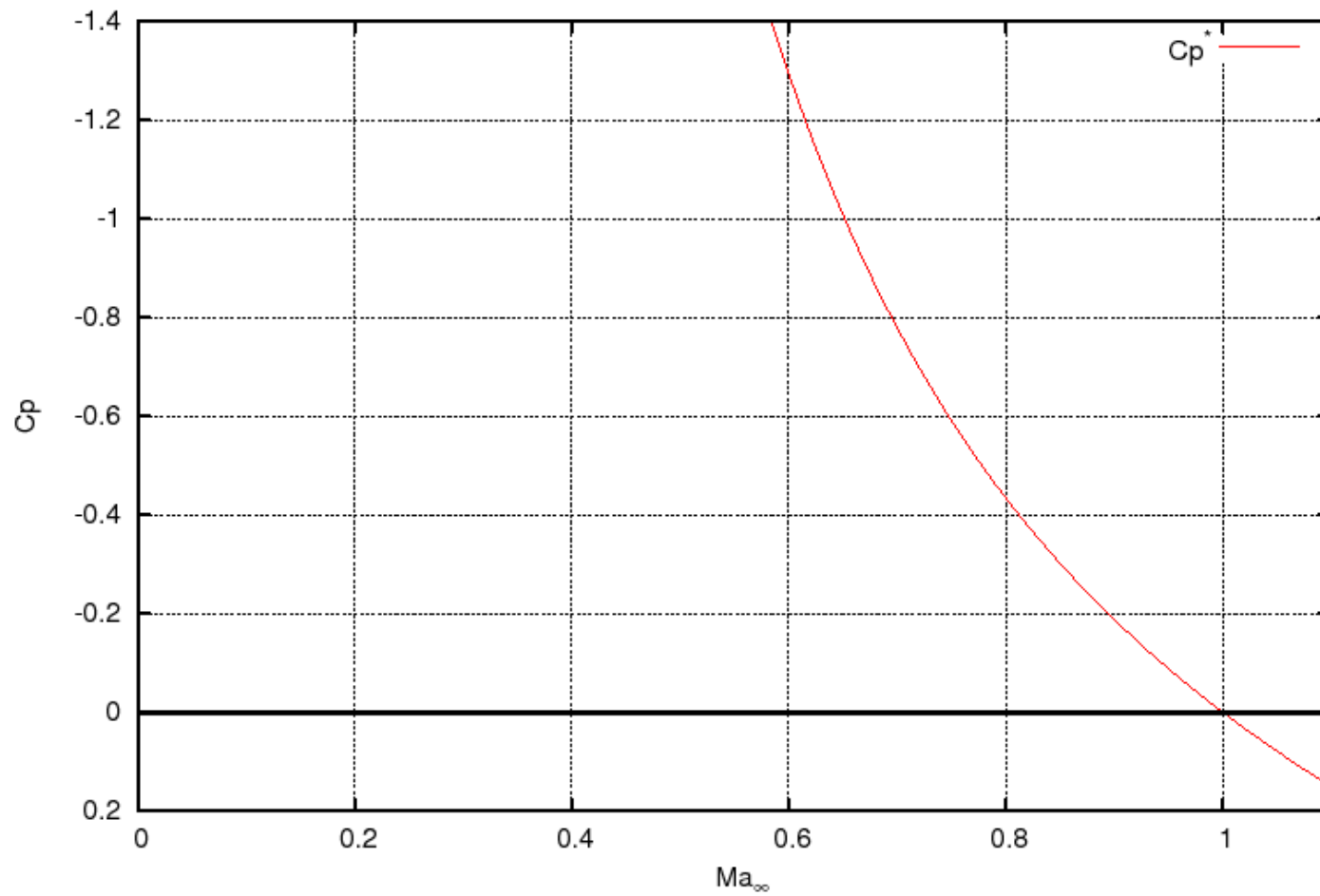
$$C_p = \frac{p - p_\infty}{\rho_\infty U_\infty^2 / 2} = \frac{p_\infty}{\rho_\infty} \frac{\left(\frac{p}{p_\infty} - 1 \right)}{\frac{1}{2} U_\infty^2} = kRT_\infty \frac{\left(\frac{1 + \frac{k-1}{2} Ma_\infty^2}{1 + \frac{k-1}{2} Ma^2} \right)^{\frac{k}{k-1}} - 1}{\frac{k}{2} U_\infty^2} = \frac{\left(\frac{1 + \frac{k-1}{2} Ma_\infty^2}{1 + \frac{k-1}{2} Ma^2} \right)^{\frac{k}{k-1}} - 1}{\frac{k}{2} Ma_\infty^2}$$

Ma_∞ p_∞ U_∞  $Ma = 1$ p^* U^*

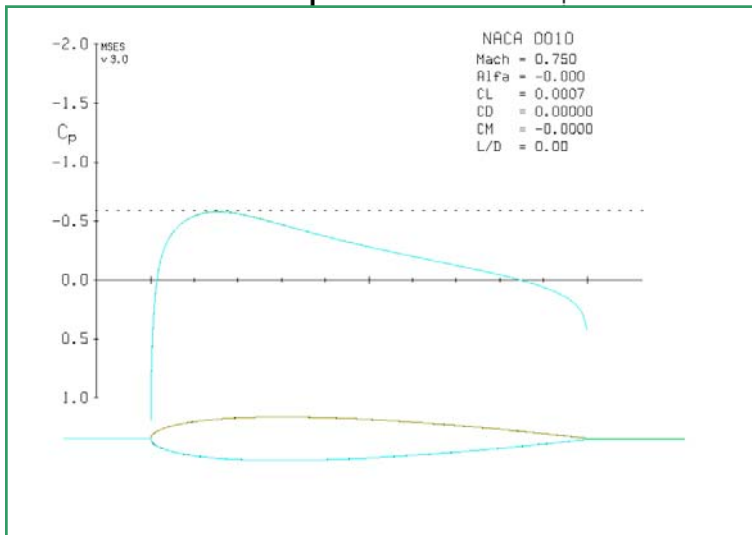
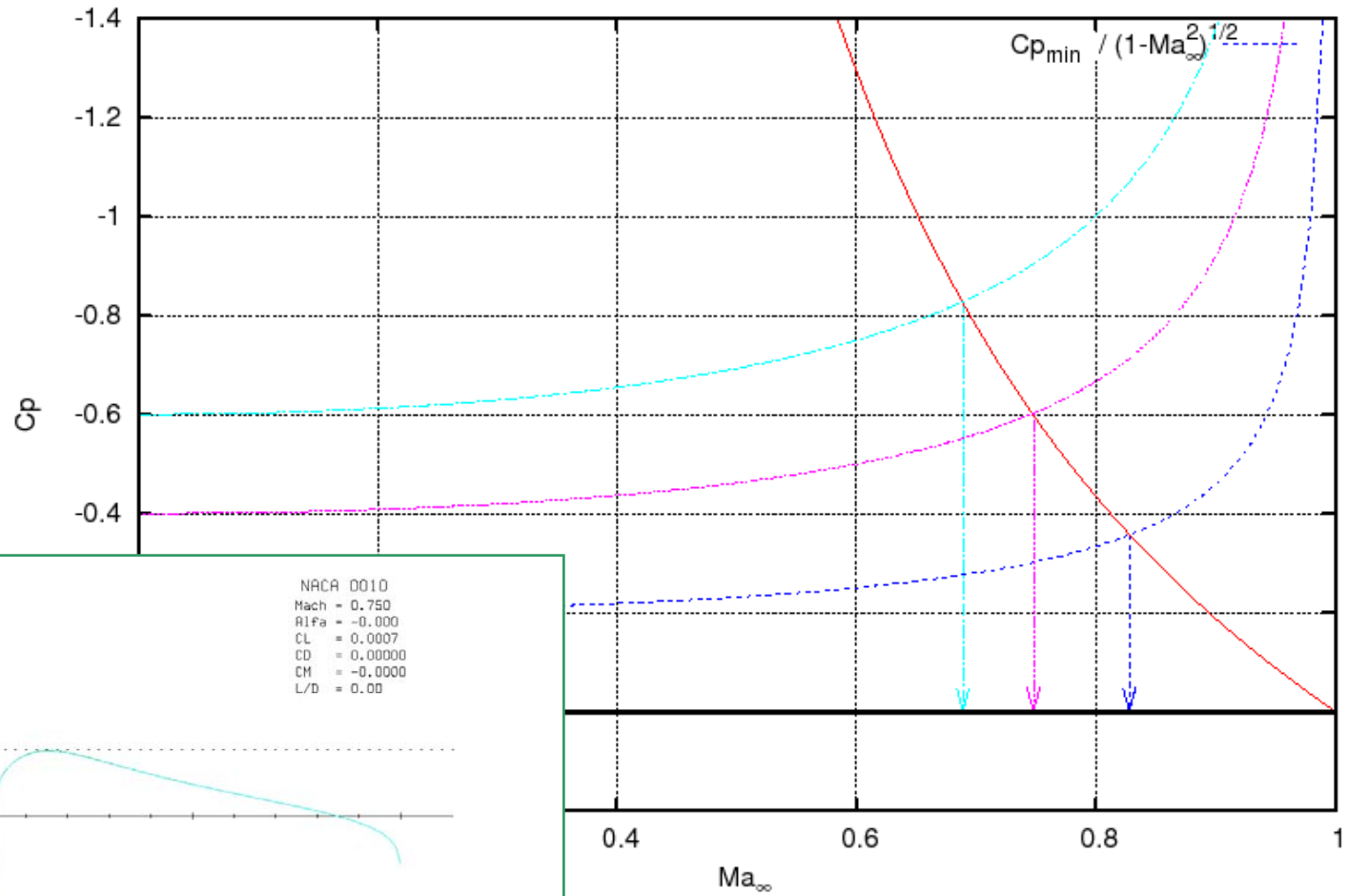
$$Cp^*(Ma_\infty) = \frac{\left(\frac{1 + \frac{k-1}{2} Ma_\infty^2}{1 + \frac{k-1}{2}} \right)^{\frac{k}{k-1}} - 1}{\frac{k}{2} Ma_\infty^2} = \frac{\left(\frac{1 + 0.2 Ma_\infty^2}{1.2} \right)^{3.5} - 1}{0.7 Ma_\infty^2}$$



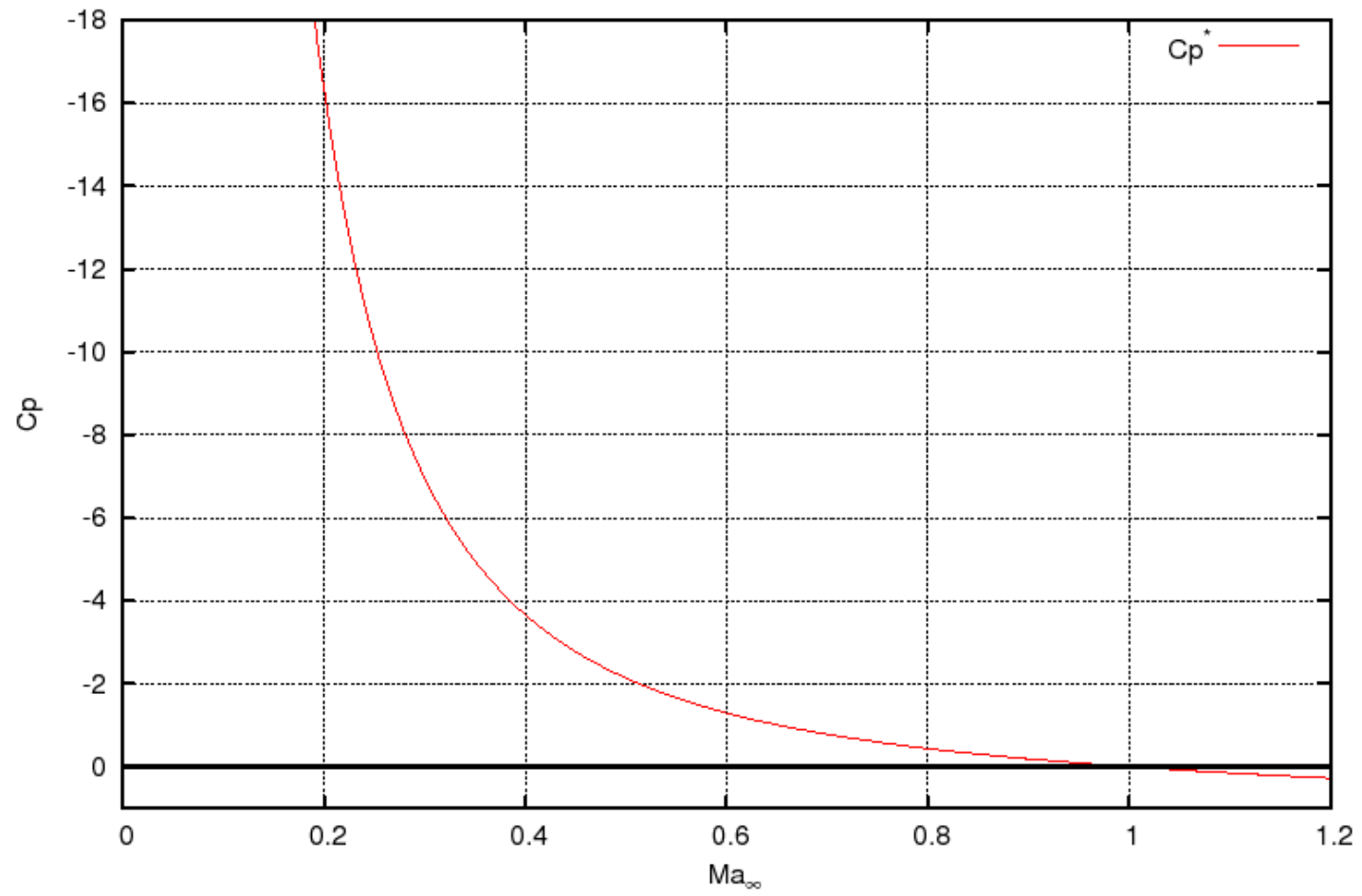
C_p^* vs Ma_∞

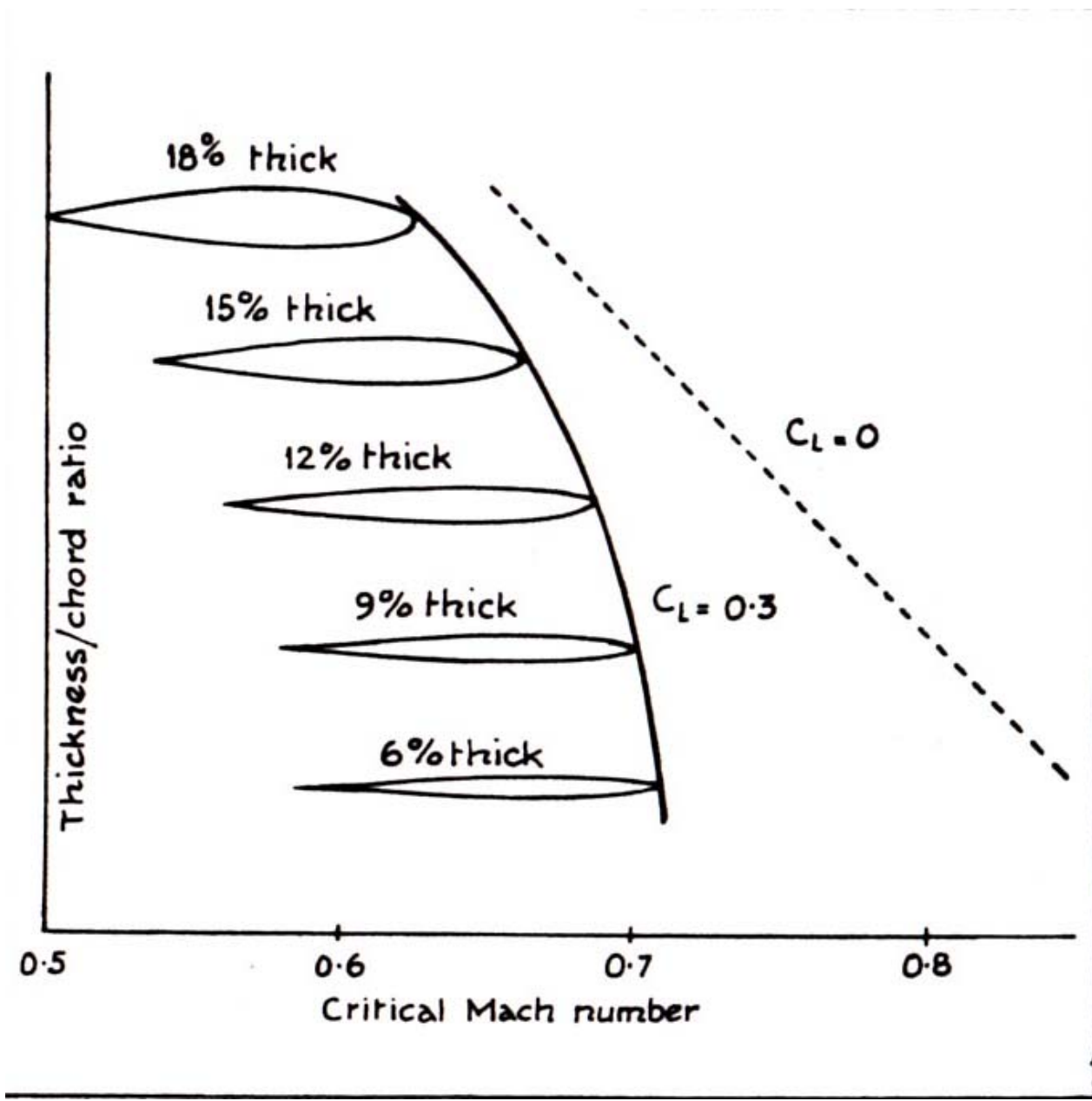


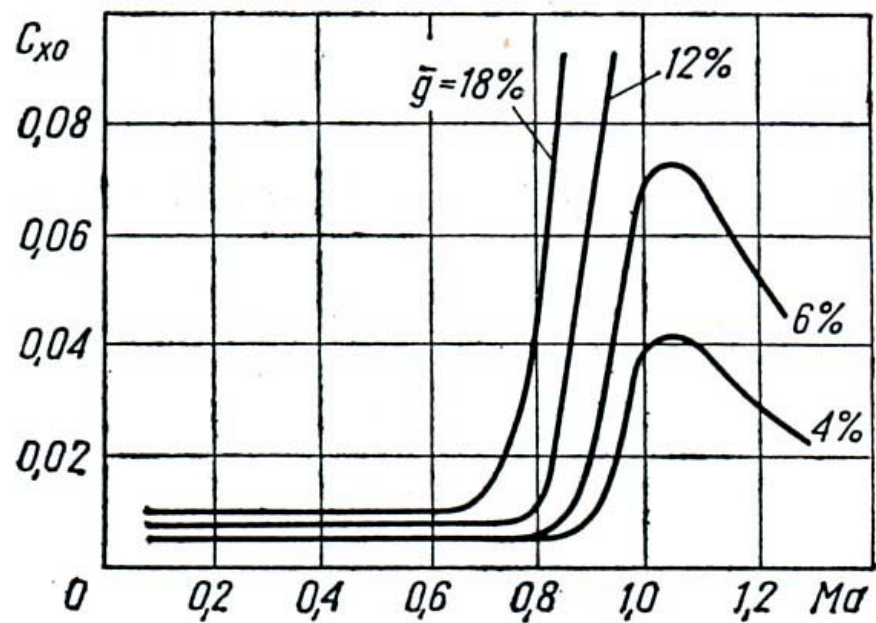
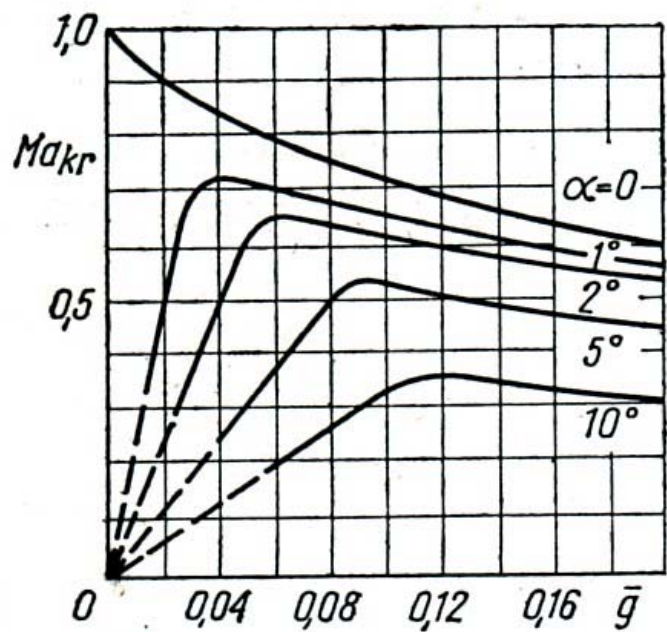
CRITICAL MACH NUMBER vs $C_{p_{min}}$



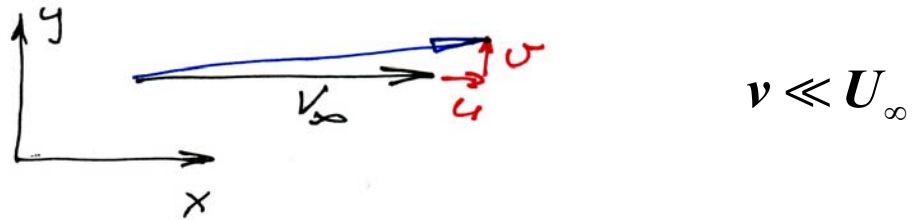
C_p^* vs Ma_∞







SUPERCritical FLOW OVER AN AIRFOIL



$$U = U_\infty + u = U_\infty + \frac{\partial \phi}{\partial x}$$

$$V = \quad v = \frac{\partial \phi}{\partial y}$$

Continuity equation:

$$\frac{\partial(\rho U)}{\partial x} + \frac{\partial(\rho V)}{\partial y} = 0 = (U_\infty + u) \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} + \gamma \frac{\partial \rho}{\partial y} + \rho \frac{\partial v}{\partial y} \simeq$$

$$U \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} = U \frac{\partial \rho}{\partial x} + \rho \operatorname{div} \vec{v} =$$

$$|S = \text{const}| = U \left. \frac{\partial \rho}{\partial p} \right|_L \frac{\partial p}{\partial x} + \rho \operatorname{div} \vec{v} =$$

$$\left. \begin{aligned} r. \text{Eulera } (x): \quad \rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial U}{\partial y} &= - \frac{\partial p}{\partial x} \\ \rho (U_\infty + u) \frac{\partial u}{\partial x} + \rho \gamma \frac{\partial u}{\partial y} &= - \frac{\partial p}{\partial x} \quad \rightarrow \quad \frac{\partial p}{\partial x} \simeq - \rho U \frac{\partial u}{\partial x} \end{aligned} \right|$$

$$= -\rho \frac{U^2}{a^2} \frac{\partial u}{\partial x} + \rho \operatorname{div} \vec{v} = \rho \left((1 - Ma_L^2) \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0$$

$$\left(1 - Ma_L^2\right) \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

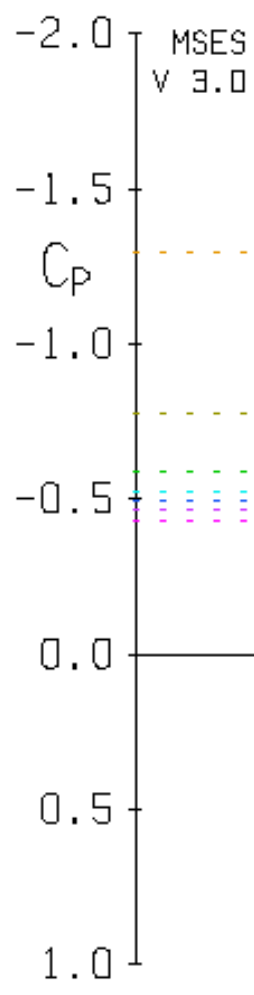
$$\left(1 - Ma_L^2\right) \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \left(1 - Ma_L^2\right) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$M_L < 1$$

$$M_L = 1$$

$$M_L > 1$$

$$\text{div} \vec{V} = q = Ma_L^2 \frac{\partial u}{\partial x}$$



NACA 0010

Alfa = 0.0000

<u>Mach</u>	<u>Alfa</u>	<u>CD</u>	<u>CM</u>
0.400	0.000	0.00000	-0.000
0.600	-0.000	0.00000	-0.000
0.700	-0.000	0.00000	-0.000
0.750	-0.000	0.00000	-0.000
0.770	-0.000	-0.00000	-0.000
0.780	0.000	0.00005	-0.000
0.790	0.000	0.00039	-0.000
0.800	-0.000	0.00129	-0.000

MSES
V 3.0

1.6

1.4

1.2

1.0

0.8

0.6

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0.0

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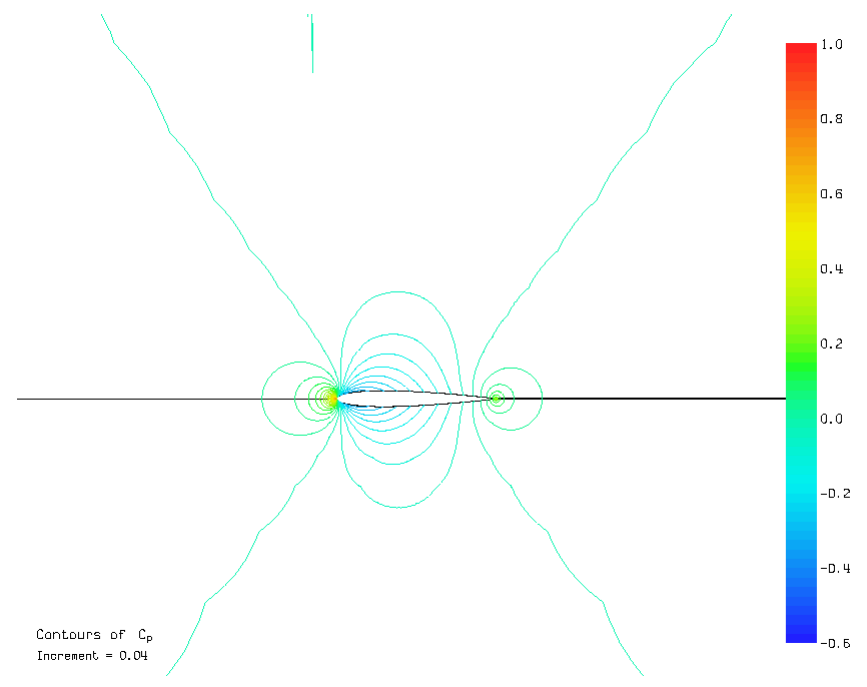
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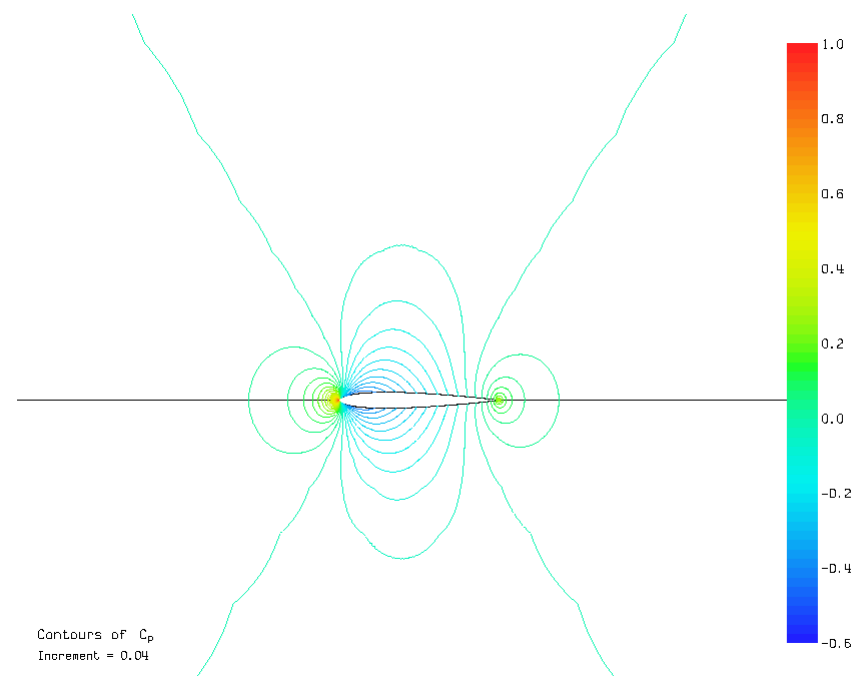
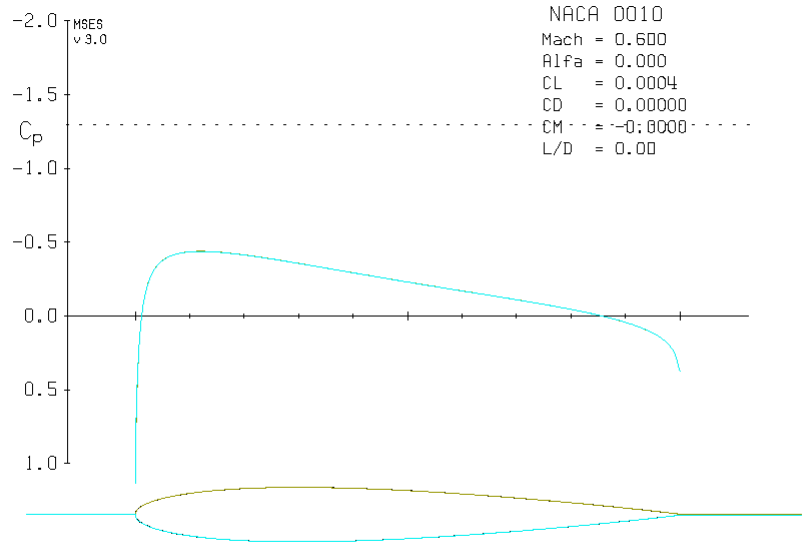
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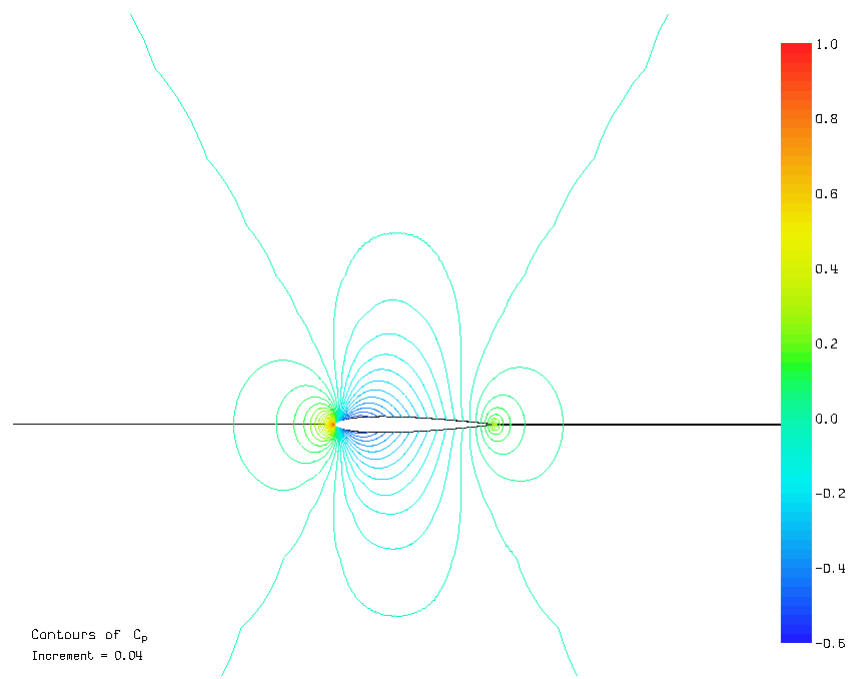
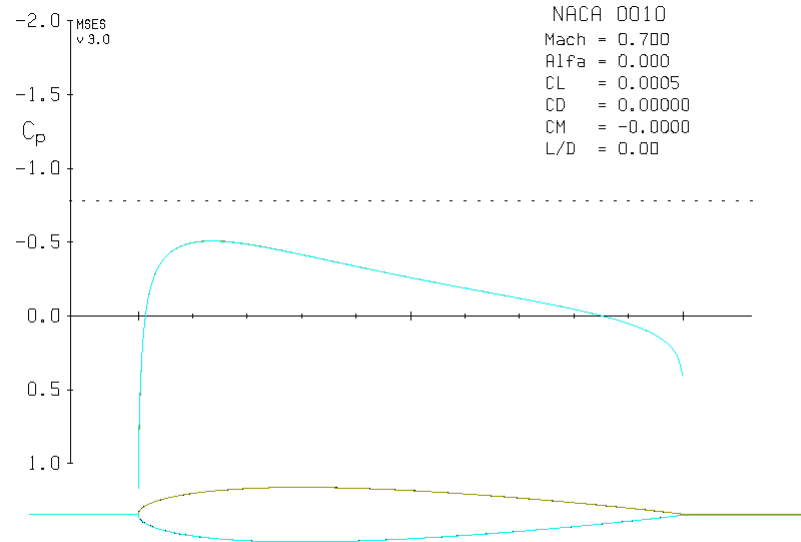


NACA 0010
Mach = 0.100
 $\alpha = 0.000$
CL = 0.0015
CD = 0.00000
CM = 0.0003
L/D = 0.00



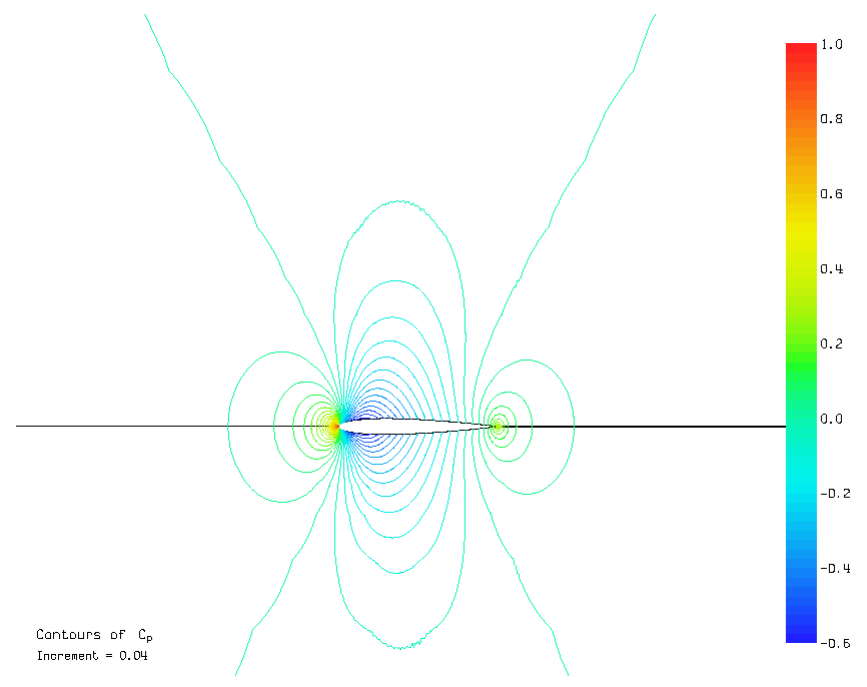
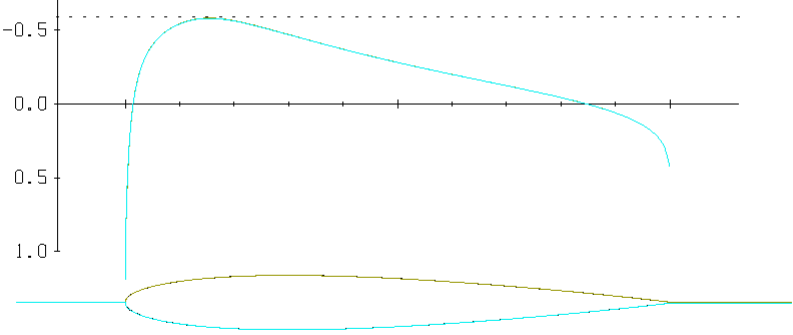
Contours of C_p
Increment = 0.04

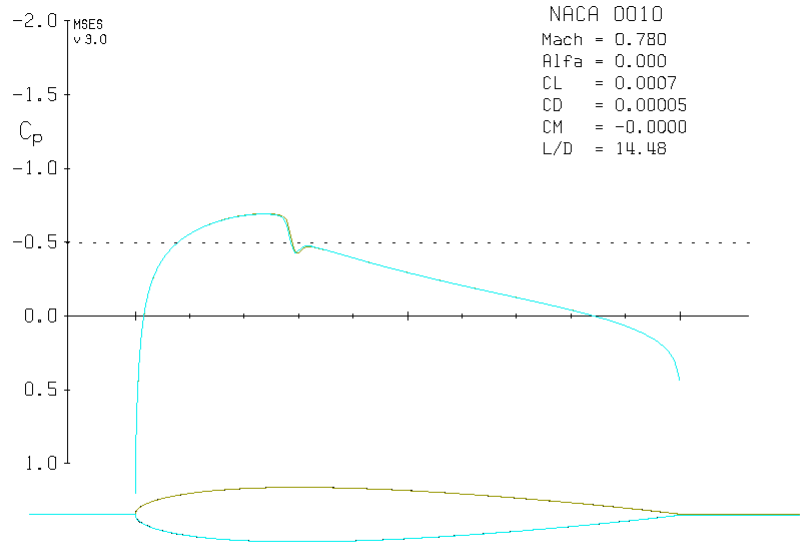




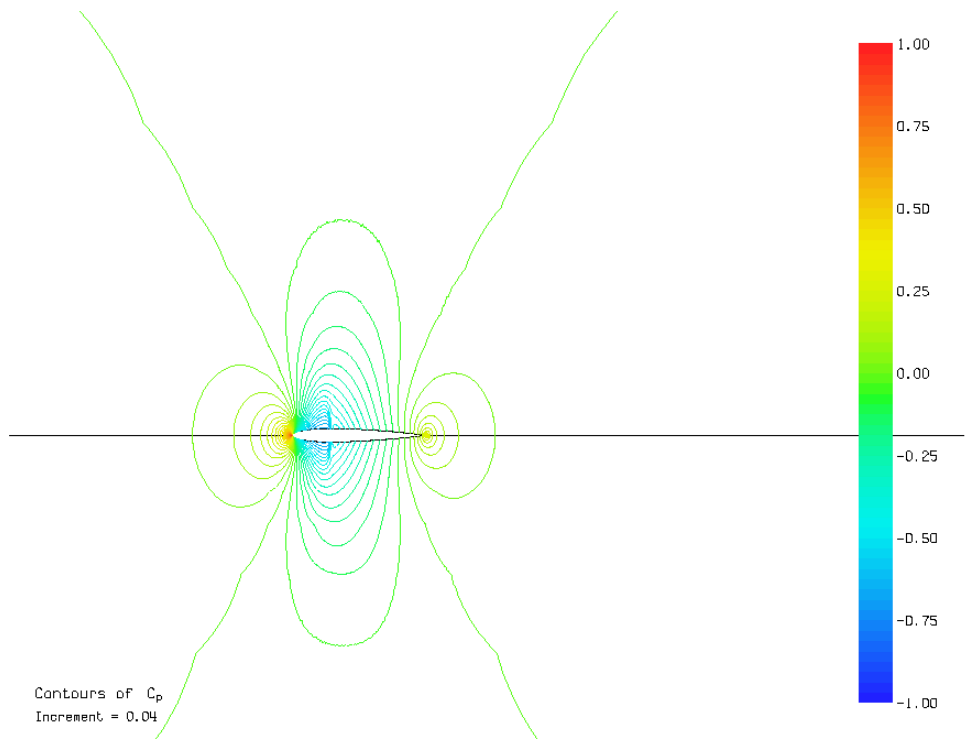
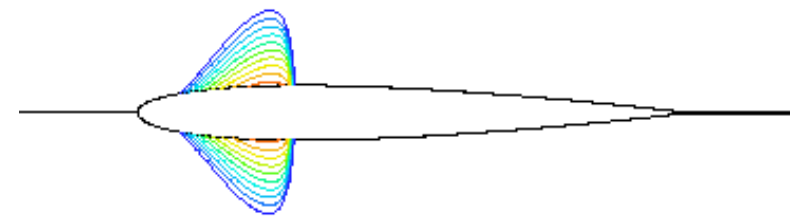
MSES
v 3.0
-2.0
-1.5
-1.0
 C_p
-0.5
0.0
0.5
1.0

NACA 0010
Mach = 0.750
 $\alpha = -0.000$
CL = 0.0007
CD = 0.00000
CM = -0.0000
L/D = 0.00





NACA 0010
 Mach = 0.780
 Alpha = 0.000
 CL = 0.0007
 CD = 0.00005
 CM = -0.0000
 L/D = 14.48

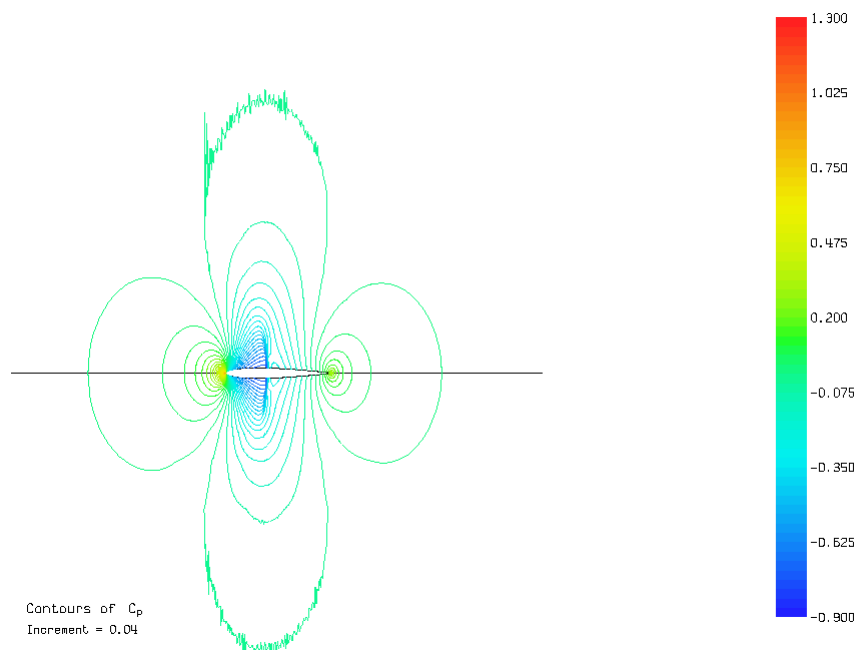
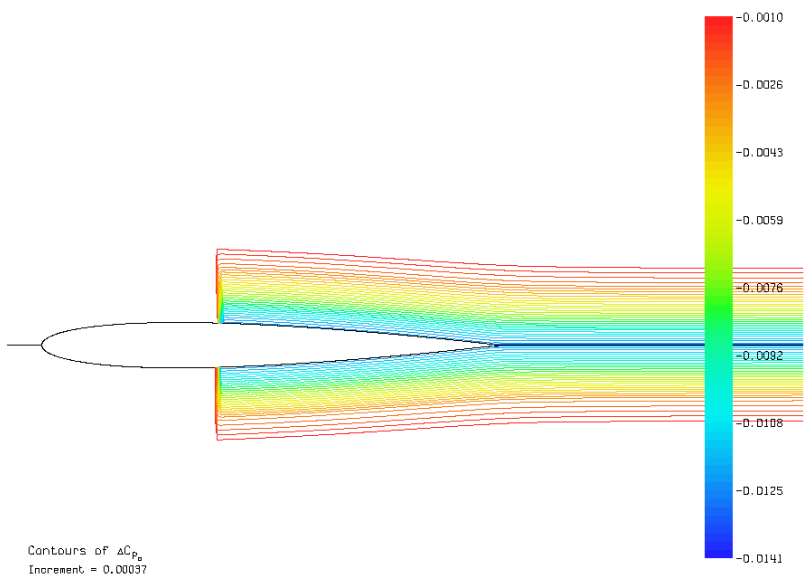
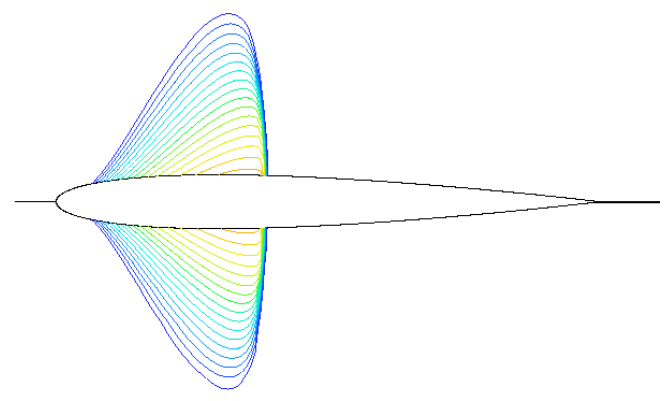
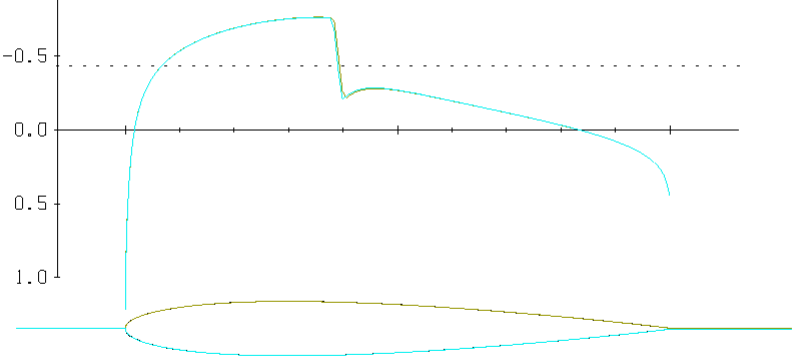


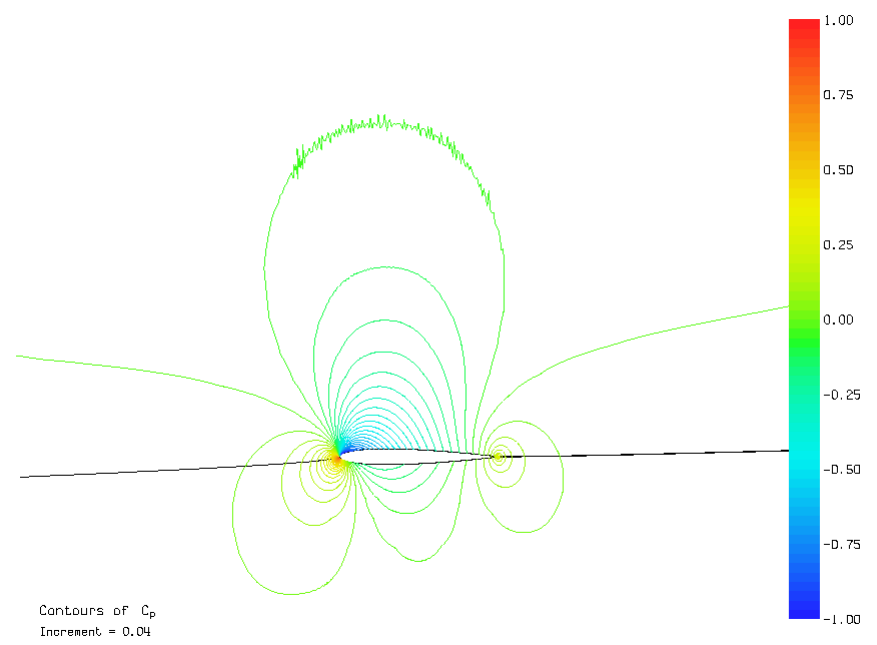
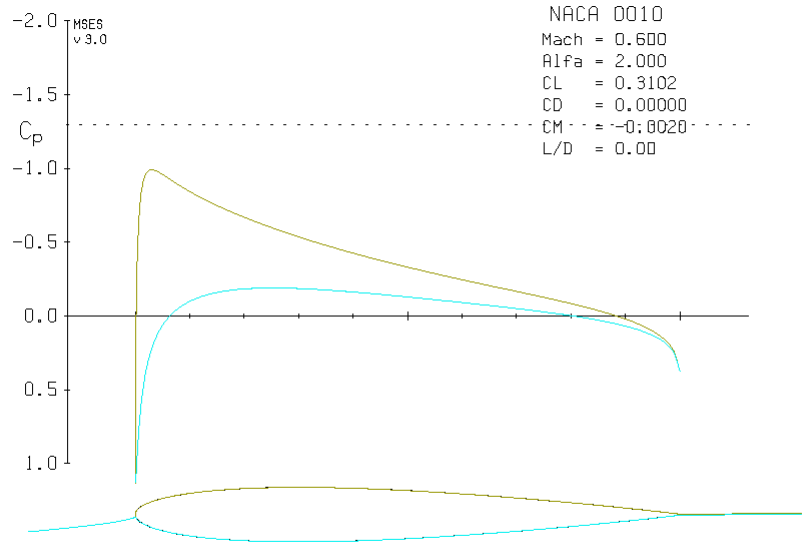
MSES
v3.0

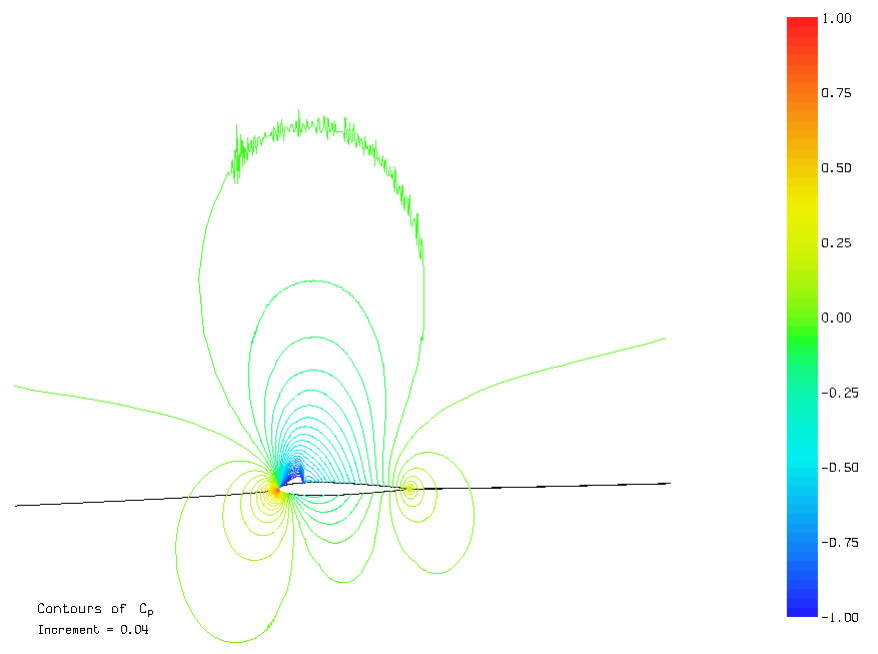
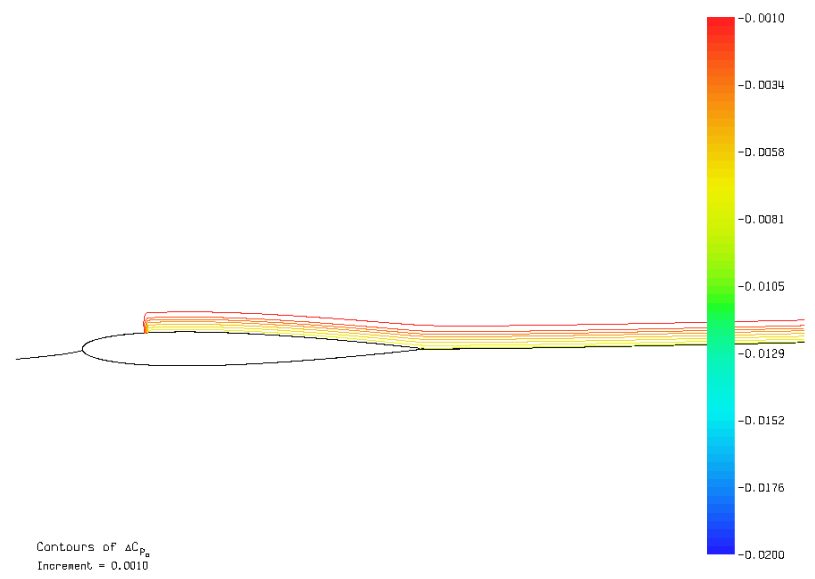
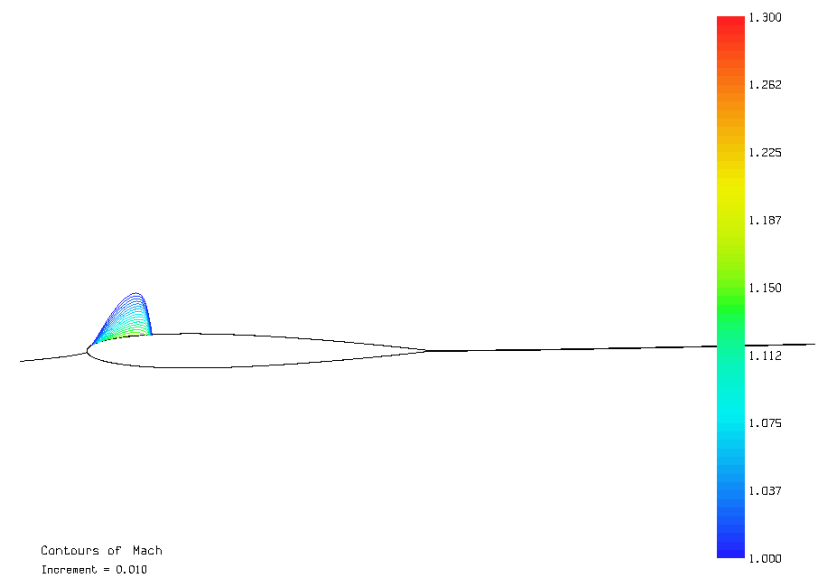
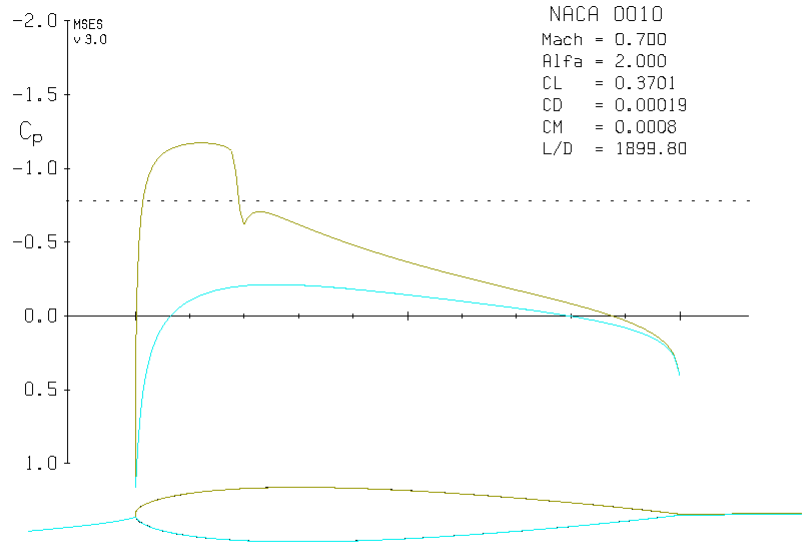
C_p

-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0

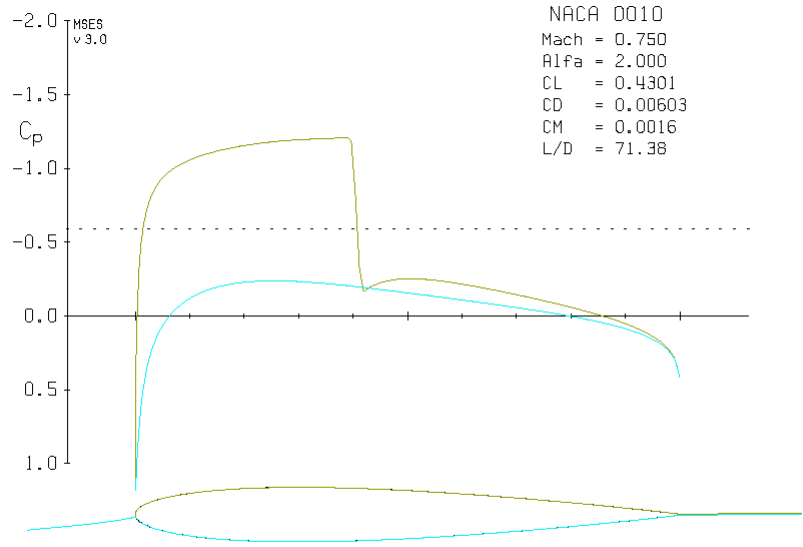
NACA 0010
 Mach = 0.800
 α = 0.000
 CL = 0.0012
 CD = 0.00127
 CM = -0.0000
 L/D = 0.97



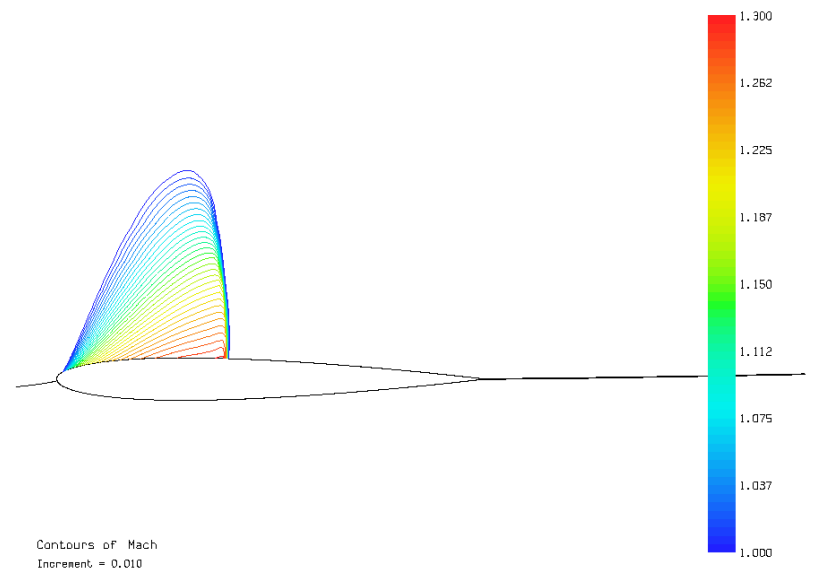




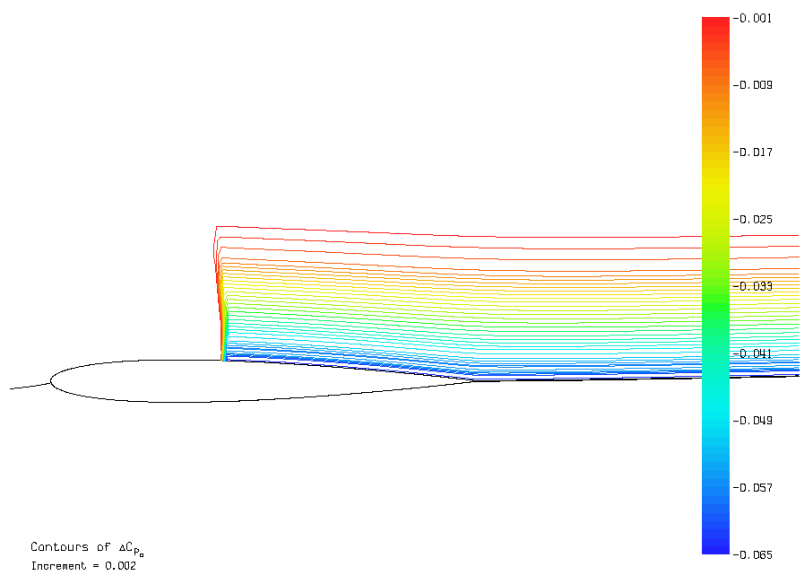
MSES
v 3.0



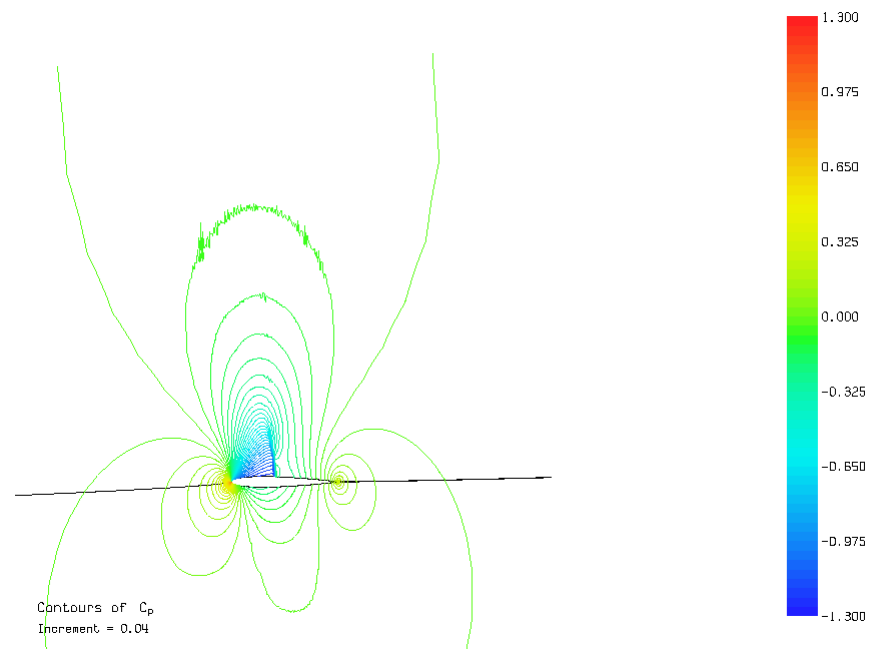
NACA 0010
Mach = 0.750
 $\alpha = 2.000$
CL = 0.4301
CD = 0.00603
CM = 0.0016
L/D = 71.38



Contours of Mach
Increment = 0.010

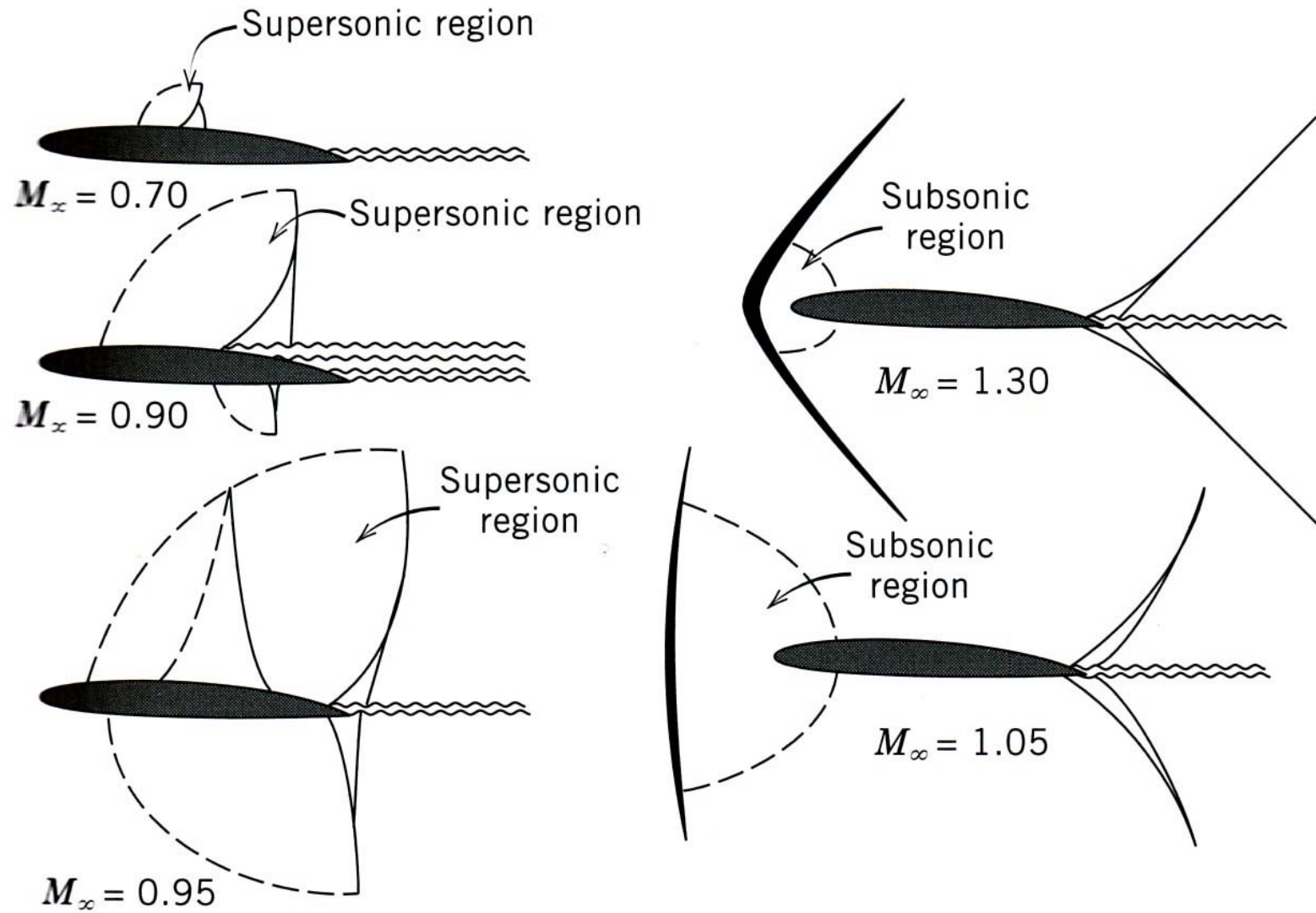


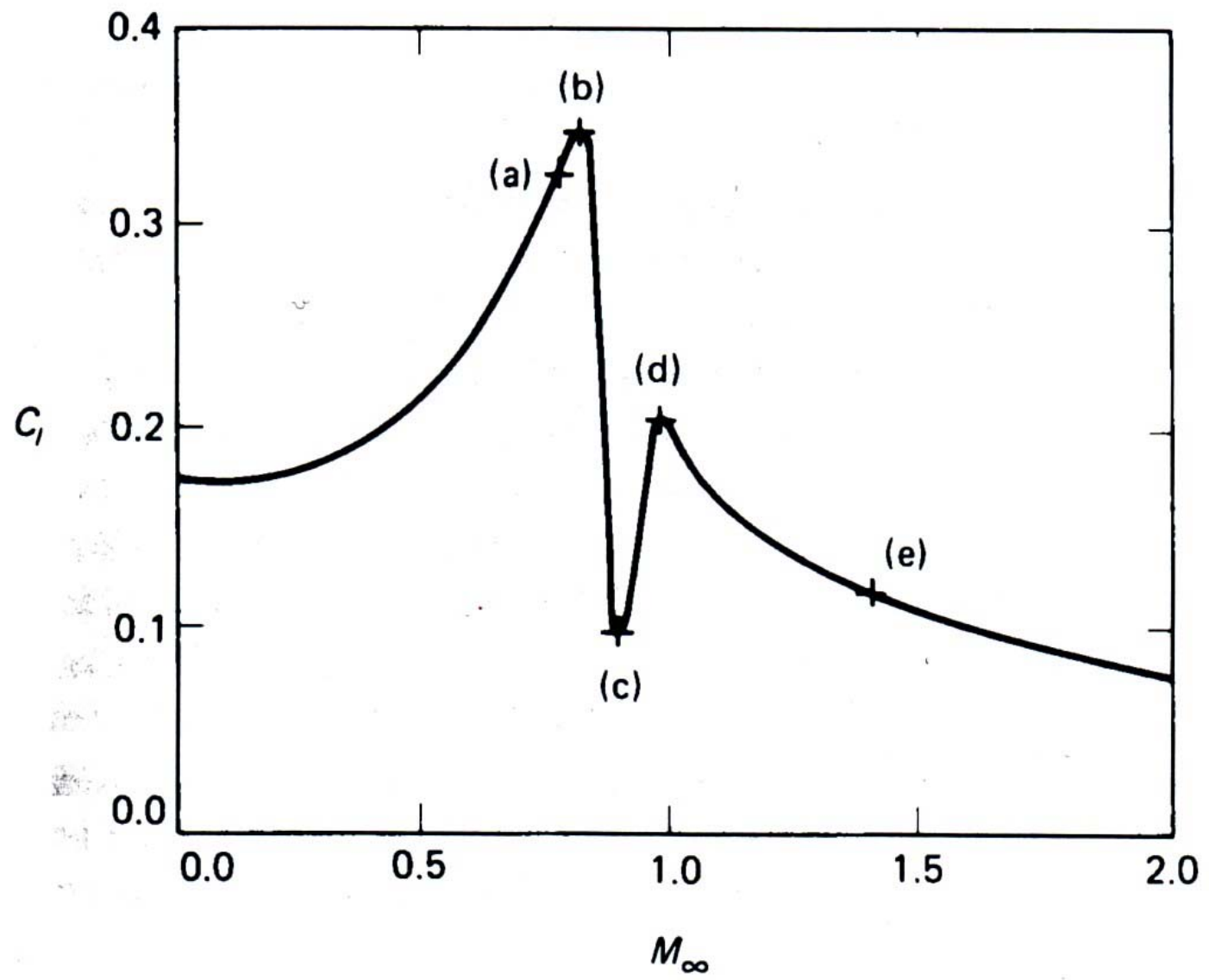
Contours of ΔC_p
Increment = 0.002

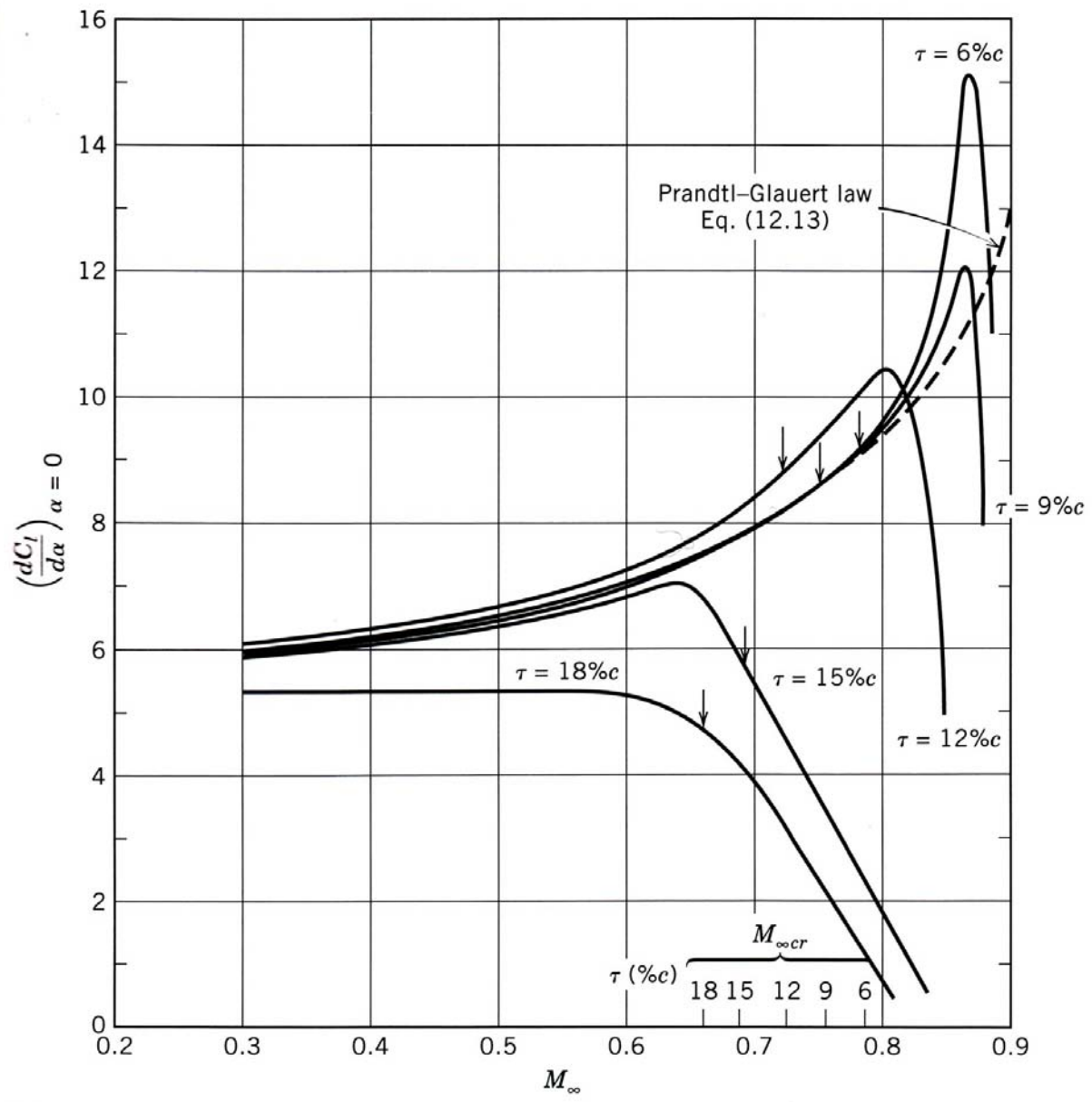


Contours of C_p
Increment = 0.04

REAL FLOW OVER AN AIRFOIL (VISCOUS FLUID)







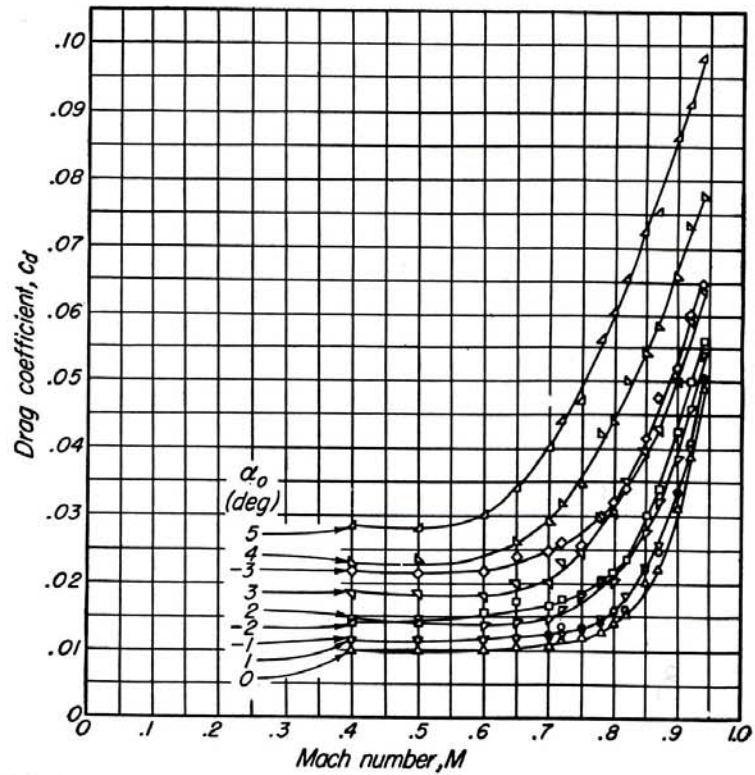


Fig. 176. Effect of compressibility on the drag of the NACA 2306 airfoil.

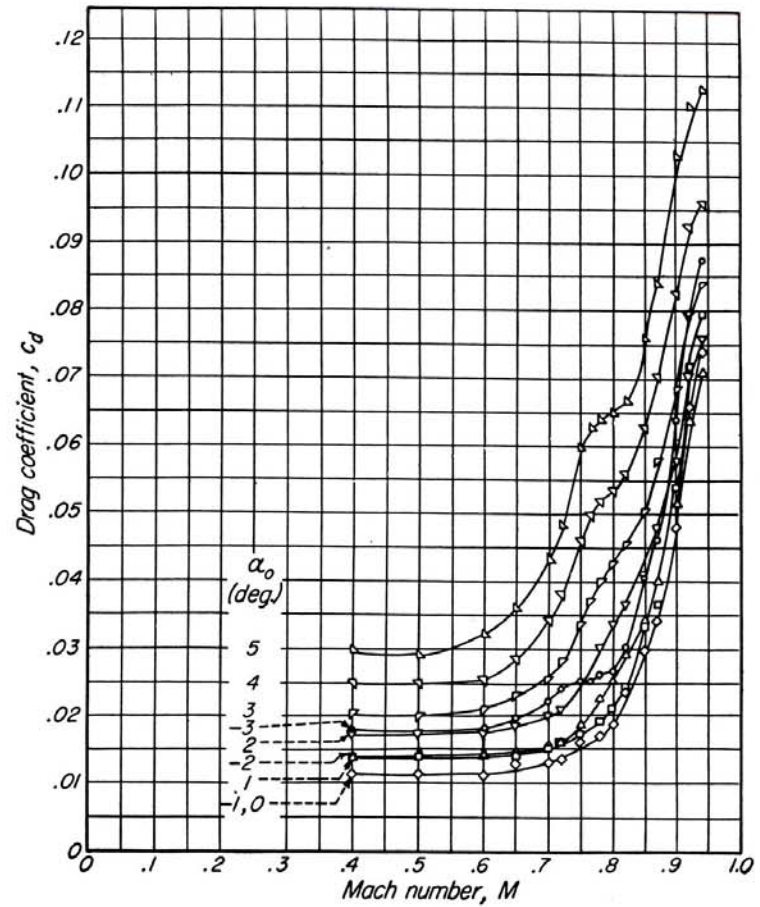
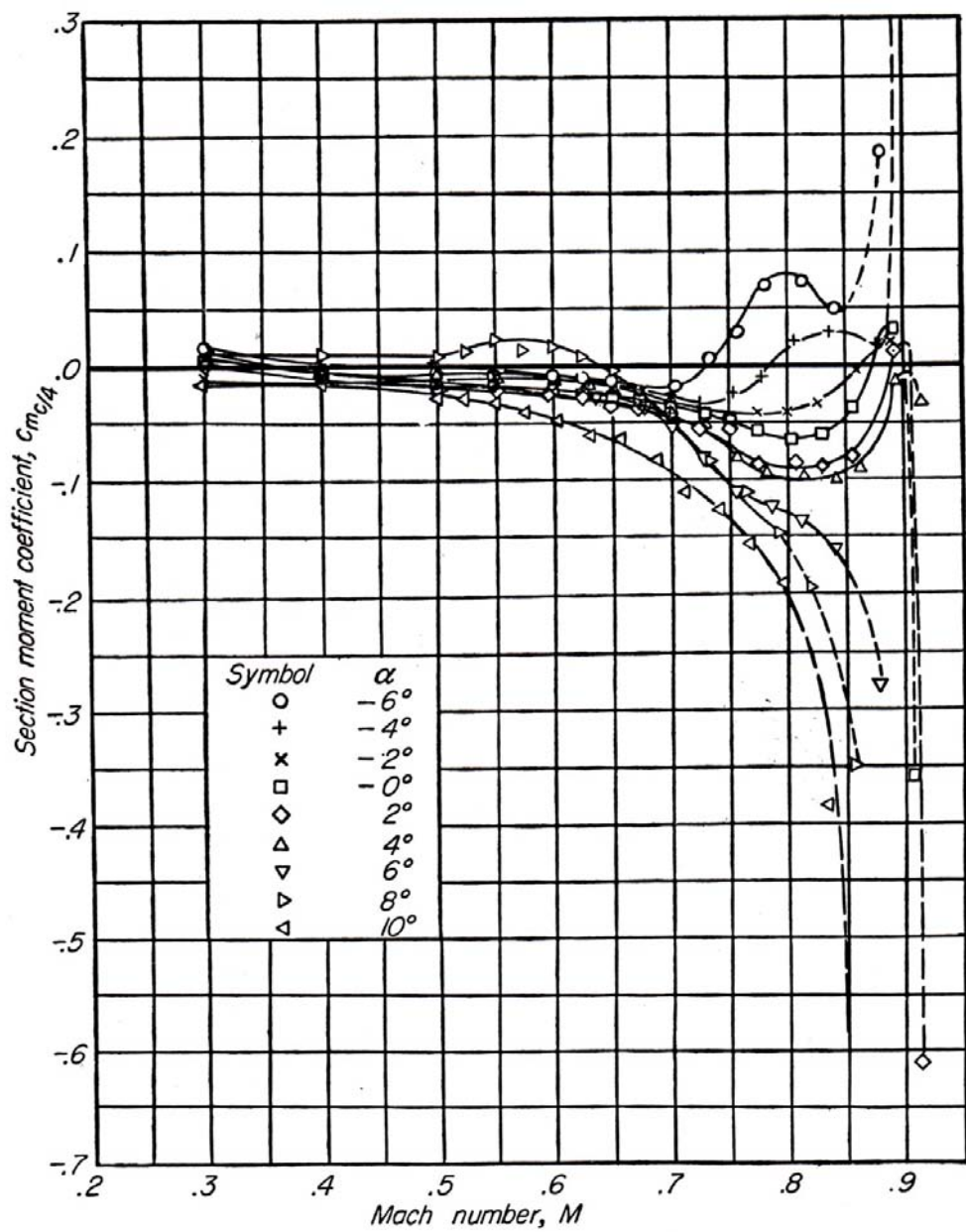
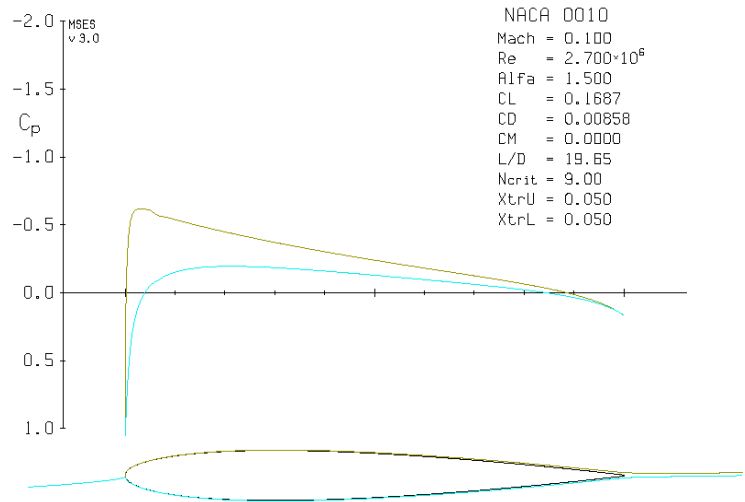
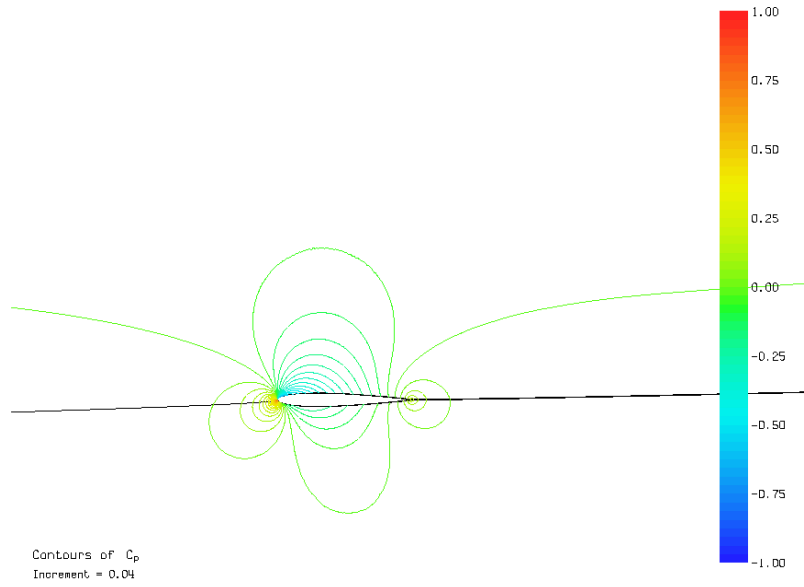


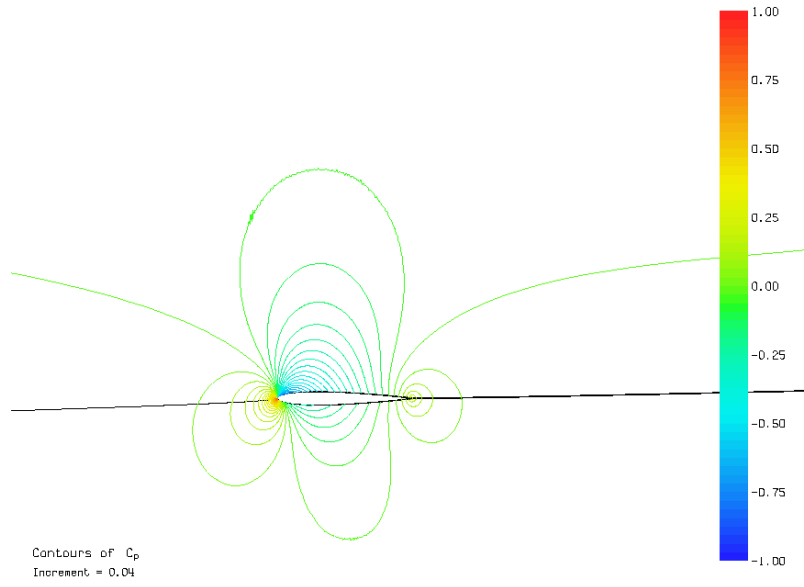
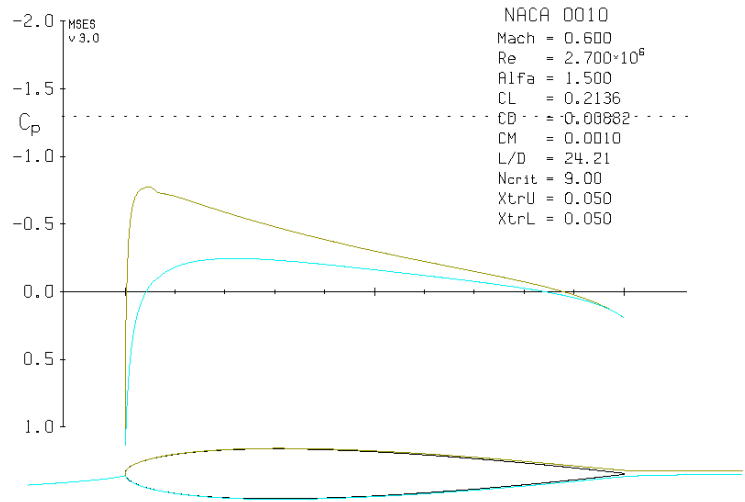
Fig. 177. Effect of compressibility on the drag of the NACA 2309 airfoil.

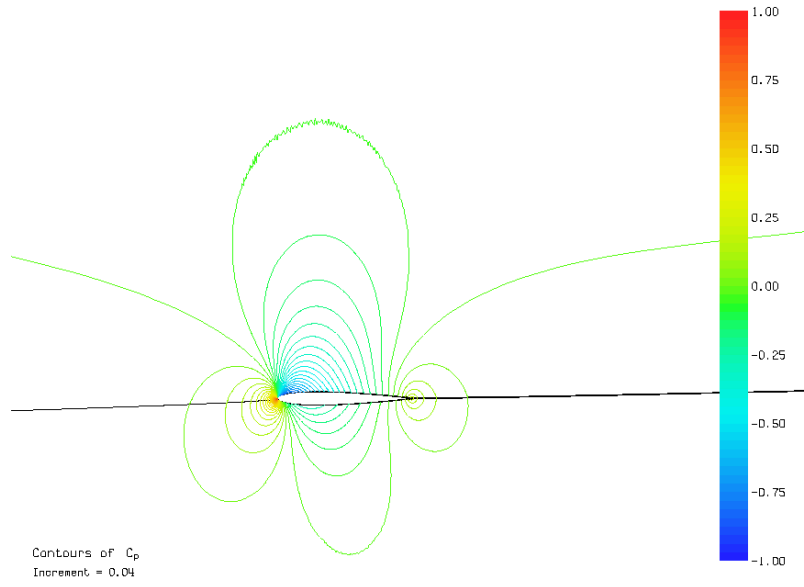
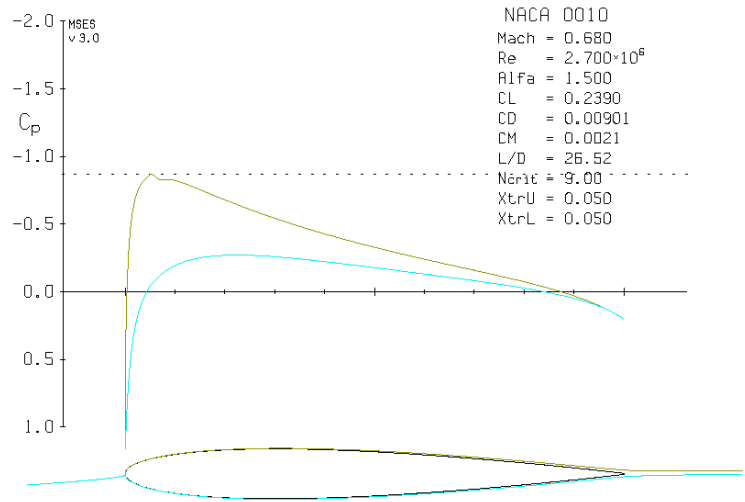


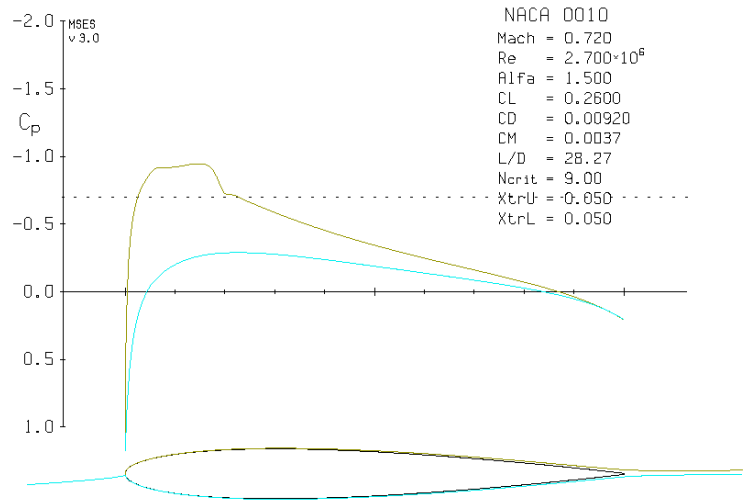


NACA 0010
 Mach = 0.100
 Re = $2.700 \cdot 10^6$
 Alfa = 1.500
 CL = 0.1687
 CD = 0.00858
 CM = 0.0000
 L/D = 19.65
 Ncrit = 9.00
 XtrU = 0.050
 XtrL = 0.050

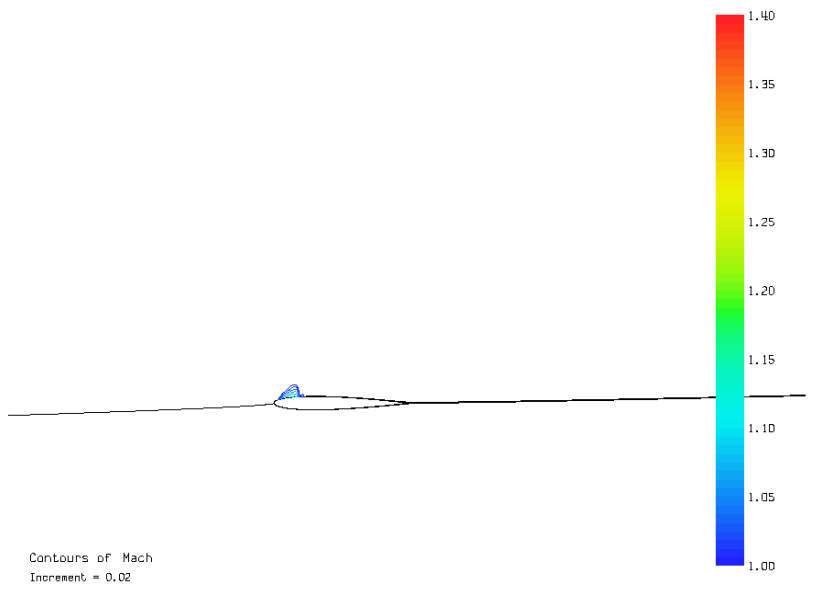
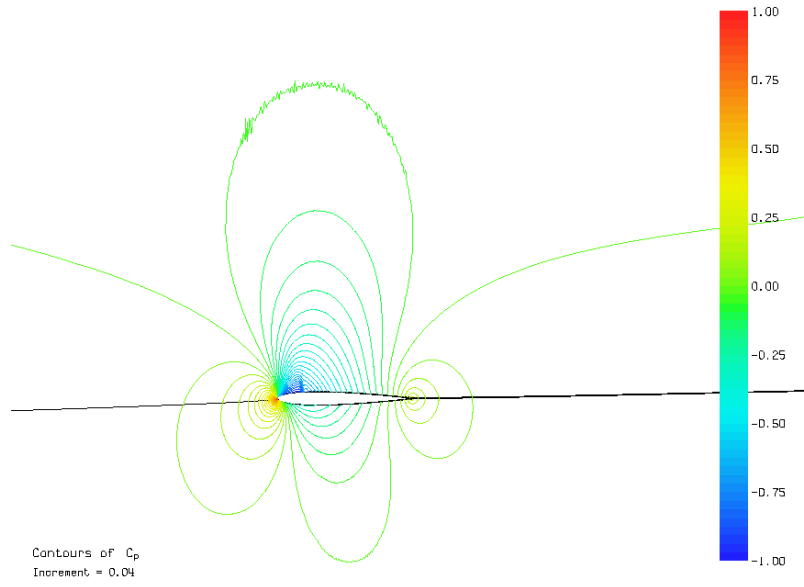


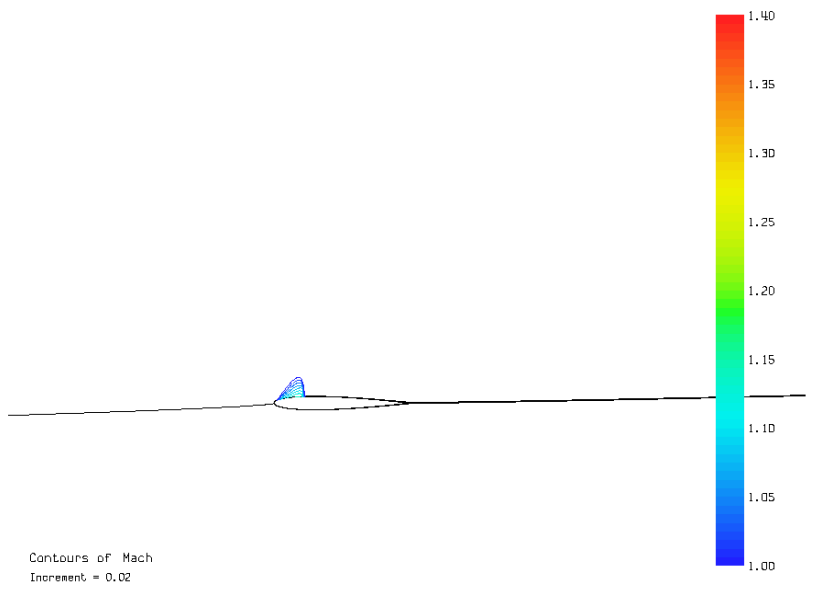
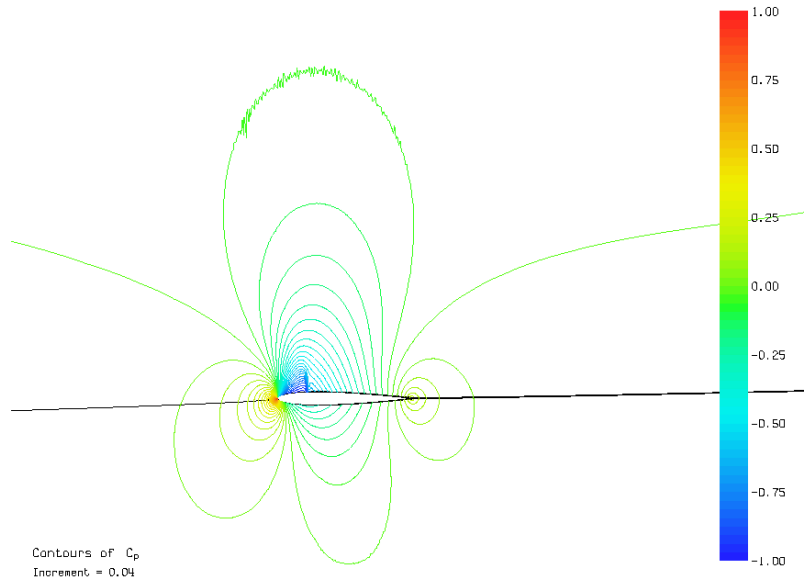
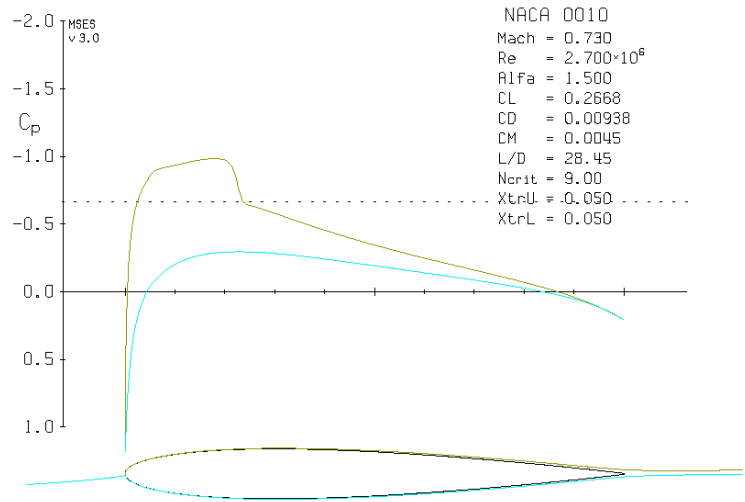


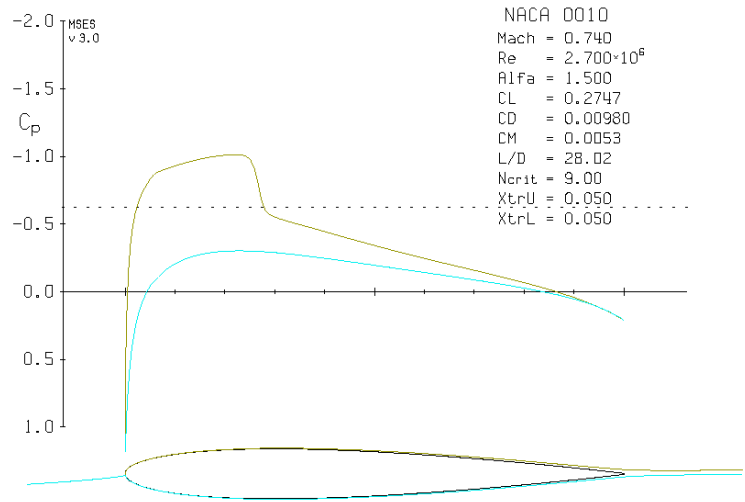




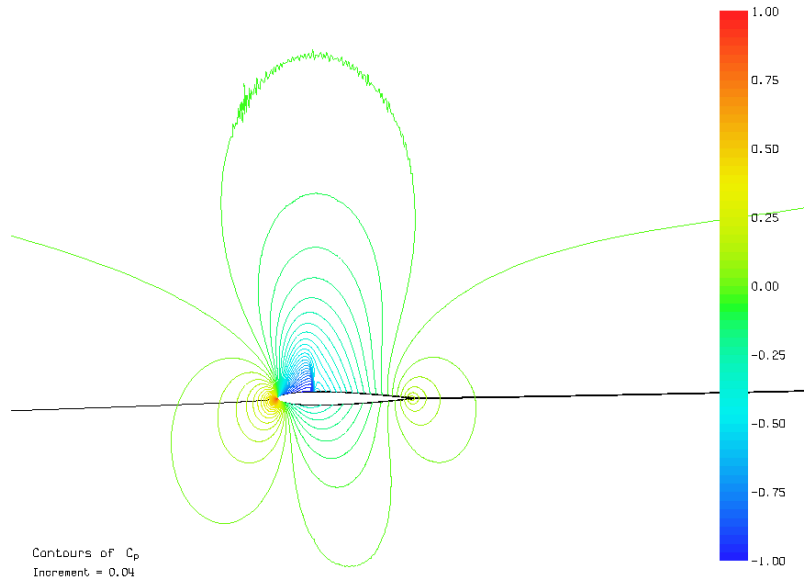
NACA 0010
Mach = 0.720
Re = 2.700×10^6
Alfa = 1.500
CL = 0.2600
CD = 0.00920
CM = 0.0037
L/D = 28.27
Ncrit = 9.00
XtrU = 0.650
XtrL = 0.050



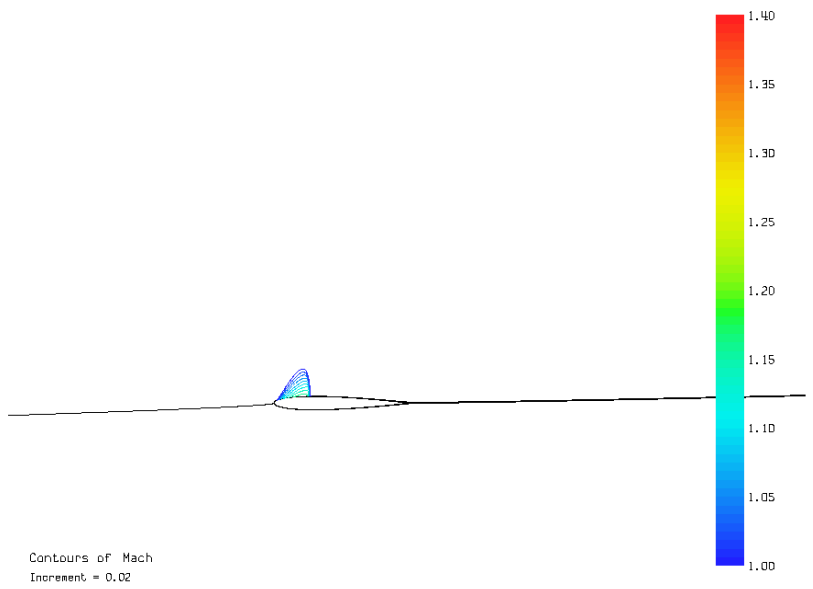




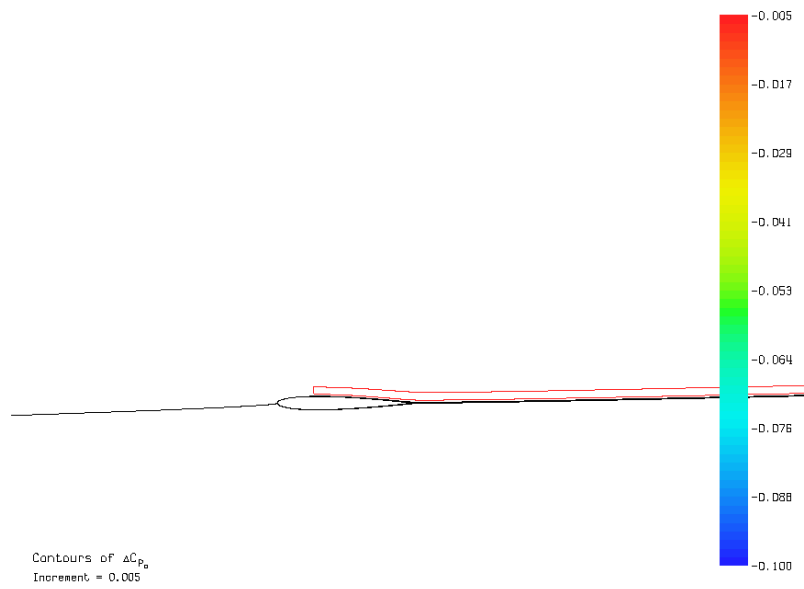
NACA 0010
 Mach = 0.740
 Re = 2.700×10^6
 Alfa = 1.500
 CL = 0.2747
 CD = 0.00980
 CM = 0.0053
 L/D = 28.02
 Ncrit = 9.00
 XtrU = 0.050
 XtrL = 0.050



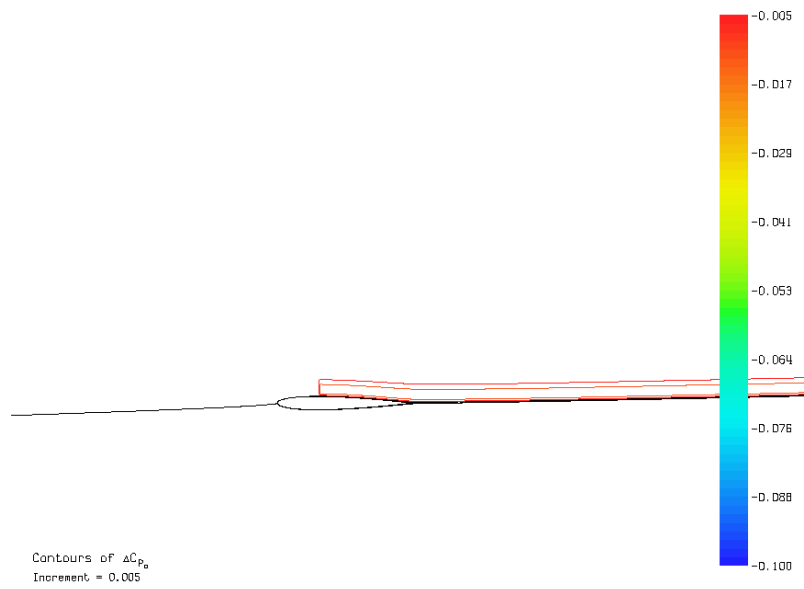
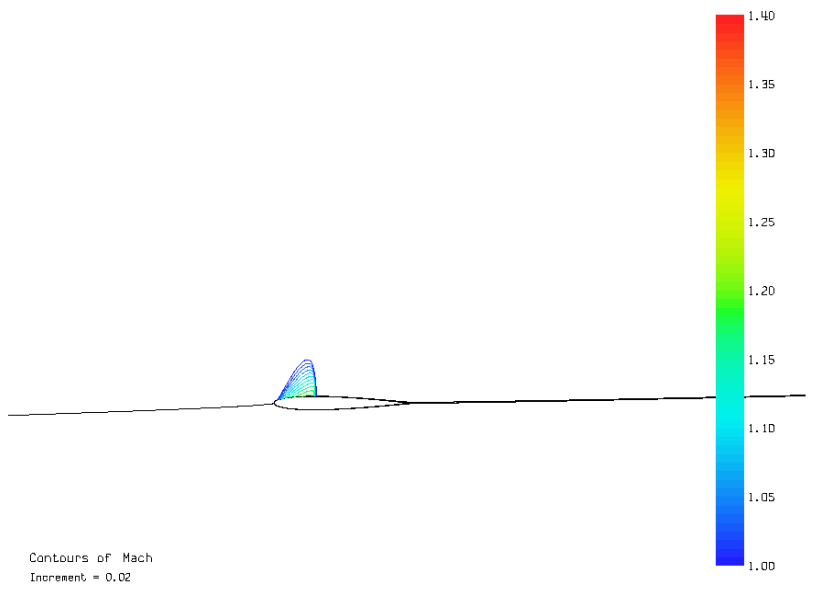
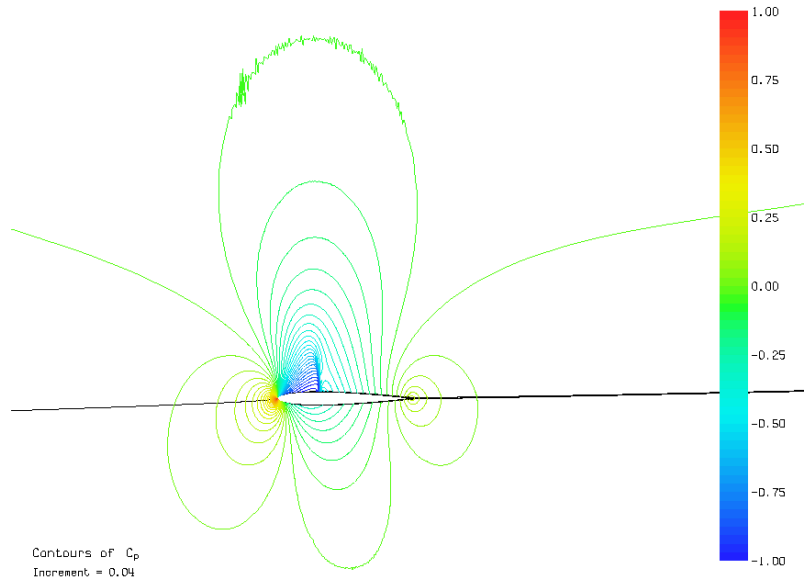
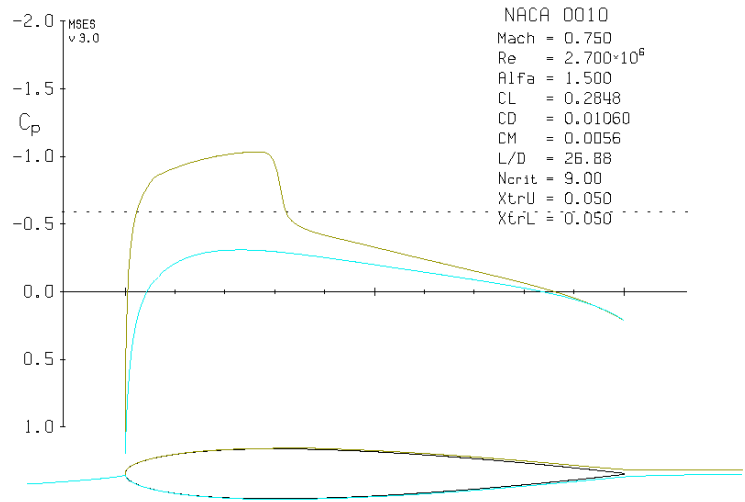
Contours of C_p
 Increment = 0.04

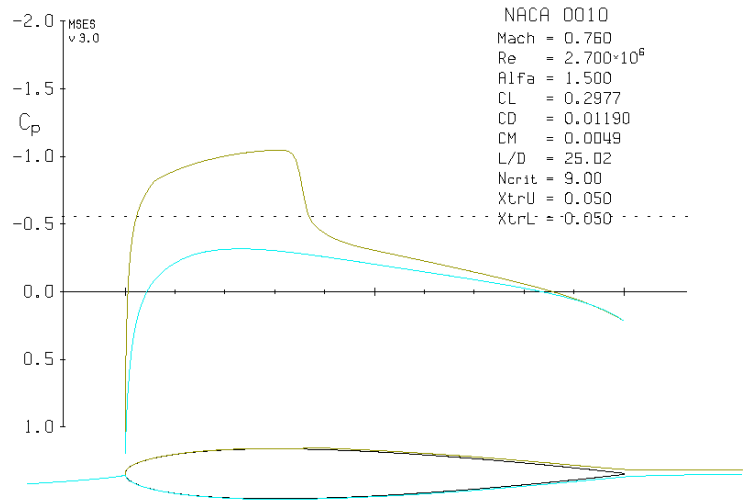


Contours of Mach
 Increment = 0.02

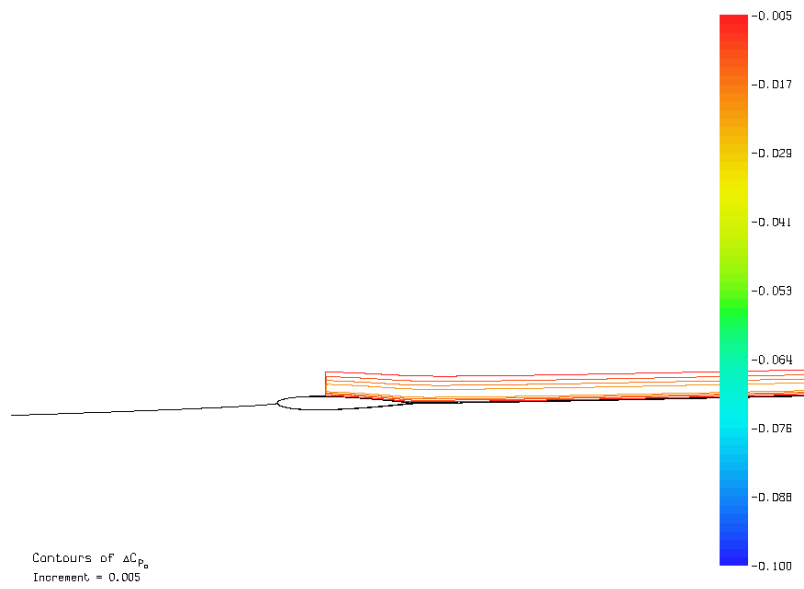
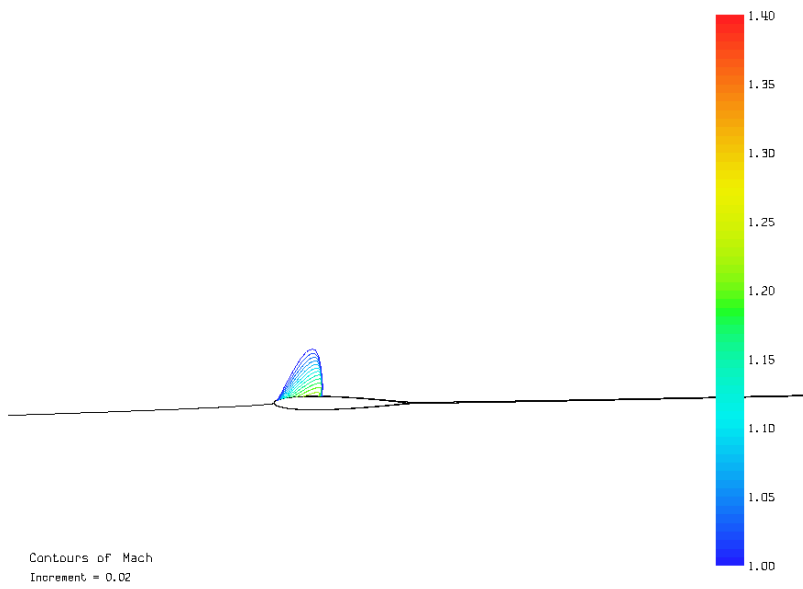
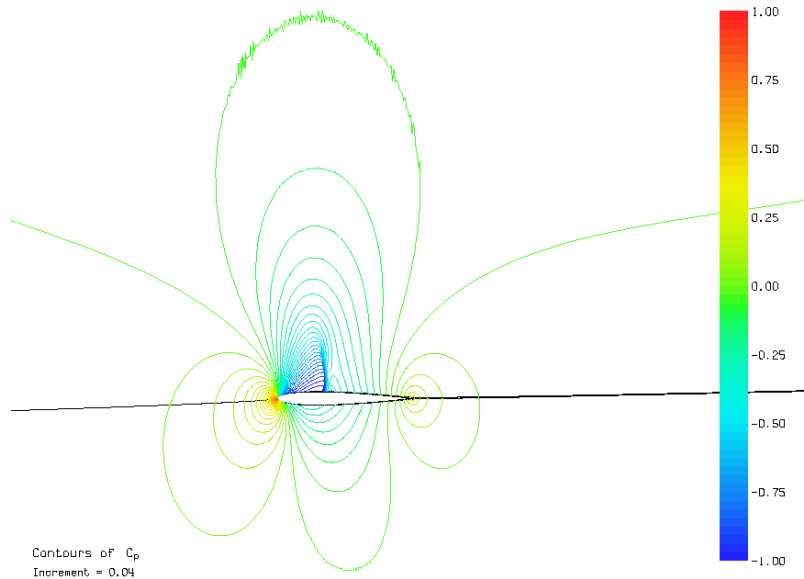


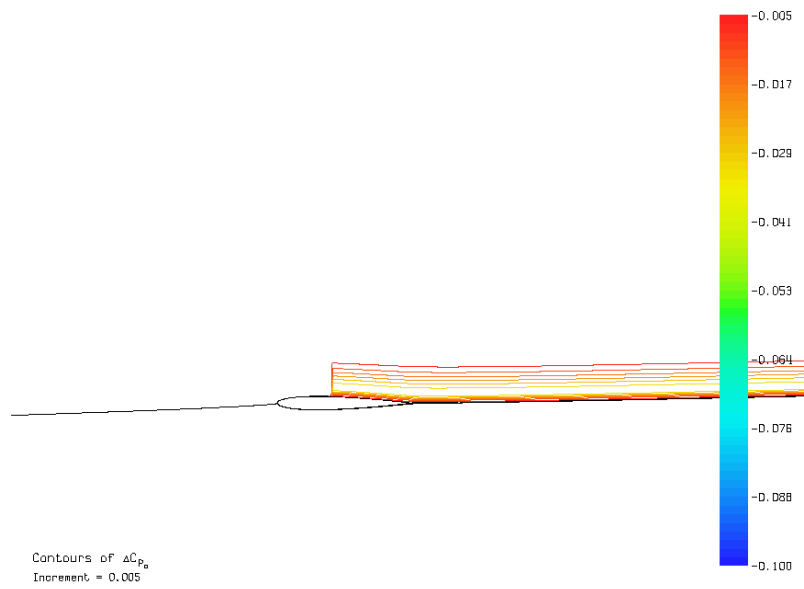
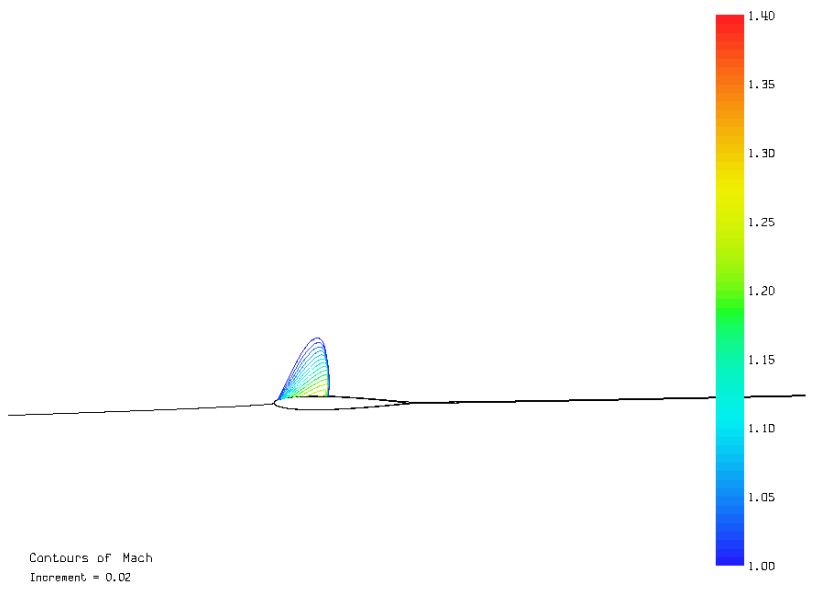
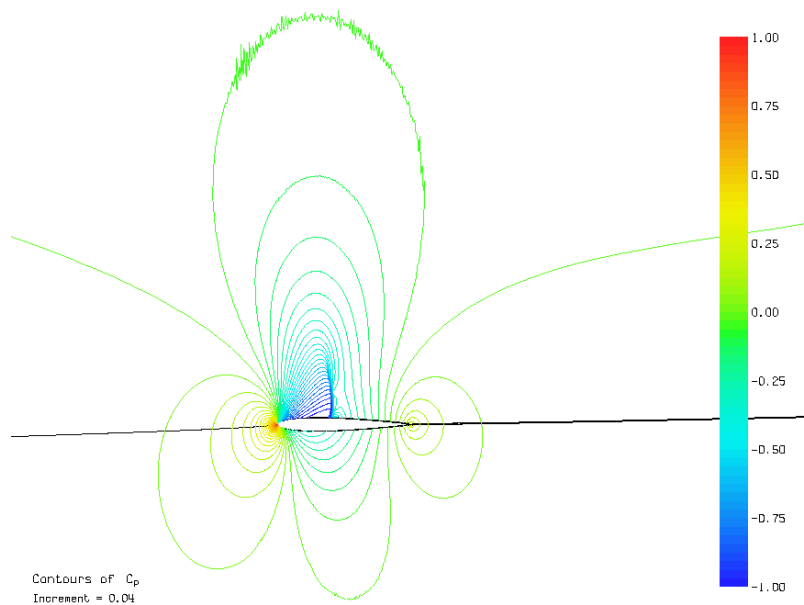
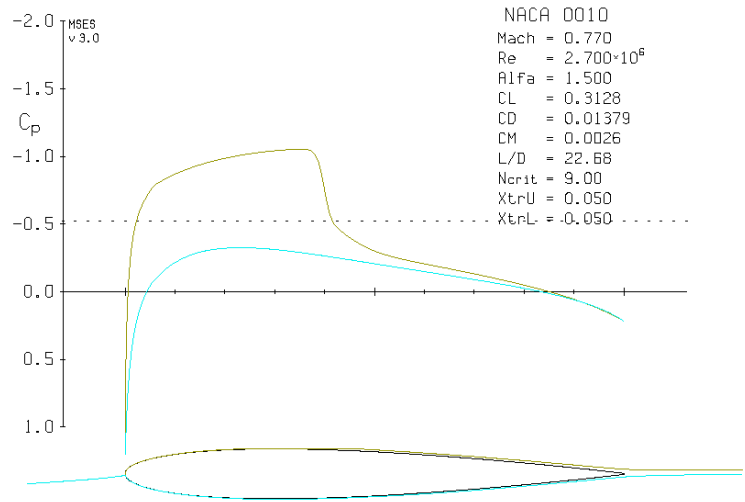
Contours of ΔC_p
 Increment = 0.005

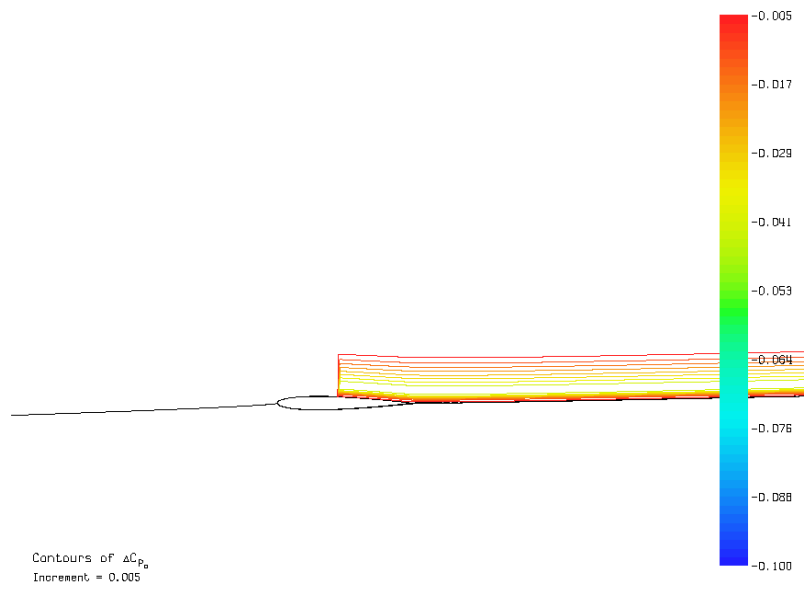
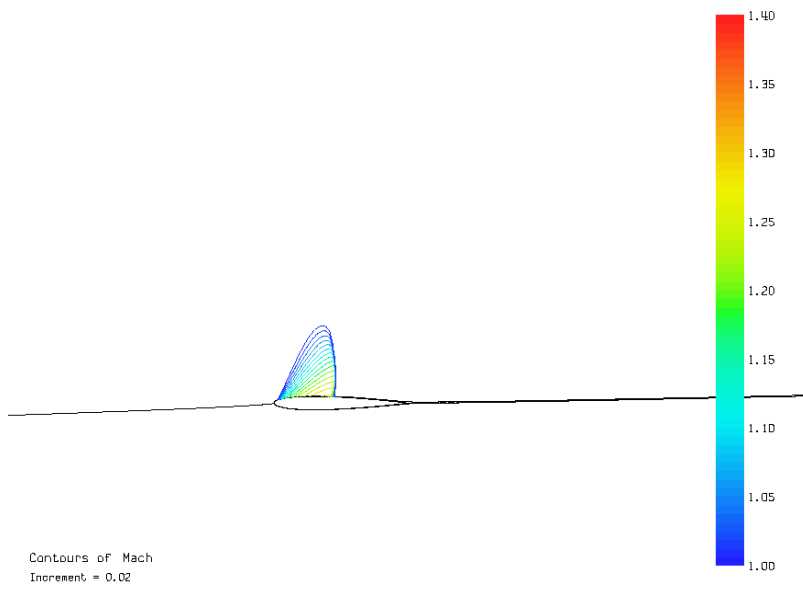
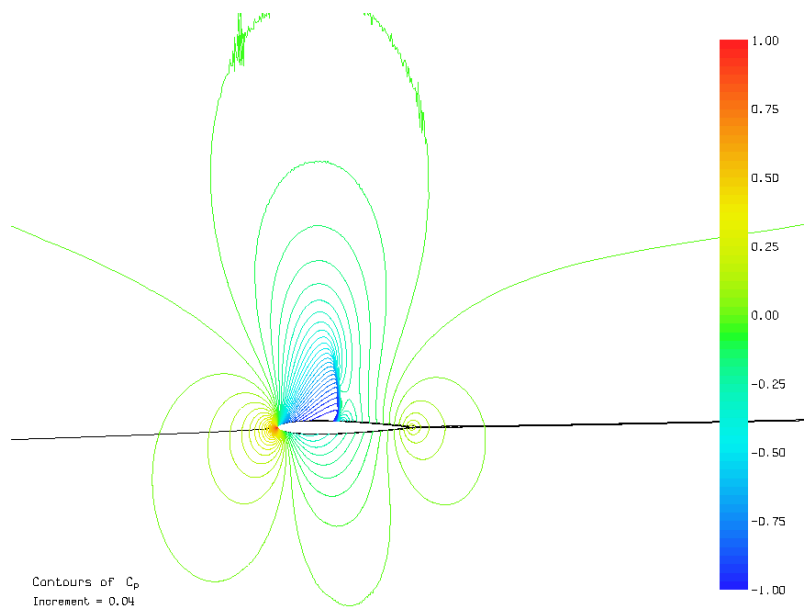
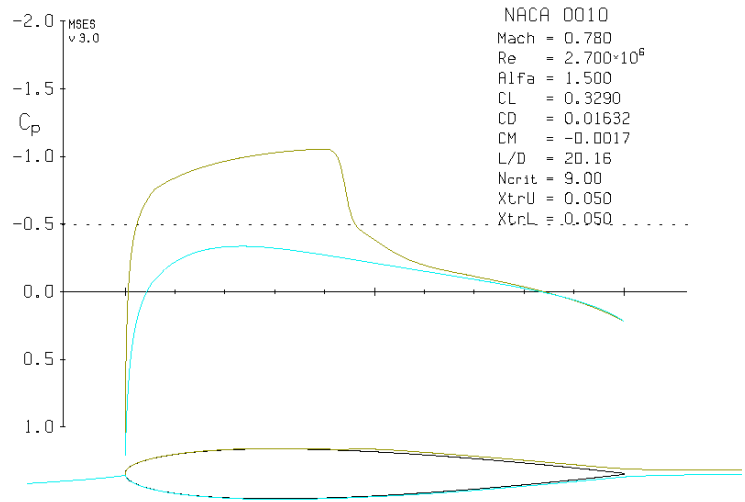


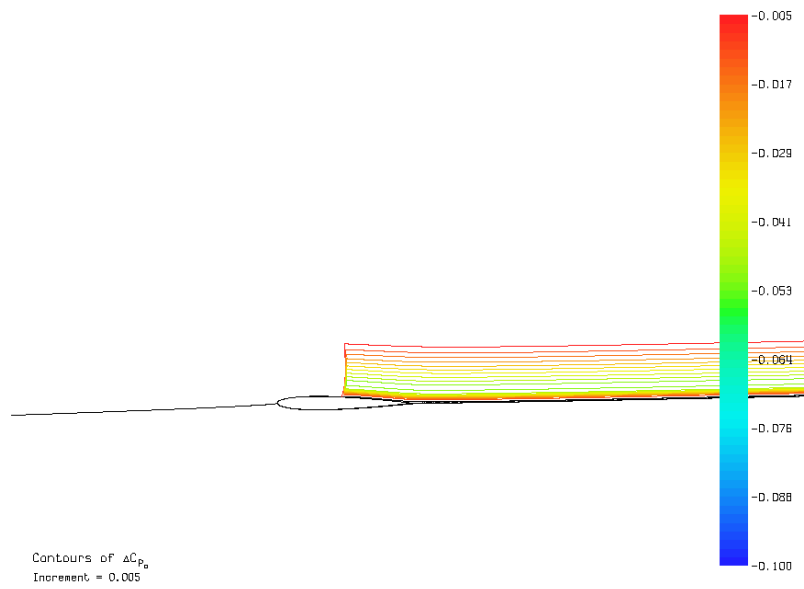
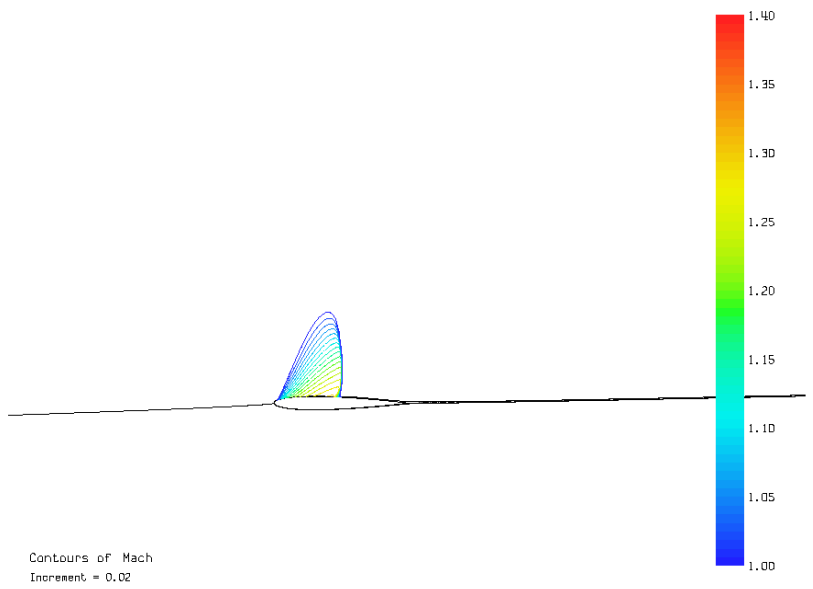
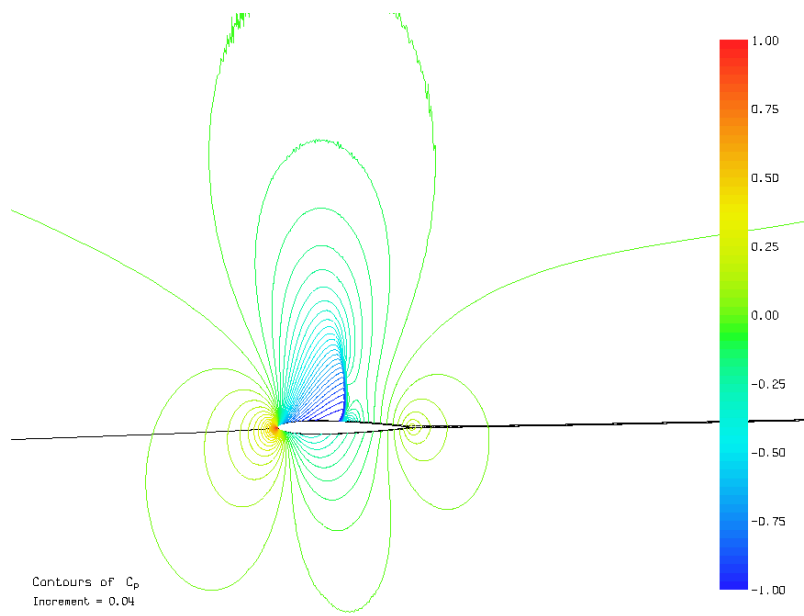
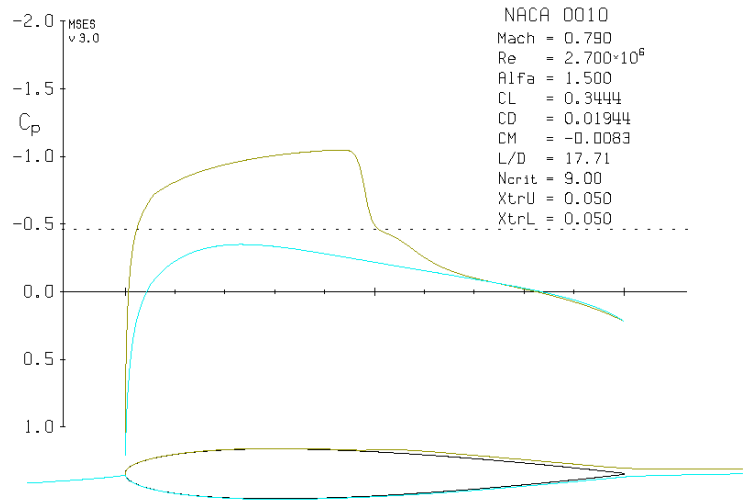


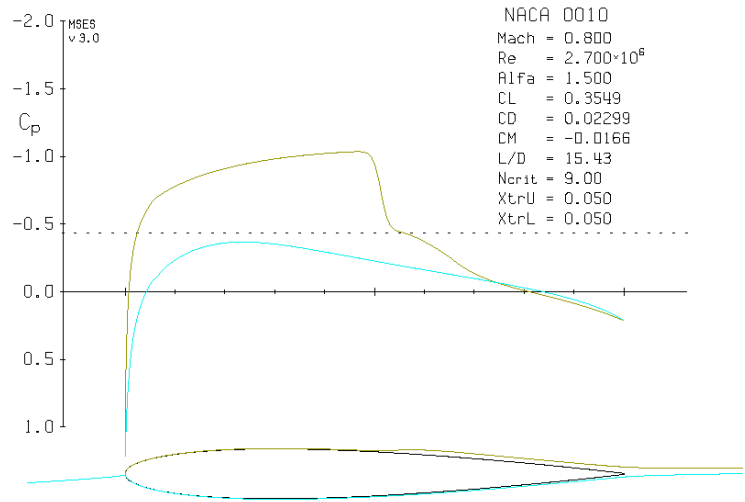
NACA 0010
 Mach = 0.760
 Re = 2.700×10^6
 Alfa = 1.500
 CL = 0.2977
 CD = 0.01190
 CM = 0.0049
 L/D = 25.02
 Ncrit = 9.00
 XtrU = 0.050
 XtrL = 0.050



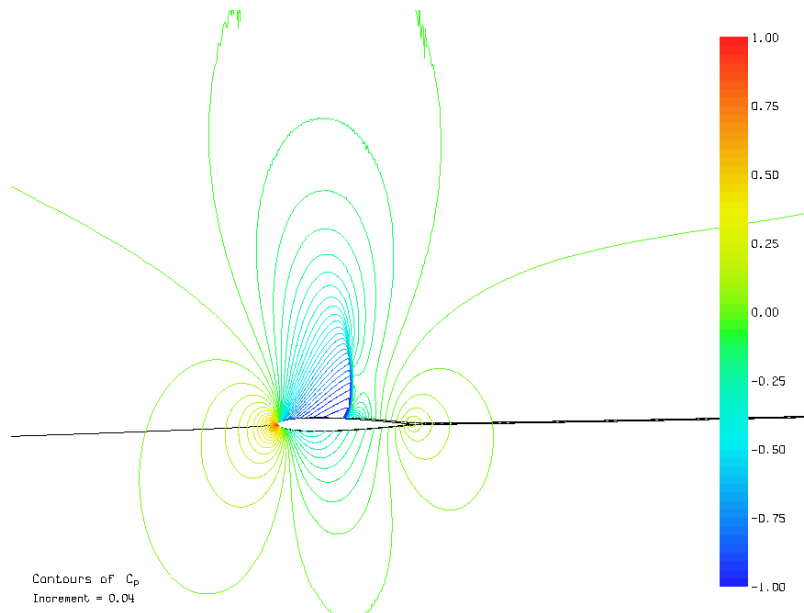




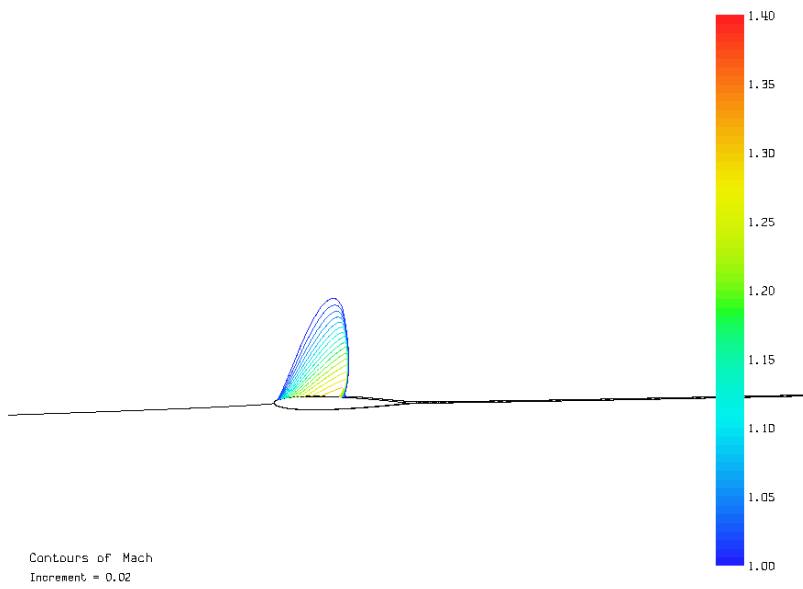




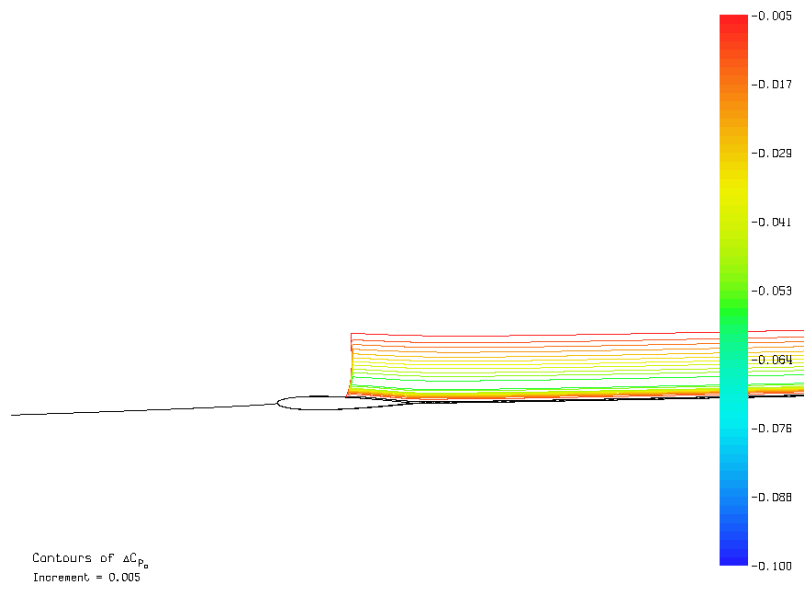
NACA 0010
 Mach = 0.800
 Re = 2.700×10^6
 Alfa = 1.500
 CL = 0.3549
 CD = 0.02299
 CM = -0.0166
 L/D = 15.43
 Ncrit = 9.00
 XtrU = 0.050
 XtrL = 0.050



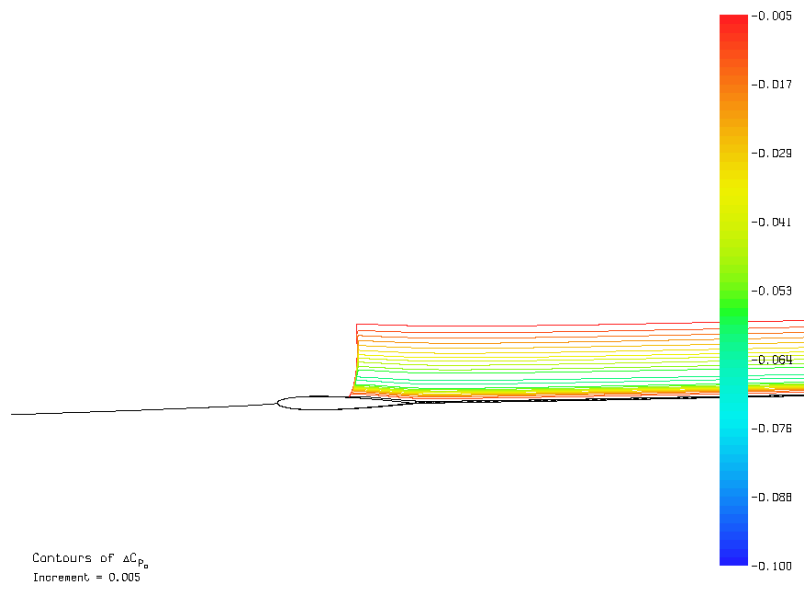
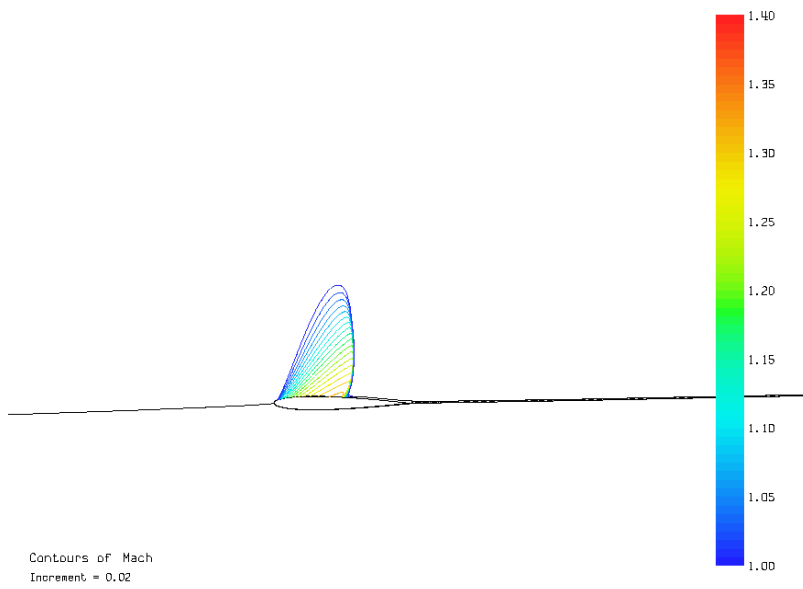
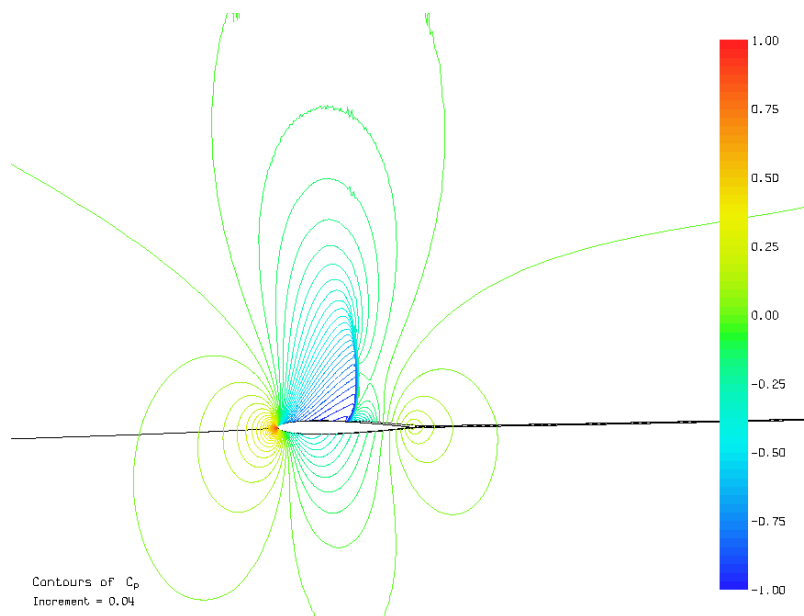
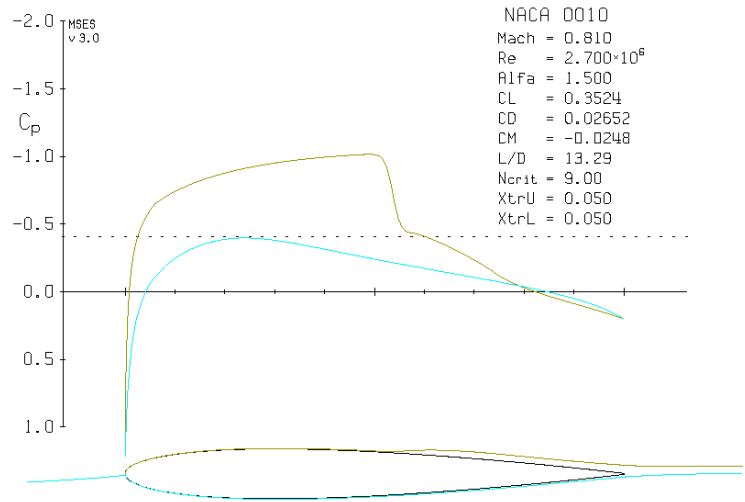
Contours of C_p
 Increment = 0.04

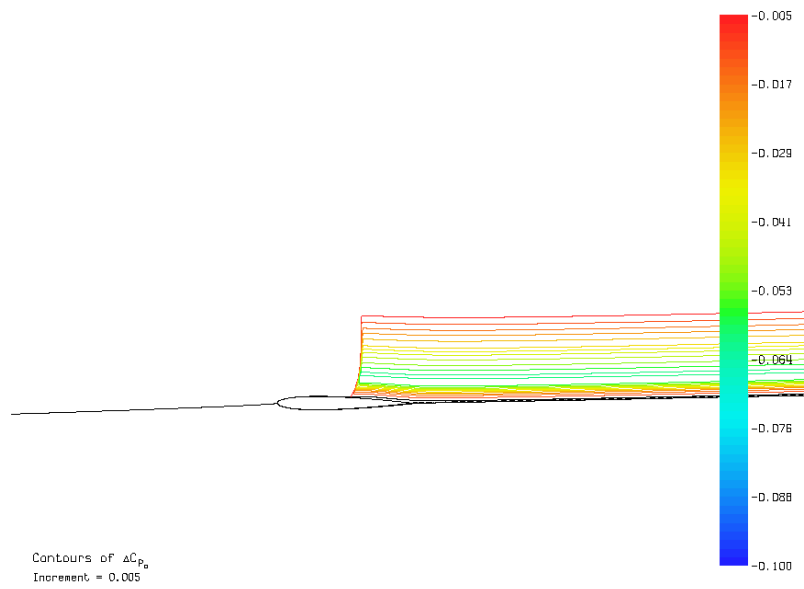
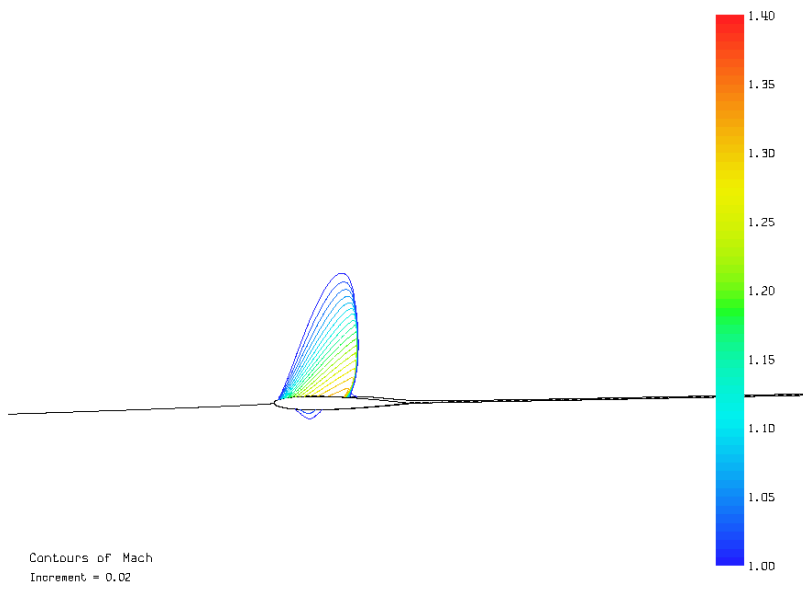
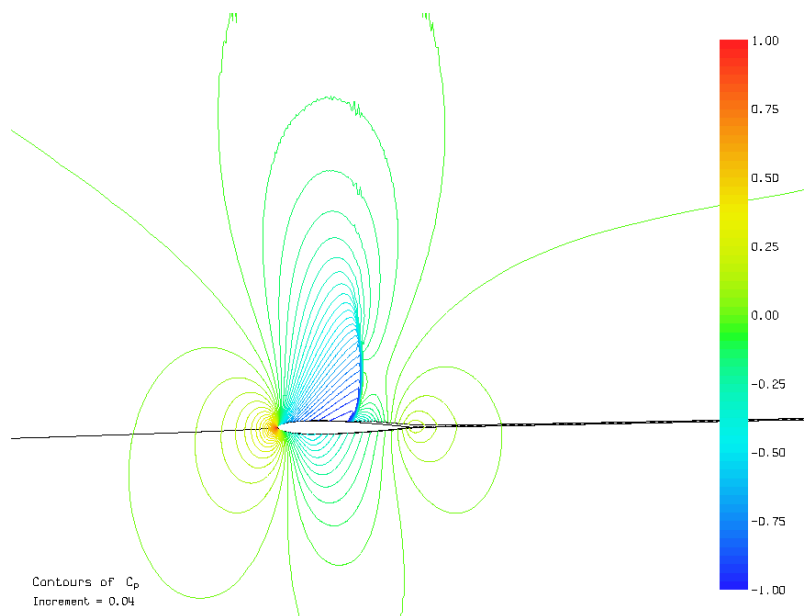
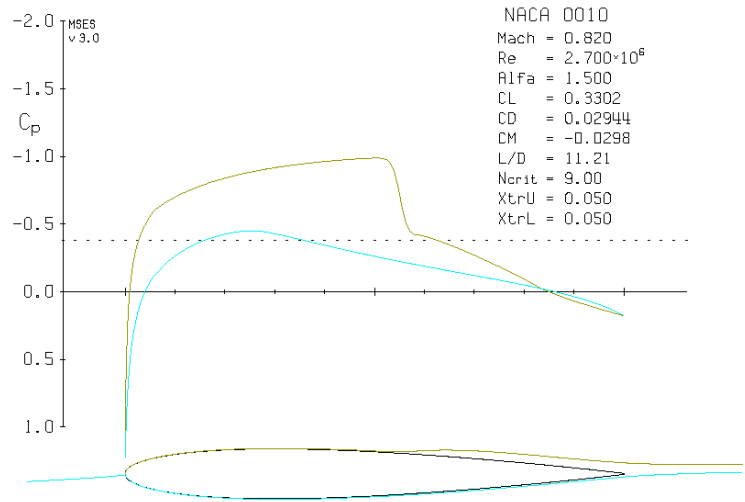


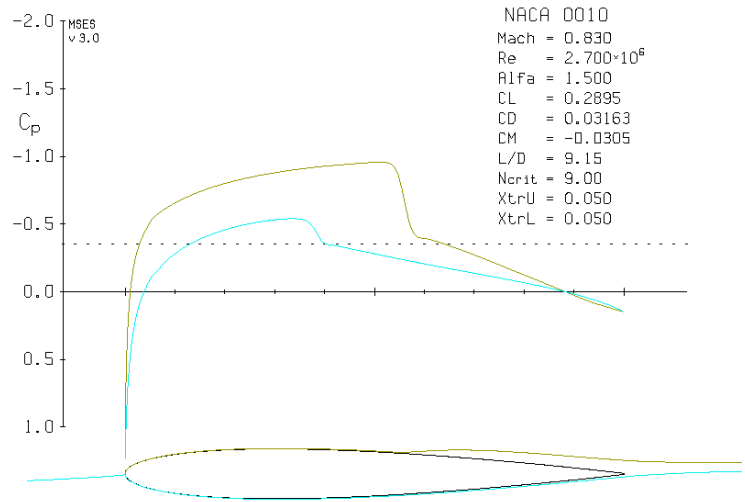
Contours of Mach
 Increment = 0.02



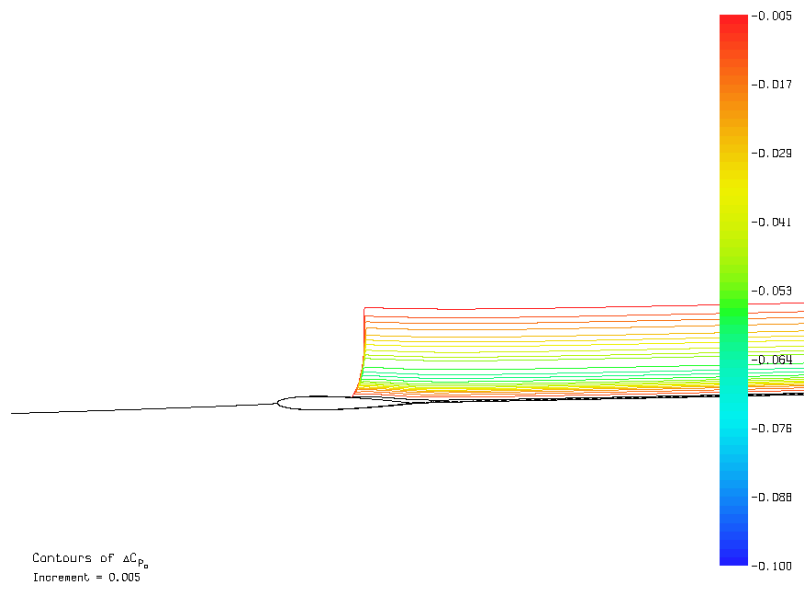
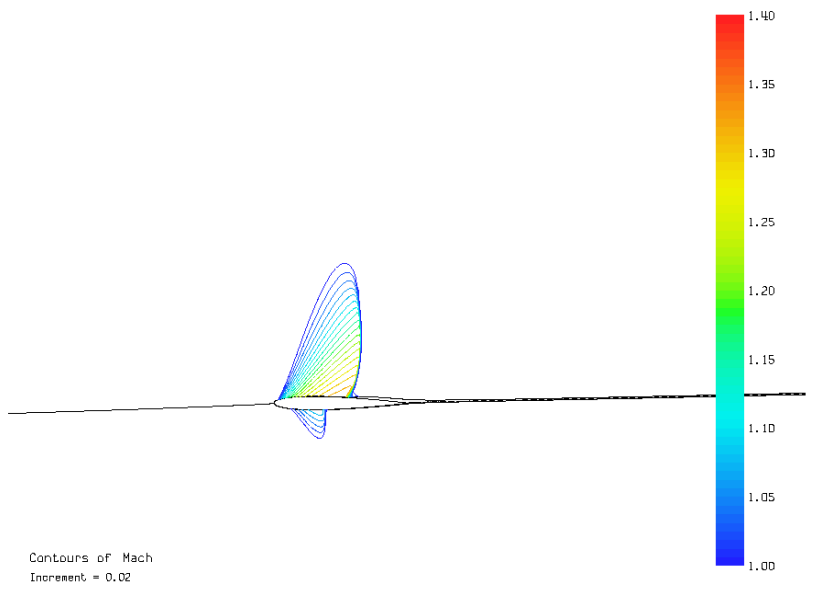
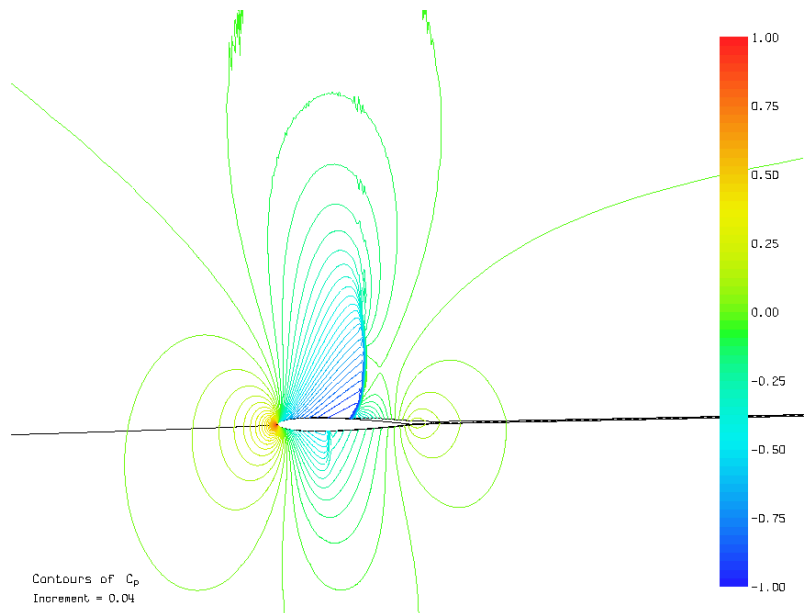
Contours of ΔC_p
 Increment = 0.005

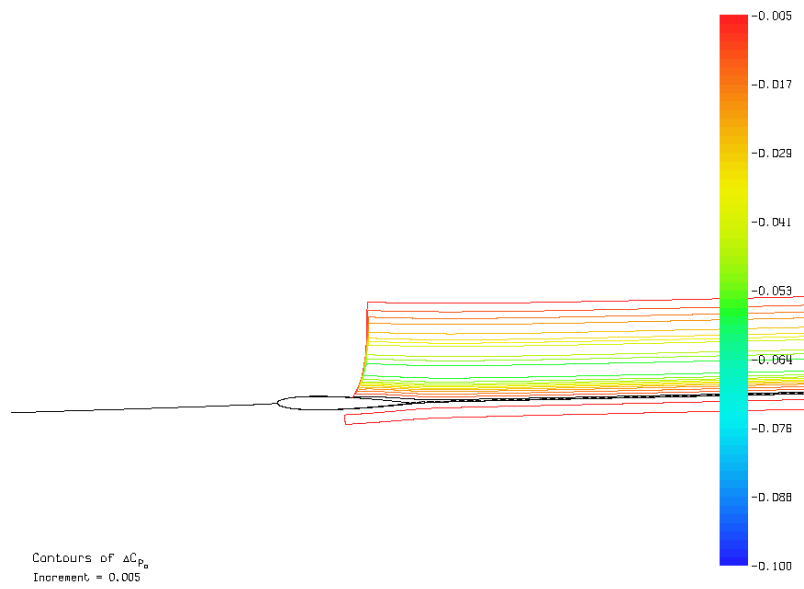
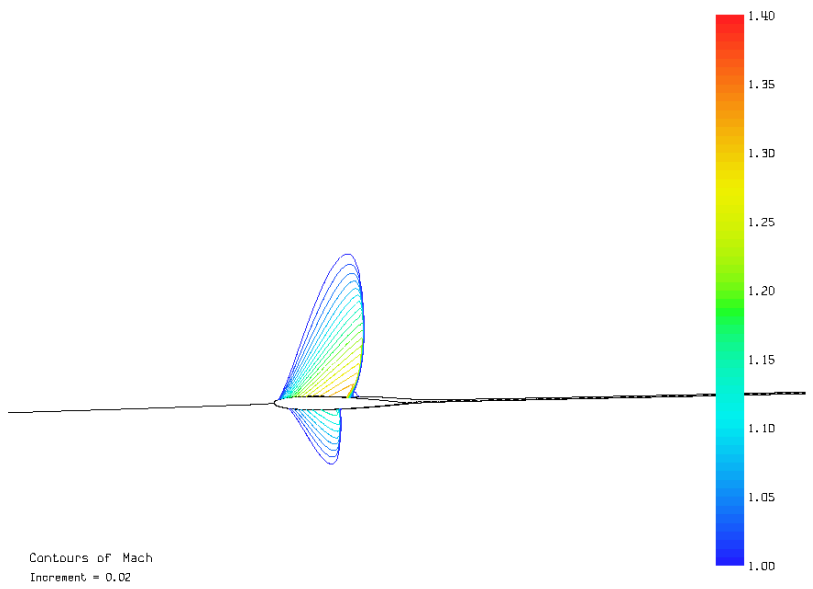
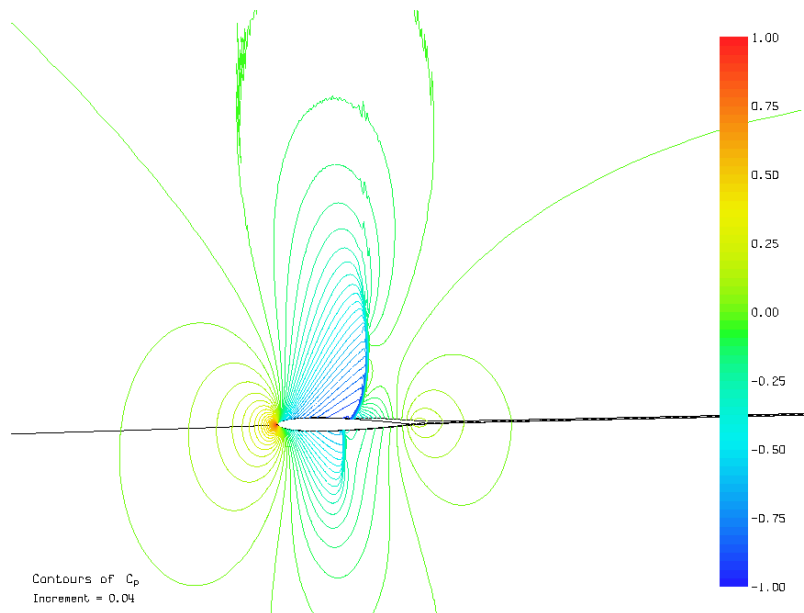
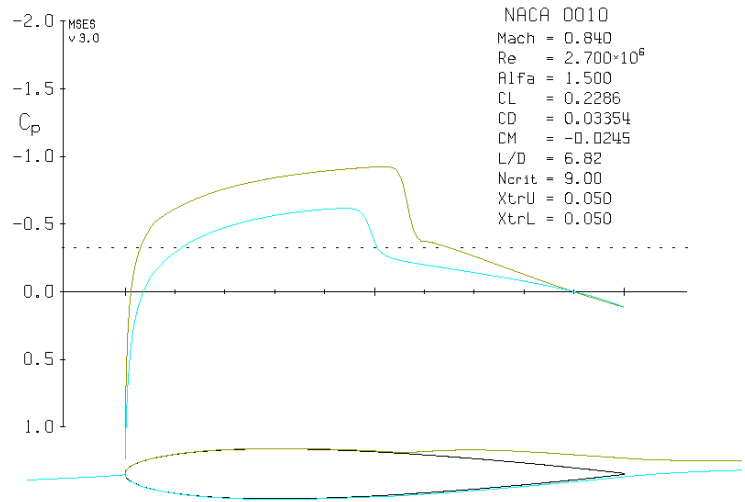


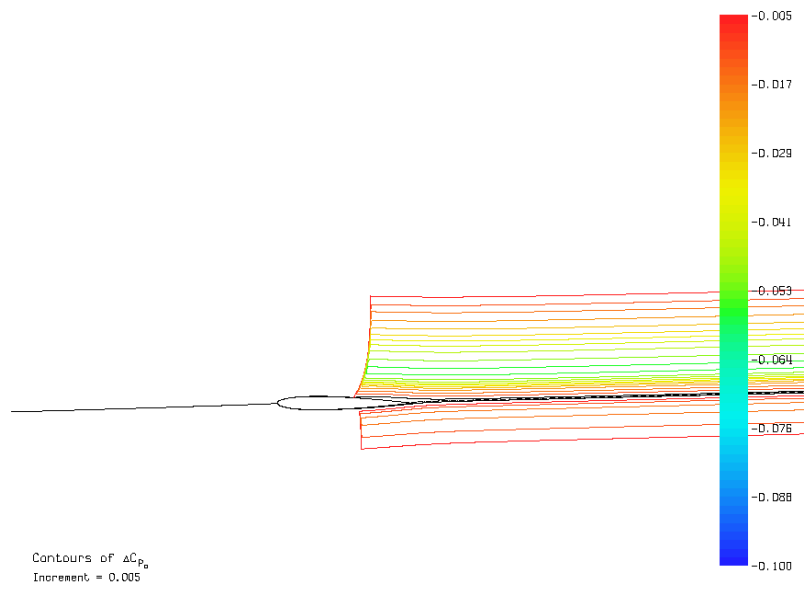
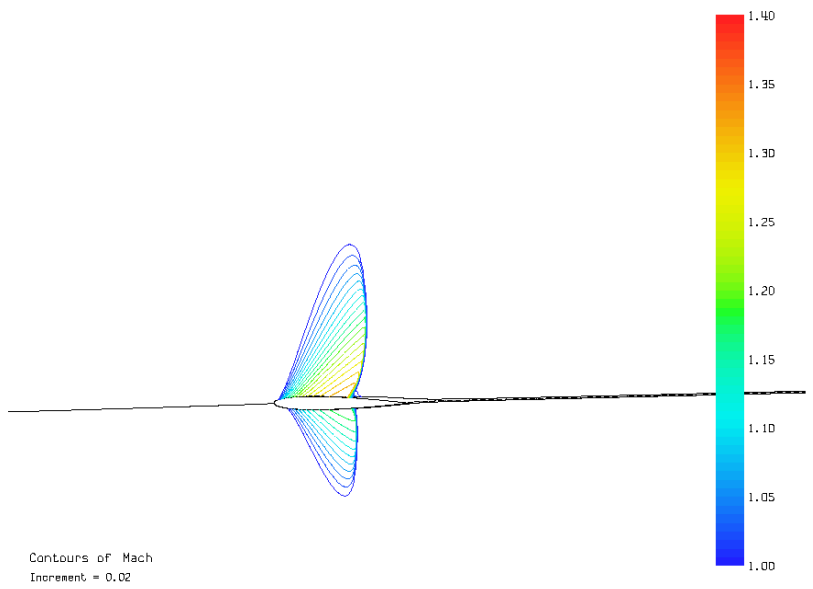
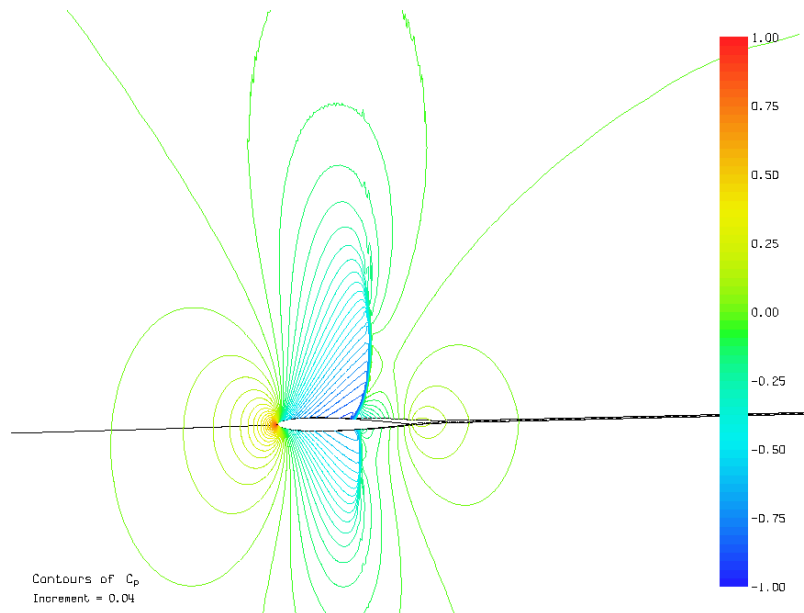
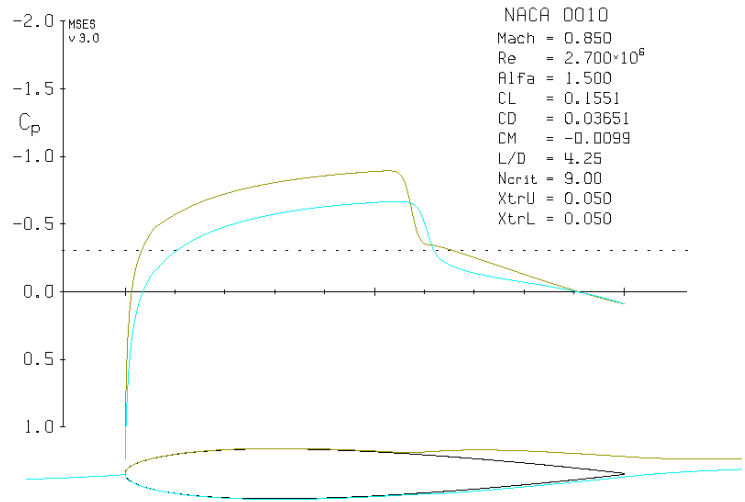


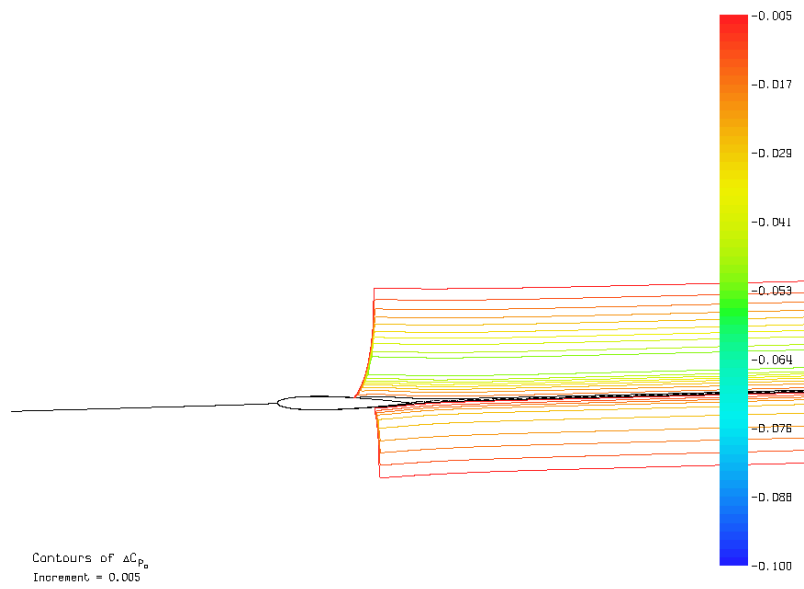
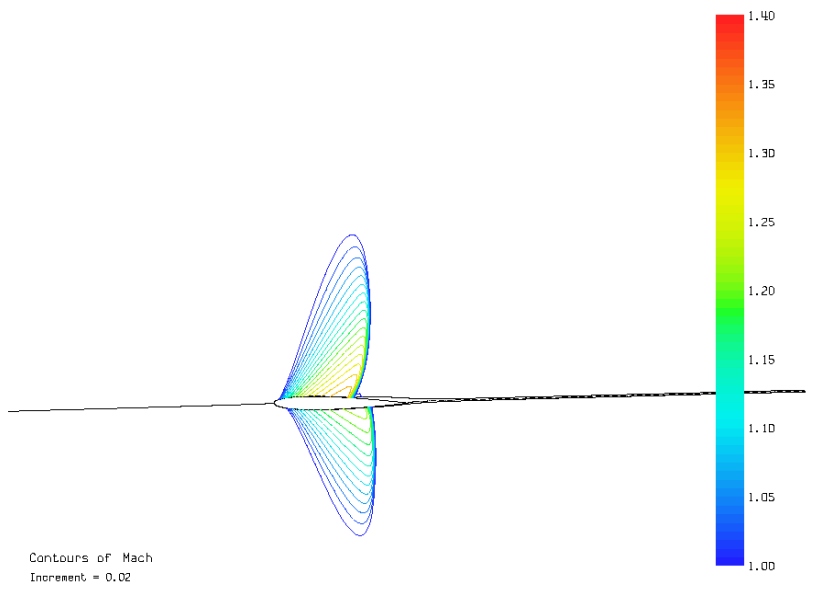
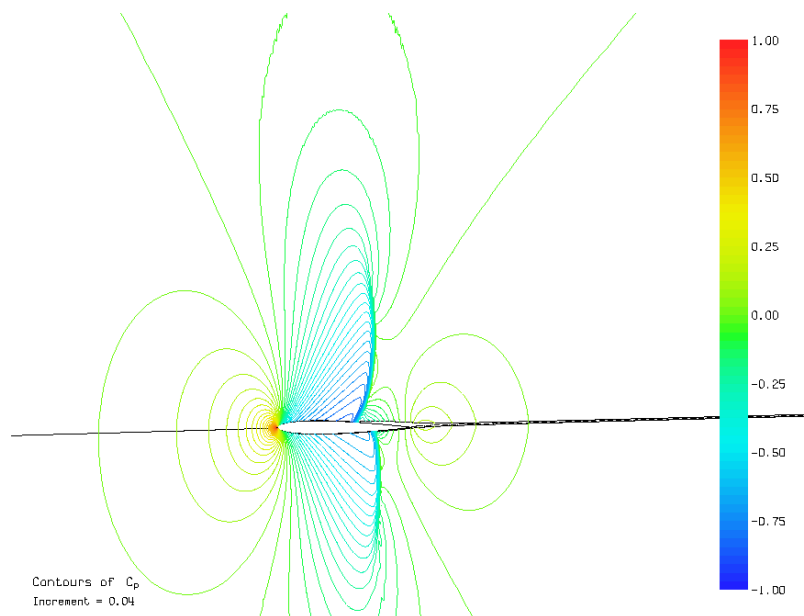
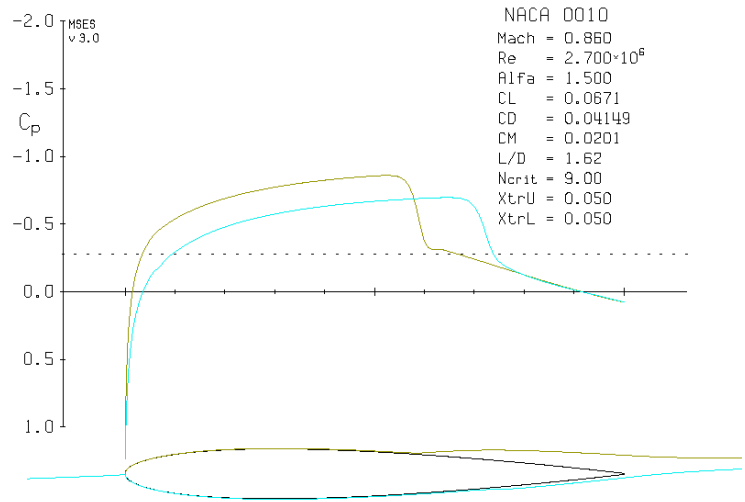


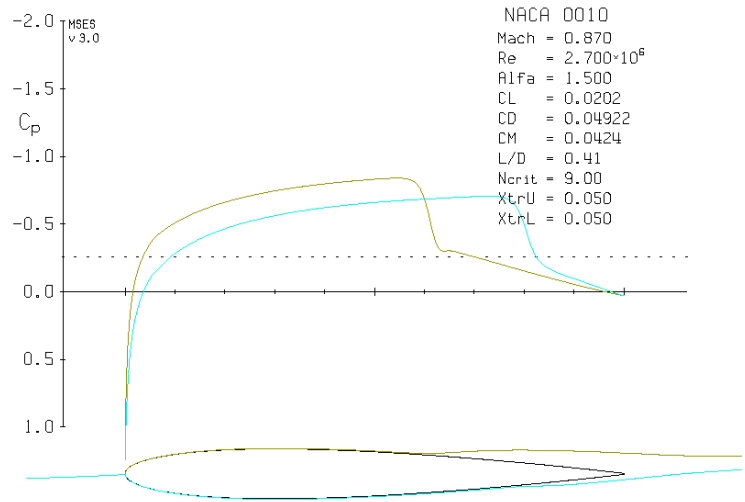
NACA 0010
 Mach = 0.830
 Re = 2.700×10^6
 Alfa = 1.500
 CL = 0.2895
 CD = 0.03163
 CM = -0.0305
 L/D = 9.15
 Ncrit = 9.00
 XtrU = 0.050
 XtrL = 0.050



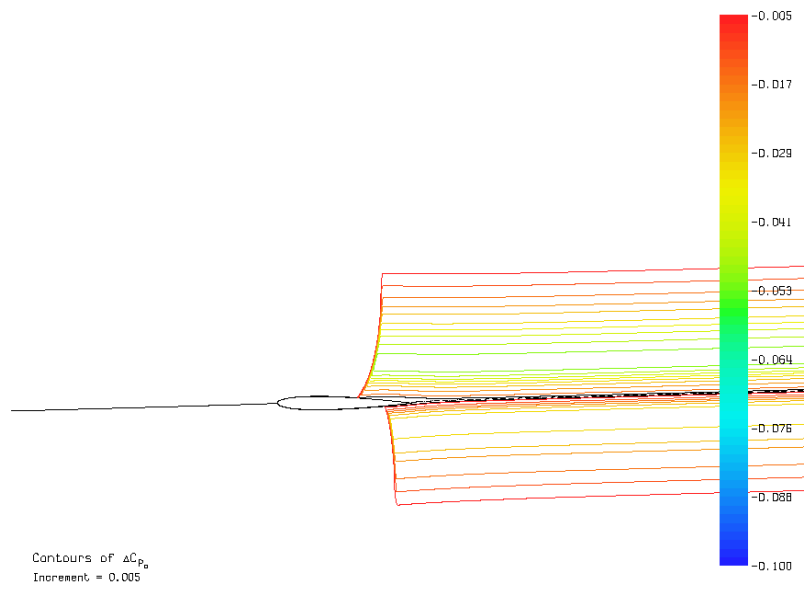
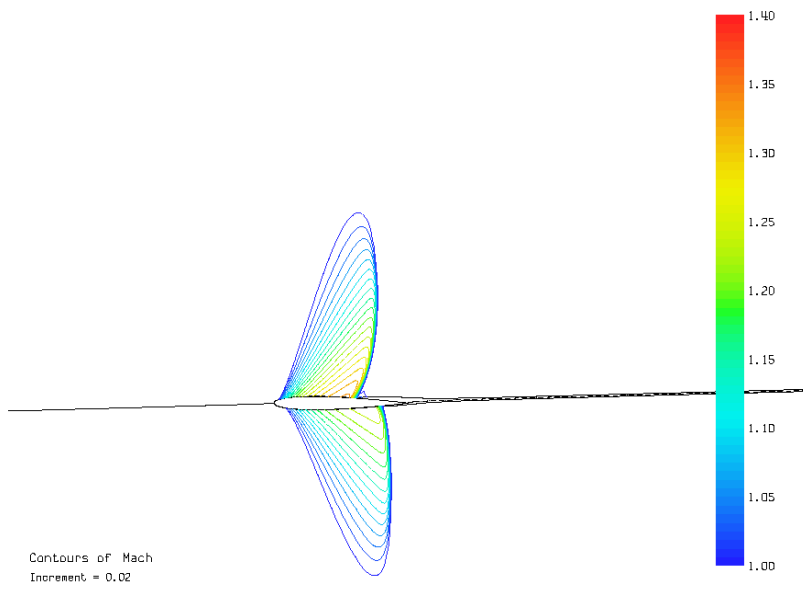
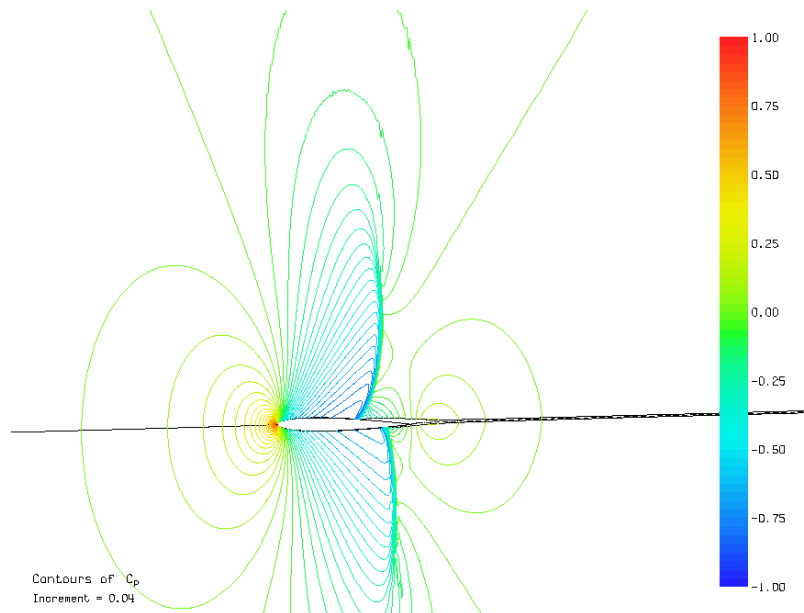


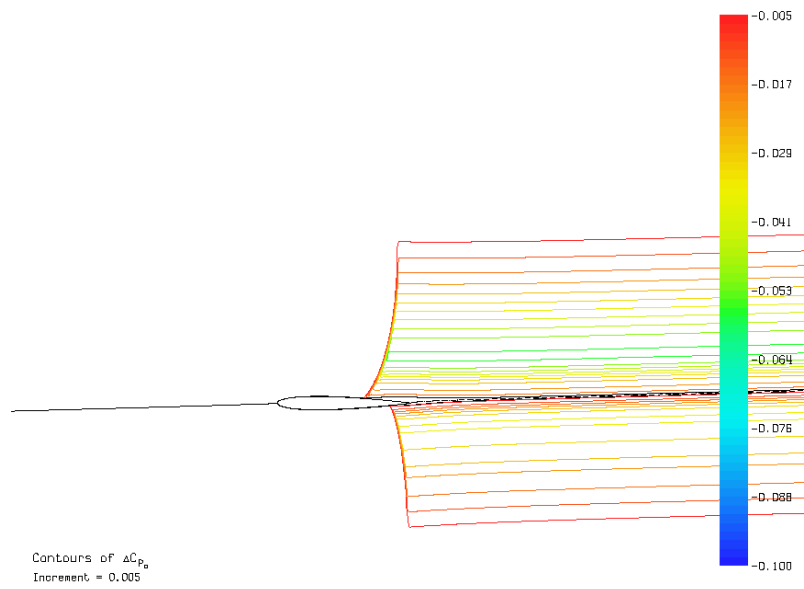
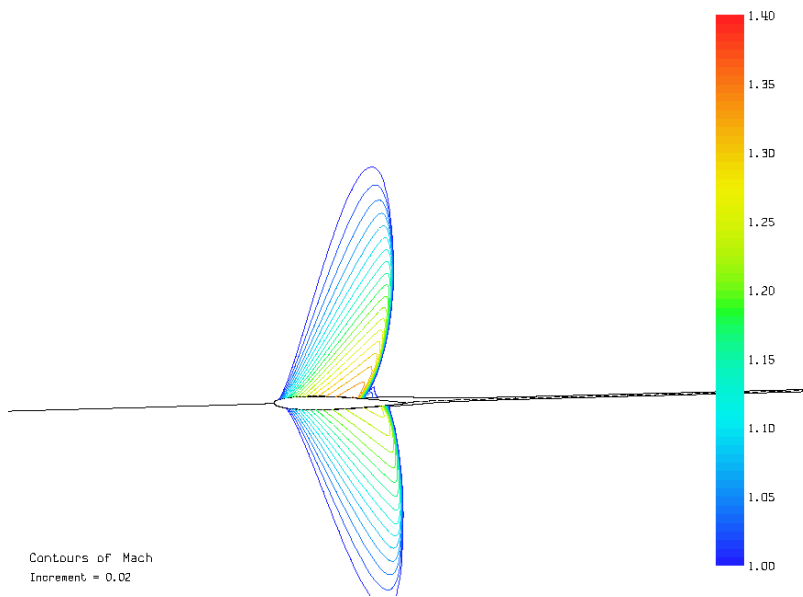
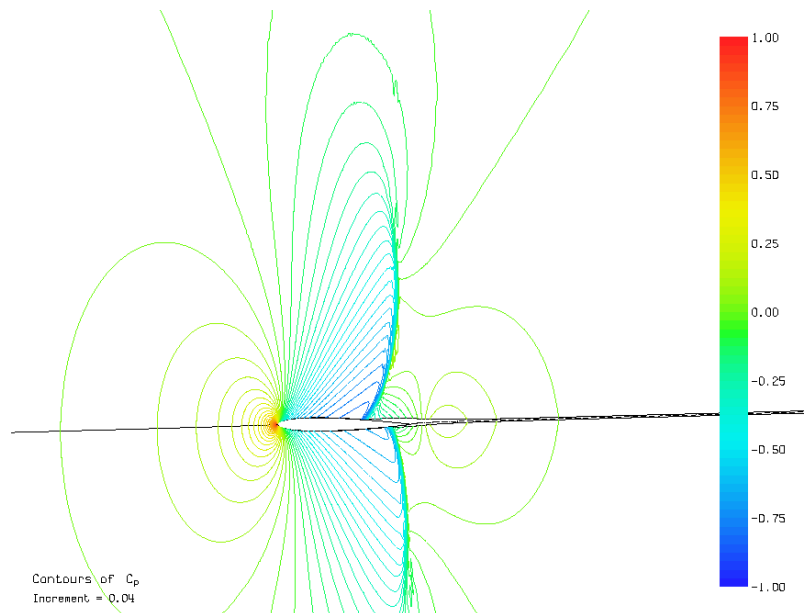
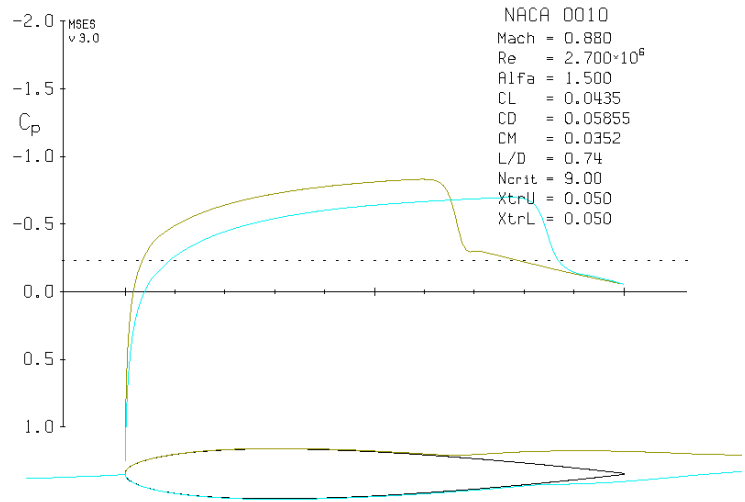


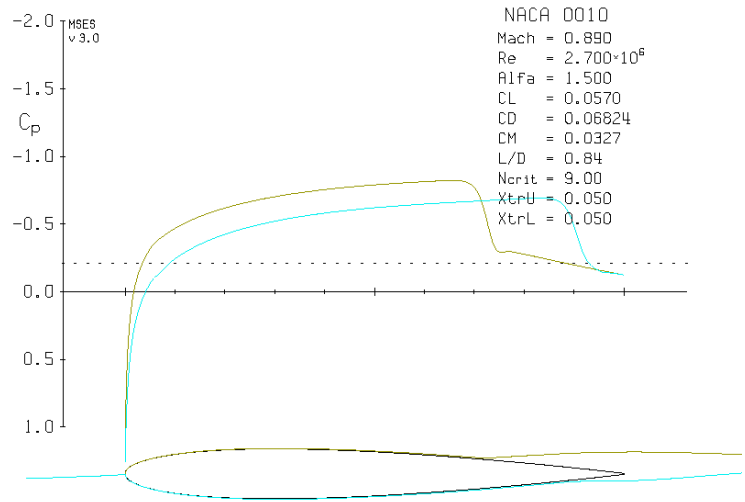




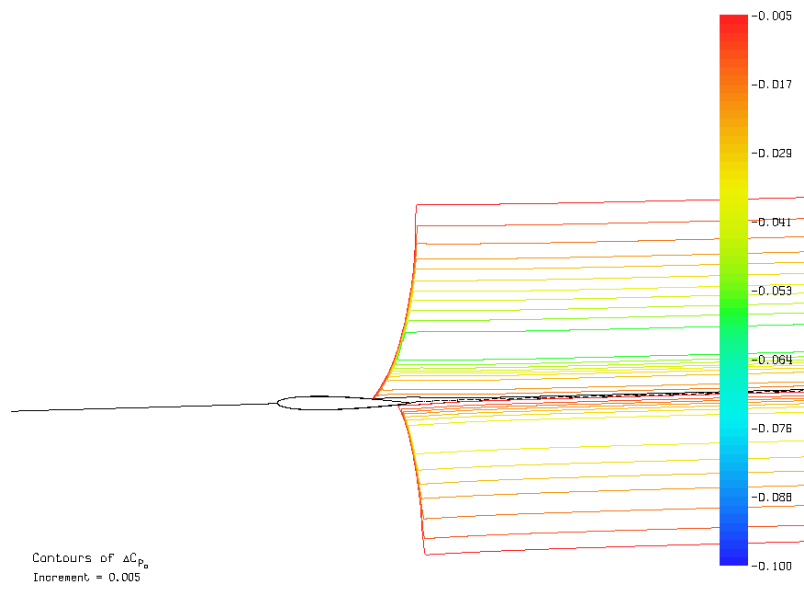
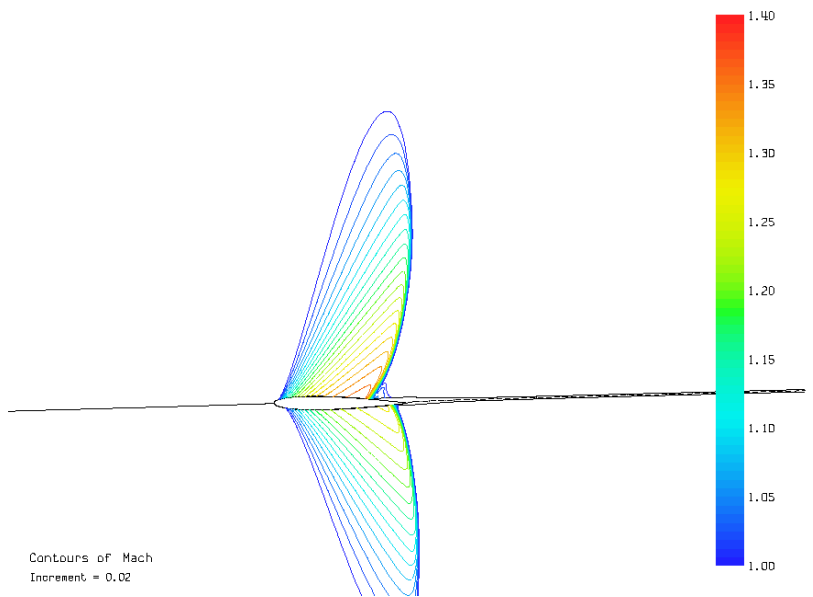
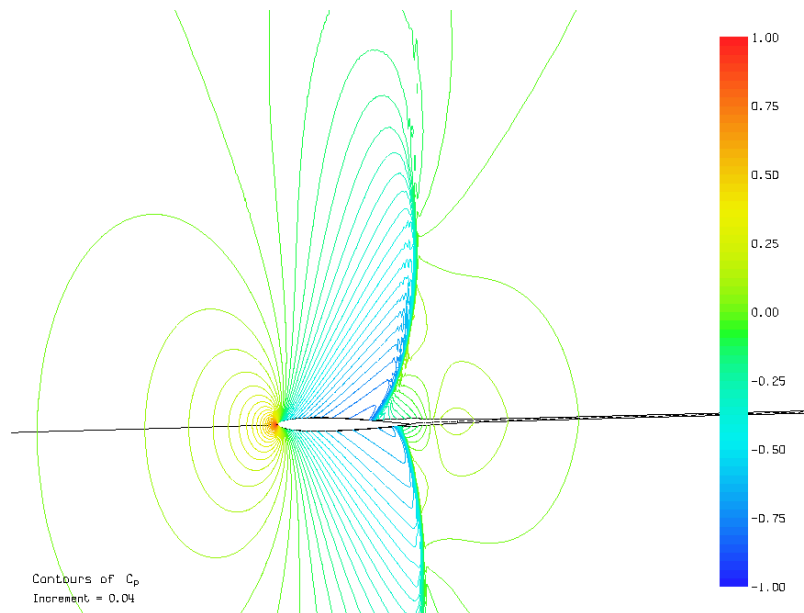
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 Re = 2.700×10^6
 Alfa = 1.500
 CL = 0.0202
 CD = 0.04922
 CM = 0.0424
 L/D = 0.41
 Ncrit = 9.00
 XtrU = 0.050
 XtrL = 0.050

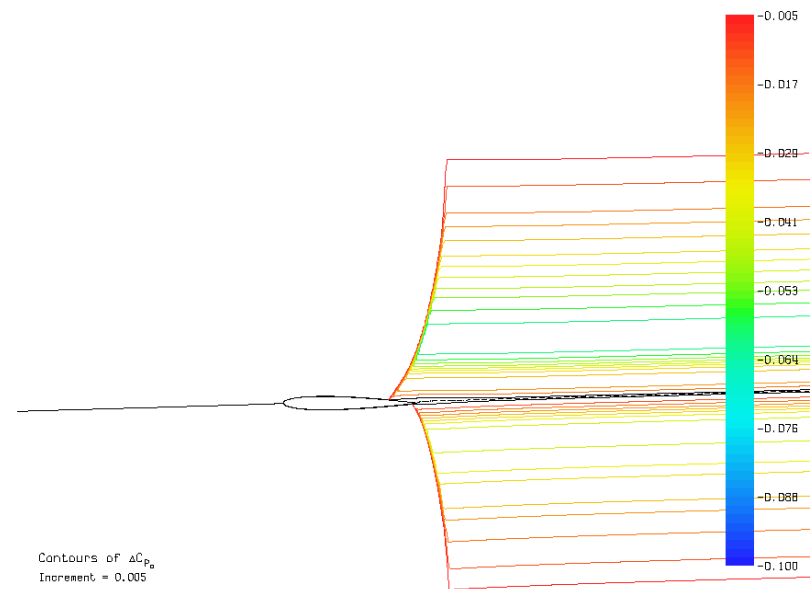
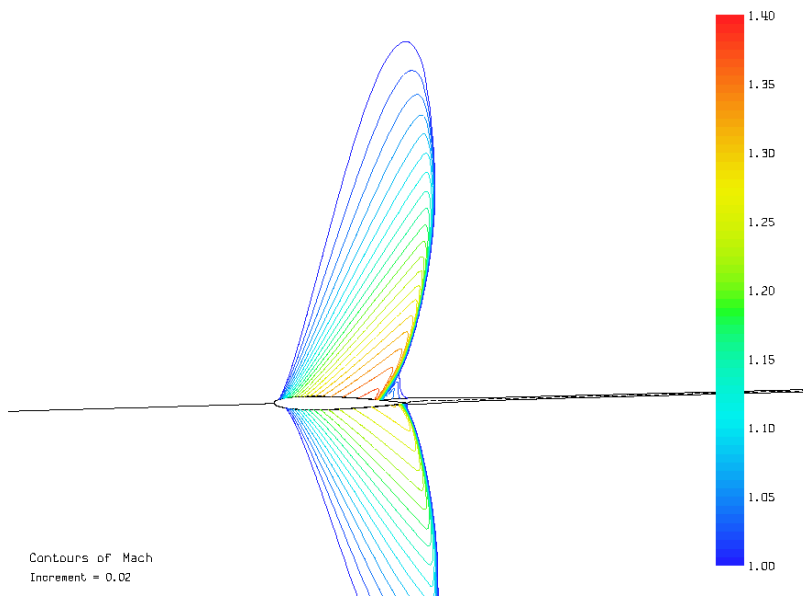
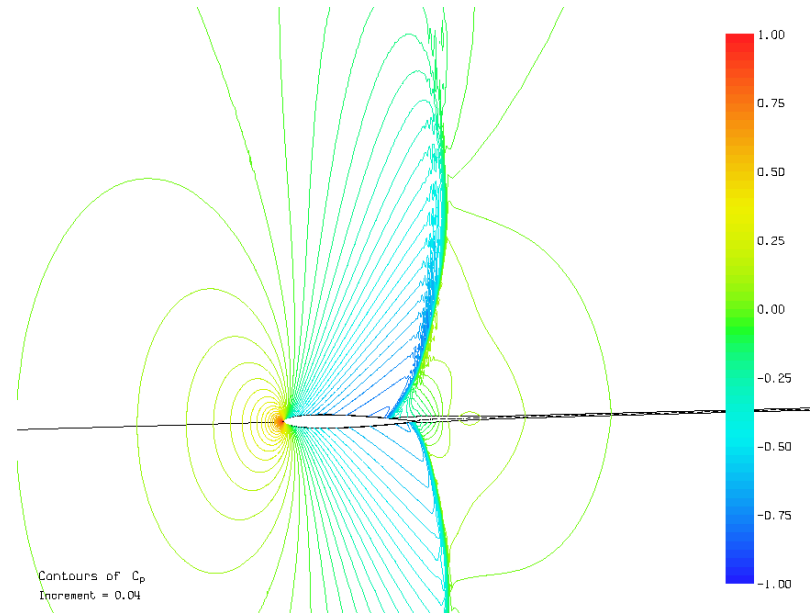
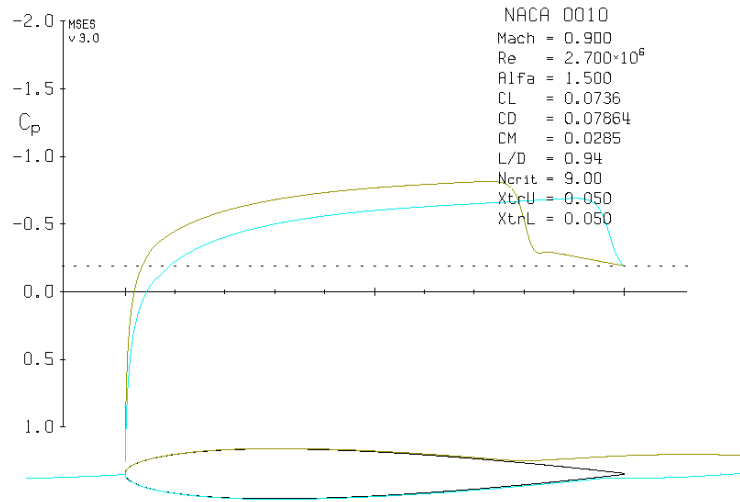


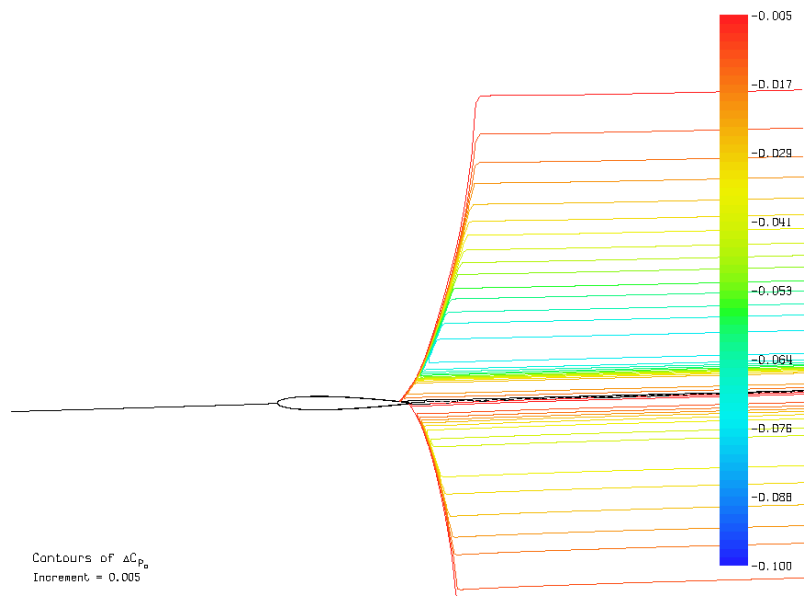
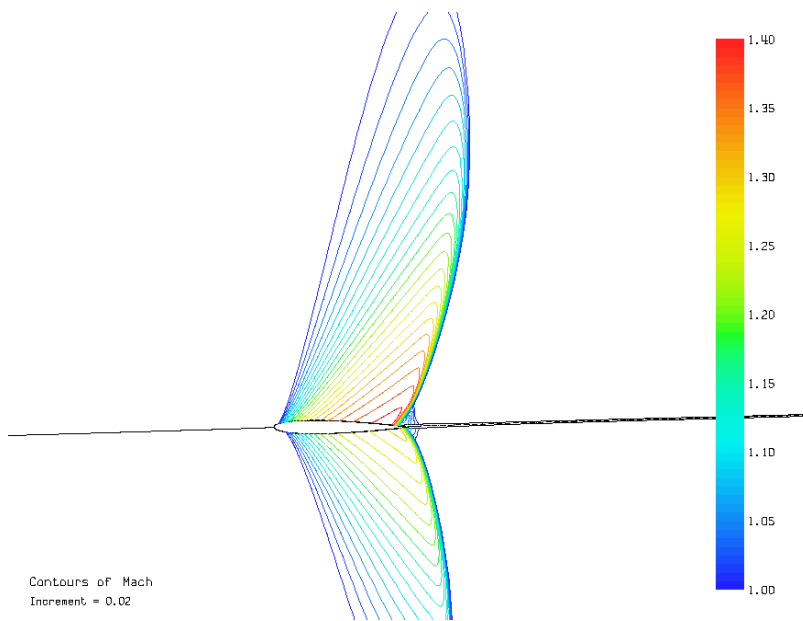
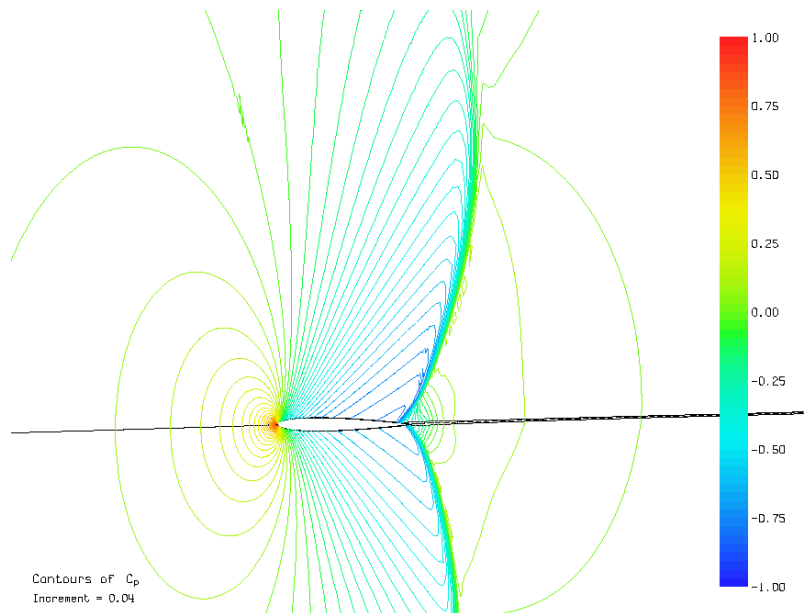
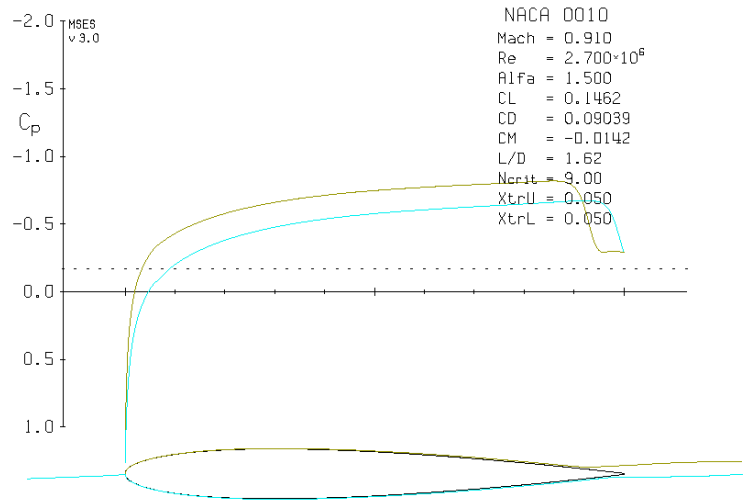


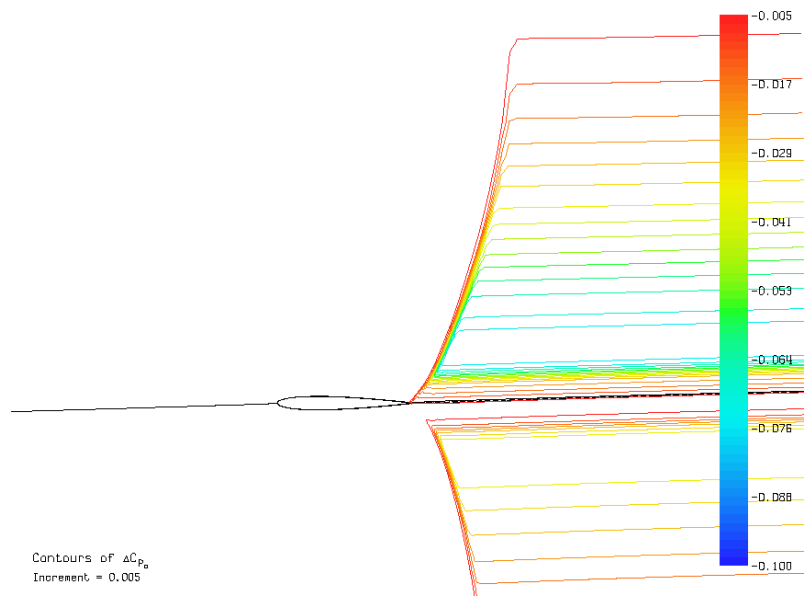
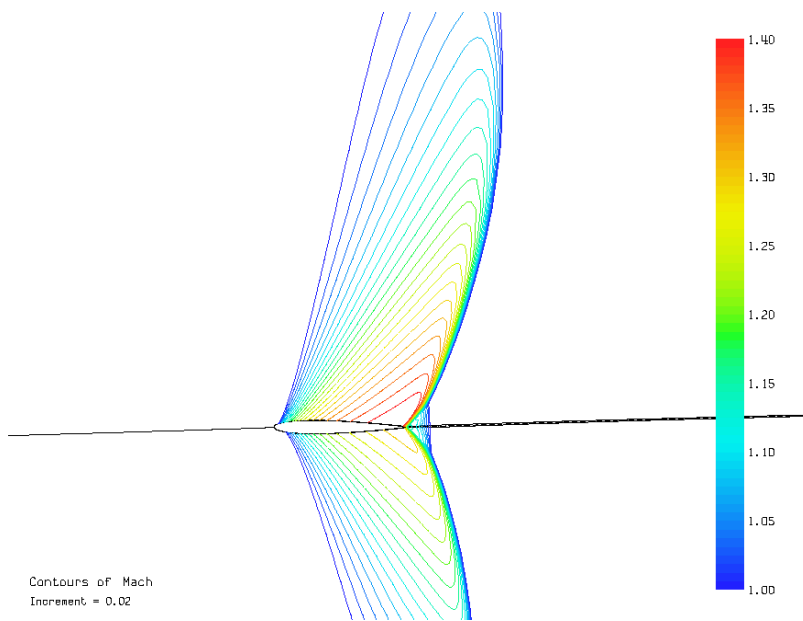
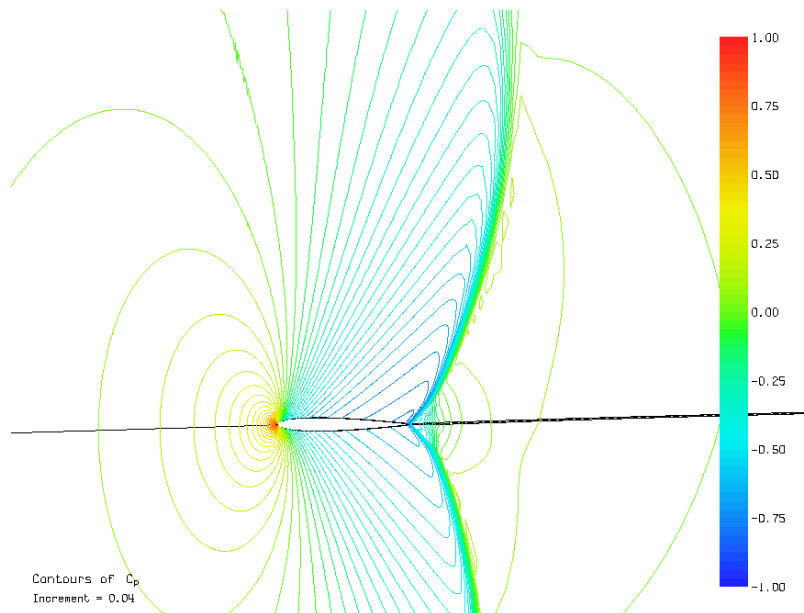
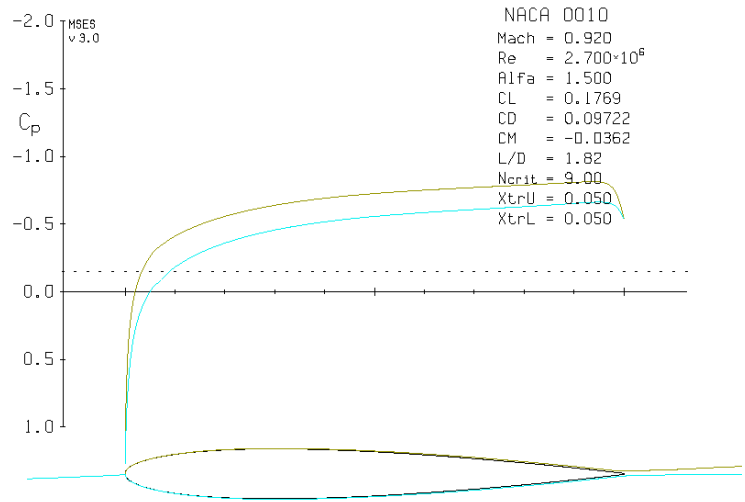


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 Alfa = 1.500
 CL = 0.0570
 CD = 0.06824
 CM = 0.0327
 L/D = 0.84
 Ncrit = 9.00
 XtrU = 0.050
 XtrL = 0.050



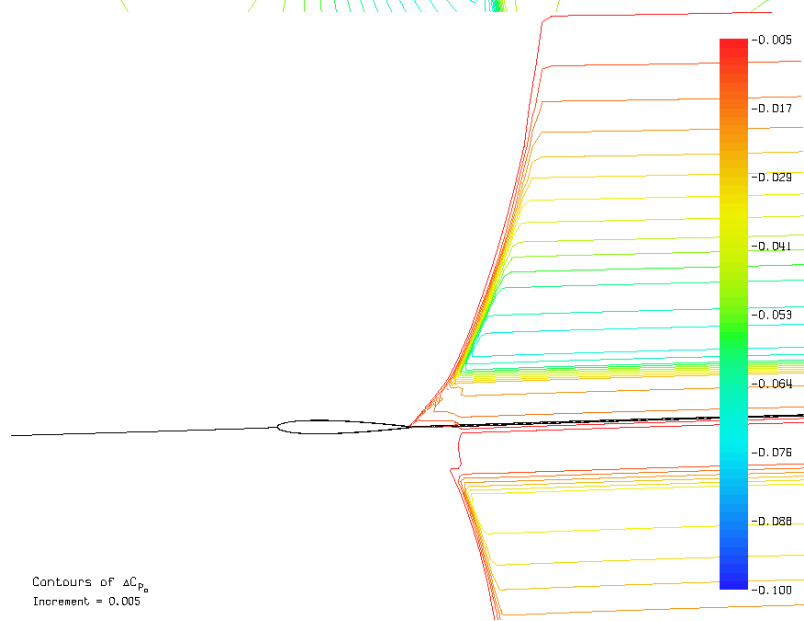
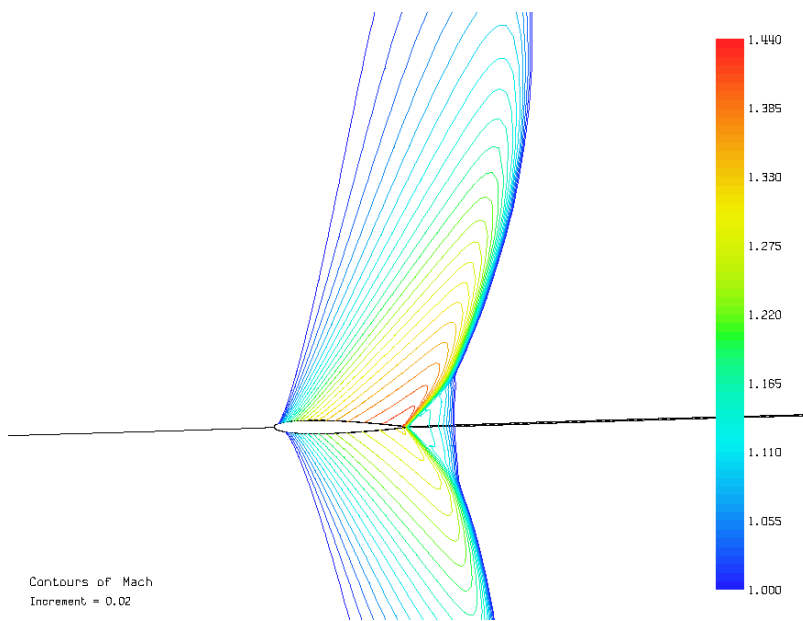
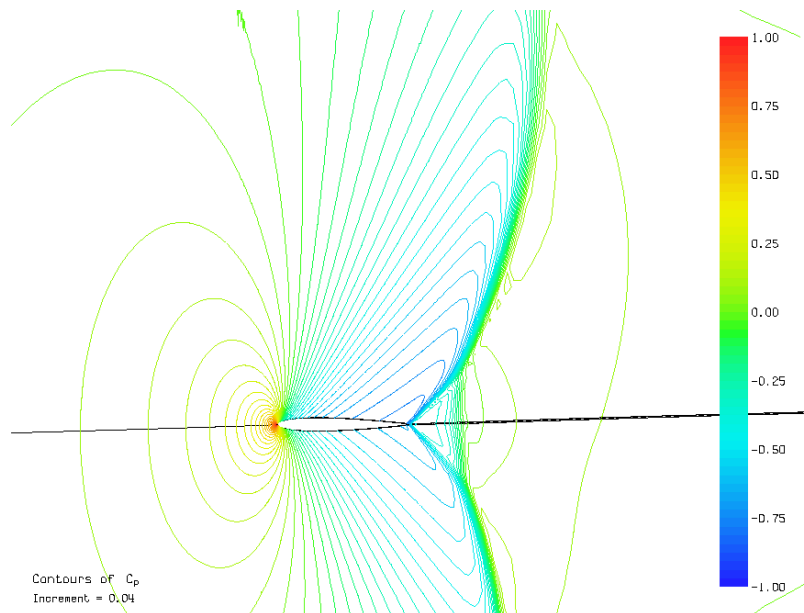
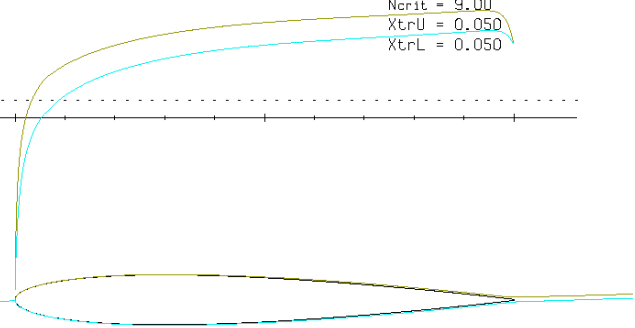


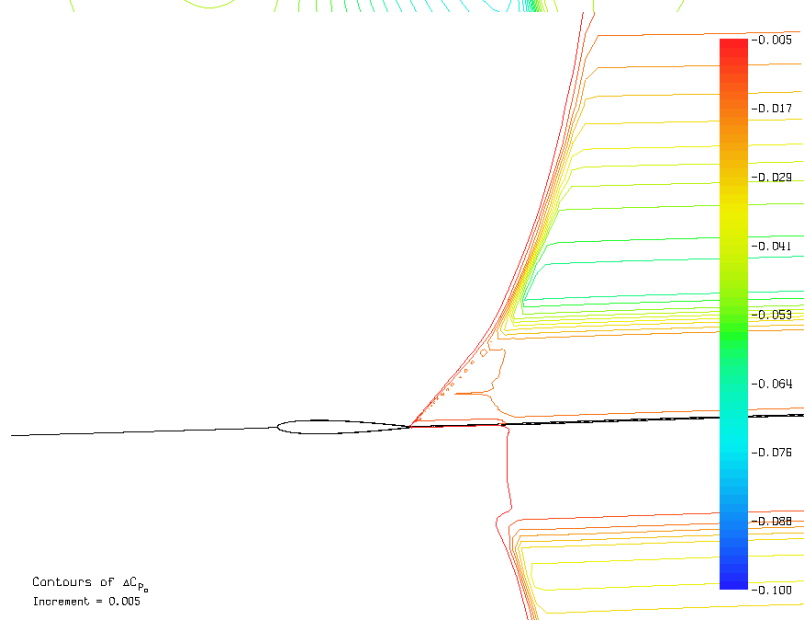
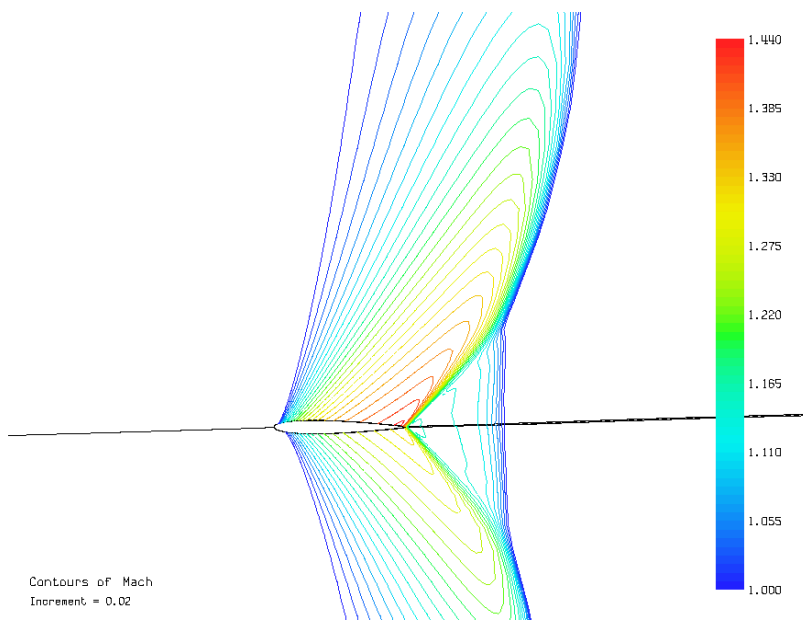
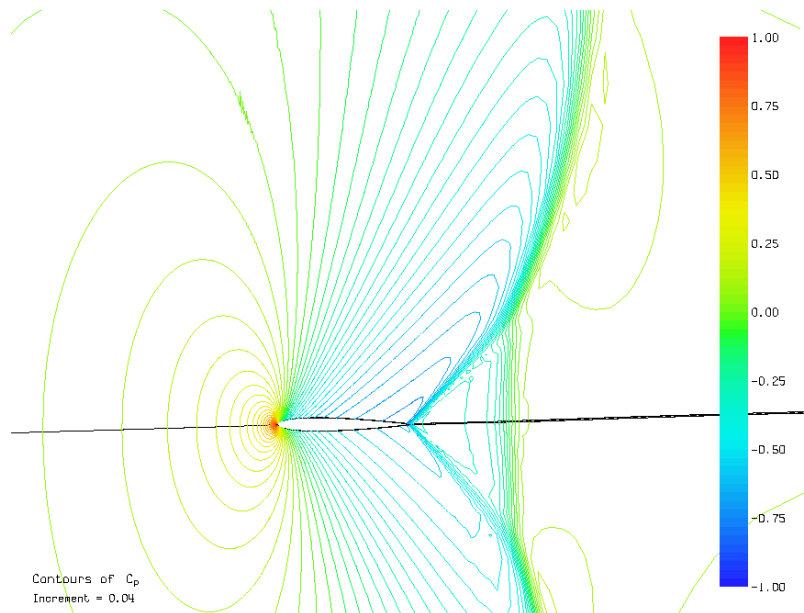
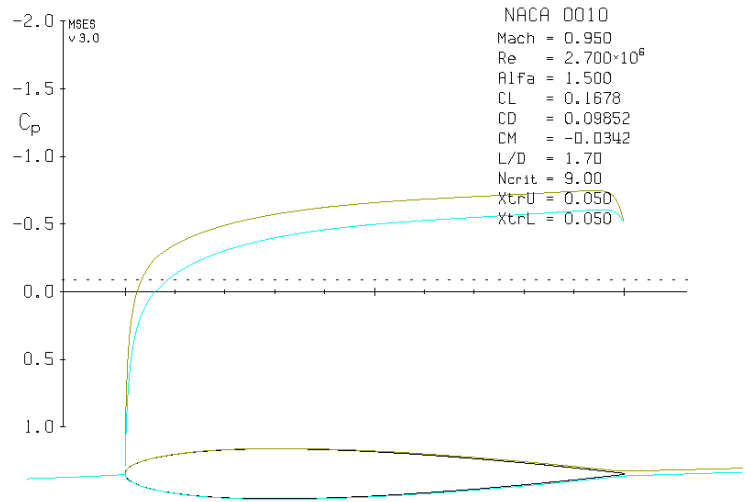


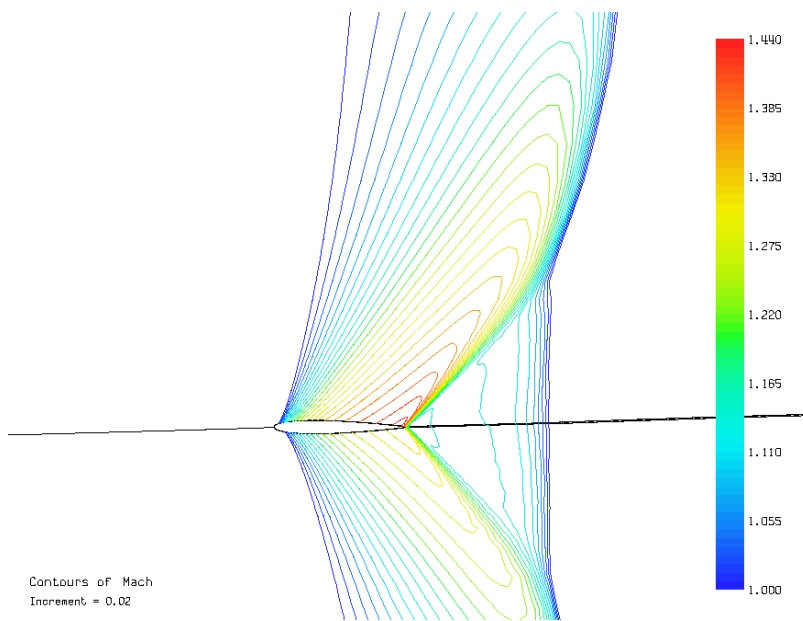
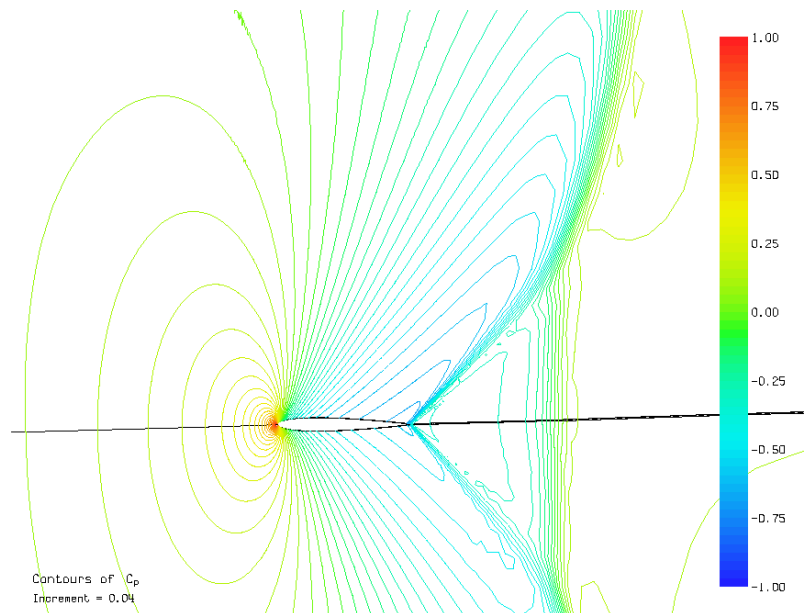
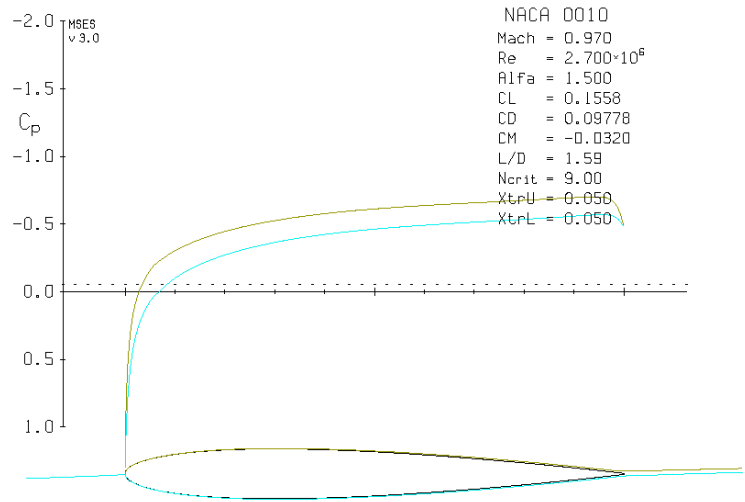


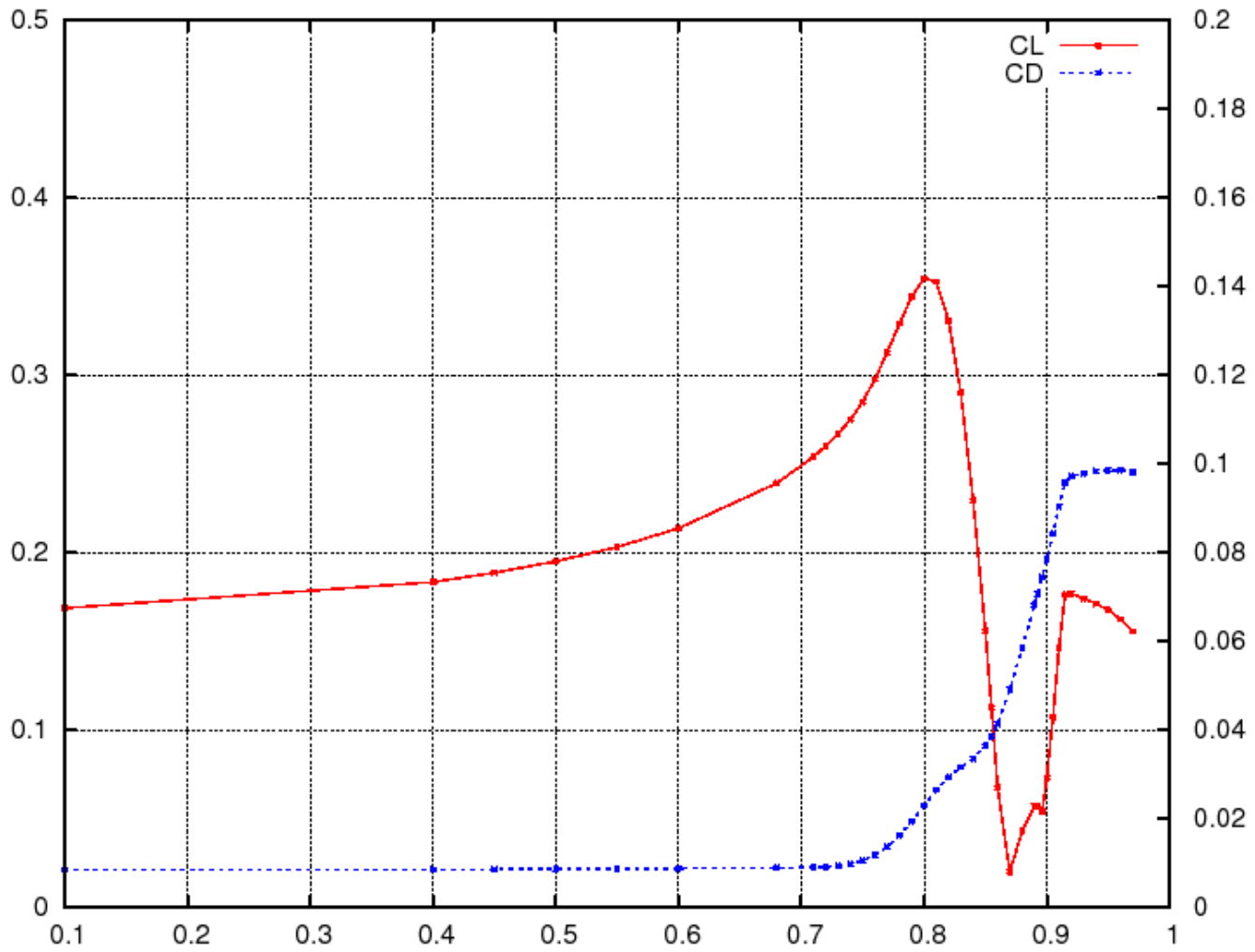
-2.0
 -1.5
 -1.0
 -0.5
 0.0
 0.5
 1.0
 C_p
 MSES v3.0

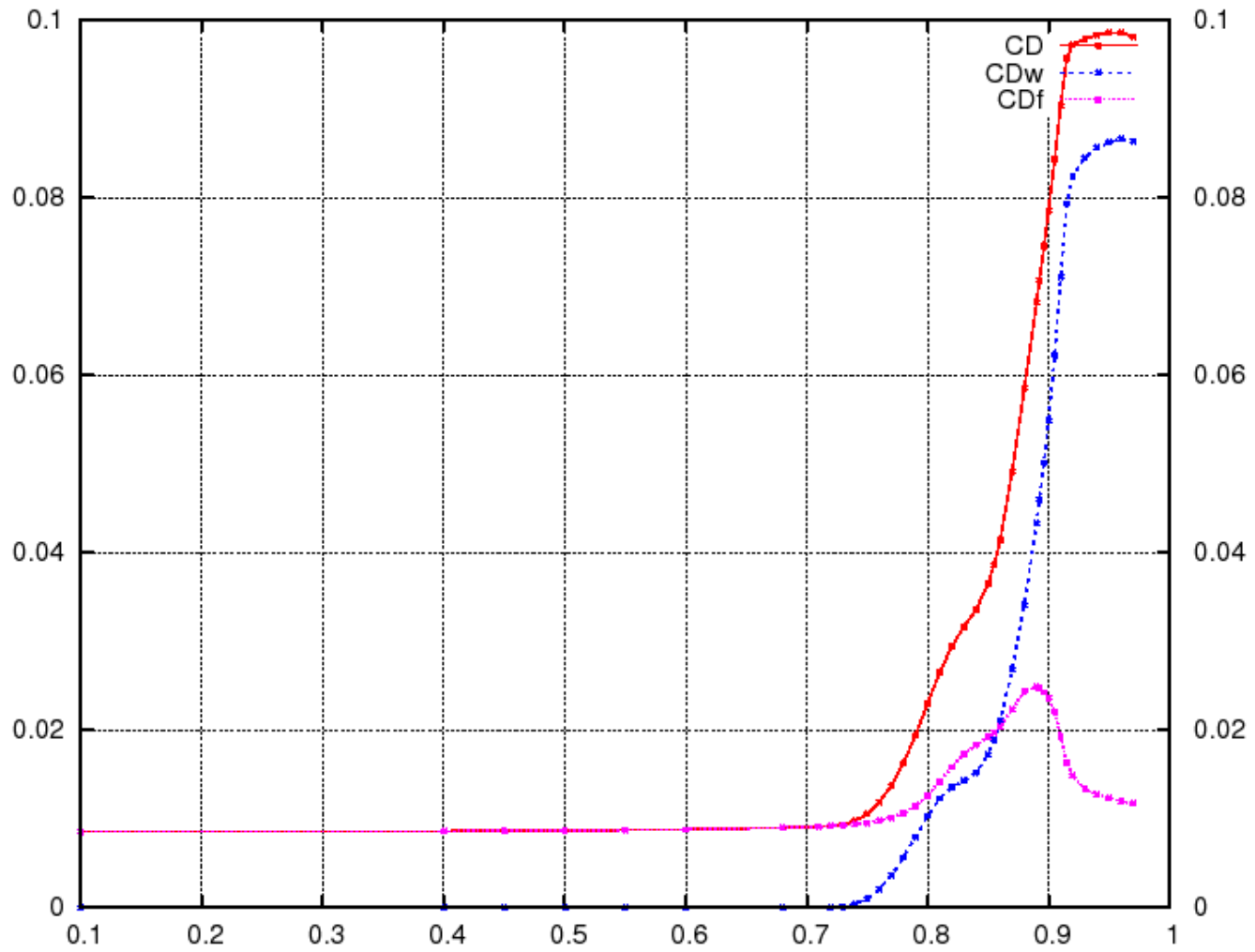
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 Re = 2.700×10^6
 Alfa = 1.500
 CL = 0.1742
 CD = 0.09784
 CM = -0.0356
 L/D = 1.78
 Ncrit = 9.00
 XtrU = 0.050
 XtrL = 0.050

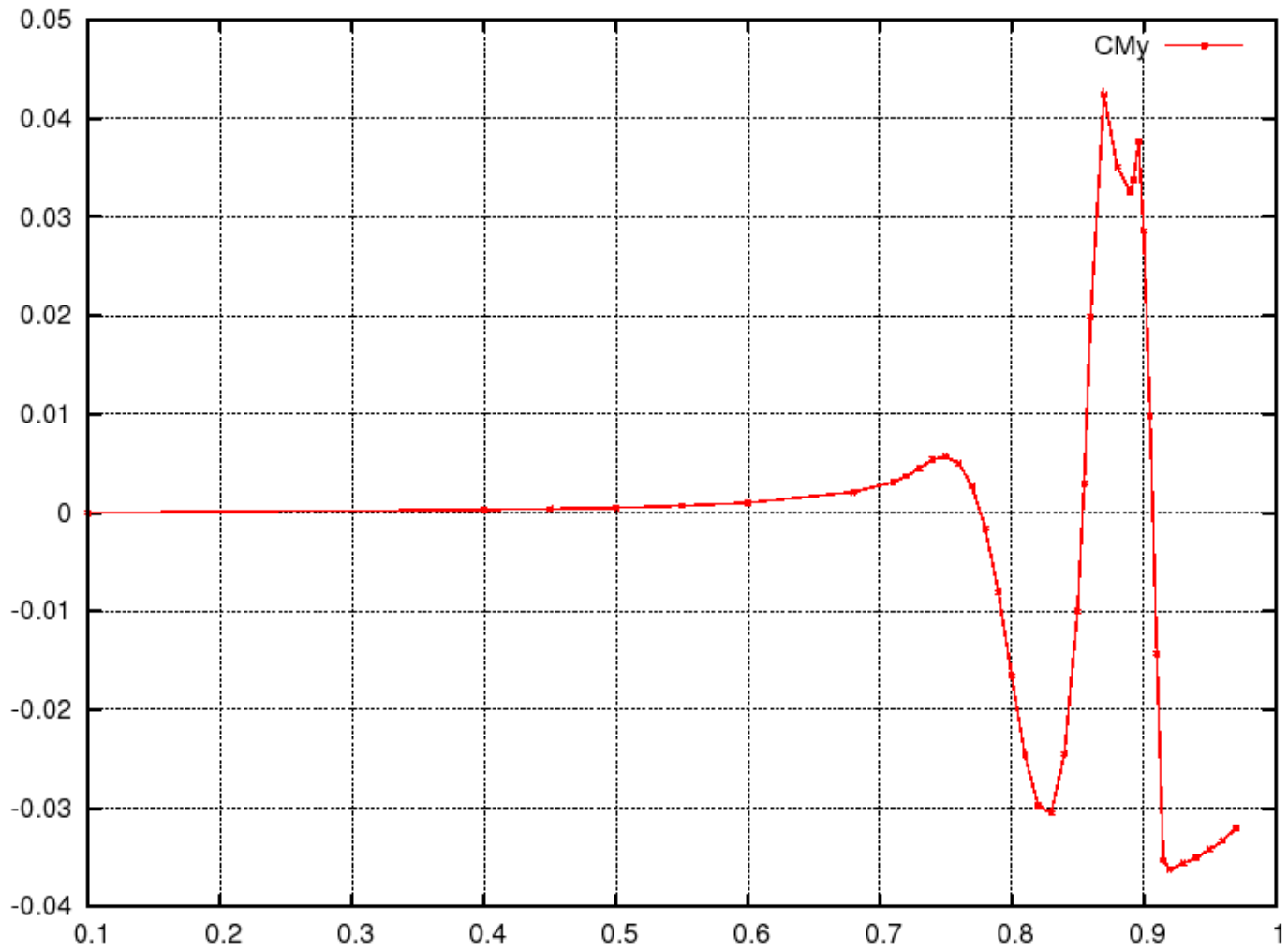


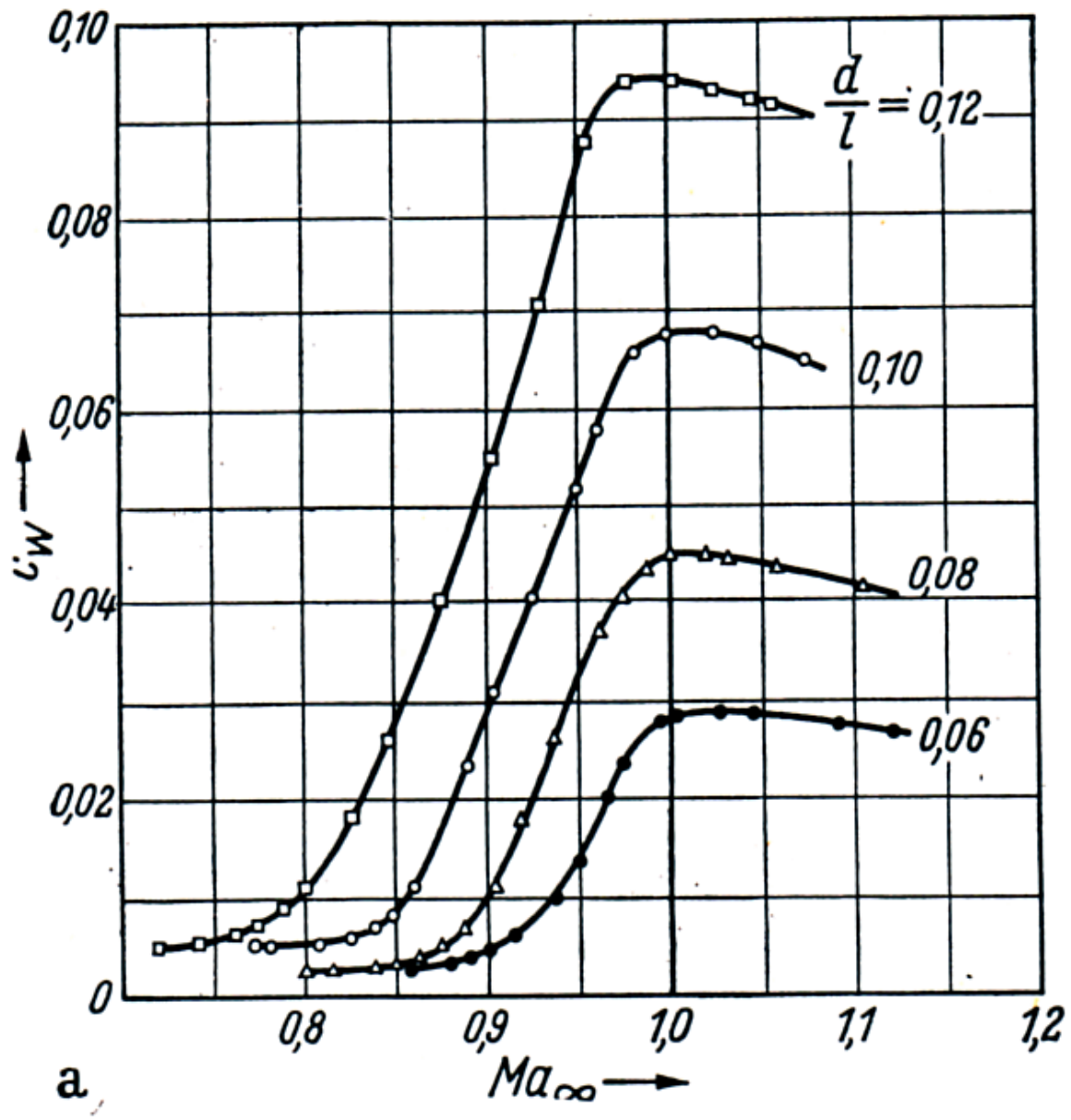


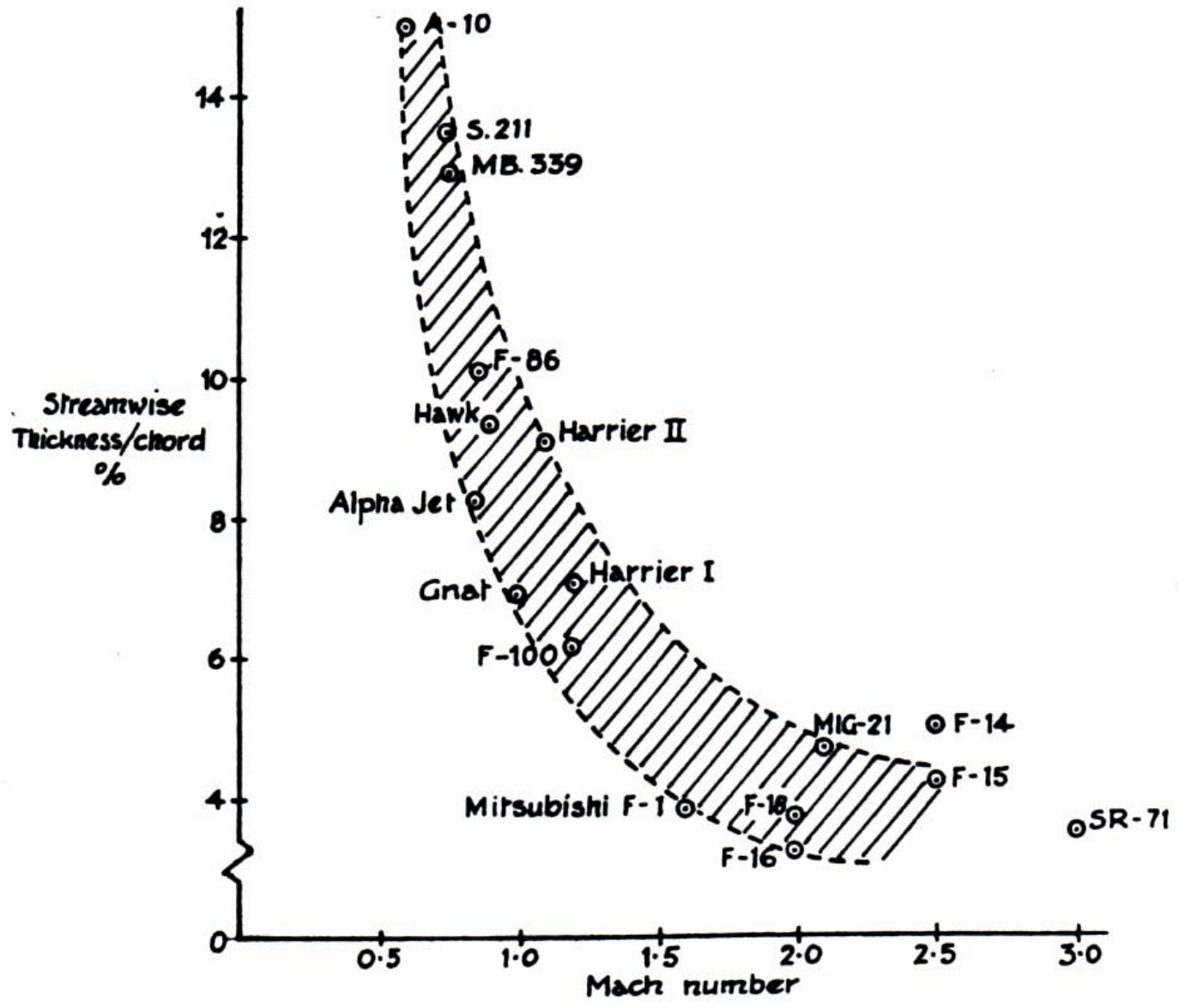


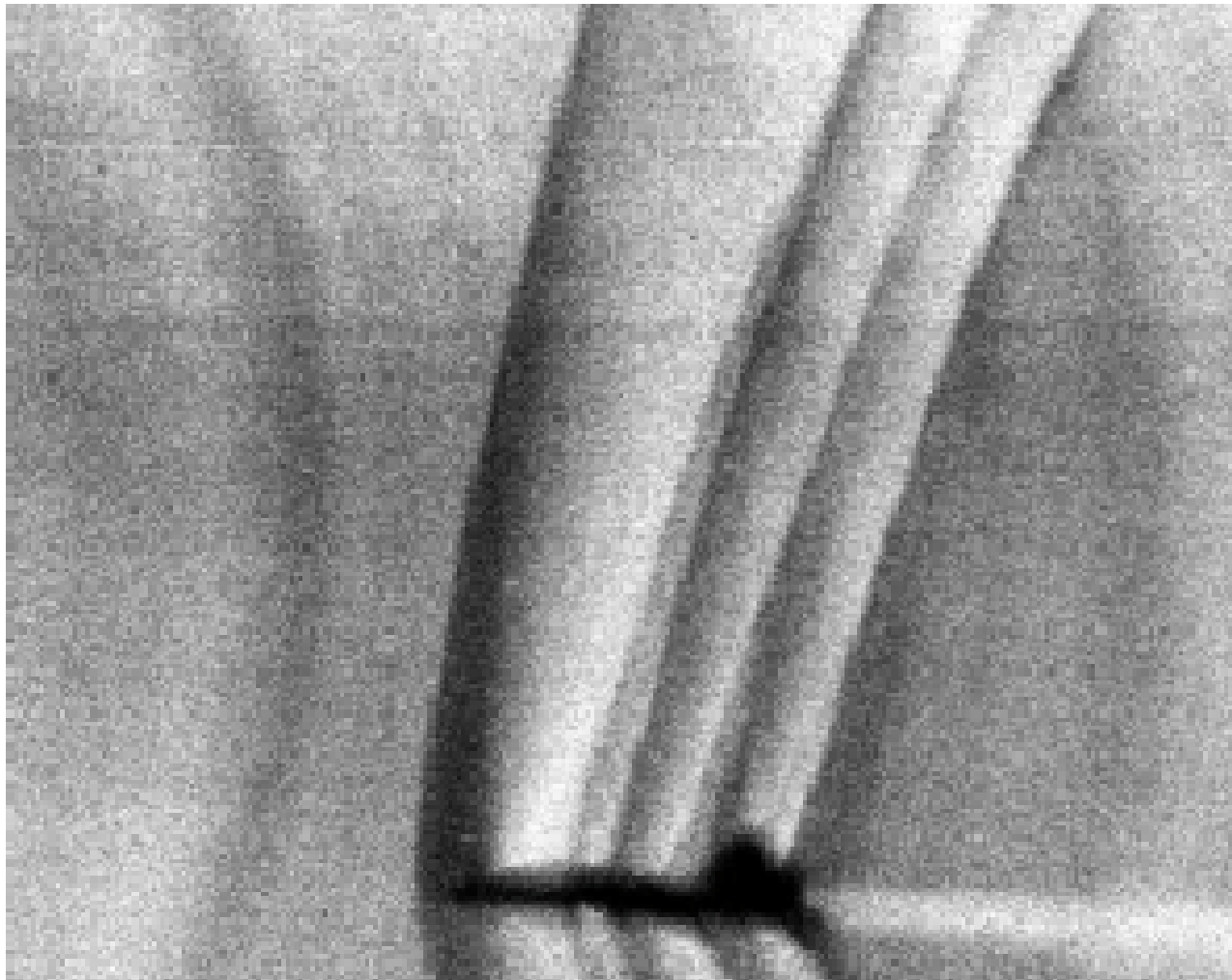












Schlieren photograph of shockwaves produced by a T-38 while flying Mach 1.1 at 13,000 feet.

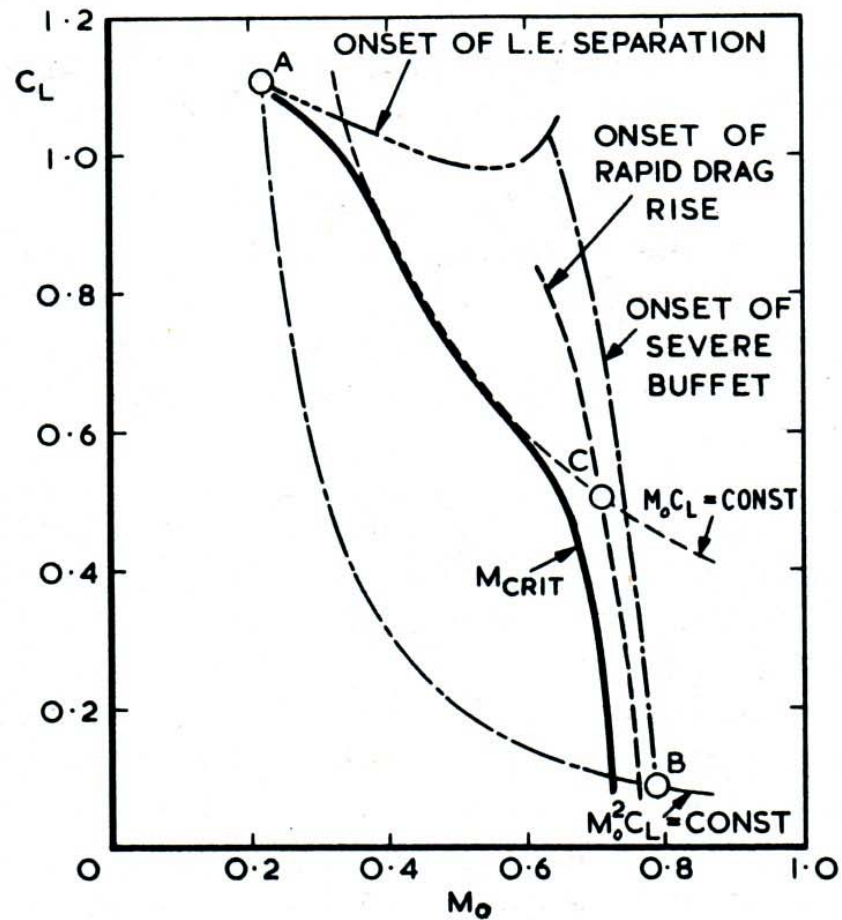


Fig. 4.59 Some significant flow boundaries on a twodimensional aerofoil. After Pearcey & Osborne (1970)

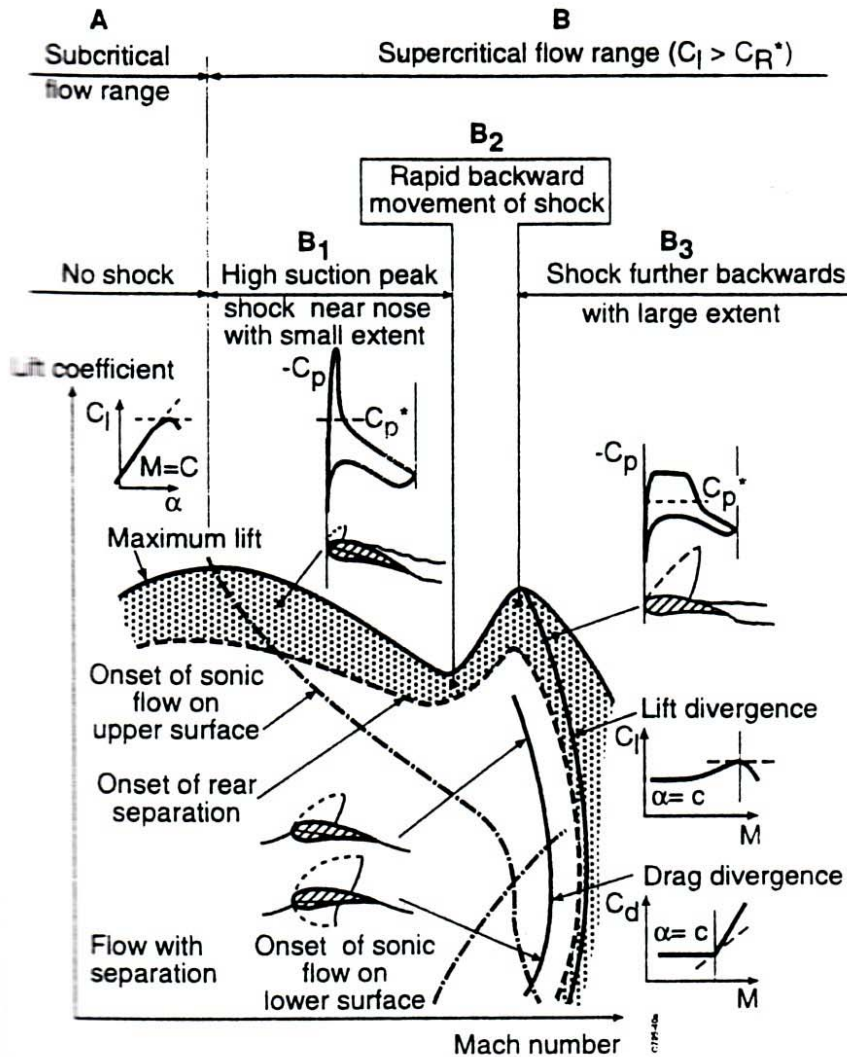
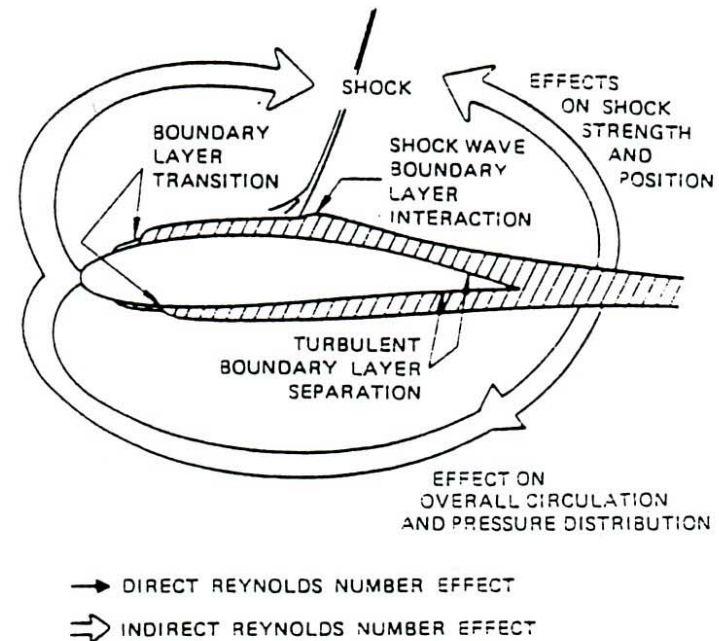
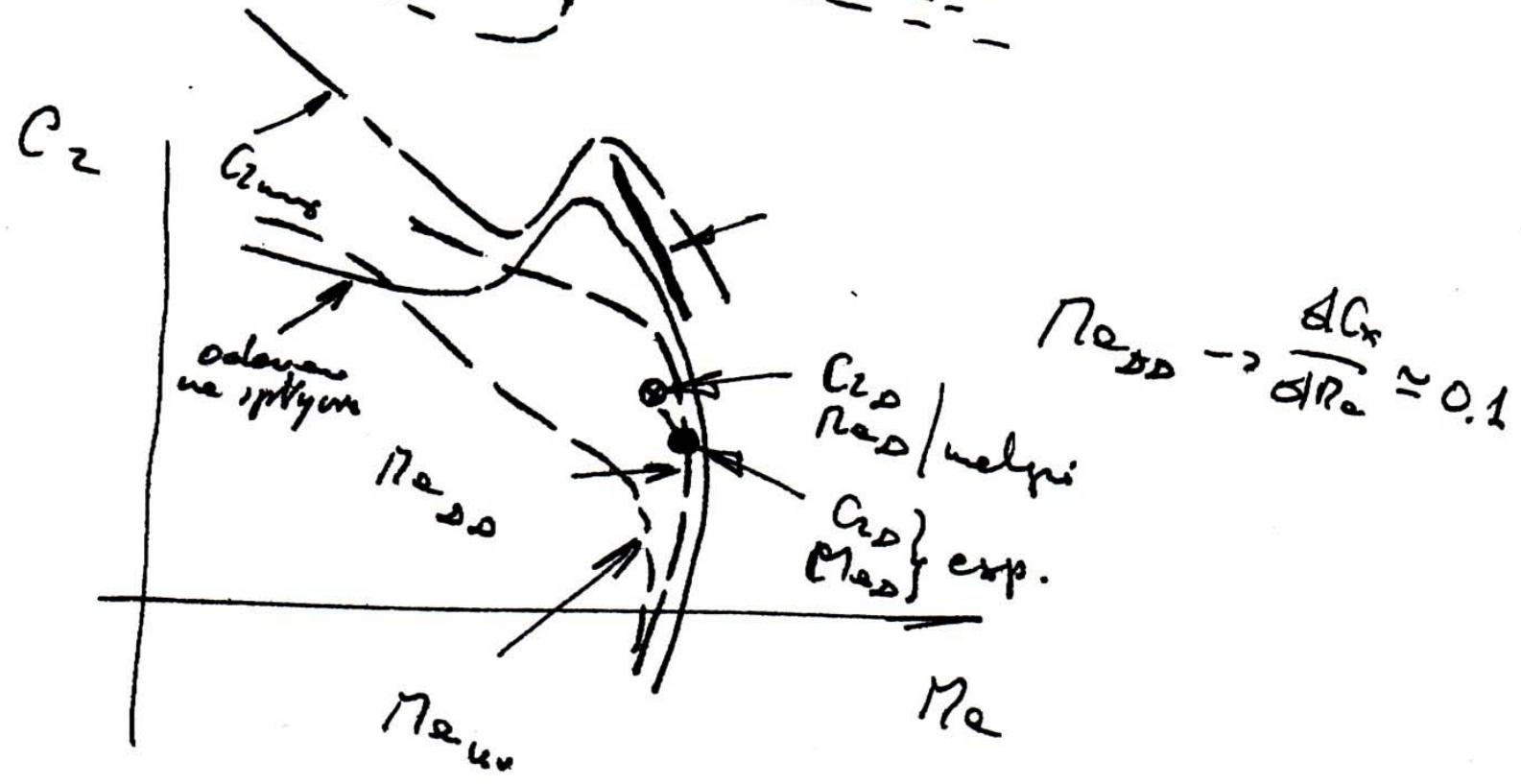


Figure 11: Airfoil characteristics in the C_L -Mach number plane.



CHARACTERISTIC	DOMINANT RE-NUMBER EFFECT	
	DIRECT	INDIRECT
LIFT AND PITCHING MOMENT		X
VISCOUS DRAG	X	
WAVE DRAG		X
DRAG DIVERGENCE		X
BOUNDARY LAYER SEPARATION	X	
BUFFET BOUNDARY	X	X

Figure 12: Schematic representation of direct and indirect Reynolds number effects (Elsenaar, 1989)

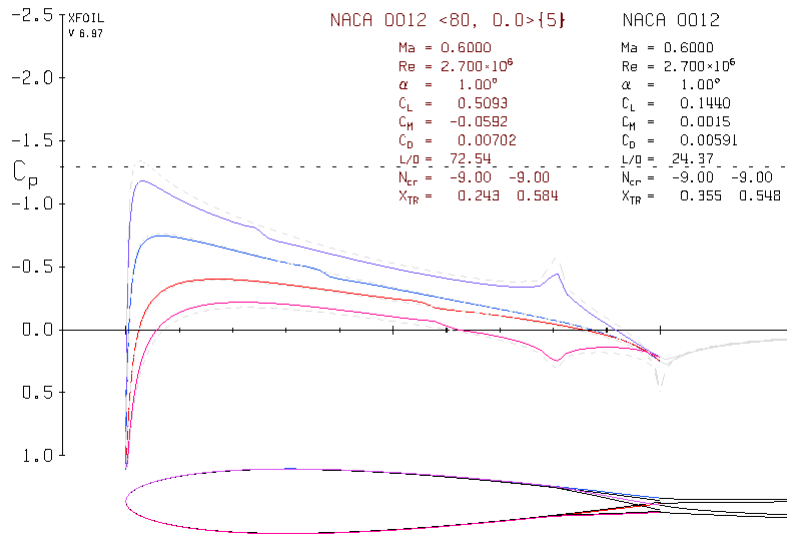


XFOIL
v 6.97

NACA 0012 <80, 0.0>{5}

Ma = 0.6000	Ma = 0.6000
Re = 2.700*10 ⁶	Re = 2.700*10 ⁶
α = 1.00°	α = 1.00°
C _L = 0.5093	C _L = 0.1440
C _H = -0.0592	C _H = 0.0015
C _D = 0.00702	C _D = 0.00591
L/D = 72.54	L/D = 24.37
N _{cr} = -9.00 -9.00	N _{cr} = -9.00 -9.00
X _{TR} = 0.243 0.584	X _{TR} = 0.355 0.548

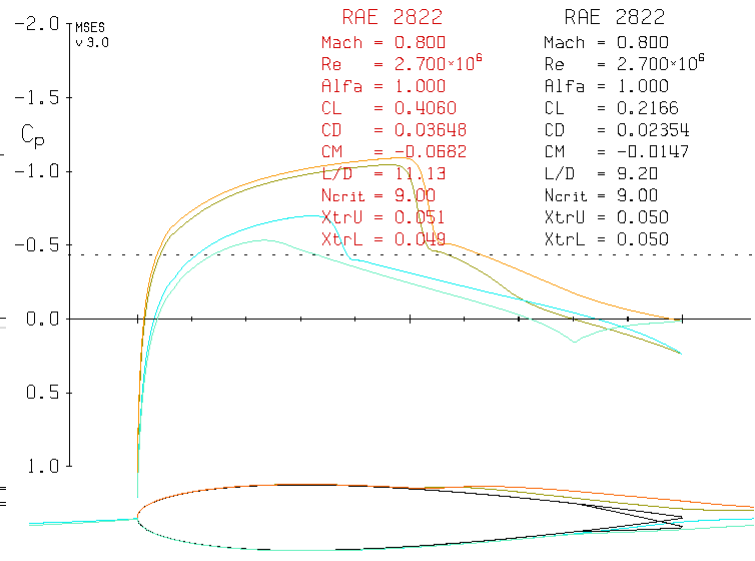
NACA 0012

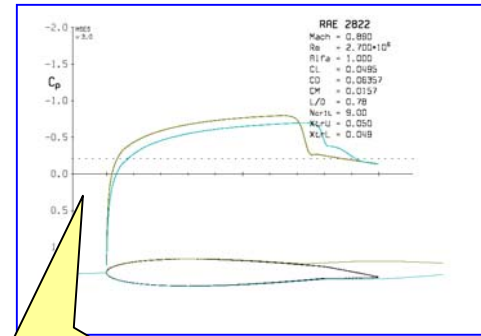
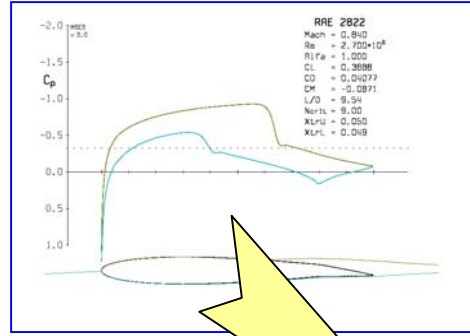
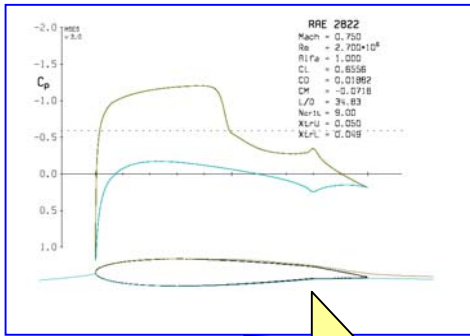


MSES
v 3.0

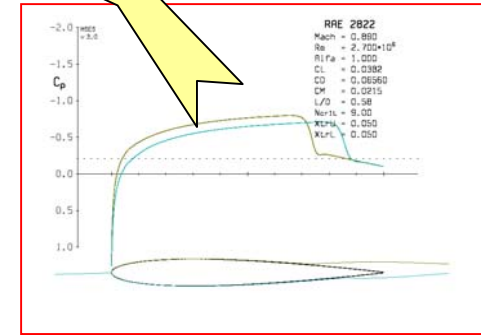
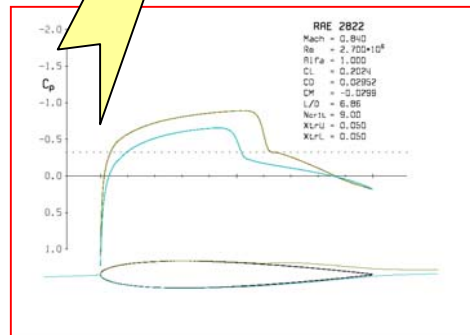
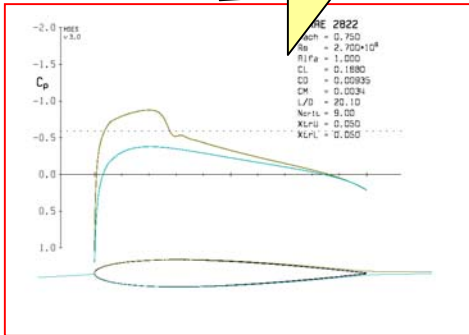
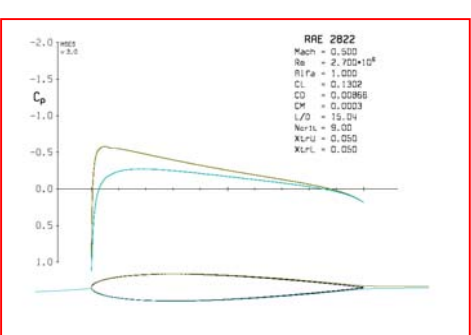
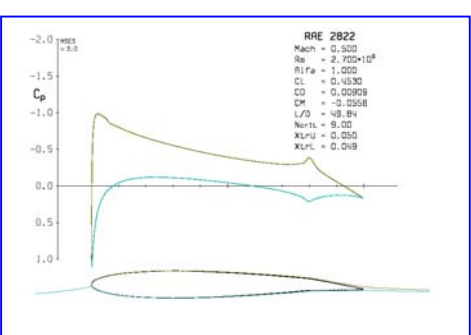
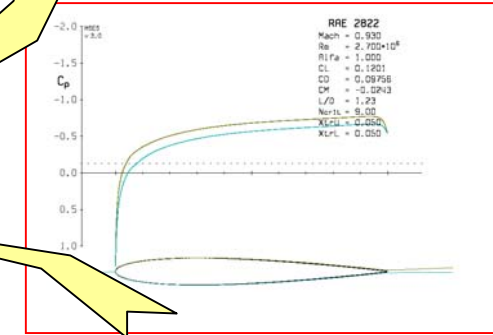
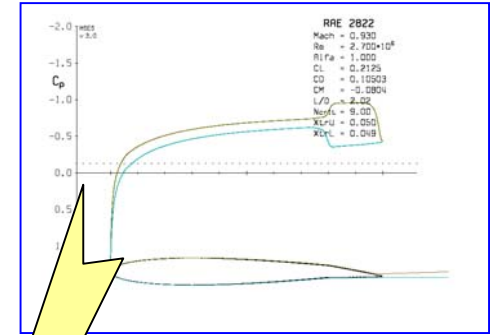
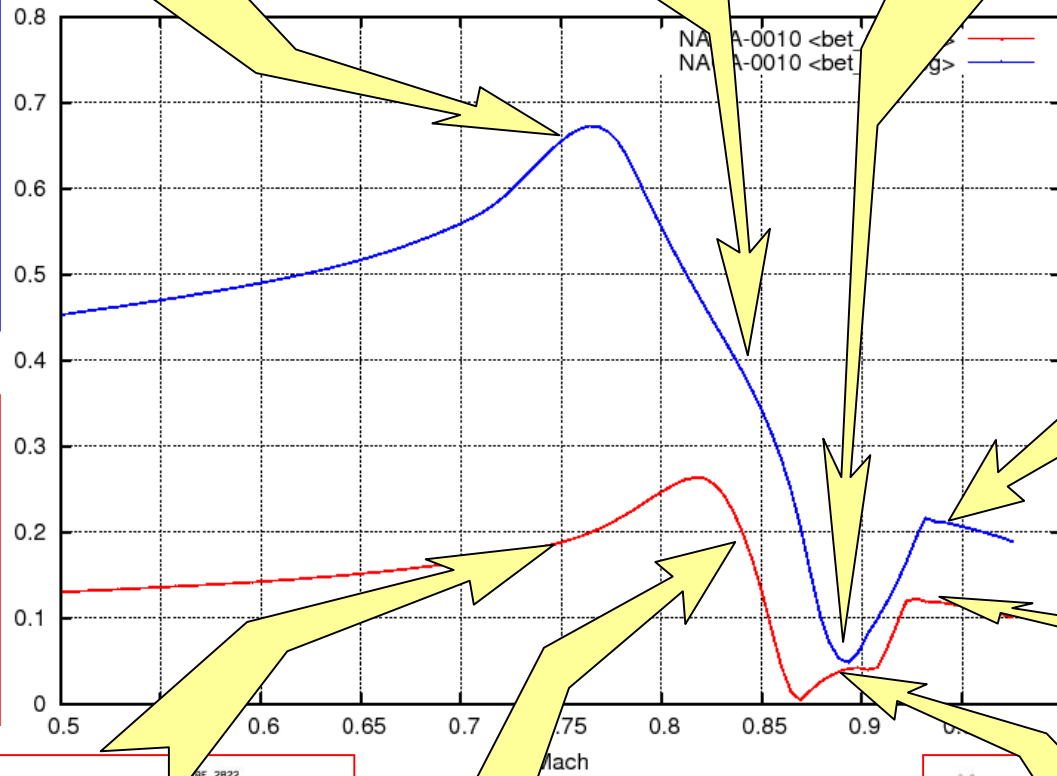
RAE 2822

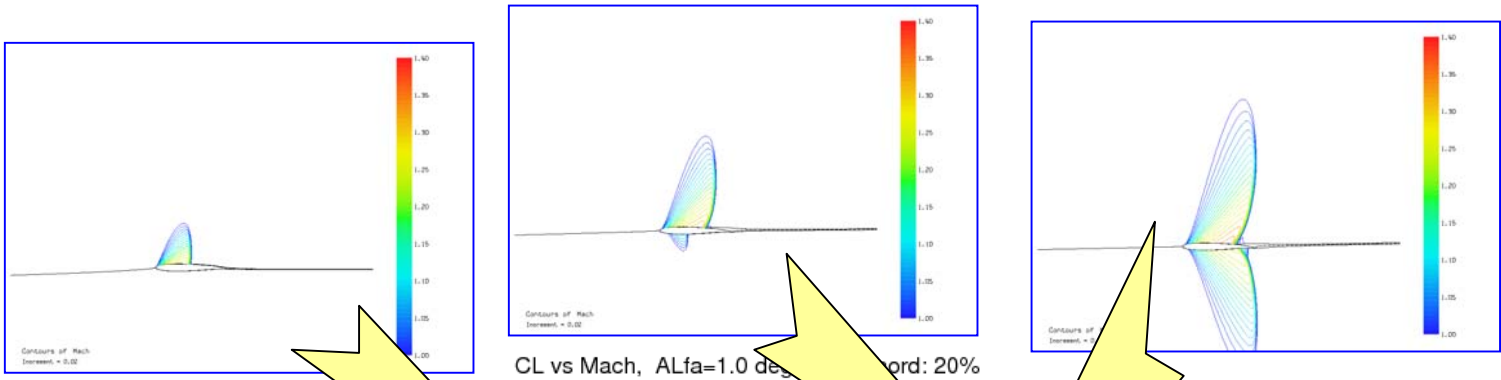
Mach = 0.800	Mach = 0.800
Re = 2.700*10 ⁶	Re = 2.700*10 ⁶
Alfa = 1.000	Alfa = 1.000
C _L = 0.4060	C _L = 0.2166
C _D = 0.03648	C _D = 0.02354
C _M = -0.0682	C _M = -0.0147
L/D = 11.13	L/D = 9.20
N _{crit} = 9.00	N _{crit} = 9.00
X _{trU} = 0.051	X _{trU} = 0.050
X _{trL} = 0.049	X _{trL} = 0.050



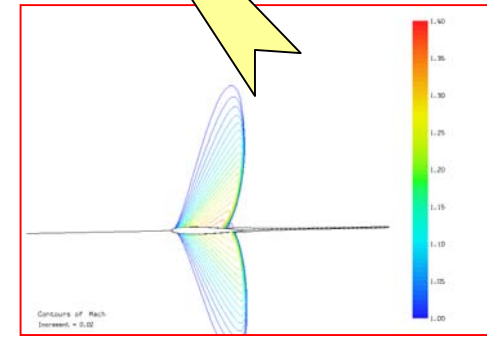
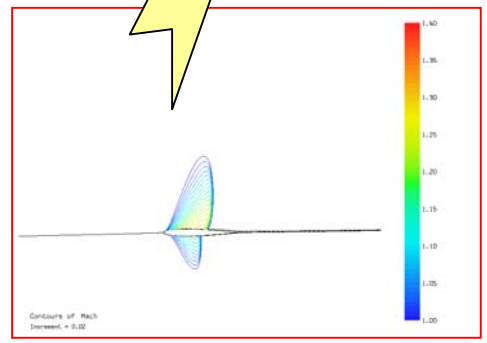
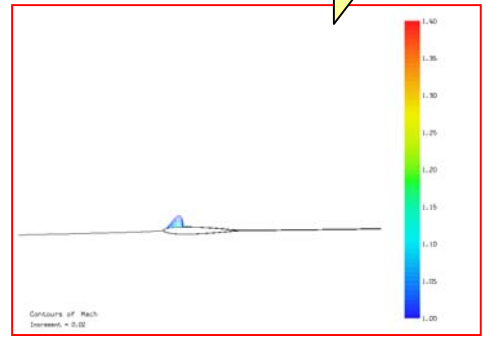
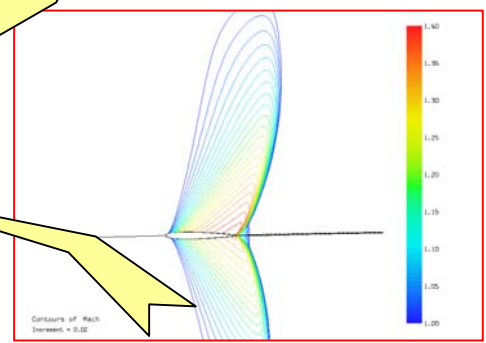
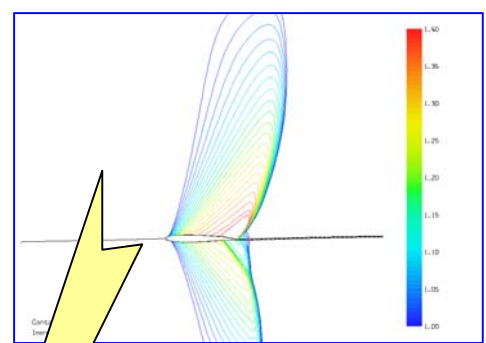
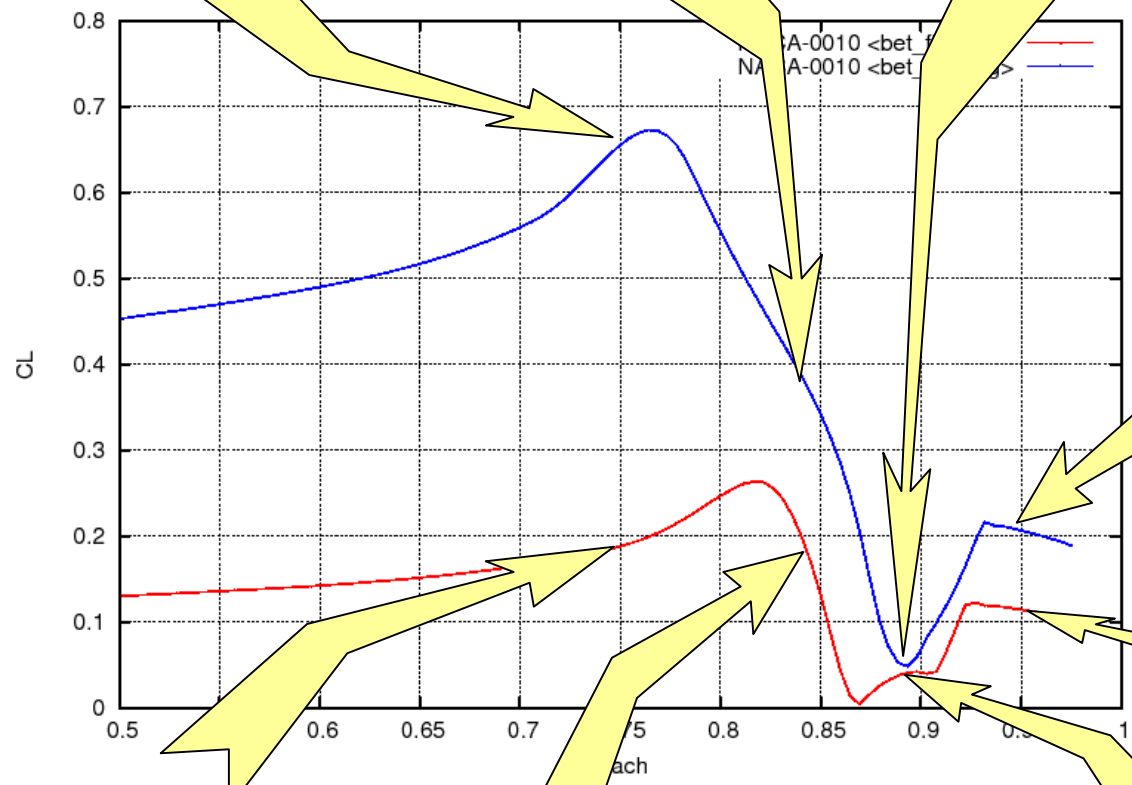


CL vs Mach, ALfa=1.0 deg, $Re = 2.7 \times 10^8$

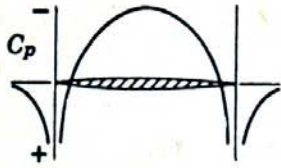




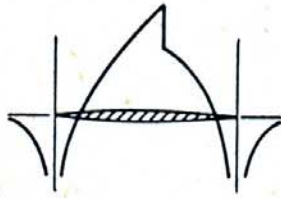
CL vs Mach, ALfa=1.0 deg, chord: 20%



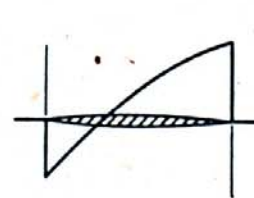
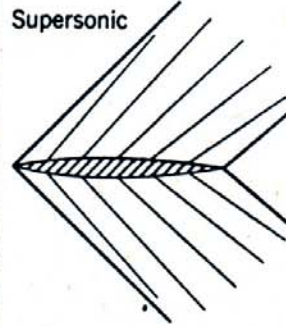
Subsonic



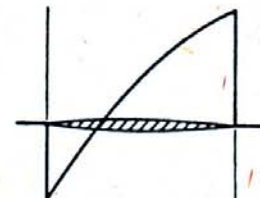
Transonic

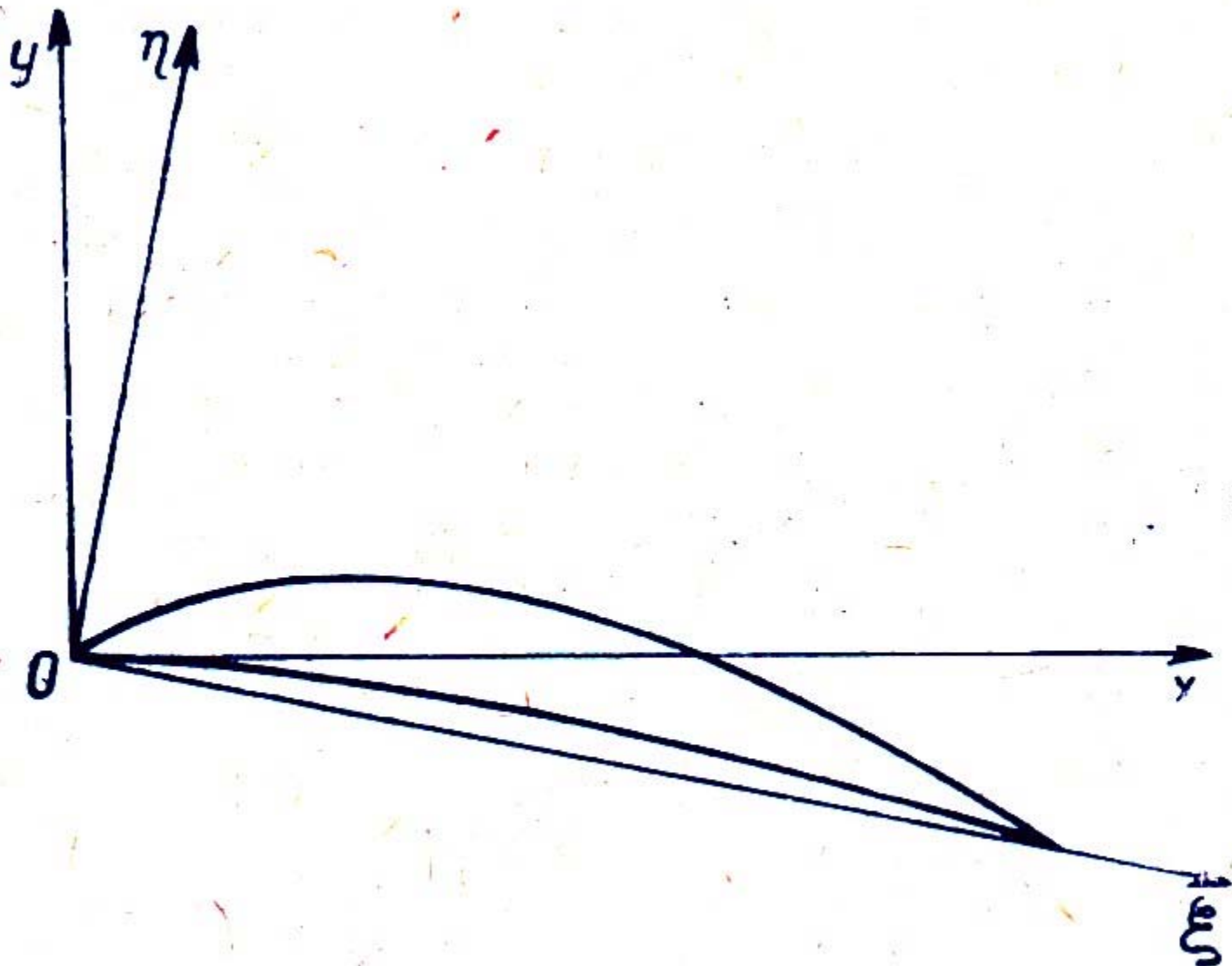


Supersonic

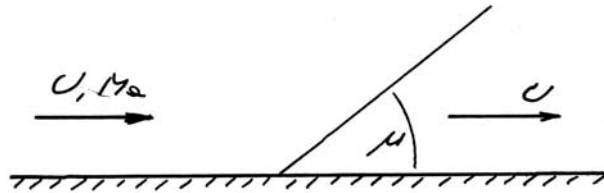


Hypersonic



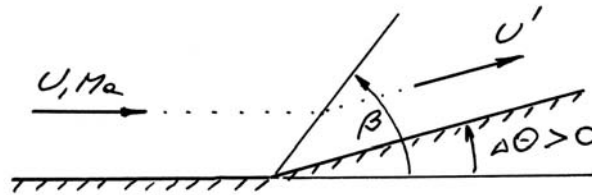


SUPERSONIC FLOW- SMALL DISTURBANCES

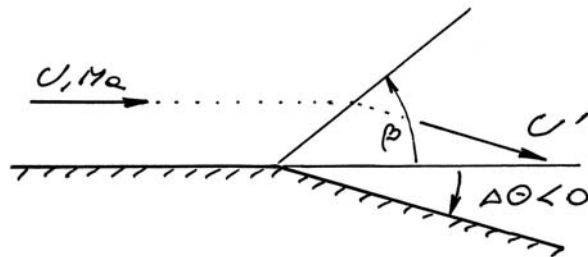


$$\mu = \arcsin\left(\frac{1}{Ma}\right)$$

$$\Delta S = 0 \quad \beta = \mu$$



$$\Delta S > 0 \quad \beta > \mu$$



$$? \quad \Delta S < 0 \quad \beta ?$$

$$V_s \equiv V'_s = V \cos(\beta) = V' \cos(\beta - \theta)$$

$$V' = V + \Delta V$$

$$V \cos(\beta) = (V + \Delta V) \left(\cos(\beta) \underbrace{\cos(\Delta\theta)}_{\sim 1} + \sin(\beta) \underbrace{\sin(\Delta\theta)}_{\sim \Delta\theta} \right)$$

$$\Delta V \cos(\beta) + V \sin(\beta) \Delta\theta \approx 0$$

$$\frac{\Delta V}{V} = -\tan(\beta) \cdot \Delta\theta$$

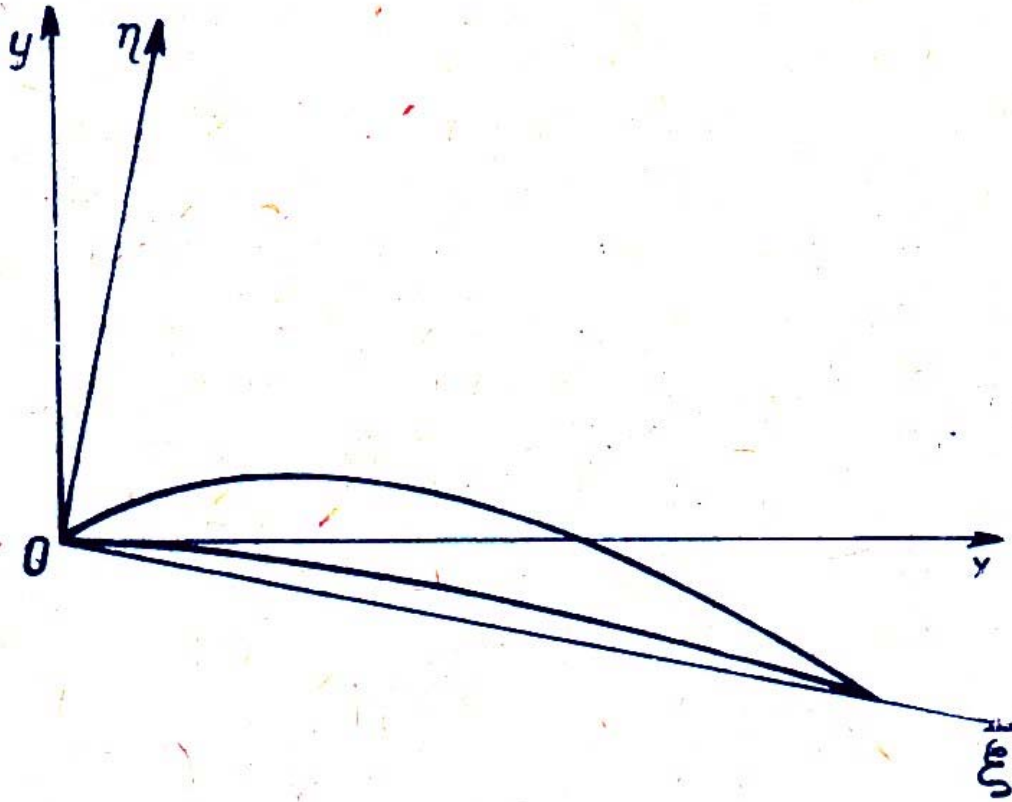
$$|\Delta\theta| \ll 1 \rightarrow \Delta S = 0 \rightarrow \beta = \mu = \arcsin\left(\frac{1}{Ma}\right)$$

$$\sin \mu = \frac{1}{Ma_\infty}$$

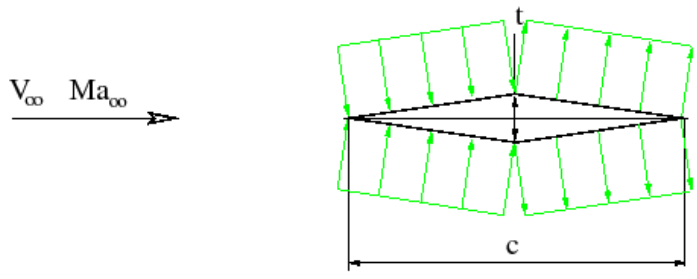
$$\cos \mu = \frac{\sqrt{Ma_\infty^2 - 1}}{Ma_\infty}$$

$$\tan \mu = \frac{1}{\sqrt{Ma_\infty^2 - 1}}$$

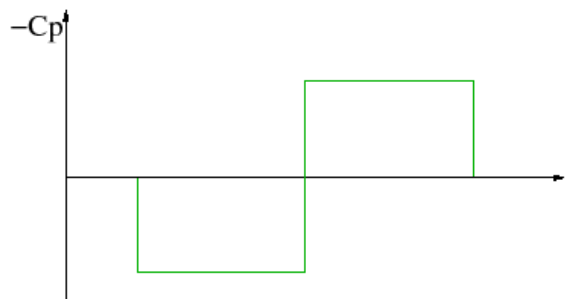
$$C_p = -2 \frac{u}{U_\infty} \simeq -2 \frac{\Delta V}{V} = 2 \tan(\mu) \cdot \Delta\theta = \frac{2 \cdot \Delta\theta}{\sqrt{Ma_\infty^2 - 1}}$$



$$C_p^{U/L} = \pm \frac{2y'}{\sqrt{Ma_\infty^2 - 1}}$$



$$y' = \pm t / c = \pm \bar{t}$$

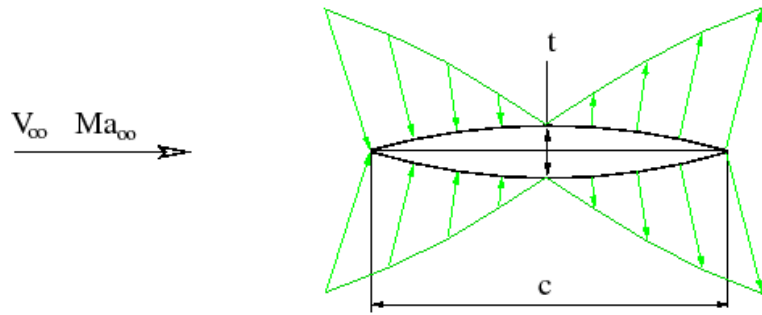


$$Cp^{U/L} = \pm \frac{2 \cdot \bar{t}}{\sqrt{Ma_\infty^2 - 1}}$$

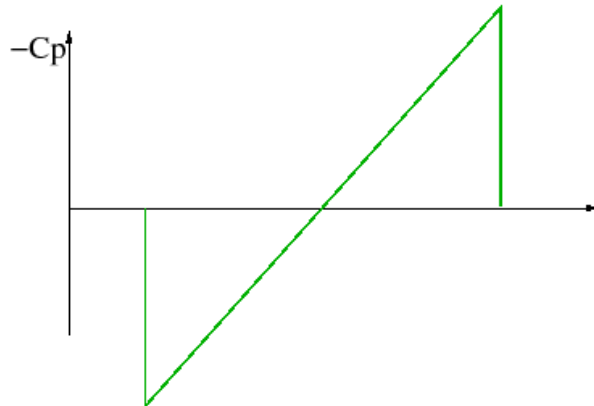
$$C_L = \frac{1}{c} \int_0^c (Cp^L - Cp^U) dx = \int_0^1 (Cp^L - Cp^U) d(x/c) = 0$$

$$C_D = \frac{1}{c} \int_0^c \left(Cp^U \frac{dy}{dx} \Big|_U - Cp^L \frac{dy}{dx} \Big|_L \right) dx = \int_0^1 \left(Cp^U \frac{dy}{dx} \Big|_U - Cp^L \frac{dy}{dx} \Big|_L \right) d(x/c) =$$

$$= \dots = \frac{4 \cdot \bar{t}^2}{\sqrt{Ma_\infty^2 - 1}}$$



$$y' = \pm 2 \cdot \bar{t} \cdot (1 - 2\bar{x})$$

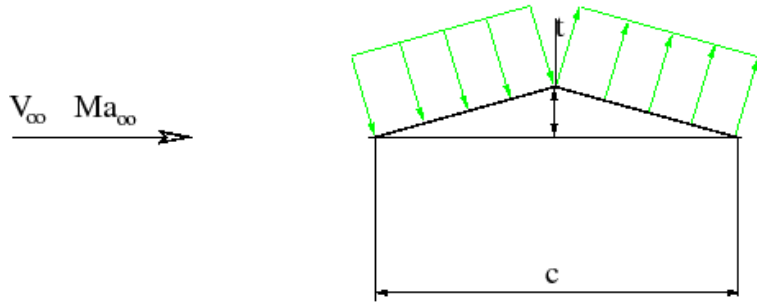


$$Cp^{U/L} = \pm \frac{4 \cdot \bar{t} (1 - 2\bar{x})}{\sqrt{Ma_\infty^2 - 1}}$$

$$C_L = \frac{1}{c} \int_0^c (Cp^L - Cp^U) dx = \int_0^1 (Cp^L - Cp^U) d(x/c) = 0$$

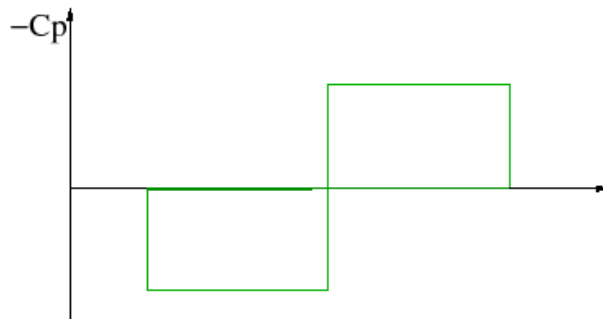
$$C_D = \frac{1}{c} \int_0^c \left(Cp^U \frac{dy}{dx} \Big|_U - Cp^L \frac{dy}{dx} \Big|_L \right) dx = \int_0^1 \left(Cp^U \frac{dy}{dx} \Big|_U - Cp^L \frac{dy}{dx} \Big|_L \right) d(x/c) =$$

$$= \dots = \frac{5.333 \cdot \bar{t}^2}{\sqrt{Ma_\infty^2 - 1}}$$



$$y' = \pm 2 \cdot t / c = \pm 2 \cdot \bar{t}$$

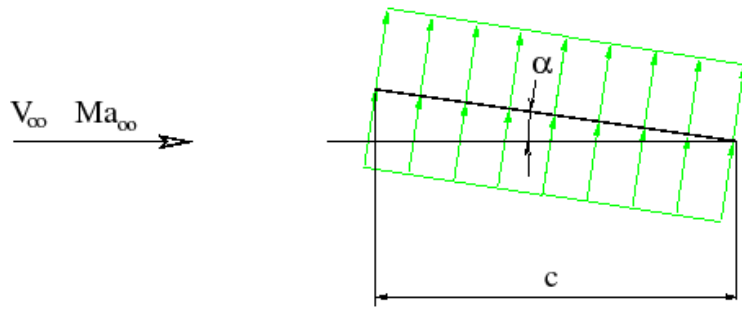
$$Cp^U = \pm \frac{4 \cdot \bar{t}}{\sqrt{Ma_\infty^2 - 1}}$$



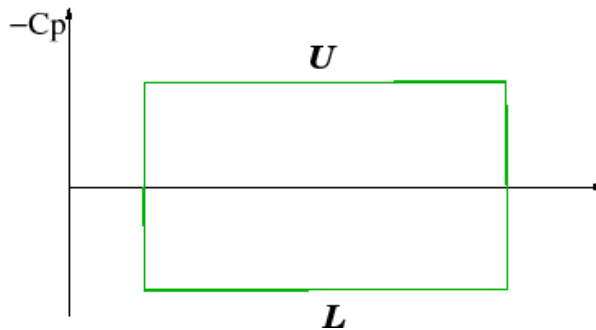
$$C_L = \frac{1}{c} \int_0^c (Cp^L - Cp^U) dx = \int_0^1 (Cp^L - Cp^U) d(x/c) = 0$$

$$C_D = \frac{1}{c} \int_0^c \left(Cp^U \frac{dy}{dx} \Big|_U - Cp^L \frac{dy}{dx} \Big|_L \right) dx = \int_0^1 \left(Cp^U \frac{dy}{dx} \Big|_U - Cp^L \frac{dy}{dx} \Big|_L \right) d(x/c) =$$

$$= \dots = \frac{8 \cdot \bar{t}^2}{\sqrt{Ma_\infty^2 - 1}}$$



$$y' = \alpha$$

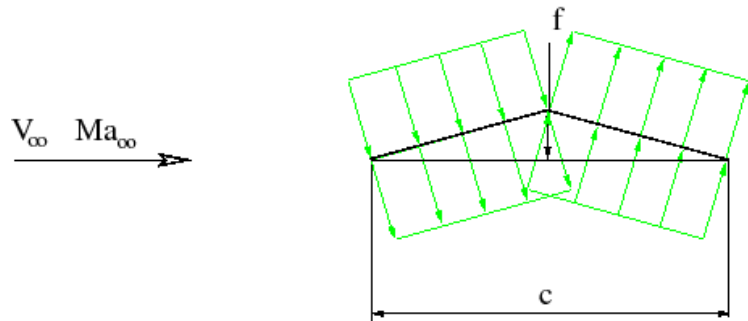


$$C_p^{U/L} = \pm \frac{2 \cdot \alpha}{\sqrt{Ma_\infty^2 - 1}}$$

$$C_L = \frac{1}{c} \int_0^c (C_p^L - C_p^U) dx = \int_0^1 (C_p^L - C_p^U) d(x/c) = \frac{4 \alpha}{\sqrt{Ma_\infty^2 - 1}}$$

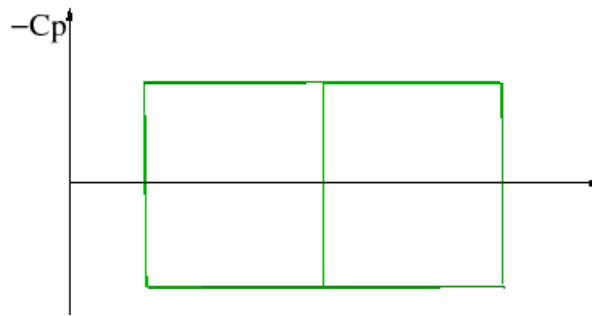
$$C_D = \frac{1}{c} \int_0^c \left(C_p^U \frac{dy}{dx} \Big|_U - C_p^L \frac{dy}{dx} \Big|_L \right) dx = \int_0^1 \left(C_p^U \frac{dy}{dx} \Big|_U - C_p^L \frac{dy}{dx} \Big|_L \right) d(x/c) =$$

$$= \dots = \frac{4 \alpha^2}{\sqrt{Ma_\infty^2 - 1}} = \frac{\sqrt{Ma_\infty^2 - 1}}{4} \cdot C_L^2$$



$$y' = \pm 2 \cdot f / c = \pm 2 \cdot \bar{f}$$

$$Cp^{U/L} = \pm \frac{4 \cdot \bar{f}}{\sqrt{Ma_\infty^2 - 1}}$$



$$C_L = \frac{1}{c} \int_0^c (Cp^L - Cp^U) dx = \int_0^1 (Cp^L - Cp^U) d(x/c) = 0$$

$$C_D = \frac{1}{c} \int_0^c \left(Cp^U \frac{dy}{dx} \Big|_U - Cp^L \frac{dy}{dx} \Big|_L \right) dx = \int_0^1 \left(Cp^U \frac{dy}{dx} \Big|_U - Cp^L \frac{dy}{dx} \Big|_L \right) d(x/c) =$$

$$= \dots = \frac{16 \cdot \bar{f}^2}{\sqrt{Ma_\infty^2 - 1}}$$

$$C_p^{U/L} = \pm \frac{2y'}{\sqrt{Ma_\infty^2 - 1}}$$

$$y^{U/L} = \pm y_t + y_c - x \cdot \alpha$$

$$C_p^{U/L} = \frac{2}{\sqrt{Ma_\infty^2 - 1}} (y'_t \pm y'_c \mp \alpha)$$

$$C_L = \frac{1}{c} \int_0^c (Cp^L - Cp^U) dx = \frac{2}{\sqrt{Ma_\infty^2 - 1}} \int_0^1 (-2y'_c + 2\alpha) d(x/c) =$$

$$= \frac{4\alpha}{\sqrt{Ma_\infty^2 - 1}} - \frac{4}{\sqrt{Ma_\infty^2 - 1}} \int_0^1 y'_c \cdot d(x/c) \equiv \frac{4\alpha}{\sqrt{Ma_\infty^2 - 1}}$$

$$C_D = \frac{1}{c} \int_0^c C_p^U y'_U \cdot dx - \frac{1}{c} \int_0^c C_p^L y'_L \cdot dx =$$

$$= \frac{2}{\sqrt{Ma_\infty^2 - 1}} \int_0^1 (y'_t + (y'_c - \alpha))(y'_t + (y'_c - \alpha)) d(x/c)$$

$$- \frac{2}{\sqrt{Ma_\infty^2 - 1}} \int_0^1 (y'_t - (y'_c - \alpha))(-y'_t + (y'_c - \alpha)) d(x/c) =$$

$$= \frac{2}{\sqrt{Ma_\infty^2 - 1}} \int_0^1 \left\{ [y'_t + (y'_c - \alpha)]^2 + [y'_t - (y'_c - \alpha)]^2 \right\} d(x/c) =$$

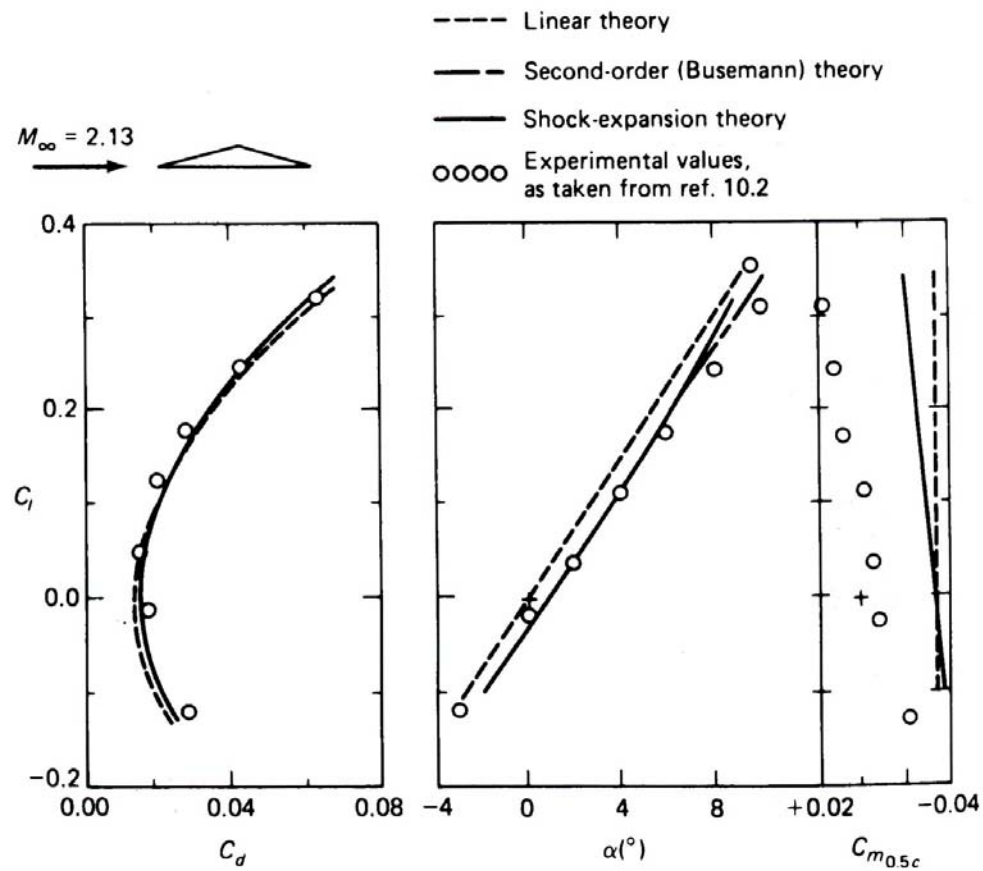
$$C_D = \dots = \frac{2}{\sqrt{Ma_\infty^2 - 1}} \int_0^1 \left\{ \left[y'_t + (y'_c - \alpha) \right]^2 + \left[y'_t - (y'_c - \alpha) \right]^2 \right\} d(x/c) =$$

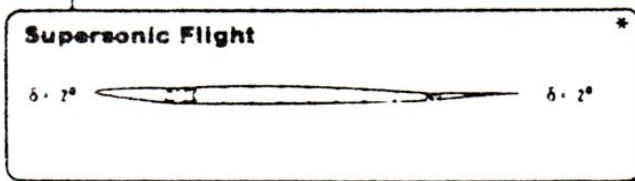
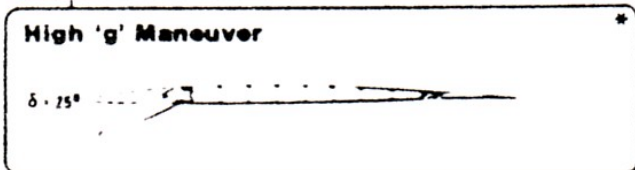
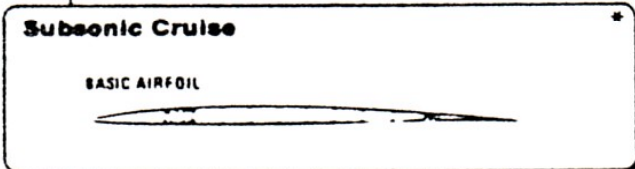
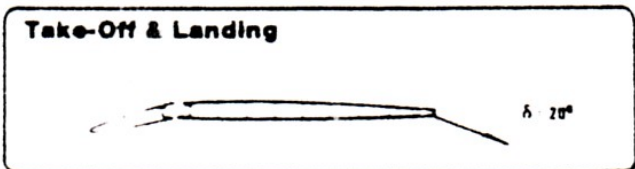
$$= \frac{4}{\sqrt{Ma_\infty^2 - 1}} \int_0^1 (y'_t)^2 d(x/c) + \frac{4}{\sqrt{Ma_\infty^2 - 1}} \int_0^1 (y'_c - \alpha)^2 d(x/c) =$$

$$= \frac{4}{\sqrt{Ma_\infty^2 - 1}} \int_0^1 \left[(y'_t)^2 + (y'_c)^2 \right] d(x/c) + \frac{4\alpha^2}{\sqrt{Ma_\infty^2 - 1}} =$$

$$= \underbrace{\frac{4}{\sqrt{Ma_\infty^2 - 1}} \int_0^1 \left[(y'_t)^2 + (y'_c)^2 \right] d(x/c)}_{C_{DW0}} + \underbrace{\frac{\sqrt{Ma_\infty^2 - 1}}{4} C_L^2}_{C_{DWL}}$$

$$C_p = \frac{2}{\sqrt{Ma_\infty^2 - 1}} y' + \left[\frac{(k+1) Ma_\infty^4 - 4(Ma_\infty^2 - 1)}{2(Ma_\infty^2 - 1)^2} \right] (y')^2$$

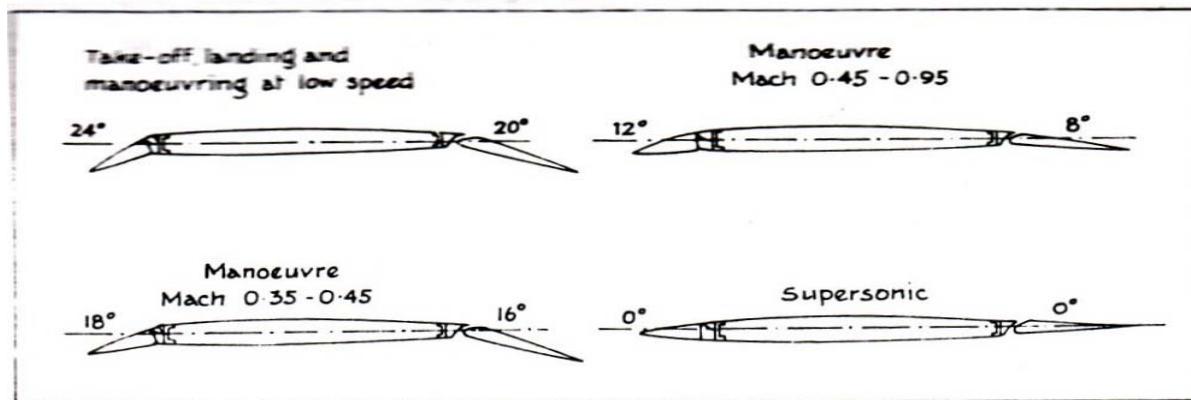


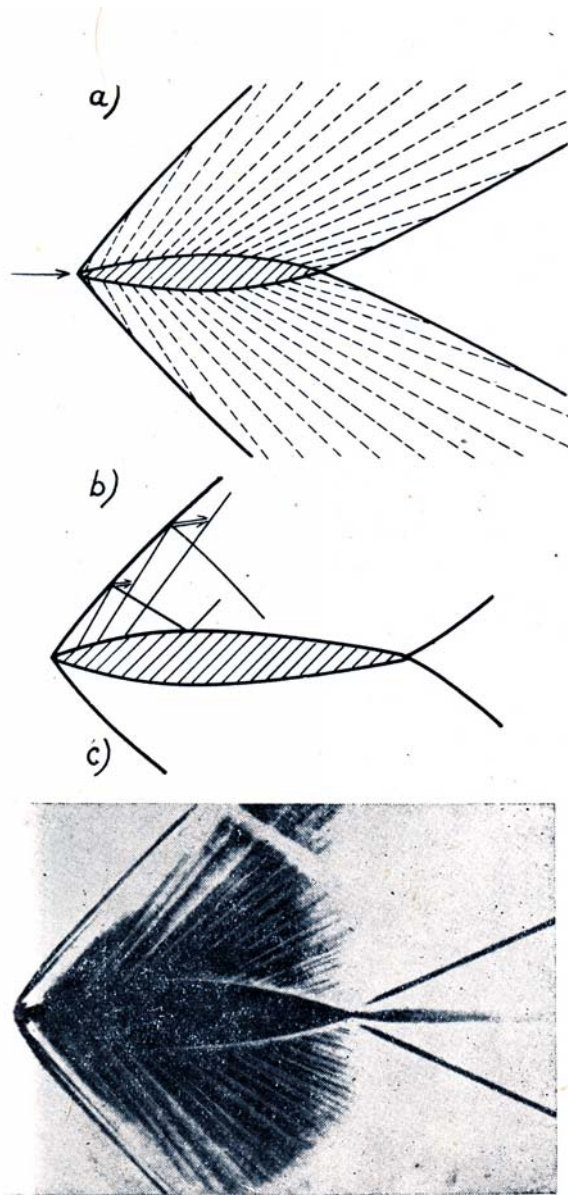


VARIABLE CAMBER SHAPES AIRFOIL TO MATCH FLIGHT CONDITION

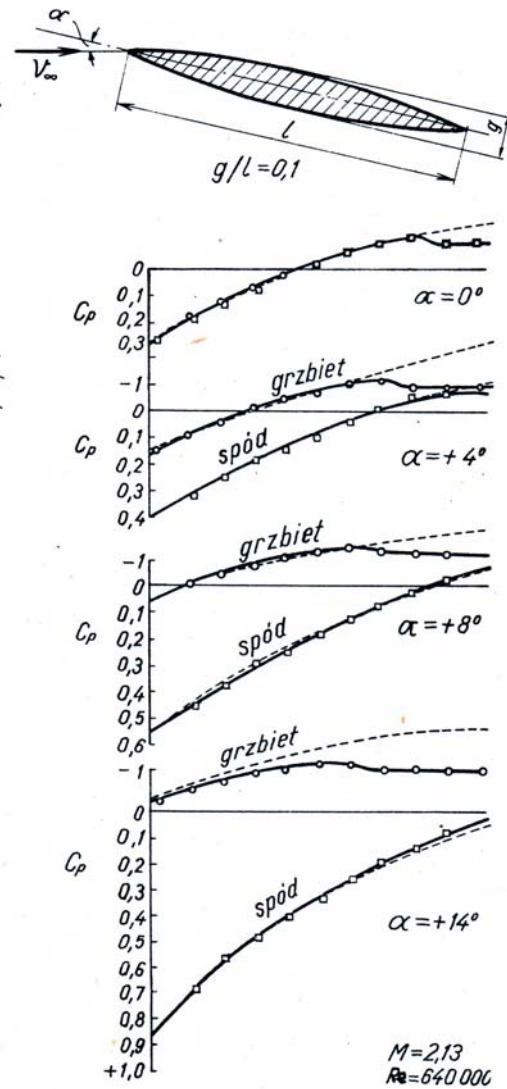
- Maximizes Lift/Drag Ratio
- Improves Directional Stability
- Minimizes Buffet

* LEADING EDGE FLAP AUTOMATICALLY PROGRAMMED FOR BEST FLAP POSITION (MAX LIFT/DRAG AS A FUNCTION OF MACH NUMBER AND ANGLE OF ATTACK)

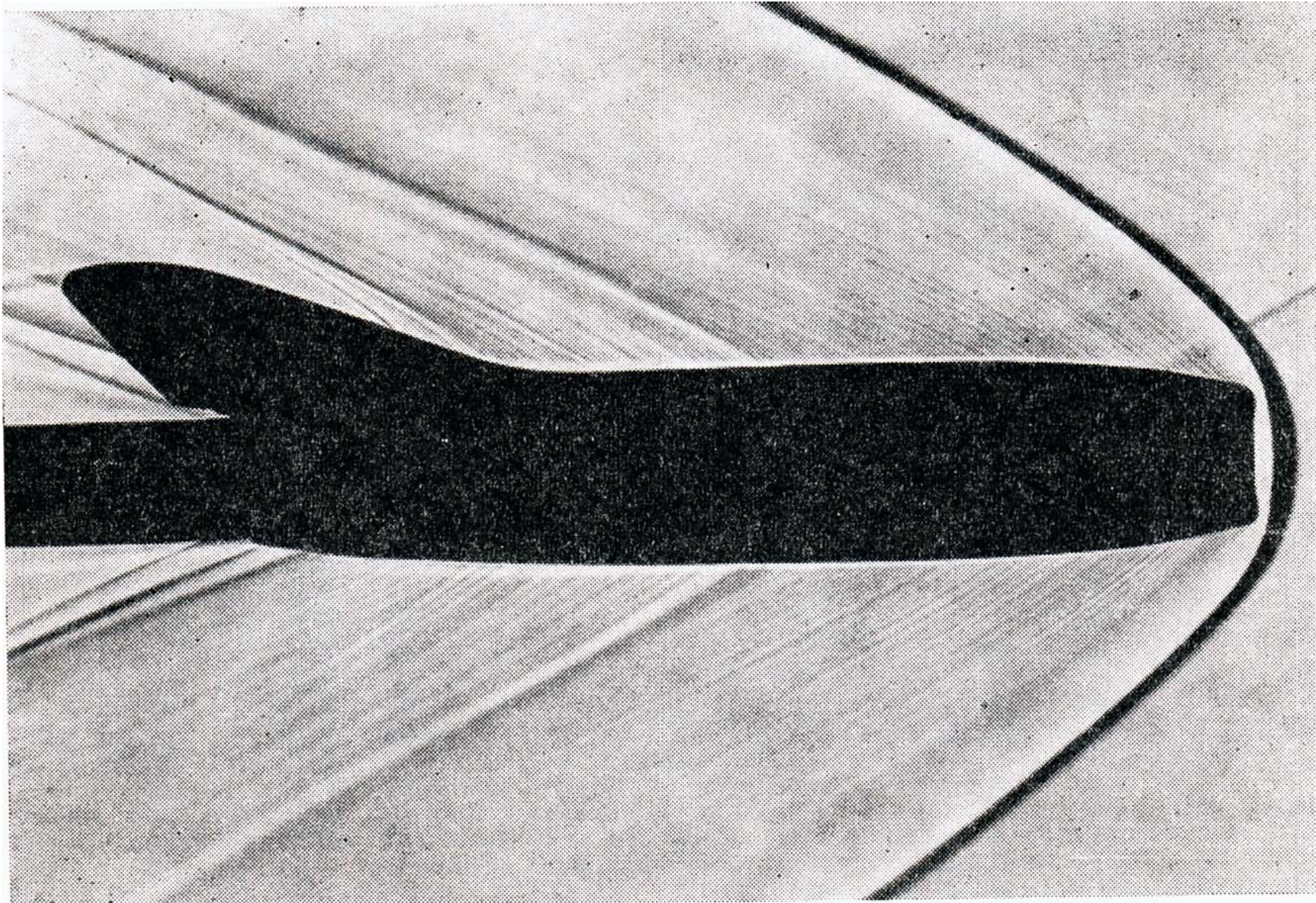


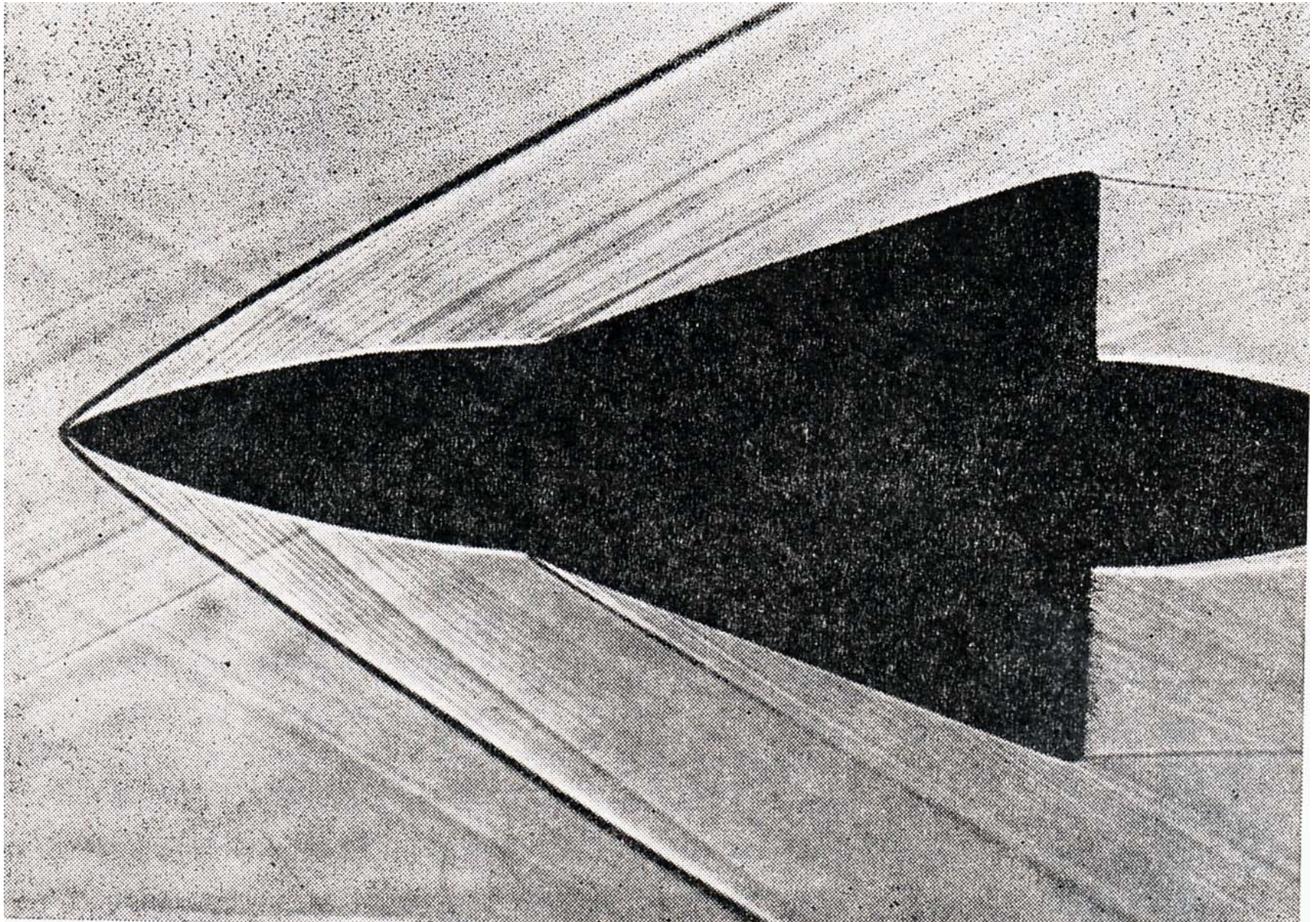


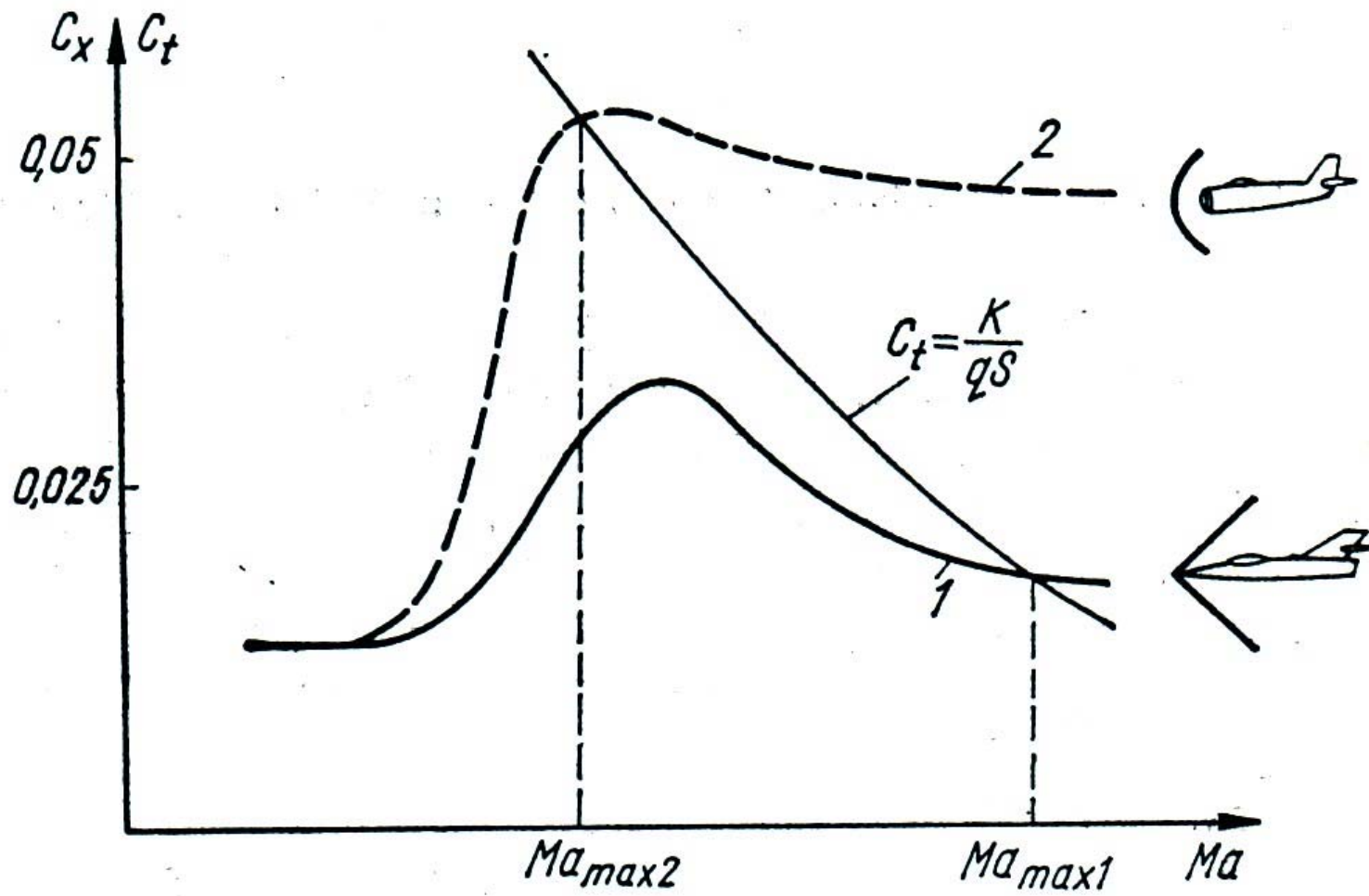
Rys. VI-71



Rys. VI-72







$$L = \frac{\rho_{\infty} V_{\infty}^2}{2} \cdot S \cdot C_L = k p_{\infty} \frac{\rho_{\infty} V_{\infty}^2}{k p_{\infty}} \frac{V_{\infty}^2}{2} \cdot S \cdot C_L = \frac{k p_{\infty}}{2} V_{\infty}^2 / a_{\infty}^2 \cdot S \cdot C_L = \frac{k p_{\infty}}{2} \cdot Ma_{\infty}^2 \cdot S \cdot C_L$$

$$\frac{L/S}{p_{\infty}} = \frac{k}{2} \cdot (Ma_{\infty}^2 \cdot C_L)$$

$$C_p = \frac{1}{\frac{k}{2} Ma_{\infty}^2} \left[\left(\frac{1 + \frac{k-1}{2} Ma_{\infty}^2}{1 + \frac{k-1}{2} Ma^2} \right)^{\frac{k}{k-1}} - 1 \right]$$

$$C_{p_{VAC}} = |Ma \rightarrow \infty| = -\frac{2}{k Ma_{\infty}^2}$$

$$C_{p_{STAG}} = |Ma \rightarrow 0| = \frac{2}{k Ma_{\infty}^2} \left[\left(1 + \frac{k-1}{2} Ma_{\infty}^2 \right)^{\frac{k}{k-1}} - 1 \right]$$

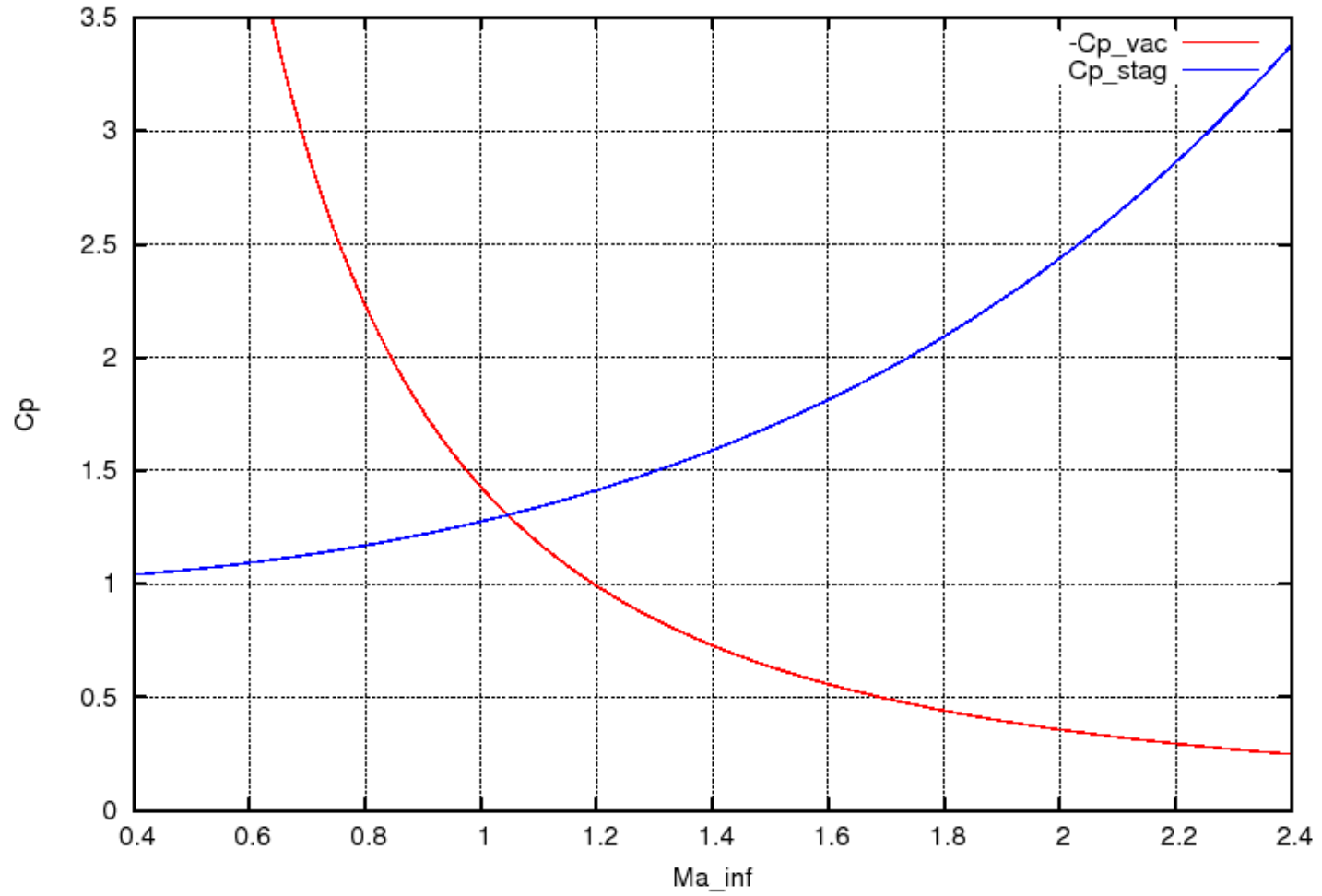
$$C_{L_{MAX}} = C_{p_{STAG}} - C_{p_{VAC}} = \frac{2}{k Ma_{\infty}^2} \left(1 + \frac{k-1}{2} Ma_{\infty}^2\right)^{\frac{k}{k-1}}$$

$$C_{L_{MAX}} Ma_{\infty}^2 = \frac{2}{k} \left(1 + \frac{k-1}{2} Ma_{\infty}^2\right)^{\frac{k}{k-1}} = |AIR| = \frac{1}{0.7} \left(1 + 0.2 Ma_{\infty}^2\right)^{3.5}$$

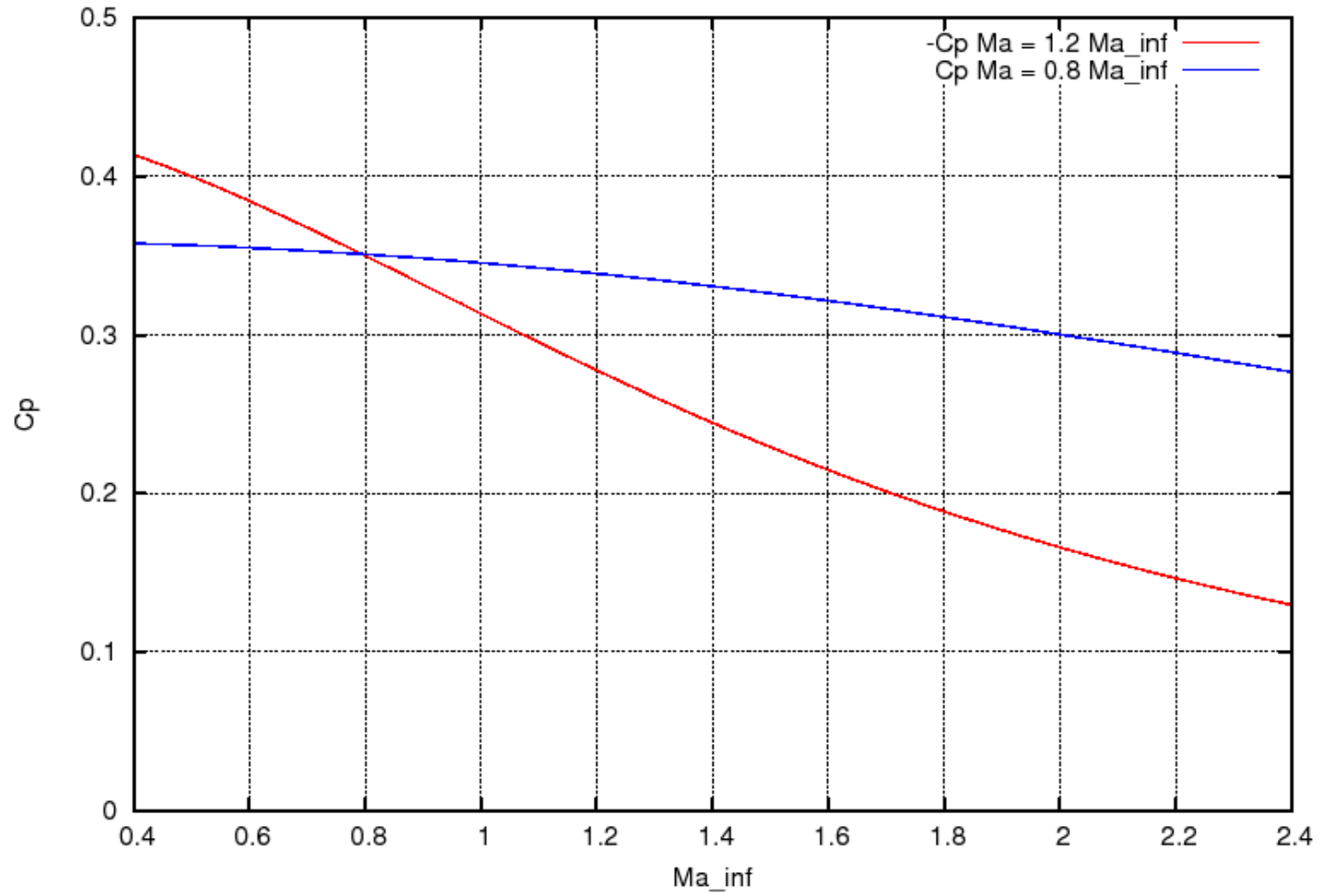
$$\left(\frac{L/S}{P_{\infty}}\right)_{MAX} = \left(1 + 0.2 Ma_{\infty}^2\right)^{3.5} / 0.7$$

$$\left(\frac{L/S}{P_{\infty}}\right)_{MAX} \approx 0.5 \rightarrow C_{L_{MAX}} Ma_{\infty}^2 \approx 0.7 \quad (1.2)$$

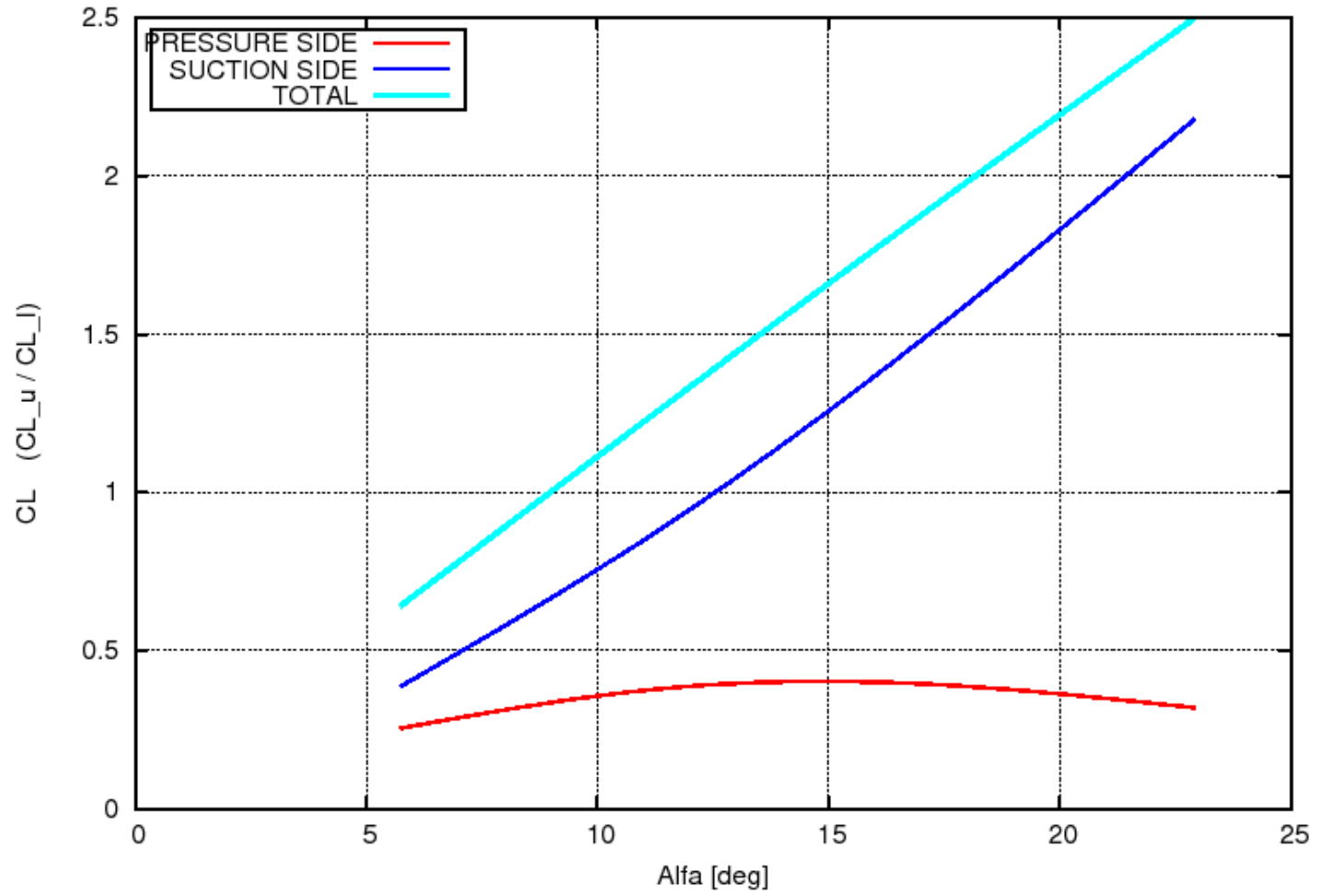
Cp LIMITS



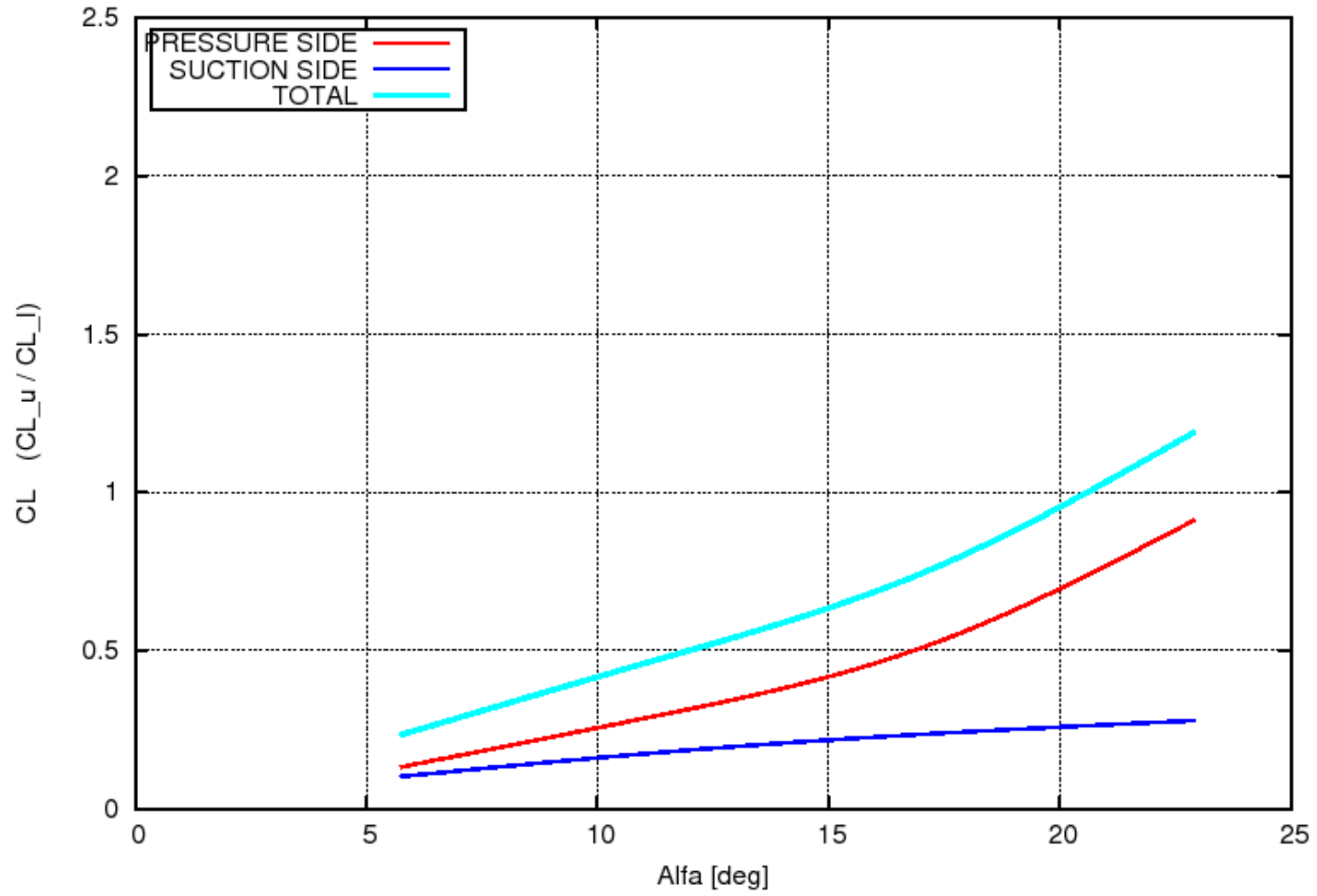
Cp LIMITS



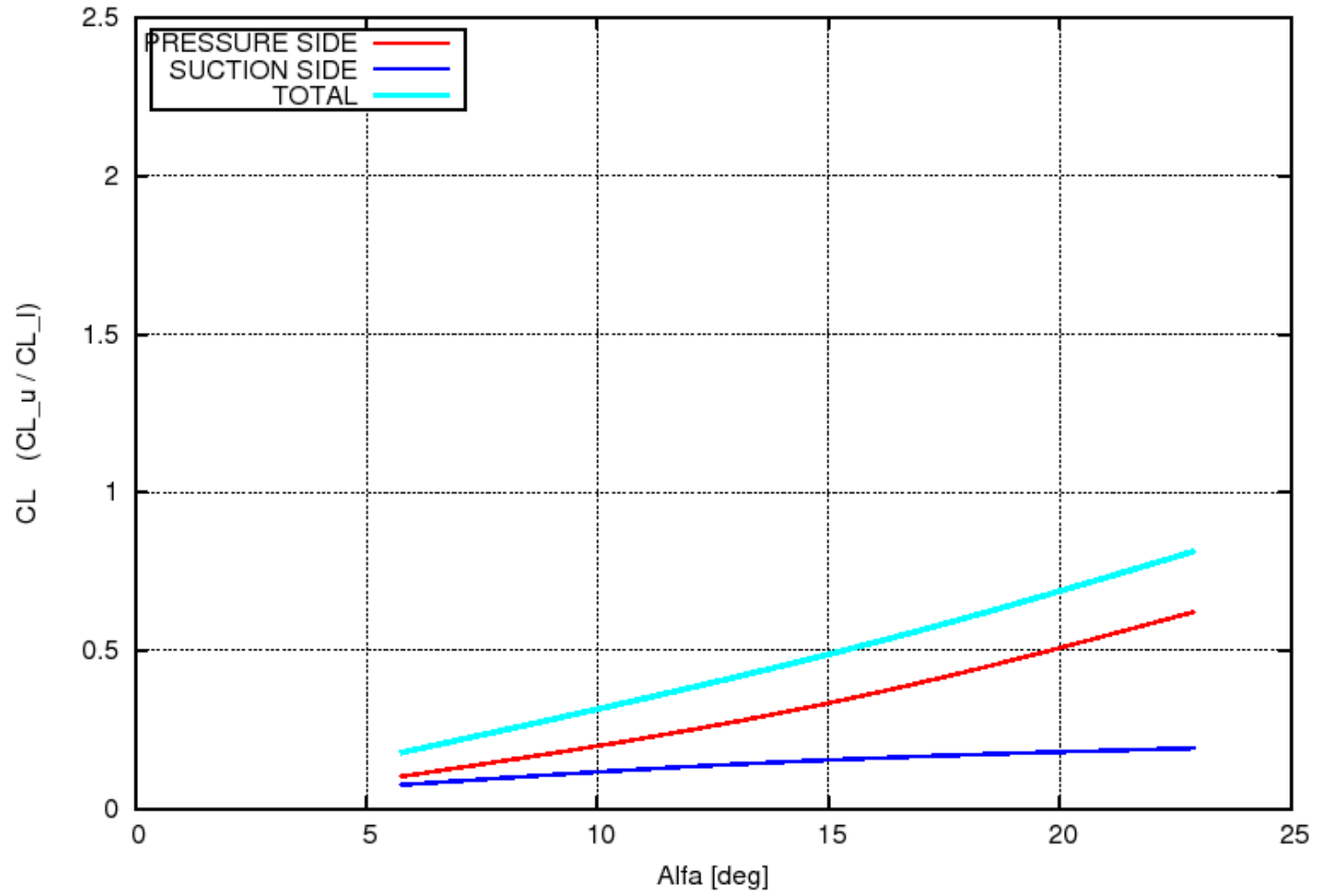
MACH_inf = 0.0



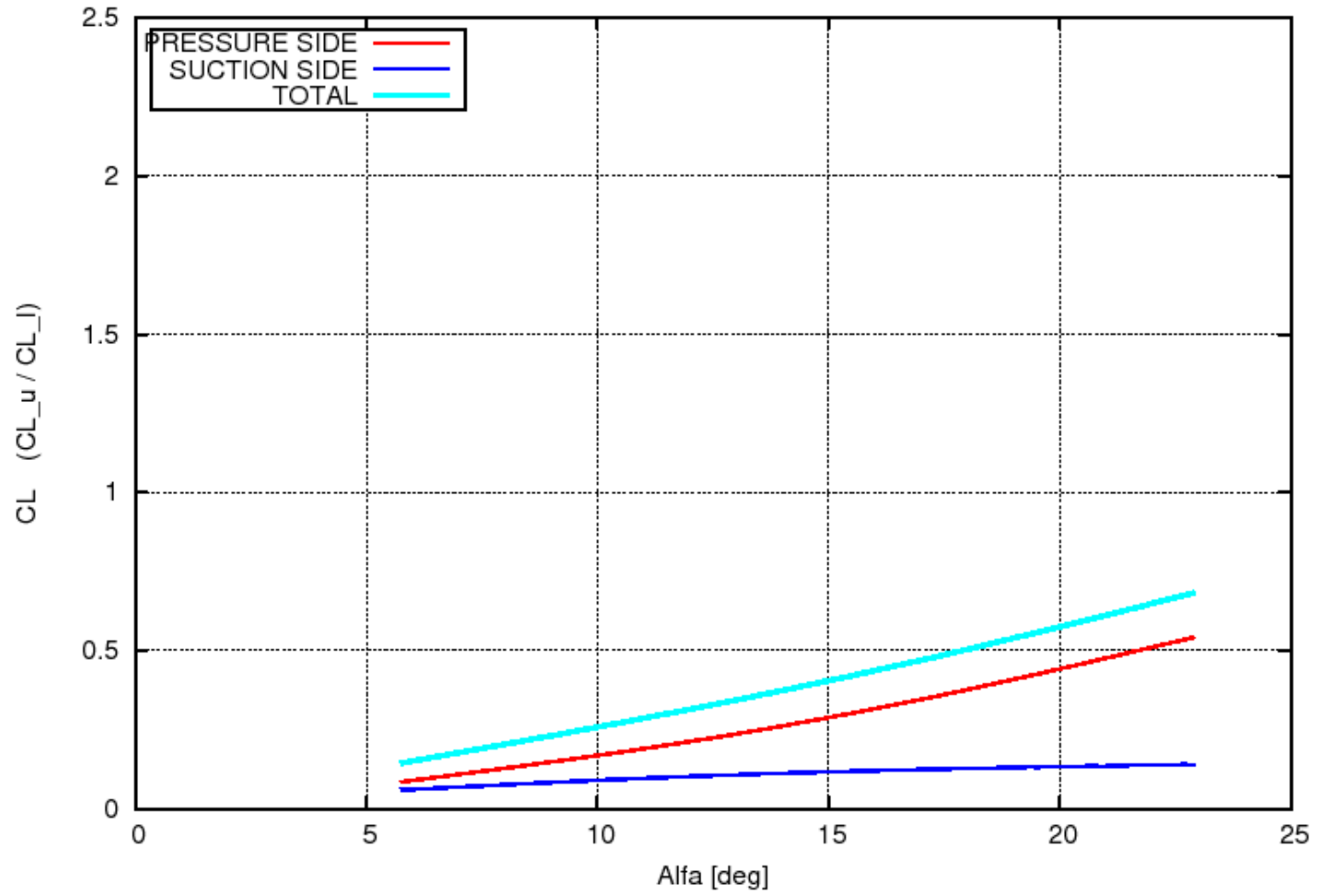
MACH_inf = 2.0



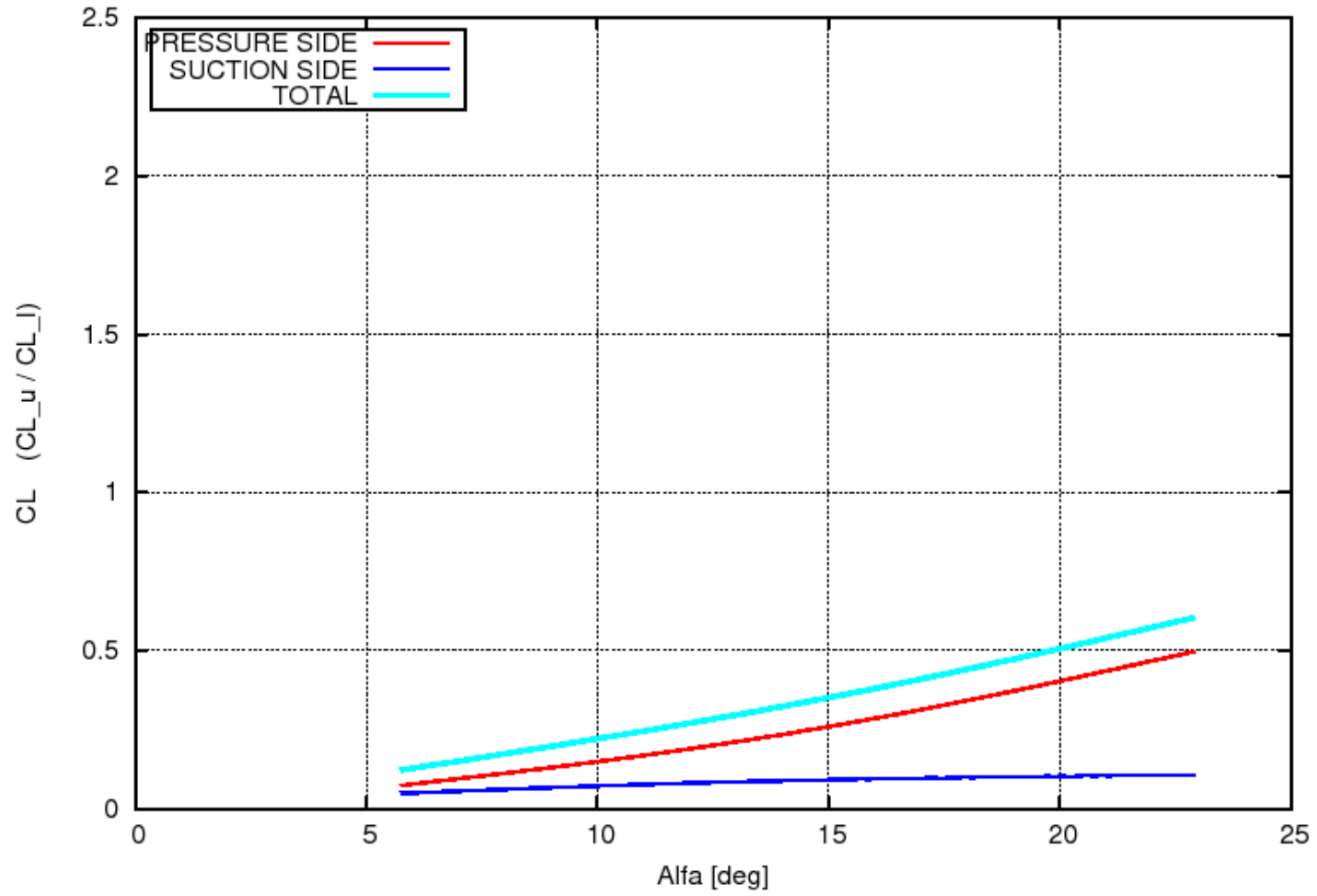
MACH_inf = 2.5



MACH_inf = 3.0



MACH_inf = 3.5



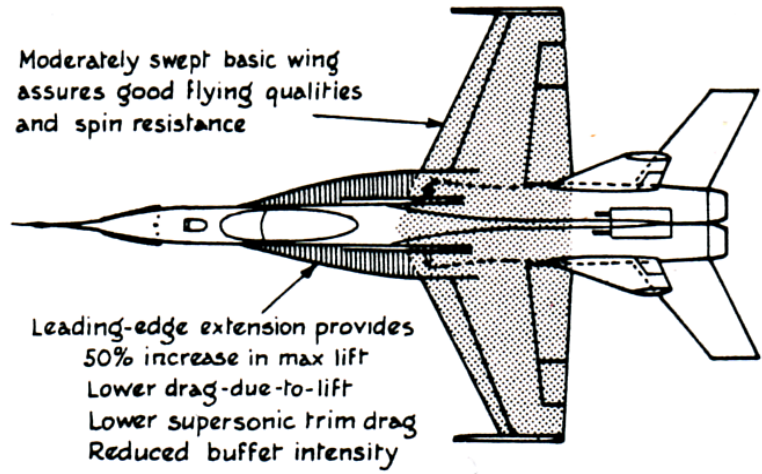
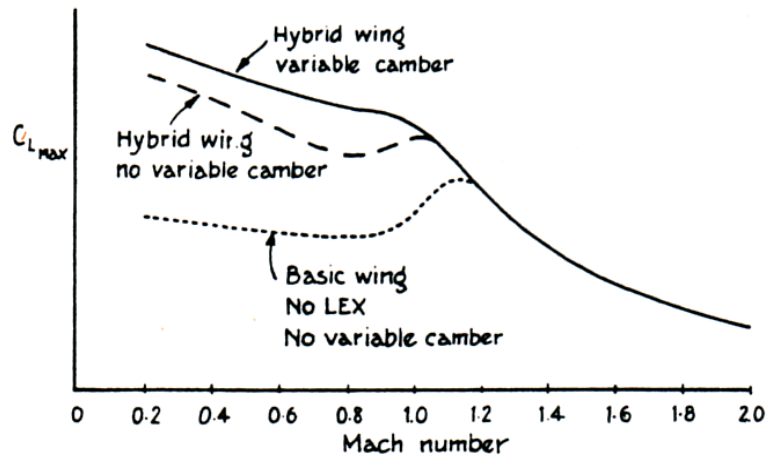


Fig 84 Effect of hybrid wing and variable camber on maximum lift.²⁷



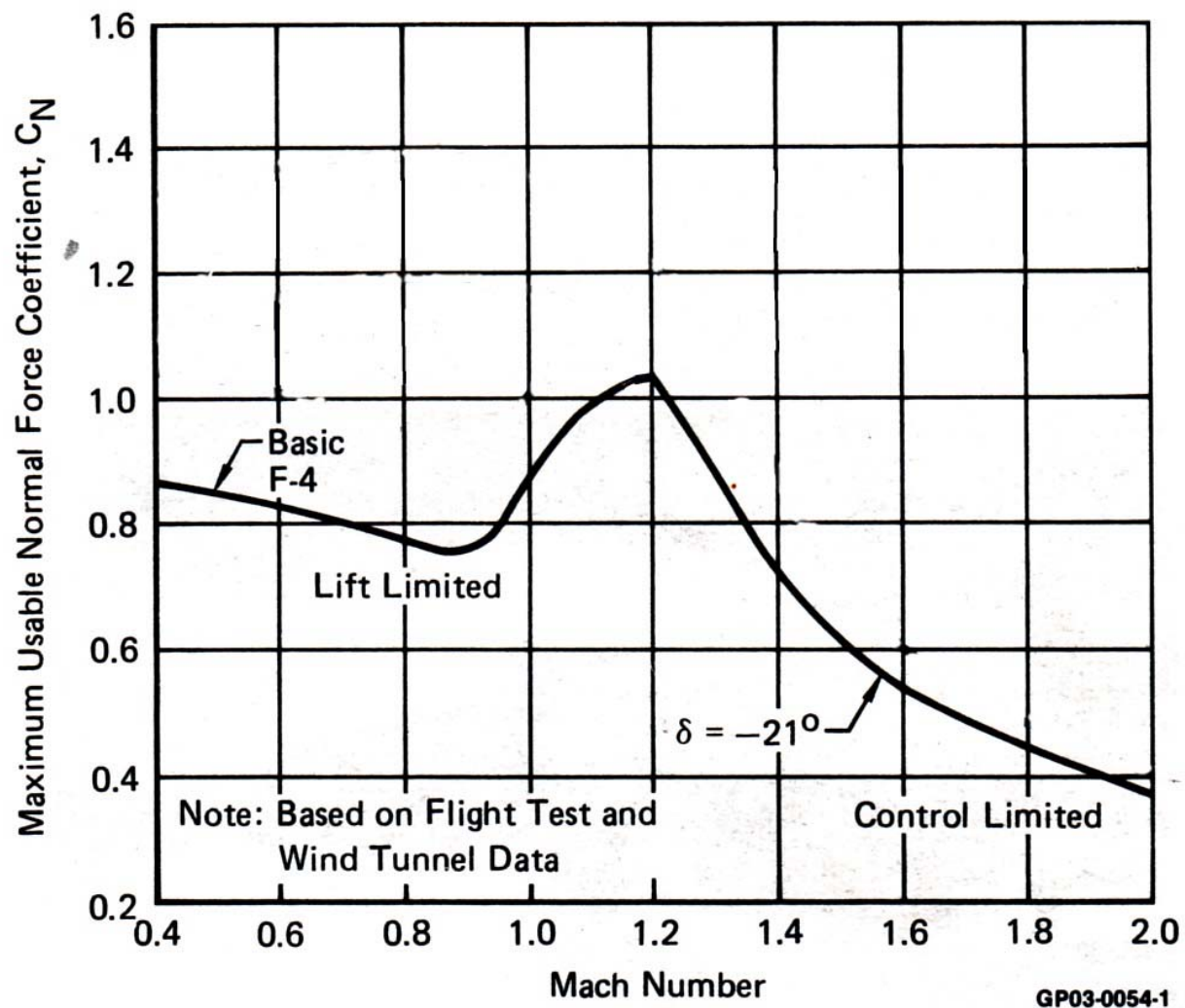
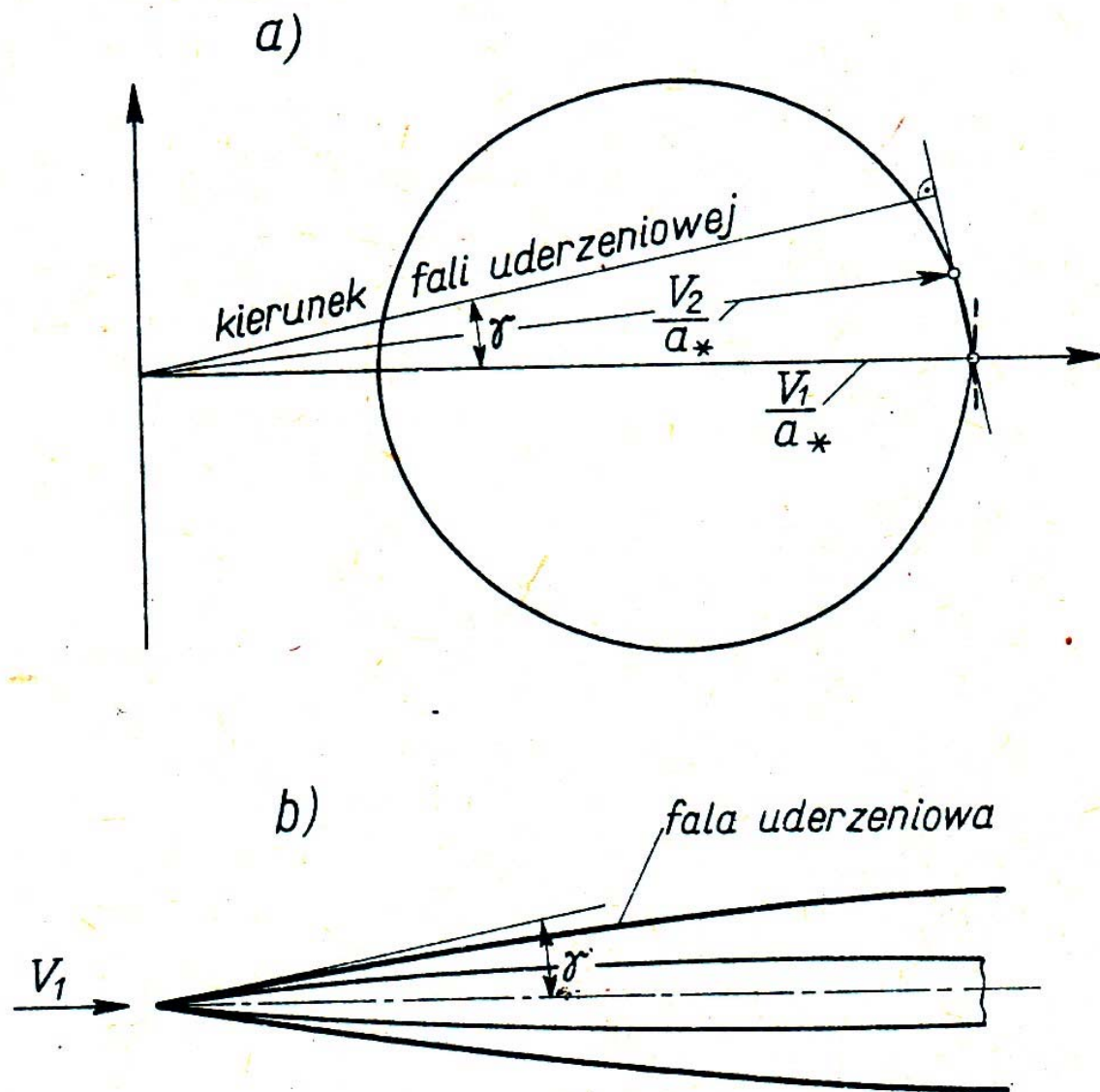
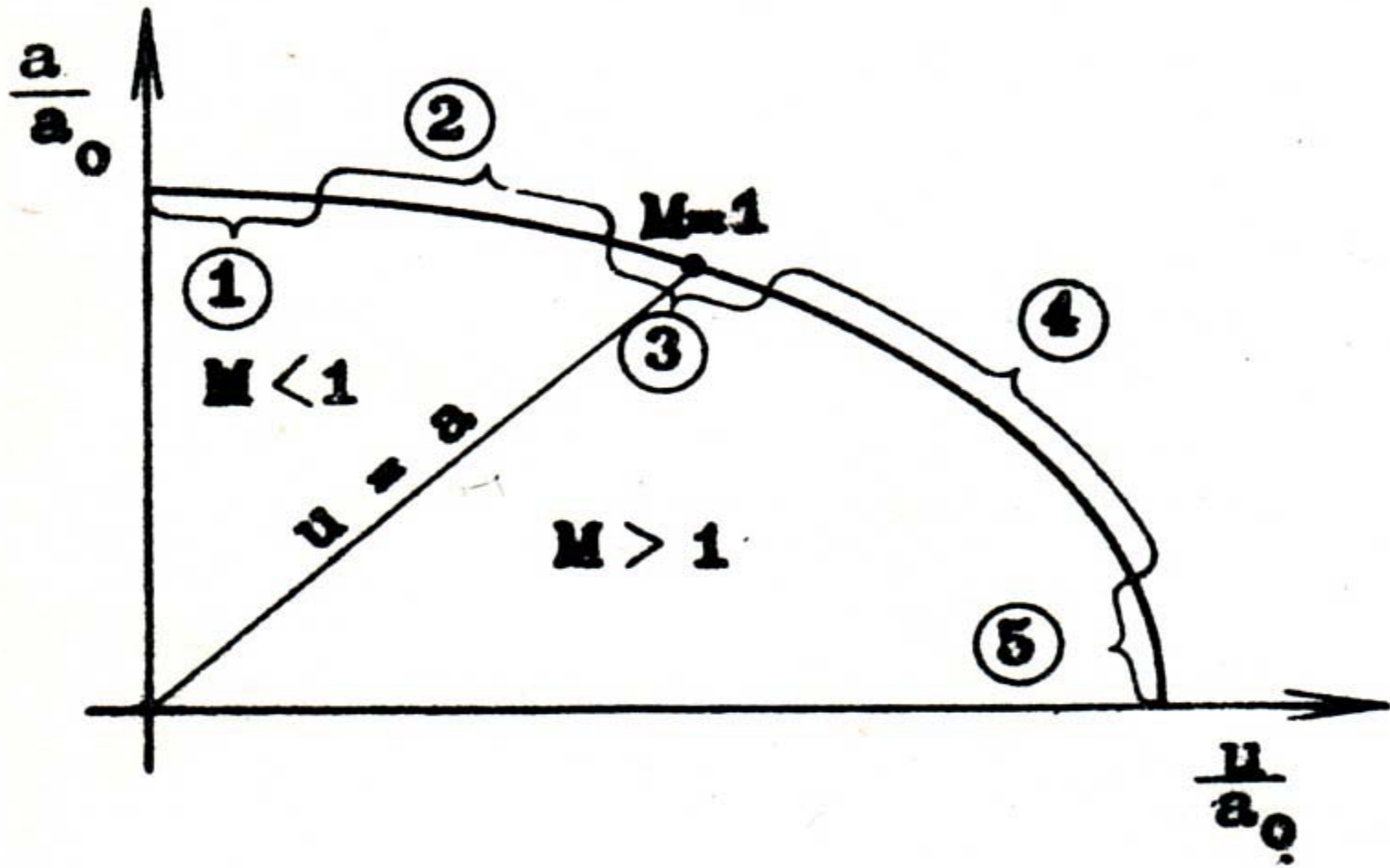


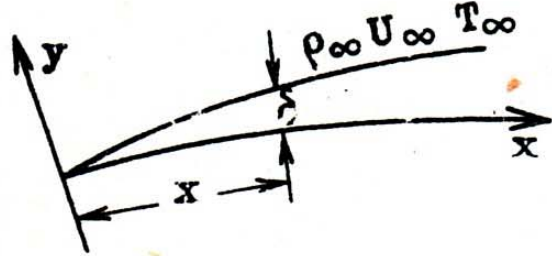
Fig. 24 Model F-4 Maximum Usable Normal Force
C.G. = 31% c



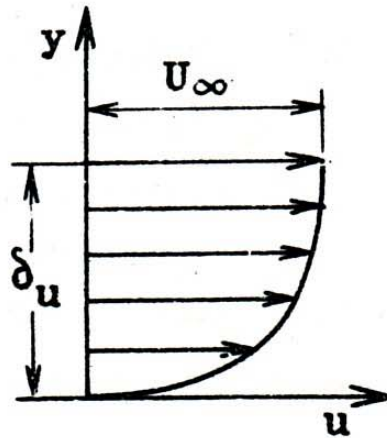
Rys. 10.48. Hipersoniczny opływ cienkiej bryły



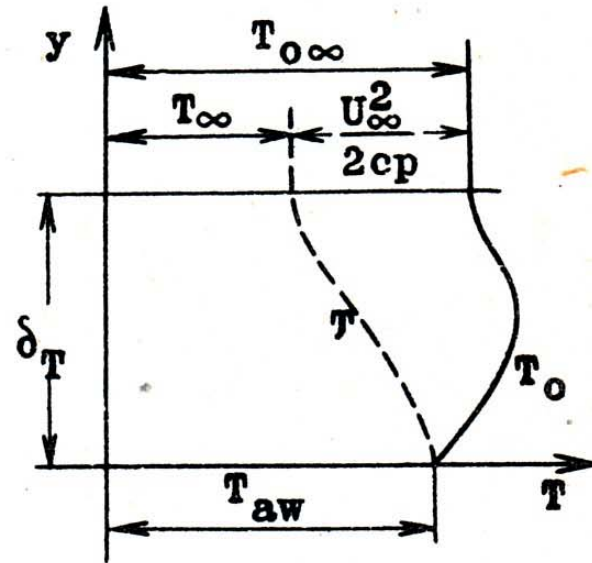
$U, a, T ?$



a/



b/



c/

$$r = \frac{T_{aw} - T_{\infty}}{T_{0\infty} - T_{\infty}} = \frac{T_{aw} - T_{\infty}}{U_{\infty}^2 / (2C_p)}$$

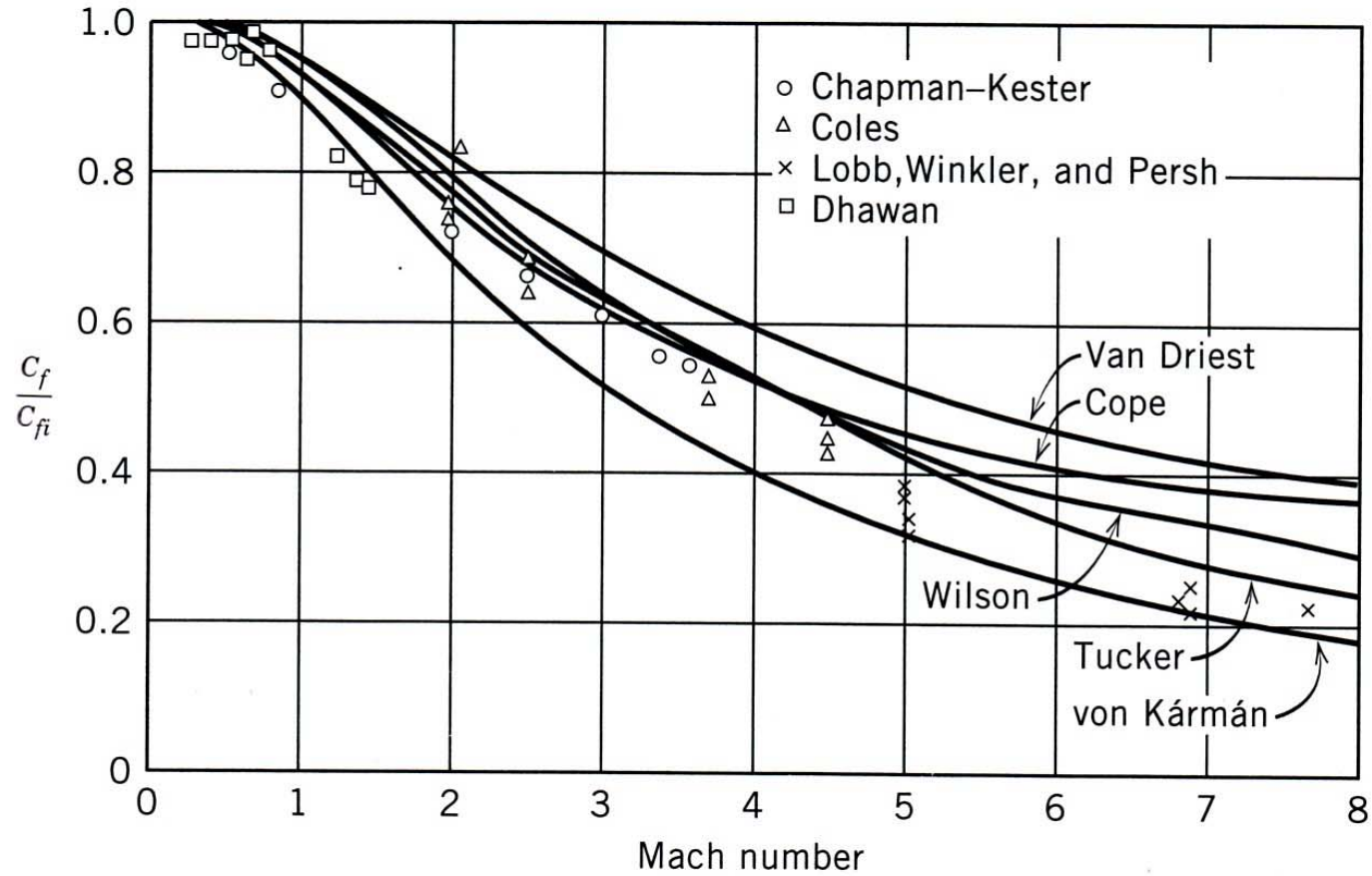


Fig. 18.24. Ratio of compressible to incompressible friction coefficient for turbulent boundary layer on a flat plate as a function of free-stream Mach number.

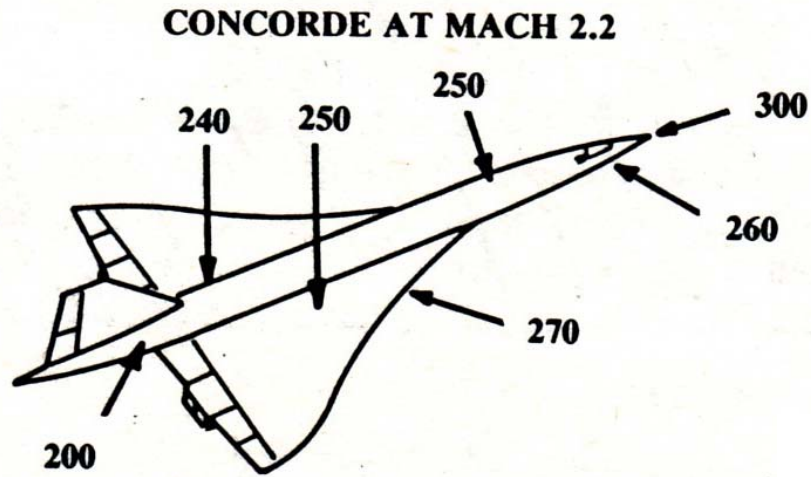
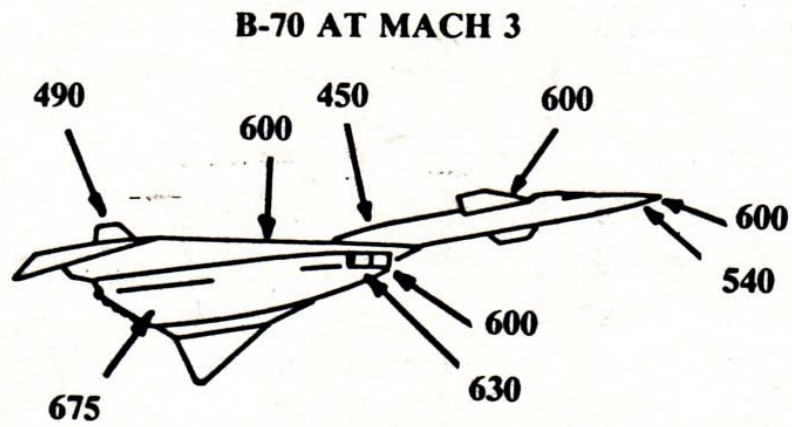
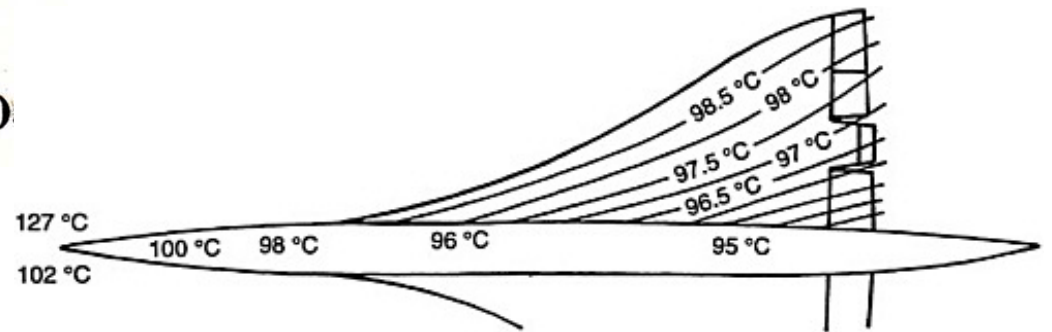


Fig. 14.21 Supersonic skin temperatures ($^{\circ}\text{F}$)



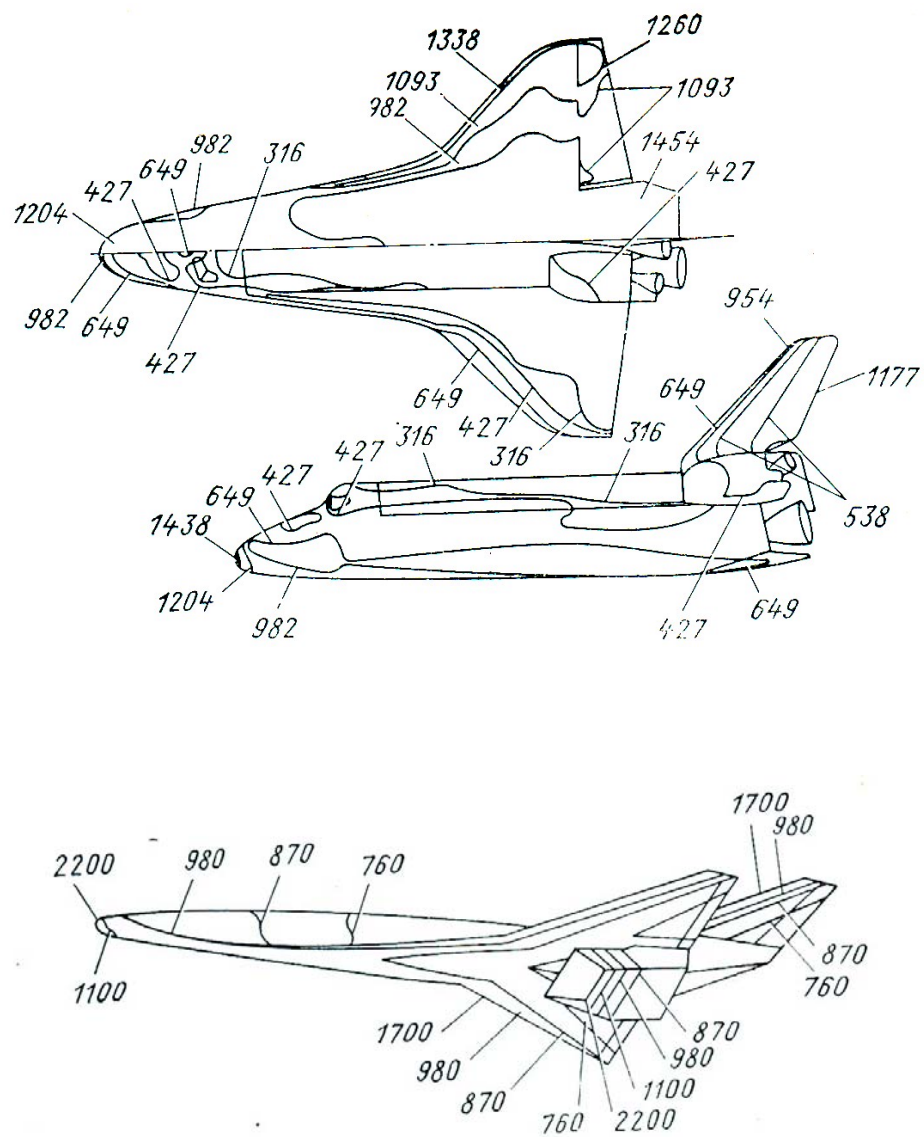


Рис. 12.12. Установившаяся температура на поверхности самолета в градусах Цельсия при длительном полете ($V = 2400$ м/с; $H = 34$ км)